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# Letter to the Editor—A Classroom Example of Linear Programming (Lesson Number 2)

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# Letters to the Editor

## A CLASSROOM EXAMPLE OF LINEAR PROGRAMMING (LESSON NUMBER 2)

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GEORGES BRIGHAM'S discussion* of the application of linear programming to the problem of blending feed illustrates the use, rather than the abuse, of optimization theory to find a practical answer With your permission, this author would like to present some similar experiments in solving feed blending problems, indicating some traps for the unwary novice and pointing out some uses of the dual solution

### FORMULATION AND RESTRICTIONS

THE experience to be described was encountered in the blending of a dairy pellet feed Some of the ingredients were much the same as those of the reference (Beet-pulp, Corn, Oats, Linseed, Minerals, etc ), but the main purpose of the formulation was to consider the possible use of such exotic ingredients as Safflower Seed, Coconut, Peanut Skins, Fish, etc After much consultation with the customer, the restrictions to be considered were grouped into four categories

I *Nutrient Restrictions*
- A Legal ('tag') requirements
- B 'Good feed' restrictions
- C Customer desirability

II *Ingredient Restrictions*
- A Palatability
- B Mechanical handling requirements
- C Storage (or spoilage) limitations

III *Proprietary Considerations*

IV *The 100 Per Cent Composition Restriction*

The tag requirements are those feed characteristics which must be listed by law (total digestible nutrients, total dry matter, fibre, etc ) Even though these requirements are only vaguely related to the well-being of a cow, there is strong pressure to set and maintain 'desirable' minimums or maximums on these characteristics These restrictions we may take as fixed

* GEORGES BRIGHAM, "A Classroom Example of Linear Programming" (Letter to the Editor), *Opns Res* 7, 524–533 (1959)

There are also certain restrictions, which although not required by law, are recognized as characteristics of a good feed, i e, high protein, minimum levels of calcium, phosphorus, and nitrogen, limits on fat content, etc Now here the question of formulations becomes slightly hazy, one must ask, "what sort of nutritive results are just tolerable?" rather than "what results would you like?" If this is not done, then one may find (after costly hours of machine time) that the customer has postulated a problem with so many restrictions that no feasible solution exists! Here is where over-zealousness on the part of the analyst does not pay off

A third nutrient factor of importance occurs at the selling point Many customers will not buy a feed that does not contain what they consider to be a key ingredient, here the analyst should point out that if this ingredient is not in the (unconstrained) minimum-cost solution, the use of Linseed (or whatever) will cost the miller money The choice is then whether to accept this loss, or to embark on a customer-education program

Ingredient restrictions are usually made on the basis of palatability and handling ability For example, since cows like a certain minimum of salt, it is possible to establish that minimal amount per ton, and then consider additional salt only as a possible mineral contribution

Handling restrictions depend upon the storage and processing equipment available, and whether the feed is to be pressed into pellets, cakes, etc Even here, however, it may not be clear which restrictions are important in the formulation For instance, in one case a bound was set on the amount of molasses that could be used, based on pelletability requirements Now molasses is cheap, has many desirable ingredients, and the cows just love it—so that it was no surprise to find that the optimal solution included as much molasses as possible, and would have included more, without the upper bound However, after presenting this feed to the miller, it was rejected on the ground that, even if he could manufacture it in pellet form, the farmers would not buy it because it would make the manure too soft for use as fertilizer! Clearly, we did not consider the whole 'system' problem!

Proprietary considerations are important because of the miller's desire to give his feed a unique characteristic in terms of color, smell, feel, or the inclusion of a 'magic' ingredient As we shall see later, it is possible to find out how much these considerations are costing him

The last restriction is the '100 per cent composition,' or simplex restriction, that the sum of the ingredients to be used in making a ton of feed must total exactly a ton Here is a case in which the inexperienced analyst can make the same mistake we did What does this restriction imply? It does not mean that the sum of all the *nutrients* in a bag of feed should total 100 per cent, as we shall see, this formulation is rather costly But what it does mean is that the sum of *everything* put in a bag of feed should total 100 per cent, for this reason it is worthwhile to consider the possible addition of a zero or low-cost 'filler' ingredient to act as slack in this restriction To carry this possibility to an extreme, a minimal-cost feed might turn out to be a pill, embedded in a block of inert matter!

Actually, this extreme result will never occur because of other weight-contribution restrictions Furthermore, it is often possible to find an acceptable filler material which will provide money-saving bulk at an extremely low cost Since such material must usually be declared on the tag, one must find an appropriate euphemistic description—as in 'cob with corn,' a cob with at least two kernels of corn! Some of our greatest savings in feed formulation have been found by adding slack to this restriction

## THE SOLUTION

IF THE analyst has been at all successful in keeping the constraint space as large as possible, he will usually find that he is able to save the miller a fair amount of money over his current feed, Broadview's experience with a certain class of dairy feeds has produced savings of 10 to 15 per cent The usual objections to the solution, however, are "That's totally different from what I'm blending now—how can I be sure that the cows will like it?", "What happens if the market changes?", "Can I trade some of this saving for more calories?" (Here he's obviously thinking about a big advertising campaign—"More calories, and it costs less, too!")

There is no escaping the fact that the first optimal feed is likely to be quite different in composition from the current one, even if the nutritive values are similar The biggest difference is usually in terms of the number of ingredients, for example, the miller was in one case using fourteen ingredients, where the number of critical restrictions was ten Our solution, of course, used ten ingredients

As far as the difference in ingredients is concerned, one procedure for phasing into an optimal feed is to take a linear combination of present and proposed feeds, until either current stocks of ingredients are used up, or the cows are happy with the change

Mathematically, this amounts to moving (slowly) from the interior of a convex polyhedron to the appropriate vertex, the intermediate solutions are non-optimal, but have much to recommend them, in terms of customer good-will

Of course, uncertainty in price structure always presents a problem If fluctuations are statistical in nature, the longterm optimal policy is to use the average ingredient costs However, the planning of a feed in terms of variable costs, limited storage facilities, quantity discounts, etc is a dynamic optimization problem that can only be solved in certain limited cases A practical solution is to make several cost-variation studies at the same time the main solution is run, most computer programs have special features that enable one to do this economically These results, together with the dual results mentioned below, indicate which ingredient prices are the most sensitive and give a rough idea of how often the feed blending should be rerun

Usually customer reluctance will limit an actual change in composition to once every few months The analyst who wises to maximize his return from the customer should remember that the most dramatic results come with the first formulation, successive reruns, while indicating different mixes, may not save enough money for the miller to make them a frequent occurrence (Of course, this service need not be costly if the computer program possesses a start-up feature that uses the old solution) Charges for analytical services rendered should be made accordingly!

## USE OF THE DUAL SOLUTIONS

ONE VERY interesting result of linear programs not elaborated in the previous article is the dual solution With a little explanation, these results can provide a feed-blending customer with answers to several important questions

Consider first the dual variables these are incremental costs associated with *increasing* each restriction coefficient By multiplying each dual price by its coefficient and adding, we obtain the dual total cost that, of course, must be equal to the feed (primal) total cost A list such as Table I can be constructed Here one can see the result of a restriction upon total (opportunity) cost, the composition and calorie restrictions are the most 'costly' while the fibre maximum causes a drop in cost It should be pointed out to the customer that restrictions which are inoperative in the final solution do not contribute to this opportunity cost (the 'complementary slackness' principle)

TABLE I

CONTRIBUTION OF RESTRICTIONS TO TOTAL COST

| Restriction | Contribution to total cost per ton |
|---|---|
| 100% composition | $13 13 |
| 585 calories/lb, minimum | 17 24 |
| 10% barley, minimum | 4 90 |
| 13% digestible protein, minimum | 4 28 |
| 4% coconut, minimum | 2 88 |
| 16% fibre, maximum | −1 58 |
| 4% fat, minimum | 1 43 |
| Other restrictions | 0 61 |
| Total | $42 89 |

A more meaningful tabulation results if we show that these duals are related to the effect on total cost of changing each restriction coefficient by 1 per cent of its value, Table II

TABLE II

EFFECT OF 1 PER CENT CHANGES IN RESTRICTION COEFFICIENTS

| Possible 1% changes | Expected savings per ton |
|---|---|
| Use 99% feed & 1% inert matter | 13 1¢ |
| Decrease calorie min to 579 calories/lb | 17 2¢ |
| Decrease barley min to 9 9% | 4 9¢ |
| Decrease digestible protein min to 12 87% | 4 3¢ |
| Decrease coconut min 3 96% | 2 8¢ |
| Increase fibre max to 16 16% | 1 6¢ |

It is important to stress that these savings only hold incrementally at the current solution point But, in practice the duals do give the customer a good idea of what each restriction is 'costing' him, and if these restrictions are at all flexible, they indicate important savings One obvious factor in Tables I and II is the opportunity cost associated with the 100 per cent composition requirement (These data were taken from a formulation made without considering the possibility of an inert, 'slack' ingredient) It is interesting also to note, in answer to the miller's last question, it is possible (at least incrementally) to add 1 per cent of inert matter and also increase the calorie content by about 4 4 calories per pound, at the *same* total cost!

The other way in which the dual solution can be of help to a feed-blending mill is through the slackness of the dual equations For each ingredient used in the primal solution, the dual equation is 'tight,' that is, it is satisfied as an equality However, if an ingredient is *not* used, one obtains a dual inequality of the form $\sum_i a_{ij} y_i \leq c_j$, where $a_{ij}$ is the nutrient-ingredient matrix, the $y_i$ are the dual variables, and $c_j$ is the cost of the $j$th ingredient

TABLE III

DUAL ANALYSIS OF INGREDIENT COST SHOWING USABILITY COST TO WHICH MARKET PRICE MUST DROP BEFORE INGREDIENT MUST BE INCLUDED IN LEAST-COST SOLUTION

| Ingredient not currently used | Current market price (per cwt) | Usability (dual) cost (per cwt) |
|---|---|---|
| NuSweet | \$2 80 | \$2 7966 |
| Oats | 2 60 | 2 41 |
| Ground barley (above minimum) | 2 45 | 2 21 |
| Linseed (above minimum) | 3 20 | 2 80 |
| Corn | 2 85 | 2 41 |
| Alfalfa | 2 50 | 1 73 |
| Coconut (above minimum) | 3 50 | 2 49 |
| Soya | 4 50 | 3 00 |
| Meat | 5 45 | 3 07 |
| Fish | 6 40 | 3 31 |

The left hand side of the equation may be interpreted as a usability cost, $c_j'$, since the criterion for re-solving the problem and including the $j$th ingredient is that $c_j$ should drop to or below the usability cost $c_j'$ For each ingredient not currently in the mix, one may draw a comparison, as in Table III

The table provides a ready check on which ingredient costs must be watched in a changing market, and when the solution should be re-run It also indicates that future problem solutions probably need not include fish, meat, soya, etc, since the chance of their being used is small, in this way, some computer time can usually be saved

Finally, it is not usually realized that the dual variables can be used to evaluate 'special deals' Suppose the customer has a chance to buy a carload of spoiled grain, with sub-standard nutrient ingredient coefficients $a_{i0}$, at an extremely low cost, $C_0$ By using the current dual variables, $y_i$, one may easily test

if $\sum_i a_{i0} y_i \leq C_0$, don't buy,

if $\sum_i a_{i0} y_i > C_0$, re-run solution and then decide

The decision to buy should be made only after the solution re-run, since the projected saving may not be worth the trouble of taking advantage of the special deal, or there may be certain risk factors involved

## SUMMARY

OF COURSE, this discussion does not exhaust the capabilities of linear programming in the feed industry One may improve the feed (say, maximize calories per pound) for the same cost, or more emphasis may be placed on the whole system of mill operation by expanding the size of the problem Also, many computer programs offer experimentation facilities of one kind or another

In summary, our personal experience suggests the following points

1 The formulation is the thing Many of the customer's requirements are not 'strict' inequalities, the bigger the space of alternatives, the more money you can save him

2 Evaluate the computer solution costs realistically, remembering that the customer will be most interested in the first, large savings—not in re-runs which provide incremental improvement in profits Charge accordingly

3 Explain and use the dual—it's a saleable item

4 Don't be afraid to recommend working compromises between present practices and computer solutions

And remember that it isn't a good operations-research problem unless the analyst also learns something

# THE USE OF ROW VALUES IN SOLUTION OF THE TRANSPORTATION PROBLEM

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A COMPUTATIONAL procedure for solving the transportation problem by hand that ordinarily has been overlooked is one involving only the computation of row values (or column values if fewer columns than rows) after each iteration instead of both row and column values as required by Dantzig's method [1] The use of row values alone has proven in our experience to permit a faster hand solution of the problem because of the time saved by not having to compute the column values after each iteration This saving becomes particularly significant in cases where either the number of rows or columns is much greater than the other The procedure has permitted expansion of the problem and allowed consideration of various ramifications to an extent which would not have been practical by other methods