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Weight or the Value of Knowledge[1]

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PREAMBLE

Frank Ramsey's note 'Weight or the Value of Knowledge' is a gem. In the note Ramsey proves that when knowledge or information is free it pays in expectation to acquire it. Collecting evidence means that one is gambling with expectation but this is always preferable to not gambling, on the assumption that information is free (see Ramsay's figure below). Ramsey also shows how much the increase in weight is. For a Bayesian, but also for a non-Bayesian, this is an important theorem since it tells us why we should continue to collect new evidence, *i.e.* continue to make new observations. Several proofs of this theorem can be found in the literature (*locus classicus* is I. J. Good's article 'On the Principle of Total Evidence' [1967] and the discussion in L. J. Savage's [1954] *The Foundation of Statistics*; an early non-Bayesian discussion of the problem can be found in C. S. Peirce's, 'Note on the theory of the economy of research' (1876) [1958]; a discussion of Ramsey's note can be found in B. Skyrms [1990] and N.-E. Sahlin [1990]).

For those interested in the history of probability theory it is a well-known fact that Ramsey in his celebrated paper 'Truth and Probability' (1926) [1990] laid the foundations of the modern theory of subjective probability. He showed how people's beliefs and desires can be measured by use of a betting method and that if a number of intuitive principles of rational behaviour are accepted, a measure used to represent our 'degrees of belief' will satisfy the laws of probability. He was the first one to prove the so-called Dutch book theorem. However, developing the theory of subjective probability he also laid the foundations of modern utility theory and decision theory (*i.e.* twenty years before J. von Neumann and O. Morgenstern [1947] developed their utility theory in *Theory of Games and Economic Behavior* and almost thirty years before L. J. Savage [1954] developed his Bayesian decision theory in *The Foundations of Statistics*). Astoundingly enough, this note shows that Ramsey also had a proof of the value of collecting evidence, years before the works of Good [1967], Savage [1954] and others.

I have not altered Ramsey's unorthodox notation for probabilities, which is taken from J. M. Keynes' *A Treatise on Probability* [1921] (the word 'weight' in the title of Ramsey's note alludes to Keynes' book). When Ramsey, for example, writes $aK/h=p$, this should be read as the probability of a and K given h equals p. But except for this detail the note reads easily.

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[1] This note is part of the Frank Ramsey Collection, document numbers 005-20-01 and 005-20-03, Archives of Scientific Philosophy in the Twentieth Century, Hillman Library, University of Pittsburgh. Quoted by permission of the University of Pittsburgh.

Lemma. Suppose a is an unknown proposition. $\phi(p)$ is expectation of advantage in regard to a if I expect it with probability p.

Then $$\phi''(p) > 0.$$

[*Proof.*] Suppose $x_1, x_2, \ldots$ are variables I can alter by my action.

Good to me if $$a = f_1(x_1, x_2, \ldots)$$ $$a = f_2(x_1, x_2, \ldots)$$

If I expect a with prob[ability] p I act so as to maximize

$$pf_1 + (1-p)f_2 \qquad \text{for fixed given } p$$

i.e. I make $$p\frac{\partial f_1}{\partial x_r} + (1-p)\frac{\partial f_2}{\partial x_r} = 0 \qquad r = 1, 2 \ldots \tag{1}$$

and $$pd^2f_1 + (1-p)d^2f_2 < 0 \tag{2}$$

My expectation of advantage

$$\phi(p) = pf_1 + (1-p)f_2 \qquad \text{with } x\text{'s det[ermined] by (1)}$$

$$\phi'(p) = f_1 - f_2 + \sum_r \left\{p\frac{\partial f_1}{\partial x_r} + (1\text{-}p)\frac{\partial f_2}{\partial x_r}\right\}\frac{dx_r}{dp}$$

$$= f_1 - f_2 \qquad \text{whatever } p.$$

$$\phi''(p) = \frac{d}{dp}\{f_1 - f_2\}$$

Now differentiating (1) w.r.t. p.

$$\frac{\partial f_1}{\partial x_r} - \frac{\partial f_2}{\partial x_r} + \sum_s \left\{p\frac{\partial^2 f_1}{\partial x_r \partial x_s} + (1-p)\frac{\partial^2 f_2}{\partial x_r \partial x_s}\right\}\frac{dx_s}{dp} = 0$$

$$\phi''(p) = \frac{df_1}{dp} - \frac{df_2}{dp} = \sum_r \left\{\frac{\partial f_1}{\partial x_r} - \frac{\partial f_2}{\partial x_r}\right\}\frac{dx_r}{dp}$$

$$= -\sum_r \sum_s \frac{dx_r}{dp}\frac{dx_s}{dp}\left\{p\frac{\partial^2 f_1}{\partial x_r \partial x_s} + (1-p)\frac{\partial^2 f_2}{\partial x_r \partial x_s}\right\} > 0 \qquad \text{by (2).}$$

Value of Knowledge

~~Suppose now I know h and expect a with prob[ability] p.~~

Expectation of advantage $= \phi(p)$.

How much is it worthwhile to find out K?

where say $K/h = \lambda$ say $\mu = 1 - \lambda$

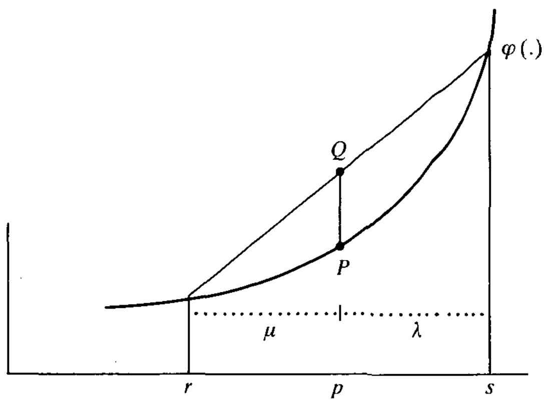
$$\frac{a}{Kh} = r, \qquad \frac{a}{\bar{K}h} = s$$

$$p = \frac{a}{h} = \frac{aK}{h} + \frac{a\bar{K}}{h} = r\lambda + s\mu.$$

Expectation of advantage if I find out K

$$= \lambda\phi(r) + \mu\phi(s)$$

gain $= PQ$ must be [positive] (unless $p = r = s$, K irrelevant)



expression for gain

$$= \lambda\phi(p + r - p) + \mu\phi(p + s - p) - \phi(p) \qquad \lambda + \mu = 1,\ r\lambda + \mu s = p$$

$$= (\lambda + \mu - 1)\phi(p) + \{\lambda(r - p) + \mu(s - p)\}\ \phi'(p)$$

$$+ \lambda\frac{(r-p)^2}{2}\ \phi''(\xi) + \mu\frac{(s-p)^2}{2}\ \phi''(\zeta)$$

$$= \tfrac{1}{2}\{\lambda(r - p)^2 + \mu\,(s - p)^2\}\phi''(\zeta).$$

Increase in weight say

$$= \lambda(r - p)^2 + \mu(s - p)^2$$

$$= \lambda\mu^2(r - s)^2 + \mu\lambda^2(r - s)^2 = \lambda\mu(r - s)^2.$$

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