The Problem of Learning to Teach
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Reviewed work(s):
Source: *The American Mathematical Monthly*, Vol. 82, No. 5 (May, 1975), pp. 466-476
Published by: Mathematical Association of America
Stable URL: http://www.jstor.org/stable/2319737
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THE PROBLEM OF LEARNING TO TEACH

I. THE TEACHING OF PROBLEM SOLVING — BY P. R. HALMOS

The best way to learn is to do; the worst way to teach is to talk.

About the latter: did you ever notice that some of the best teachers of the world are the worst lecturers? (I can prove that, but I'd rather not lose quite so many friends.) And, the other way around, did you ever notice that good lecturers are not necessarily good teachers? A good lecture is usually systematic, complete, precise — and dull; it is a bad teaching instrument. When given by such legendary outstanding speakers as Emil Artin and John von Neumann, even a lecture can be a useful tool — their charisma and enthusiasm come through enough to inspire the listener to go forth and do something — it looks like such fun. For most ordinary mortals, however, who are not so bad at lecturing as Wiener was — nor so stimulating!— and not so good as Artin — and not so dramatic! — the lecture is an instrument of last resort for good teaching.

My test for what makes a good teacher is very simple: it is the pragmatic one of judging the performance by the product. If a teacher of graduate students consistently produces Ph. D.'s who are mathematicians and who create high-quality new mathematics, he is a good teacher. If a teacher of calculus consistently produces seniors who turn into outstanding graduate students of mathematics, or into leading engineers, biologists, or economists, he is a good teacher. If a teacher of third-grade "new math" (or old) consistently produces outstanding calculus students, or grocery store check-out clerks, or carpenters, or automobile mechanics, he is a good teacher.

For a student of mathematics to hear someone talk about mathematics does hardly any more good than for a student of swimming to hear someone talk about swimming. You can't learn swimming technique by having someone tell you where to put your arms and legs; and you can't learn to solve problems by having someone tell you to complete the square or to substitute sin u for y.

Can one learn mathematics by reading it? I am inclined to say no. Reading has an edge over listening because reading is more active — but not much. Reading with pencil and paper on the side is very much better — it is a big step in the right direction. The very best way to read a book, however, with, to be sure, pencil and paper on the side, is to keep the pencil busy on the paper and throw the book away.

Having stated this extreme position, I'll rescind it immediately. I know that it is extreme, and I don't really mean it — but I wanted to be very emphatic about not going along with the view that learning means going to lectures and reading books. If we had longer lives, and bigger brains, and enough dedicated expert teachers to have a student/teacher ratio of 1/1, I'd stick with the extreme view — but we don't. Books and lectures don't do a good job of transplanting the facts and techniques of

Talks given at the Annual Meeting in San Francisco, January 17, 1974, at a joint AMS-MAA Panel discussion.

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the past into the bloodstream of the scientist of the future — but we must put up with a second best job in order to save time and money. But, and this is the text of my sermon today, if we rely on lectures and books only, we are doing our students, and their students, a grave disservice.

What mathematics is really all about is solving concrete problems. Hilbert once said (but I can't remember where) that the best way to understand a theory is to find, and then to study, a prototypal concrete example of that theory, a root example that illustrates everything that can happen. The biggest fault of many students, even good ones, is that although they might be able to spout correct statements of theorems, and remember correct proofs, they cannot give examples, construct counterexamples, and solve special problems. I have seen many students who could state something they called the spectral theorem for Hermitian operators on Hilbert space but who had no idea how to diagonalize a 3×3 real symmetric matrix. That's bad — that's bad learning, probably caused, at least in part, by bad teaching. The full-time professional mathematician and the occasional user of mathematics, and the whole spectrum of the scientific community in between — they all need to solve problems, mathematical problems, and our job is to teach them how to do it, or, rather, to teach their future teachers how to teach them to do it.

I like to start every course I teach with a problem. The last time I taught the introductory course in set theory, my first sentence was the definition of algebraic numbers, and the second was a question: are there any numbers that are not algebraic? The last time I taught the introductory course in real function theory, my first sentence was a question: is there a non-decreasing continuous function that maps the unit interval into the unit interval so that length of its graph is equal to 2? For almost every course one can find a small set of questions such as these — questions that can be stated with the minimum of technical language, that are sufficiently striking to capture interest, that do not have trivial answers, and that manage to embody, in their answers, all the important ideas of the subject. The existence of such questions is what one means when one says that mathematics is really all about solving problems, and my emphasis on problem solving (as opposed to lecture attending and book reading) is motivated by them.

A famous dictum of Pólya's about problem solving is that if you can't solve a problem, then there is an easier problem that you can't solve — find it! If you can teach that dictum to your students, teach it so that they can teach it to theirs, you have solved the problem of creating teachers of problem solving. The hardest part of answering questions is to ask them; our job as teachers and teachers of teachers is to teach how to ask questions. It's easy to teach an engineer to use a differential equations cook book; what's hard is to teach him (and his teacher) what to do when the answer is not in the cook book. In that case, again, the chief problem is likely to be "what is the problem?". Find the right question to ask, and you're a long way toward solving the problem you're working on.

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What then is the secret — what is the best way to learn to solve problems? The answer is implied by the sentence I started with: solve problems. The method I advocate is sometimes known as the "Moore method," because R. L. Moore developed and used it at the University of Texas. It is a method of teaching, a method of creating the problem-solving attitude in a student, that is a mixture of what Socrates taught us and the fiercely competitive spirit of the Olympic games.

The way a bad lecturer can be a good teacher, in the sense of producing good students, is the way a grain of sand can produce pearl-manufacturing oysters. A smooth lecture and a book entitled "Freshman algebra for girls" may be pleasant; a good teacher challenges, asks, annoys, irritates, and maintains high standards — all that is generally not pleasant. A good teacher may not be a popular teacher (except perhaps with his *ex*-students), because some students don't like to be challenged, asked, annoyed, and irritated — but he produces pearls (instead of casting them in the proverbial manner).

Let me tell you about the time I taught a course in linear algebra to juniors. The first hour I handed to each student a few sheets of paper on which were dittoed the precise statements of fifty theorems. That's all — just the statements of the theorems. There was no introduction, there were no definitions, there were no explanations, and, certainly, there were no proofs.

The rest of the first hour I told the class a little about the Moore method. I told them to give up reading linear algebra (for that semester only!), and to give up consulting with each other (for that semester only). I told them that the course was in their hands. The course was those fifty theorems; when they understood them, when they could explain them, when they could buttress them with the necessary examples and counterexamples, and, of course, when they could prove them, then they would have finished the course.

They stared at me. They didn't believe me. They thought I was just lazy and trying to get out of work. They were sure that they'd never learn anything that way.

All this didn't take as much as a half hour. I finished the hour by giving them the basic definitions that they needed to understand the first half dozen or so theorems, and, wishing them well, I left them to their own devices.

The second hour, and each succeeding hour, I called on Smith to prove Theorem 1, Kovacs to prove Theorem 2, and so on. I encouraged Kovacs and Herrero and all to watch Smith like hawks, and to pounce on him if he went wrong. I myself listened as carefully as I could, and, while I tried not to be sadistic, I too pounced when I felt I needed to. I pointed out gaps, I kept saying that I didn't understand, I asked questions about side issues, I asked for, and sometimes supplied, counterexamples, I told about the history of the subject when I had a chance, and I pointed out connections with other parts of mathematics. In addition I took five minutes or so of most hours to introduce the new definitions needed. Altogether I probably talked 20 minutes out of each of the 50-minute academic hours that we were together. That's a lot — but it's a lot less than 50 (or 55) out of 50.

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It worked like a charm. By the second week they were proving theorems and finding errors in the proofs of others, and obviously taking pleasure in the process. Several of them had the grace to come to me and confess that they were skeptical at first, but they had been converted. Most of them said that they spent more time on that course than on their other courses that semester, and learned more from it.

What I just now described is like the "Moore method" as R. L. Moore used it, but it's a much modified Moore method. I am sure that hundreds of modifications could be devised, to suit the temperaments of different teachers and the needs of different subjects. The details don't matter. What matters is to make students ask and answer questions.

Many times when I've used the Moore method, my colleagues commented to me, perhaps a semester or two later, that they could often recognize those students in their classes who had been exposed to a "Moore class" by those students' attitude and behavior. The distinguishing characteristics were greater mathematical maturity than that of the others (the research attitude), and greater inclination and ability to ask penetrating questions.

The "research attitude" is a tremendous help to all teachers, and students, and creators, and users of mathematics. To illustrate, for instance, how it is a help to me when I teach elementary calculus (to a class that's too large to use the Moore method on), I must first of all boast to you about my wonderful memory. Wonderfully bad, that is. If I don't teach calculus, say, for a semester or two, I forget it. I forget the theorems, the problems, the formulas, the techniques. As a result, when I prepare next week's lecture, which I do by glancing at the prescribed syllabus, or, if there is none, at the table of contents of the text, but never at the text itself, I start almost from scratch — I do research in calculus. The result is that I have more fun than if I had it all by rote, that time after time I am genuinely surprised and pleased by some student's re-discovery of what Leibniz probably knew when he was a teenager, and that my fun, surprise, pleasure, and enthusiasm is felt by the class, and is taken as an accolade by each discoverer.

To teach the research attitude, every teacher should do research and should have had training in doing research. I am not saying that everyone who teaches trigonometry should spend half his time proving abstruse theorems about categorical teratology and joining the publish-or-perish race. What I am saying is that everyone who teaches, even if what he teaches is high-school algebra, would be a better teacher if he thought about the implications of the subject outside the subject, if he read about the connections of the subject with other subjects, if he tried to work out the problems that those implications and connections suggest — if, in other words, he did research in and around high-school algebra. That's the only way to keep the research attitude, the question-asking attitude, alive in himself, and thus to keep it in a condition suitable for transmitting it to others.

Here it is, summed up, in a few nut shells:

The best way to learn is to do — to ask, and to do.

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The best way to teach is to make students ask, and do. Don't preach facts — stimulate acts.

The best way to teach teachers is to make them ask and do what they, in turn, will make their students ask and do.

Good luck, and happy teaching, to us all.

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II. THE PROBLEM OF LEARNING TO TEACH — BY E. E. MOISE

It was a real pleasure to listen to Professor Halmos's talk. Seldom have I heard so much to agree with, and so much to applaud. He has given us a beautiful description of our task as teachers. And the description implied — as it had to — a wholesale rejection of the naive empiricism and naive behaviorism which have become an endemic plague in much of the educational world.

In the present state of our knowledge, teaching is an art. In mathematics, at least, attempts to turn it into a science have been retrogressive, in every case that I know of. Even when mathematics is taught poorly or only passably, we take for granted that students will have an opportunity to react to it in different ways, and to learn it at different levels, according to their own talents, temperaments, and motivations. At least, we *used* to take this for granted, until various people found ways to put a stop to it. I believe that the ultimate caricature of good mathematical teaching is linear error-free programming. Under this scheme, instead of taking care to ensure that every student is provided with the most stimulating challenges that he can react to successfully, people use their best efforts to create a situation in which nobody is faced with any challenge at all.

Certain ways of using "modules" have the same vice in a milder form. Some schools are now using a scheme under which courses are split up into small parts (the "modules"), with a standard test for each of them. When a student had passed the test on one module, he is ready to move on to the next. In some schools, at least, the rules prescribe that a student's grade at the end of the year is based on the number of modules that he has completed. Since the tests are of such a sort that almost any student can pass them eventually, the moral conveyed by all this is that an A-student is one who acquires a C-knowledge of mathematics at high speed. I suppose it is possible for a student in such a program to analyze ideas in depth, and to spend lots of time working on hard problems. But to behave in such a way, the student would have to resist the suggestions conveyed to him by the people who are receiving pay on the ground that they are promoting the student's intellectual development.

One of the difficulties with the pseudoscientific "learning theorists" is that they concentrate their attention on those aspects of the learning process that are capable of being meticulously observed and measured. Such a proceeding is not valid, or

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even safe; we simply don't know enough about learning processes to do anything predicated on the notion that our knowledge is complete. Curiously, there is empirical evidence against the validity of the empiricists' conception of learning.

In the early 1960's, Dr. Lyn Carlsmith (Harvard Educational Review, vol. 34 (1964), pp. 3–21) found a group of 20 male students, in the Harvard College class of 1964, whose fathers had gone overseas when their sons were no more than six months old, and had not returned until at least two years later. She then took a carefully matched control group of 20 male students whose fathers had not been absent in their early childhood. The SAT test was given to both groups. This test is in two parts, mathematical and verbal. Ordinarily, the difference M − V of the mathematical and verbal scores M and V is positive for boys and negative for girls. The control group conformed to this expectation: in 18 cases out of 20, M − V was positive. But in the "father-absent" group, M − V was positive in only 7 cases out of 20. For a smaller group of 18 doctors' sons, similarly matched, the results were even more striking: in the control group, M − V was positive in 7 cases out of 9, while in the father-absent group, M − V was positive in only *one* case out of 9.

Further study of larger samples confirmed all this. Apparently, M − V diminishes sharply as the duration of the father's absence increases; and the absence of the father in the *first six months* of a boy's life makes a significant difference in the relation between his SAT scores twenty years later.

These results are hard to reconcile with two views now widely held, namely, (1) intellectual capacities that seem to be purely cognitive really are, and (2) these capacities are acquired in ways that are readily accessible to empirical study. I believe that both these notions are not just inexact but very wide of the mark. It would be interesting to know just what it is that fathers teach their baby boys, and how the fathers go about it.

Obviously this study left important questions unanswered. For example, did the absence of the father inhibit the growth of mathematical faculties, or promote the growth of verbal ability, or both? (There is the prior question whether the "mathematical" part of the SAT test measures the sort of ability that produces a mathematician.) The study reminds us, however, of something that we should have known all along, that some of the most important learning processes go on when nobody is looking, and that they go on in ways that are very hard to keep track of. It is simplistic to suppose that people remember what they are told, and understand the things that are explained to them clearly. More commonly, people remember what interests them, and understand the things that they enjoy understanding. Thus intellectual development is linked with development of personality, and the refinement and enlargement of esthetic perceptions is a vital part of intellectual growth. This sort of growth does not lend itself to mechanization.

The processes by which people learn to teach are equally obscure. Some years ago — or so the story goes — a class in Social Relations at Harvard played an elaborate prank on their section man. The section man was in the habit of pacing back

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and forth while talking. In a secret caucus, the class agreed on a imaginary line, down the middle of the classroom. When the teacher moved to the left of the line, the class became eager and alert. When he stepped to the right of the line, the class became apathetic. When the class had taught the teacher to stay to the left of the center line, they gradually moved the line, until at the end of two or three weeks they had the teacher boxed into a corner. He had no idea of what was going on.

This was different, in a way, from the usual process under which students teach teachers to teach. But I think that the main difference is the students knew what they were doing. Ordinarily, I believe the process is unconscious for everybody.

This brings us, at last, to the question that I was supposed to be discussing at the outset: granted that teaching is an art, learned by experience, what can we do to help people to learn it? It seems to me that beginning teachers can probably get a great deal of help from policies which could easily be carried out in most departments.

One of the greatest troubles, I believe, in the initial teaching experience, is that the learning of teaching is virtually solitary. At the places that I know about, senior faculty members visit each teaching fellow's classes about once a semester. Even for purposes of evaluation, these procedures are perfunctory, and their value as teacher training is nil. It is hard to think of another art that people are expected to learn in such a way, with no significant help in the form of knowledgeable criticism.

This suggests that we should try to turn the learning of teaching into a group activity. I propose the following scheme. Beginning teachers would be organized in groups of about five, with *identical* teaching assignments, preferably a single course. They would share an office, so that they could conveniently discuss the problems that they all faced. Schedules would be arranged so that they could visit one another's classes. They would all meet, at least once a week, in a sort of "teaching seminar," to discuss what was going on. Each would have full responsibility for his own section, pacing the course to suit himself, subject only to the loose constraints imposed by the place that the course was supposed to fill in the curriculum. Each would make up his own assignments, and write his own hour tests and final examination. If a highly skilled senior faculty member formed part of the group, or met with them as an advisor, this would no doubt be helpful; but I believe that the senior man ought to be an advisor and not a boss. Classroom visits by peers would be much more frequent than visits by the advisor.

I see reason to hope that this sort of consultative effort would improve and vastly accelerate the process by which teachers learn by experience. Some features of it may need further explanation.

(1) I believe that there are such things as pedagogical principles. But even if we agree on what these are, they are hard to demonstrate, or even to convey, by abstract statements, and the art of putting them into practice takes quite a while to learn. I think that discussions of pedagogic questions are of immediate practical utility in proportion to their specificity. Hence an arrangement under which beginning teachers

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would discuss not the general problems of education but rather, at a given moment, the problem of teaching a particular topic at a particular stage in a particular course. Under these conditions, I think that general ideas will emerge, in such forms that their meanings will be clear and the extent of their validity will be evident. This is why I think it vital for the teaching assignments to be identical; we need a situation in which the people discussing teaching problems have the same problems on their minds.

(2) If the group has a supervisor who tells everybody exactly what to do, he will almost certainly be telling some of them the wrong things. It is no part of any teacher's job to duplicate the performance of any other teacher however skilled. Teaching is an interpersonal relation, and optimal styles depend on the personalities of individuals. Such styles do and should change in response to class reactions.

(3) Moreover, if all important decisions are made by some higher authority, the beginning teacher will be less likely to come to grips, in his own mind, with the sort of problem that he will have to solve for himself in future years when the boss is gone. Hence the proposal that beginning teachers have full responsibility for their own courses, at a time when they have the benefits of consultation and criticism.

It seems likely that this scheme would amply repay the effort that it would require. Obviously, the only way to find out is to try it. I believe, however, that it involves at least one important hazard and has at least two important limitations.

First, it may be that working under observation, even by peers, will make people over-cautious, in an attempt to avoid the possibility of looking foolish. Probably this danger can be minimized if people are clearly aware of it. It seems especially important for the advisor to be aware of it, and for him to be of a gentle disposition.

Second, the whole scheme, in the form described, deals with fairly traditional teaching, in which the general content and method of the course are taken as given. This means that the skills acquired are only the beginning of professional maturation. The best courses that I know of were of the teacher's own design, and in some cases they were improvisations, whose outcome was not known even to the teacher at the outset. I believe, however, that fairly conventional teaching is a natural first step in professional development. This is a limit to the problems that one man can think about in one semester.

Finally, I don't think that we ought to feel complacent about our present lack of an adequate theory of teaching. If we had such a theory, we would be better off, and I think that one of the tasks of the coming generation is to create one. I have no idea of the form that such a theory might take. Perhaps its most likely inventors are people each of whom has a sophisticated and creative grasp both of mathematics and of psychology.

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III. THE PROMOTION OF PARTICIPATION — BY GEORGE PIRANIAN

I address this to teachers of graduate and undergraduate students, to teachers in junior colleges, and perhaps to high-school teachers. Teachers in elementary schools already know what I have to say.

My colleagues have discussed ways to stimulate classroom participation. Paul Halmos has talked about participation by students, and Ed Moise proposes to inject life into the teachers. I shall try to reinforce their message with a story, and I'll mention a few relevant technicalities.

In 1967–68, The University of Michigan let me teach a section of the honors course in calculus. Because of a long period without freshman contacts, the prospect filled me with fear; but the students were a lovable lot, and we soon developed effective cooperation.

We had a solid book. Unfortunately, the author had taken himself a bit too seriously, and consequently the text was on the dreary side. To compensate, I regularly assigned special problems. For example, I asked the students to prove or disprove that if a real-valued function on the line is continuous at a point, then it is continuous throughout some neighborhood of that point. The subsequent classroom discussion of such a problem could chew up an entire period. But the course ran well, and I was so pleased that at the end I asked one of the girls to grade papers for me during her sophomore year.

In June, Addison-Wesley sent me a copy of Joseph Kitchen's *Calculus of One Variable*. Because the book looked lively, I thought we should try it, and to show my affection for the grader, I wrote to the publisher and requested that he send a copy to her home.

In September, when Lisa came to my office, I asked her opinion, and she said "It's just like the book we used, except that the Piranian problems are already in it." The students bought the book, and I looked forward to a great year.

After one week, I felt apprehensive, and soon I sensed the cold shadow of failure. Despite the excellent text and the bright students, the class sat glued to the runway. And then it happened that Kitchen skipped a point I consider important, and this forced me to devise a special problem for the occasion. The consequence was dramatic. With a roar of the engines and a slight shudder of the fuselage, we took off for the white clouds in the blue sky.

The moral is simple: no matter how sound, complete, and clear my text or lecture notes may be, the students should know that I'm developing the course especially for them, and that I'm turning myself inside-out in their behalf. For example, I must not assign homework by opening the text to page 93 and saying "for next time, try problems 3, 7, 10, 16, 19, and let me see, 21; class dismissed." We'll come back to this in a few minutes.

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I must not give the impression of a man hired to teach as many students as possible and wired to do it with maximum industrial efficiency. I must indulge in extensive participation; the best way to achieve this is to recognize that this year's students require a new course, and that regardless of the cost, my section deserves special treatment. You can't teach with the left hand; you can't teach with the right hand. Like playing volley-ball, swimming, or racing a small sailboat, the job requires both hands, both arms, and the muscles of the legs and the torso.

The job takes more. I can preach an eloquent sermon on the gospel according to Darboux and Riemann, or give a spirited performance on Cantor sets, or use both hands and feet in a glorious axiomatic fugue, and yet reap substantial failure. No man can please all the people all the time, and no style of teaching is effective for all students. Therefore, successful teaching requires cooperation from the class.

You and I would know how to live, if we were young again. Meanwhile, multitudes of boys and girls suffer from awkwardness, uncertainty, and hesitation. Ask a dozen of your students with how many of their classmates they are acquainted, and you'll be astonished to learn about the bleakness and academic isolation in which some of them exist. A few years ago, I hit upon an unobtrusive way of sending a bit of mature wisdom across the generation gap. Early each term, I distribute a dittoed sheet listing the Ann Arbor addresses and telephone numbers of the entire class. This may encourage collaboration on homework; but it does not produce the miserable situation in which Archibald copies Merthiolate's paper fifteen minutes before it is due. Half of the class may meet for a great jam session. Leaders emerge, and the strong give guidance to the weak. I should share my salary with four or five students. They do some of my most difficult work, and I receive credit for their success. The kids learn to communicate, and when the homework is done, they may be so full of social steam that they go jogging together. If a few hundred of us were to trot from the Hilton to the Fisherman's Wharf, San Francisco would notice our physical condition and our social cohesion.

I've come back to homework. I do not know how to present mathematical ideas so effectively that students can take possession of them simply by sitting at my feet and smelling my socks. Let me change to a slightly less offensive metaphor: after grazing in my lush pastures, the students must ruminate; they must dedicate substantial time to the chewing of the cud. That's why we need homework.

Suppose now that our calculus text has a set of problems on integration by parts, a set on masses and centroids, a set on cylindrical and spherical coordinates. In each set, the problems range from the trivial to bread-and-butter drill, and they may end with a few important stinkers.

This is a reasonable arrangement of the text. A natural way of running the homework show is to assign problems from Set 23 today, problems from Set 24 tomorrow, and so forth. This is efficient for the teacher, for the students, and for the grader, and it is consistent with the principle of orderly progress. Nevertheless, the practice is a manifestation of pedagogic brutality. The poor boy who can barely manage

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Problem 7 never gets the benefit of Problems 10, 16, and 19, except during a discussion that he endures passively because in his inexperienced view it comes too late to be of any use.

A more effective assignment for tomorrow might look like this:

Problem 18 in Set 22,

Problems 15 and 16 in Set 23,

Problems 9 and 12 in Set 24,

Problems 1 and 4 in Set 25.

Under this plan, the difficult problem comes after a week of experience with easier exercises in the same topic. The student profits from repeated exposure, and the teacher has several opportunities for clarifying the basic principles and demonstrating the necessary technique. A tough piece of meat calls for slow cooking, and a difficult idea requires thought on several consecutive days. Use the scheme of staggered assignments, tell the students that you've carefully programmed the homework for maximum effectiveness, and make certain that you're telling the truth.

Staggered homework is a small technicality; but it makes a difference. Also, it illustrates the dictum that genius is the capacity for taking trouble.

I urge the mathematical community to strengthen its pedagogical effort, not by buying new gadgets, not by creating new committees of experts, but by intensification of personal effort. Let each man assume the responsibility for teaching with greater vitality. If this reduces his rate of publication by thirty percent, so much the better. It will be good news for libraries, and it will help save Mathematical Reviews.

In the deliberations among the elders, the first question about a man should be how well he teaches, the second question, how good his publications are — never, how numerous.

I do not say this because we should create more mathematicians; there are enough of us. Nor am I concerned with the problem of generating stronger enrollment in mathematics classes to prevent economic dislocation of superannuated fuddy-duddies. There's one commodity that the world needs above everything else, and for which we'll never develop a satisfactory substitute. We need good men and women. As teachers, we have the desperately urgent task of communicating to the young some of the intellectual values of civilized mankind. We have the task of inspiring students to rise to the highest level of excellence that they can attain. Our survival depends on our collective success. I apologize for ending on such a serious note; but we face a problem of the utmost importance.

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