

WHAT PRICE SPEED?^x

Specific Power Required for Propulsion of Vehicles

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INTRODUCTION

The history of technique and engineering testifies to the irresistible urge of humanity toward increasing the speed of locomotion. Means of locomotion on the ground, on the surface of, and within water, through the air, and perhaps through empty space, compete in an ever-growing effort toward higher velocities. Obviously, there are limitations for every type of locomotion. At a certain speed, any particular type becomes so inefficient and uneconomical that it is unable to compete with other more appropriate types.

It is difficult to find a measure of the comparative economy of locomotion, since it is impossible to find a general measure for the value of speed in human life. Obviously, speed has a quite different value in war and peace, in transportation of persons, and of cargo. The appreciation of speed depends upon our whole philosophy of life, that is, on factors far beyond the scope of engineering science.

In this short study, the problem of comparative merits of various means of locomotion is considered merely from an engineering point of view. The power required for transportation of unit weight is used as a measure for the comparison. Evidently for a definite system of locomotion, the minimum of power necessary for transportation of unit weight is determined by the physical laws of the resistance of the medium, the efficiency of the method of propulsion, the unit weight and fuel consumption of the particular type of power plant, and many

other factors. Nevertheless, it appears that if one throws all data together, a general trend, almost a kind of universal law, can be found for the power required per unit gross weight of the vehicle as a function of maximum speed. The demonstration of this general trend is the subject of the present contribution. One has to realize that the material is necessarily approximate and incomplete, and the conclusions are of a rather tentative nature.

The data for power, weight, and maximum speed are taken, in general, from publications; the data concerning the products of the Fiat concern, from records of this firm. No classified material was used in the plotting of the diagrams.

The authors present their study in this incomplete form to encourage a more complete compilation of statistical material by persons or organizations which are in a better position to do such work. They will be especially glad if manufacturers would speak up and show examples in which the power-weight ratio lies below the minimum curves shown in the accompanying diagrams.

A preliminary examination of the material has shown that it appears justified to separate the data for individual vehicles from the data for trains. It is known that, especially in the case of fast trains, the average resistance per vehicle is substantially smaller than that of a single vehicle. In the first part of this paper, we restrict ourselves to single vehicles. Table 1 gives the types of single vehicles to be considered.

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TABLE 1 TYPES OF VEHICLES

I (a) Commercial ships	II (a) Motor-driven railway cars	III (a) Airships
(b) Battleships	(b) Trucks	(b) Helicopters
(c) Destroyers	(c) Automobiles	(c) Private airplanes
(d) Submarines on surface		(d) Commercial airplanes
(e) Submarines submerged		(e) Bombers
		(f) Fighters

METHOD OF PROCEDURE

The following procedure was employed in the evaluation of the material:

First, an extensive amount of material was collected for each type of vehicle and the power for unit weight (in hp per ton), calculated and plotted as a function of the maximum velocity which the vehicle can reach in level motion. The values thus obtained show large dispersion. This appears natural since not all vehicles are designed with a view to obtaining the highest velocity with a minimum power per unit weight. Other factors, such as, for example, the price of manufacturing or design criteria incompatible with minimum power, may prevail. For our purpose, it seems to be logical to determine a limiting curve for each type representing the "minimum" value of the "power per unit weight" as a function of the "maximum velocity".

Hence, from the great number of vehicles of a given class, those examples were selected which have less power for unit weight installed than other vehicles of the same class and same maximum velocity. A continuous curve is plotted through the points representing these selected examples. It must be noted that this curve does not necessarily determine the minimum of power that is required to transport one ton at a given velocity. It may occur that a vehicle built for higher velocity can be more economical at a lower speed, not using its full power, than a vehicle designed for the same lower speed as its maximum velocity. The exact meaning of the curve is the statement that, according to present experience, in order to design a vehicle of a certain type for a given maximum speed, at least as much power has to be installed as is shown by the diagram.

It must be noted also that the full power installed in the vehicle is used in the computations. This includes, of course, some power which is not used for production of thrust; for example, power used for auxiliary purposes. Also in the case of trucks and railway cars, a certain portion of the power is reserved for climb. In other words, the maximum speed indicated in the diagram is, in general, less than the full speed which could

be reached by the vehicle if the full power could be utilized on a horizontal track. These types of vehicles are often designed in such a way that the maximum power is not available for the horizontal run. To minimize this effect, such examples were selected for which the power reserve at the maximum speed is relatively small. On the other hand, the necessity of such power reserve corresponds to the nature of the vehicle and, therefore, it appears justified to use the full power for comparison with other types.

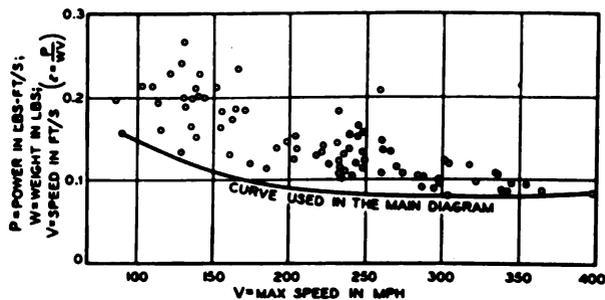


FIG. 1 EVALUATION OF DATA FOR COMMERCIAL PLANES

Fig. 1 represents an example of the evaluation of the data related to one type of locomotion, namely, commercial airplanes. The diagram contains 85 points representing 85 airplanes-old and new-with a maximum speed between 80 and 395 mph. The non-dimensional quantity ϵ plotted as ordinate is the ratio between the maximum power P and the product gross weight W times velocity V . The quantity P/V is the tractive force which would correspond to full use of the power P for propulsion with a propulsive efficiency of 100 per cent. Then $\epsilon = \frac{P}{WV}$ is the ratio between this tractive force and the weight of the vehicle. Evidently, if one compares two vehicles and considers the work necessary for a given transport performance, i.e., the transportation of the same gross weight over the same distance, this work is directly proportional to ϵ . For example, in the case of commercial aircraft, the diagram shows that an optimum for power requirement exists at a speed of approximately 320 mph. As a matter of fact, the increase of maximum speed from 200 mph (say, from the DC-3) to 360 mph (Constellation or DC-6) has been achieved by actually decreasing the work necessary for the same transport performance. The amount of work increases only slightly if the speed is raised to 400 mph.

If we consider the propulsion problem from the viewpoint of the resistance of the medium,

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MARINE VEHICLES

ϵ is the ratio between a kind of total resistance which includes, in addition to direct drag, the drag equivalent to losses in the transmission and in the propulsive mechanism and the gross weight of the vehicle. It is a kind of global friction coefficient for the vehicle. We call it the coefficient of the specific tractive force or specific resistance.

It is apparent that this coefficient is useful for the comparison of different types of transportation, since it gives an indication of the price one has to pay in power for speed, and it helps to find out whether a certain system of transportation is suitable for further speed increase without penalty of higher specific power requirement or whether one is near the economical limit. Of course the real measure of economy should be the work necessary to transport certain useful load, over a given distance. Therefore, our conclusions -- insofar as economy is concerned -- are strictly correct only if the ratio of useful load to gross weight remains constant. The authors hope that somebody will carry further the present analysis substituting the useful load for the gross weight. Since exact information concerning useful load is not readily available to the authors, they decided to use the gross weight as parameter.

In the following paragraphs we shall consider the three main classes of vehicles somewhat in detail.

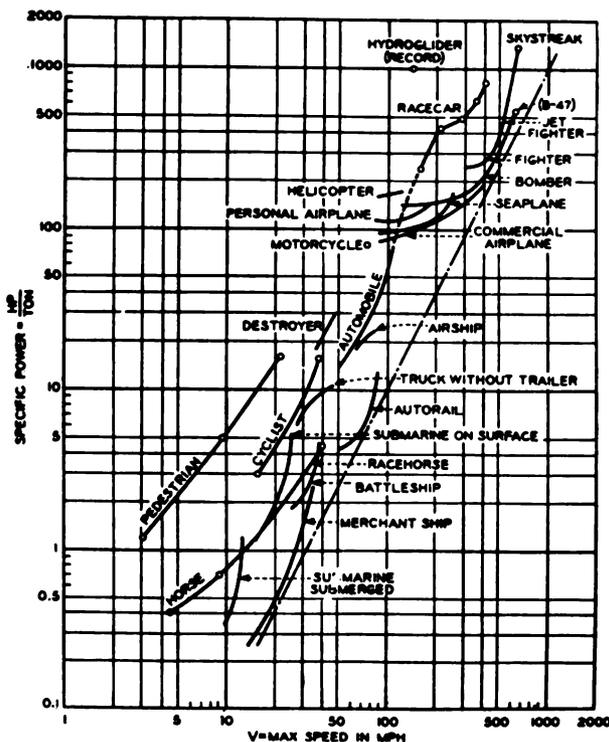


FIG. 2 SPECIFIC POWER OF SINGLE VEHICLES

Fig. 2 shows the curves representing the minimum specific power (i.e., built-in horsepower divided by gross weight), as a function of maximum speed for various types of single vehicles. Fig. 3 shows the specific tractive force, i.e., the power per unit weight divided by the maximum velocity. Both diagrams show that nautical vehicles have the lowest values for specific power or specific tractive force at least at low velocities. Evidently this fact makes ships the most economical single vehicles at low speed. The diagrams also show that for medium speeds the terrestrial and, for high speeds, the aerial vehicles represent the optimum cases.

Generally speaking, the power required for ship propulsion consists of four contributions:

- The skin friction acting on the wetted surface.
- The pressure drag produced by eddy or wake formation.
- The wave resistance.
- The air resistance of the superstructure.

The frictional resistance can be considered proportional to the square of the velocity, the density of the water, and the wetted-surface area, multiplied by a coefficient which essentially depends upon roughness of the surface, and a dimensionless parameter known as Reynolds number. For a smooth surface, the friction coefficient decreases with increasing Reynolds number, i.e., it is smaller for large boats than for small boats. For rough surface, the coefficient is essentially independent of speed. The eddy resistance behaves in general similarly to the frictional resistance. Fig. 3 shows that in the speed domain in which these two components of the resistance predominate, the specific tractive force, for example, in the case of merchant ships, shows a moderate increase with speed. For the same vehicle, the specific tractive force would be approximately proportional to the square of the speed. However, if we consider ships of different sizes, then ships of large displacement are better off because the wetted surface increases more slowly than the displacement, i.e., the gross weight. Therefore the increase of the specific tractive force in our diagram is much slower than that predicted by the quadratic law. It appears, nevertheless, that at least in the practical speed domain, the slowest ship requires the least tractive force.

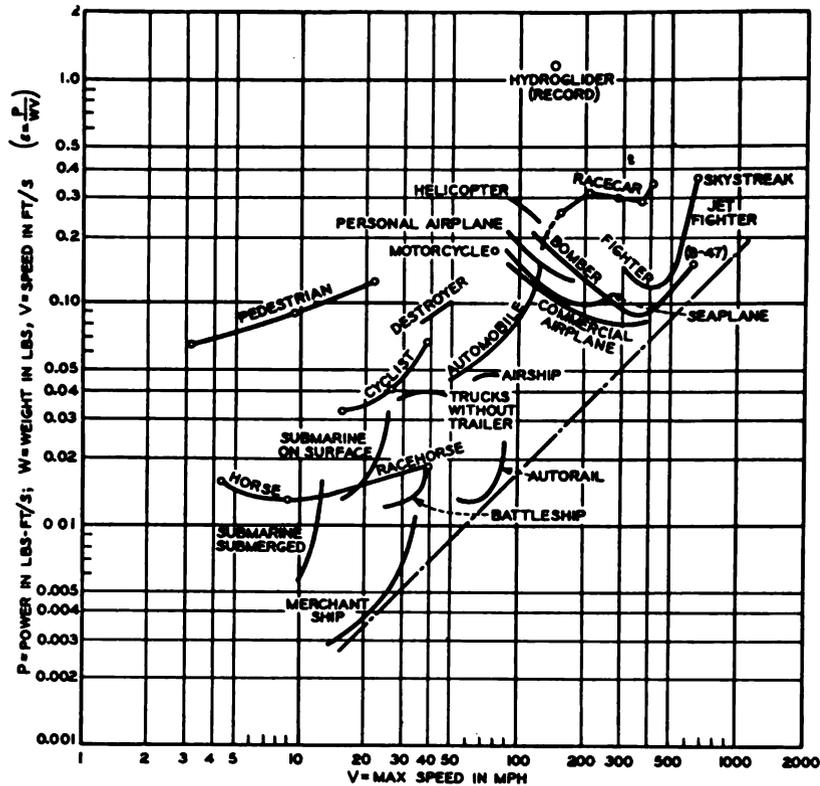


FIG. 3 SPECIFIC RESISTANCE OF SINGLE VEHICLES

On the other hand, for both merchant and battleships, the specific tractive force shows a rapid increase in the speed range between 30 and 40 mph. The reason for this increase is the increasing wave resistance which depends upon another dimensionless parameter known as Froude number. This parameter may be defined by the Formula V/\sqrt{gL} , where V is the speed, g the acceleration of gravity, and L an appropriately chosen linear dimension of the ship, for example, the length of the water line. As the speed becomes of the same order of magnitude as the velocity of propagation of the predominant wave produced by the motion of the ship (which velocity is proportional to the quantity \sqrt{gL}), the wave resistance rapidly increases. The only effective remedy against this obstacle to speed increase is to increase the length of the ship. This measure is limited, however, by several factors, one of which is the increase of structural weight due to high bending moments; furthermore, capital investment, and also the practical limits of maneuverability. Docking and the like also limit the size of a ship. The compiled data show that, for example, in the case of merchant ships, the tangent to the curve shown in Fig. 2, at the present speed limit actually reached is about 6:1, which means that for a speed increase of about 1 per cent, the built-in power per ton must be increased by about 6 per cent. A similar result is shown for battleships.

Evidently, these means of locomotion are approaching speeds at which further increases appear uneconomical.

The destroyers are shown in the diagrams operating in the speed range between 35 and 50 mph. This operation requires about 8 times larger tractive force per unit weight as required, for example, for a large commercial liner. Of course these vehicles are not built for economical operation at such high speeds. Their main feature is maneuverability in battle conditions. Their maximum speed is limited rather by cavitation characteristics of the propellers than by increasing wave resistance.

The specific tractive force required for the submarine on the surface is considerably larger than for merchant ships or battleships. This is due partly to the fact that they are not built primarily for surface locomotion and partly to the limitations of their size. The submarine in submerged state faces frictional and eddy drag only. The diagram shows a rather surprising increase of power required with increasing maximum speed. Such an increase of power does not seem necessary from a hydrodynamic point of view and must be connected with the particular specifications for design of the submarines considered. It is to be noted that recent progress in submarine design is not included in our figures.

TERRESTRIAL VEHICLES

The resistance of terrestrial vehicles can be separated into three components. These are as follows:

- (a) Rolling friction.
- (b) Air resistance of the body.
- (c) Air resistance of the rotating wheels.

In addition, since our definition of tractive force is based on total power built into the vehicle, the transmission losses are included.

In examining Fig. 3, a large disproportion in specific tractive force required for trucks and passenger automobiles, as compared to rail cars, is evident. This is explained sufficiently by the difference in the rolling friction between pneumatic tires and road, as compared to the rolling friction between steel wheels and rail. In the case of the rail car, we find that at relatively low speeds (between 50 and 70 mph) the specific tractive force is essentially independent of speed. As a matter of fact, at these speeds the greatest portion of the total resistance is contributed by the roll friction which is essentially proportional to the load on the wheels and almost independent of speed. As the relative contribution of the air resistance increases, the specific tractive force increases. The rate of increase of ϵ , however, is not determined directly by the law of air resistance which would make the tractive force proportional to the square of the speed; the increase of power at a higher rate is due to several factors, such as the necessity of a climb reserve which prevents the use of the total power built in at high-speed level run.

Similar conditions prevail in the case of automobiles. A recent analysis carried out by Romani for the French Center for Studies in Automobile Engineering shows that in the case of a nicely streamlined automobile, which is supposed to have a drag coefficient equal to about 0.3, referred to the master cross-section of the car, the air resistance becomes equal to the rolling resistance at a speed of about 40-45 mph. At higher speeds, the air resistance becomes more and more the controlling factor. For commercial passenger cars, 125 mph can be considered as the highest speed. The curve shows also that in this case the installed power per unit weight is larger than would be strictly necessary following the law of air resistance.

It is interesting to note, however, that the increase of power required is much more moderate for racing cars. This is due (1) to better streamlining, i.e., reduction of the coefficient of the air resistance; (2) to the fact that the design of race cars allows the use of full motor power in level run at maximum speed. As a matter of fact, if we compare the specific tractive force for the car holding the present speed record with a standard passenger car which has 120 mph maximum speed, we find that the ratio between the respective values of the specific resistance is equal to about 3, whereas the ratio between the square of the velocities is about 11. For vehicles of the same size and shape, whose resistance consists essentially of air resistance, the two ratios should be equal. Of course, in addition to the points mentioned in the foregoing, it must be considered that the rolling resistance is also reduced in the case of the racing car by use of special tires and roadbeds.

For motorcycles, only one point is shown in each diagram since there is little difference between the performance of different makes of motorcycles. The comparison shows that the motorcycle is a rather prodigal device for locomotion. This may be due partly to the relatively large air resistance which is produced by the wheel itself and the parasite drag of the bodies exposed to wind. It has to be pointed out, however, that the ratio of useful load to total weight is very favorable in the case of the motorcycle.

It is interesting to see that the tractive force of single trucks (without trailer) is practically independent of speed in the speed range between 30 and 50 mph where the rolling friction is the determining factor, and the curve representing trucks constitutes almost a continuation of the curve representing passenger automobiles.

AERIAL VEHICLES

Aerial vehicles are predominant in the high-speed range, especially in the speed range above 150 mph. There is one notable exception; that is, the lighter-than-air craft, the airship. In its own speed range, the airship appears several times more favorable -- at least in the sense considered in this paper -- than the helicopter. This does not mean, of course, that the airship can replace a helicopter. The latter is a short-range device, with excellent ability to take off and land almost everywhere. The airship is a long-range means of transportation requiring special landing installations. The

specific tractive force of the airship is of the same order as that of the truck or the automobile. It is also seen that in the speed range in which airships were used, the specific tractive force does not show any increase with increasing speed. One may therefore conclude that it should be possible to design airships of considerably higher speed (say, 120 mph) without any significant increase of the work necessary for a given transport performance. For this aim, the size of the airships has to be enlarged considerably and this may require the solution of new structural problems as well as new methods of propulsion; significant aerodynamic improvements may also appear feasible. At present the difficulties of handling very large ships and the need of relatively large capital investment seem to be the main impediments to the revival of airship development. On the other hand, its large cargo capacity and the comfort of travel may yet secure a place for the airship in transoceanic travel in a speed range between those of the large passenger ship and the commercial airliner.

Proceeding to some remarks concerning heavier-than-air craft -- especially airplanes -- let us consider Fig. 3, i.e., the diagram showing the specific tractive force as a function of the maximum speed. The large specific tractive force required at low speed is a characteristic feature of all airplanes. It is apparently an unavoidable consequence of the fact that the airplane has to provide its own sustentation by power. This is, of course, also true in the case of the helicopter and of the bird-imitating ornithopter which, however, has not yet reached any stage of practical application. In so far as commercial air transport is concerned, this fact is a fundamental restriction in the application of the airplane. Whereas in the domains of ship, railway, and automobile transportation, it is possible to build high-powered fast vehicles for passenger transport and low-powered slow vehicles for inexpensive cargo, the same problem has not yet been solved so far as air transport is concerned.

Let us glance for a moment at the diagram showing the power required per ton for various airplanes (Fig. 2).

For all airplanes, the minimum number of horsepowers required for 1 ton weight is greater than 110. This corresponds to a power loading of about 50 lb per hp. This does not mean that no airplane can be designed to carry more than 50 lb per hp. However, this value of the

power loading appears at the present time as a highest limit for an economically possible airplane. It also represents a reasonable limit for take-off. To be sure, jet-assisted take-off may change the take-off limitations. One sees, furthermore, that the ratio between the speed of the fastest commercial airplane and the slowest private plane in use is about 4.5:1, whereas the ratio between their specific powers is less than 2:1. There are two reasons for this fact: (1) Airplanes with low wing loading have, in general, relatively low structural efficiency. When the size of such a lightly loaded plane is increased, the empty weight goes up to such an extent that pay load and range shrink to small uneconomical items. (2) Small airplanes have in general a less favorable lift-drag ratio than large airplanes because the parasite drag constitutes a larger portion of the total drag. Whether a low-powered glider with excellent lift-drag ratio could reach sufficient popularity to make its manufacture and sale economically feasible, is yet an undecided question. Maybe if a small jet or turboprop engine could be manufactured at low cost, the present situation would change.

It has been mentioned already that the speed of commercial airliners has been greatly increased without penalty to their economy. In this development -- in addition to aerodynamic improvements, the realization of high-altitude air transport played a significant role. The spectacular increase of speed of bombarding aircraft is due mainly to the increase of size and flight altitude.

It is quite instructive to look at the diagram representing the specific tractive force for airplanes (Fig. 3). By its definition, this quantity is equal to the drag-lift ratio of the plane divided by the propulsion efficiency. The optimum ratio for commercial airliners is about 0.08, corresponding to a lift-drag ratio of 12:1, or taking into account the propeller efficiency about 14:1. This figure is proof of the rather highly developed state of aerodynamic design. In the case of fighters, the optimum occurs at a speed of about 400 mph, and the best value is 0.118, which is yet an excellent figure if one takes into account the fact that fighters are not primarily designed for economy -- that economy must be sacrificed for maneuverability and offensive armament, which means unavoidable parasite drag.

We have seen that in the case of ships, the increase of wave resistance at a certain Froude number increases greatly the power required.

In a somewhat analogous way, the airplane has to overcome the so-called compressibility effects, when one approaches sonic velocity, i.e., Mach number 1. This is clearly shown in the diagrams. For example, in Fig. 2, the horsepower-per-weight ratio increases rapidly between 500 and 600 mph speed, and a corresponding increase of the tractive-force coefficient appears in Fig. 3. It is known, however, that by the use of so-called swept-back wings, the critical Mach number can be increased, and the rapid rise of the drag delayed to a higher speed range.

The relatively favorable values for the B-47 bomber are probably due to the effect of sweep-back. The data for fighters with swept-back wings are in general not yet available for publication, but they probably would yield points between those representing the Skystreak and the B-47 bomber.

There are no published data available for airplanes with supersonic velocity at level flight.

Our representation of jet planes is not quite consistent with that of propeller-driven airplanes. For jet planes, we used the static thrust of the engine divided by the gross weight of the airplane for the calculation of the specific tractive force. Similarly, we used thrust horsepower for the calculation of the horsepower-per-ton ratio. It was felt that it is difficult to say what is the meaning of "built-in power" in the case of the jet engine. The use of the thrust-horsepower for the comparative computation gives some advantage to the jet engine, because in the case of the conventional airplanes, the shaft horsepower was used which is equal to the thrust horsepower divided by propulsion efficiency.

LIVING VEHICLES

For curiosity's sake, the authors included some data concerning the man, walking and running, the man on bicycle, and the horse. They did not attempt to discuss quantitatively the case of fish and fowl, nor that of the man swimming in water.

The main difficulty in the case of a living power plant is the estimate of the effective horsepower, which greatly depends upon the duration of the effort used in locomotion. After consulting some publications on the matter, Table 2 was compiled and the data given in the table were used for the diagrams.

TABLE 2 DATA ON LIVING POWER PLANTS

Kind of locomotion	Weight, lb	Speed, mph	Power, hp
MAN—WALKING AND RUNNING			
Walking.....	155	3	0.084
Marching fast.....	135	9	0.30
100-yard runner.....	122	22.4	0.90
MAN—ON BICYCLE			
Pleasure trip.....	185	15.5	0.25
Speeding on highway.....	160	25.0	0.47
On racetrack.....	155	38.1	1.01
HORSE			
With carriage, at fast step.....	3500	4.5	0.64
With carriage, trotting.....	2650	9.0	0.85
Racehorse in gallop, with jockey..	1000	38.5	2.0

It has to be realized that these data are somewhat arbitrary. Nevertheless, one can make interesting observations; for example, the power-per-weight ratio is almost identical for the fastest racehorse and the fastest battleship at about the same maximum speed. It is the belief of the authors that it would be of interest to analyze sport records concerning swimmers, marathon walkers, runners, horse and dog races, and the like, from point of view of the performance of the human and animal power plant.

COMPOSITE VEHICLES

Composite vehicles are in use on the ground, on the water surface, and in the air. We want to restrict ourselves to the terrestrial vehicles. The tugboat is employed mostly for river and canal transportation, and at low speeds; it is rather impractical on the high sea. The glider train has been used as an aerial composite vehicle, but the advantages of such an arrangement are not very great, in so far as saving of power is concerned. It has been suggested that aerodynamic advantages could be realized by coupling airplane wings end by end because the tip losses would be essentially reduced. Similar effect is realized in formation flight of birds and also of airplanes. There are, however, not many data available on this subject.

Fig. 4 represents the curves for power per ton at maximum speed for tractors and trucks with trailers and various types of trains. The reduction of power required per unit weight is due to several reasons: (1) Especially in the case of fast trains, the air resistance of the complete train is considerably less than the sum of the air resistances of the single cars used separately. (2) The concentration of the propulsive power in the locomotive allows the employment of large power-plant units, with better efficiency and lower specific weight. It is seen, however, that the trains driven by electromotors fed from power lines have higher power per weight ratio than, for example, the Diesel-electric and steam-driven passenger

trains. They are even less favorable than single rail cars. The reason is probably the speed limitation by the method of transmission of electric energy from the power line to the locomotive. Because of this limitation, the designers of such trains may not have made such effort for power economy as, for example, have the designers of Diesel-electric trains.

The fast freight train is, no doubt, the most economical type of transport in the speed range between 40 and 60 mph in so far as power required is concerned. The data show a wide dispersion, according to the different number of cars in the train and different sizes and design of the cars themselves. For American freight trains, one obtains a specific tractive-force coefficient of the order of 0.0025 at 60-mph speed, which means that a tractive force of about 5 lb is sufficient to move 1 ton weight at this considerable speed. This figure is to be compared with 0.04 for trucks at the same speed and with 0.08 for airplanes at a speed of 200-300 mph. Unfortunately, as it has been pointed out previously, it is not possible to construct airplanes with reasonable economy at low speeds in order to compete with the railroad in cargo transport. For a speed of 60-80 mph, the coefficient of tractive force would be of the order of 0.2. Consequently, the railroad men should have the consolation that the railway age is not yet terminated. The partial change over from railroad to trucks, which occurred in the recent past, is mainly due to the greater flexibility of the truck system and lesser handling costs of the cargo carried by trucks.

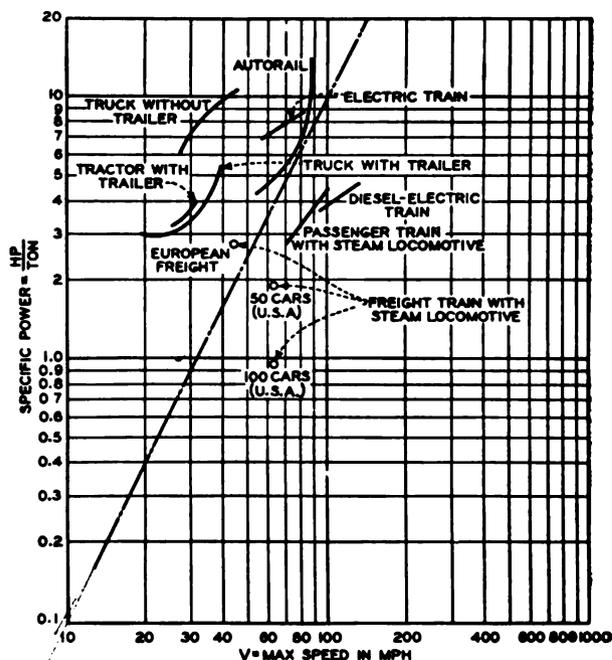


FIG. 4 SPECIFIC POWER OF CONVOYS

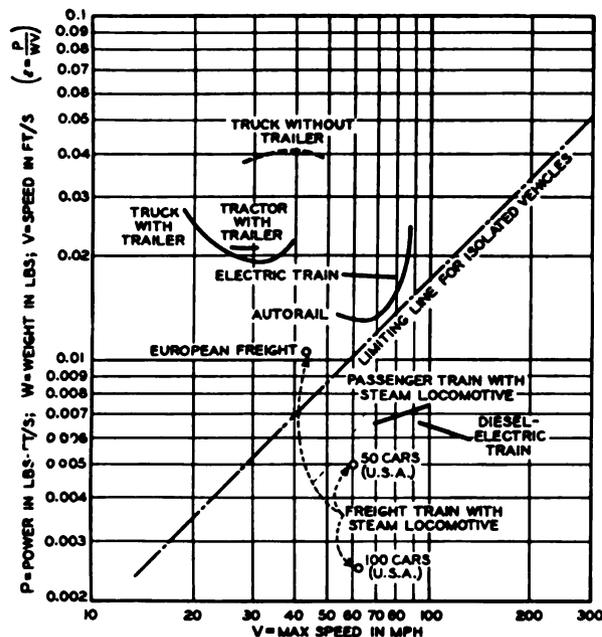


FIG. 5 SPECIFIC RESISTANCE OF CONVOYS

GENERAL DISCUSSION

Let us return to the diagrams pertinent to single vehicles. We observe in both Figs. 2 and 3 that all curves lie above a certain limiting speed beyond which the vehicle becomes uneconomical. The increase of specific resistance with speed for a given vehicle is determined by the law of resistance for the method of locomotion considered. Such resistance laws, in general, depend on various characteristic dimensionless quantities. For example, the law of resistance for ships depends upon the Reynolds number and the Froude number. The Reynolds number determines the ratio between inertial and frictional forces. It contains the density and viscosity coefficient of the fluid medium, a characteristic length of the vehicle and its speed. The Froude number represents a ratio between inertial forces in the fluid and gravity. It is composed of the velocity of the vehicle, a characteristic length, and the acceleration of gravity. The law of resistance of terrestrial vehicles depends primarily on the coefficient of rolling friction and the coefficient of the air resistance. The best values of these coefficients for practically possible shapes are more or less given. However, if we consider the law of similarity of similarly designed vehicles of different sizes, we find that the ratio between air resistance and weight depends on

a non-dimensional parameter quite analogous to the Froude number, since the weight of the vehicle increases with the third power of the length, and the air resistance with its second power. The same parameter enters also into the analysis of aerial vehicles. In addition, the Reynolds number has an influence as far as the frictional resistance of the vehicle is concerned, and the Mach number as far as compressibility effects enter into the picture.

The consideration of all these parameters, however, does not give a full explanation of the limitations of speed which we investigate in this paper. For example, theoretically speaking, the increase of size or at least the length would prevent the rapid increase of resistance per weight ratio which limits the speed of the boats by keeping the Froude number sufficiently low. Why cannot this be done? Of course, increase of size has practical disadvantages which are difficult to be put into equations. The main limitations, however, are certainly those imposed by structural considerations.

It is necessary, therefore, to investigate which dimensionless parameters should be constructed to express the similarity relations between structures of different sizes. The structural efficiency of a vehicle depends certainly on the specific weight γ of the construction material and the allowable stress σ_a for the same material. The ratio between allowable stress and the specific weight has the dimension of a length so that the quantity $\gamma L / \sigma_a$ where L represents the characteristic length of the structure, is a dimensionless quantity. As a matter of fact, if we replace the allowable stress by the ultimate stress σ_u of the material, the quantity $\gamma L / \sigma_u$ becomes the ratio between the length of the vehicle and the so-called length of rupture of the material. The length of rupture is the length of a vertically hanging rod which would break under its own weight.

If we introduce such a structural parameter into our considerations, it appears more understandable that every class of means of transportation approaches a speed limit beyond which no practical design is possible. If we eliminate the length L between the Froude number V / \sqrt{gL} and the structural parameter $\gamma L / \sigma_u = \rho g L / \sigma_a$, we obtain a new parameter which can be written in the form $V / \sqrt{\sigma_u / \rho}$. It is easily seen that this parameter is dimensionless, since the quantity σ_u / ρ has the dimension of the square of a velocity. As a matter of fact, the velocity $\sqrt{\sigma_u / \rho}$ can be given a mechanical interpretation. One can imagine a thin ring built of a material with the ultimate strength σ_u and the density ρ . If one rotates such a ring with the circumferential velocity V , the quantity $\sqrt{\sigma_u / \rho}$ represents the speed at which such a ring would break under the action of the centrifugal forces.

It is interesting to compare approximate values of the quantity $\sqrt{\sigma_u / \rho}$ for various construction materials. Such a comparison is given in Table 3 for a few materials.

In addition to the stress-density ratio, the table also incorporates the ratio between elastic modulus and density. The quantity $\sqrt{E / \rho}$ has also the dimension of a velocity. As a matter of fact, it is directly proportional to the velocity of propagation of sound in the material concerned. It is remarkable that it has almost the same value for all the material considered in the table.

Since the stresses, in general, and especially dynamic stresses, are dependent upon the speed of the vehicle, and on the other hand a limiting line, which should be valid for all types of vehicles, cannot depend upon parameters containing quantities related to specific media or specific types of vehicles, it is probable that the values of the dimensionless parameters

$$V / \sqrt{\frac{\sigma_u}{\rho}} \quad \text{and} \quad V / \sqrt{\frac{E}{\rho}}$$

TABLE 3 COMPARISON OF $\sqrt{\sigma_u / \rho}$ AND $\sqrt{E / \rho}$ FOR VARIOUS MATERIALS

Material	E, psi	Yield point, psi	Tensile strength, psi	Density ρ slugs/cu ft	$\sqrt{\sigma_u / \rho}$ fps	$\sqrt{E / \rho}$ fps
Structural steel.....	30×10^6	50000	80000	15.25	870	16800
Stainless steel.....	29×10^6	40000	80000	15.05	875	16600
Heat-treated steel.....	30×10^6	110000	150000	15.25	1190	16800
Dural 24S-T.....	10.4×10^6	48000	65000	5.4	1320	16650
Titanium (at present).....	15×10^6	90000	120000	8.7	1410	15800
Titanium (in development)...	15×10^6	150000	180000	8.7	1725	15800

have a determining influence on the limitations of speed. The elastic modulus enters in the resistance against buckling and in the flexibility of the structure, which also may introduce limitations. In certain structures, for example, thin-walled so-called monocoque structures, combinations of the ultimate stress and the elastic modulus determine the allowable ultimate load. Hence both dimensionless structural parameters may have influence on speed limitations.

According to the evidence of our collected material, the minimum value of the specific resistance ϵ of single vehicles seems to follow a trend which indicates that the over-all minimum value of ϵ is approximately proportional to the speed of the vehicle. The equation of the limiting line shown in Fig. 3 would be $\epsilon = 0.00175V$ where V is the speed in mph. However, further analysis of the various vehicle systems is necessary to decide whether or not a general law expressed in the dimensionless parameters

$$V/\sqrt{\frac{\sigma_s}{\rho}} \quad \text{and} \quad V/\sqrt{\frac{E}{\rho}}$$

can be established.

It is however, an interesting question whether, and in which way, further increases of velocity of locomotion may be possible without paying the penalty for speed.

First there is the question of how far propulsion efficiency can be increased. In most cases of vehicles of high-quality design, the propulsion efficiency is almost at the optimum limit. The improvement of thermodynamic efficiency of the power plants may radically change the range of various vehicles, but enters only indirectly into the consideration of the specific resistance or the power-weight ratio. For example, a prime motor with lower weight and lower consumption may make it possible to increase the size of a boat in order to come into a more favorable Froude-number range without becoming utterly uneconomical.

There is also the question of novel methods of propulsion: For example, the drag coefficient of surface vessels may be decreased very essentially by lifting the bulk of the floating structure above the water level and supplying sustentation by hydro-foils. We do not attempt to estimate the effect of such a radical innovation. Whether the trials until now appear promising is a question of individual judgment. In the field of aerial vehicles, the long-range rocket shot into high altitude and gliding from

the ionosphere to a distant point on the earth may represent a new method of transportation. This case does not fit easily into our computations because it does not represent level flight but a combination of ballistics and glider technique.

According to computations of H.S. Tsien, a rocket with an average speed of 4500 mph over a 3000-mile range would require a thrust of 190,000 lb of 140 sec duration. This total impulse, distributed over the total flight duration of 40 min. corresponds to an average thrust equal to 11,100 lb. The initial weight is estimated to be equal to 96,000 lb and the final weight to 19,000 lb. With 57,500 lb as average weight, the specific resistance would be equal to $\epsilon \sim 0.2$. This figure is, of course, excellent for such tremendous speed. On the other hand, the ratio between useful and total weight is extremely low as compared to level-flying aerial vehicles.

Disregarding, however, such speculative means of transportation, it appears from the considerations of this paper that probably substantial increase in speed could be realized if new materials with increased stress-density ratio could be made available. If the quantity $\sqrt{\sigma/\rho}$ could be increased essentially beyond the present limits, evidently the limiting line of the specific resistance versus speed would be displaced to higher values of the velocity. At a recent date, titanium alloys seem to give such promise. High-rate titanium alloys, manufactured at reasonable prices, may substantially change the results of the present analysis.

ED. NOTE.

It is pertinent, as a supplement to this broad treatment of locomotion in general, to point out that the specific power of cross-country vehicles, if plotted on Fig. 2, would cover the area between the curve for cyclist and the upper portion of the curve for pedestrian. It is in terms of specific resistance that even more marked departure from the Gabrielli-von Karman limiting line is apparent. Cross-country vehicles, in Fig. 3 would plot above the pedestrian curve at about the same level as the helicopter. Improved performance would require a reduction of this environmental resistance to motion by better adaptation of a vehicle form to the environment in which it operates.