High Achievers in Mathematics: What Can We Learn From and About Them?

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Success in mathematics is widely regarded as an important gate keeper for many courses and occupations. But does success in mathematics at school influence educational and career paths? Do talented mathematics students have distinctive working habits, are they attracted to a mathematics intensive field or more likely to turn to other areas? These and related issues are explored through information gained from students recognized at secondary school as high achievers in mathematics.

Review of Previous Research

The development of exceptionally talented individuals, including high achievers in mathematics, has attracted sustained and diverse research attention. The *Study of Mathematically Precocious Youth* [SMPY] founded by Julian Stanley in 1971 has spawned a huge amount of literature, ranging from publications in which the rationale for the program and early findings pertaining to SMPY participants were described (e.g., Stanley, Keating, & Fox, 1974) to more recent documentation of longer term personal growth, educational and vocational adult achievements. As noted by Lubinski, Benbow, Webb, and Bleske-Rechek (2006) many of these latter publications focus on students who "before the age 13, … scored within the top 0.01% for their age on either SAT mathematical reasoning ability (SAT-M ≥ 700) or SAT verbal reasoning ability (SAT-V ≥ 630)" (p. 194). Others to explore the development and working preferences of highly able mathematics students and of mature and successful mathematicians include Burton (2004), Csikszentmihalyi, Rathunde, and Whalen (1993), Gustin (1985) and Wieczerkowski, Cropley, and Prado (2000). The collective findings from these studies have revealed the importance of parental, educational and peer support, the mathematics students' and mathematicians' willingness to work hard, satisfaction from their mathematical achievement, their appreciation of the aesthetics of mathematics – its "succinctness, compactness, or conciseness" (Burton, 2004, pp.187-188), and the way they were "motivated by the order and beauty they see in mathematics" (Gustin, 1985, p. 330). Other findings from these works are interwoven with the reporting of data from the current study.

Aims

The overall aims of this study are to examine how exceptionally high achievers in mathematics perceive mathematics, and to gain insights into their background, motivations, work habits, and occupational choices. This information is particularly timely, given the drift away from demanding mathematics courses and the widespread concerns about the declining popularity of mathematics.

Method

Csikszentmihalyi, Rathunde, and Whalen's (1993) study of talent development and Eccles' (1985) model of academic choice were particularly influential in shaping the focus of this study. These authors highlighted personal qualities and characteristics (e.g., subject specific and broader attitudes and beliefs, expectations, motivations, self-perceptions) and environmental factors (e.g., the cultural milieu, the home, peer and educational environments) as significant predictors of success. Aspects of these factors are tapped through the Web based survey used in this study.

Sample Selection – Preamble

Purposeful sampling was used to select information-rich cases for this project. Focusing on students who have demonstrated exceptional performance in a limited domain, that is, purposeful sampling to obtain optimally intense descriptions of the phenomena of interest, is an approach frequently selected by those working within a framework of gifted education (e.g., Bloom, 1985; Csikszentmihalyi et al., 1993; Lubinski, Webb, Morelock, & Benbow (2001). Participants for the current study comprised past and current high achievers in the Australian Mathematics Competition, which is described more fully below.

The Australian Mathematics Competition [AMC].

The first AMC, held in 1978, attracted some 60,000 students from 700 schools. Since then the competition has grown substantially and, according to the Australian Mathematics Trust [AMT] (2007), it has become "the largest single event on the Australian Calendar". In recent years some 400,000 students from 40 countries have entered the competition. The AMC is open to students of all standards and each entrant receives some form of acknowledgement of success (AMT, 2007). Each year a small number of students, about 1 for every 10,000 students entered, receives a medal. "These are awarded on the judgement of the committee to students who are outstanding within their region (Australian State or Territory or other country), within their year group and internationally" (AMT, 2007). Thus the level of achievement required for receipt of a medal is as stringent as that for entry into the SMPY program, some of whose participants are described by Lubienski et al. (2006).

Between 1978 and 2006, 690 medals were awarded to secondary school students within Australia. Since some students win a medal in more than one year the actual number of recipients is less. Many of the earlier medallists can be reached as they maintain contact with the AMT through personal communications or the AMT website. For others, older address details are still available. There are few females among the medallists (Leder, Forgasz, & Taylor, 2006) even though roughly equal numbers of females and males enter the AMC, especially in the earlier secondary school years.

Sample Selection – Further Details

For this study, the potential sample was restricted to students who attended schools in Australia and were awarded a medal between 1978 and 2006. To comply with ethics requirements, potential participants were contacted as follows.

A letter, containing a brief outline of the study, support for it from the Executive Director, details about the principle investigator, and a request to complete a survey posted on the AMT Website, was sent by the AMT to medallists for whom contact details were available. For recipients of multiple medals, the letter was sent to the most recent available address. Using this approach, 420 letters were mailed. (This ensured almost full coverage of those awarded a medal over the period of interest – see footnote 2). Fifty-two letters were, however, returned as undeliverable.

At the time of writing this paper (early February 2008), the survey had been accessed by 94 individuals – 84 males, 9 females, 1 gender unknown. To ensure that the data gathered were restricted to AMC medallists[5], surveys were discarded if items relating to winning the AMC medal were not answered.

The Instrument

The survey covered five broad areas: background; school and university; career/vocation (actual or intended); work habits; and some general issues about self. Some items were open ended: for example, "What did winning a medal mean to you?"; "What are your favourite leisure pursuits?" Others were in 5-point Likert format : for example, "Once I undertake a task, I persist"; "I'd rather work alone than with others to complete a task", with possible responses ranging from strongly agree to strongly disagree. More of the survey's content can be inferred from data reported in the results section.

Data Gathering and Synthesis

SurveyMonkey (http://www.surveymonkey.com/) was used to create the online survey and to validate, collect and summarise responses.

5 Although only medallists were specifically directed to the survey site by letter, others who perused the AMT Website also had access to the survey and might have explored it out of curiosity.

Results and Discussion

Response Rate

Surveys were still being completed some two months after the original posting. This could be a reflection of the wide geographic distribution of former medallists, and possible delays in the letter being forwarded to them by their family. For this study, there are different ways of calculating the response rate. Two are given here.

As already indicated, at most 368 of the letters recommending AMC medallists to complete the survey reached their intended destination. The 79 useable responses thus represent a response rate of at least 21%. As is discussed in the next section, the full range of AMC medallists – from the beginning of the competition to the last year surveyed – were represented in the 79 completed surveys. Of the 94 who accessed the survey, 79, that is, 84%, completed a survey useable for the study. Thus most of those for whom the survey was intended found it sufficiently relevant to finish it. These response rates are within acceptable limits (see, e.g., Hamilton, 2003; Kaplowitz, Hadlock, & Levine, 2004)

Sample Details

Of the 79 useable sets of responses, 74 were completed by males; 5 by females. The low number of responses by females prevents any gender comparisons being made. Just over 40% of the respondents were multiple medallists[6]: 25 won two medals, 9 won more than 2 medals, and 45 indicated that they had won one medal. Those who completed the survey included medallists from 1979 to 2005, whose dates of birth ranged from 1960 to 1994. All but five of the respondents indicated that they had completed or were close to completing their first degree. Just under two-thirds of these had completed at least one additional degree. Thus medallists now well into their chosen career as well as students still at school or university were among those who completed the survey.

Background Information

Twenty-one of the medallists (27%) were born out of Australia. This is somewhat higher than the corresponding figure of 22.2% for Australia's population, reported in 2006 Census data (http://www.abs.gov.au/websitedbs/d3310114.nsf/Home/census). Though not directly comparable, it is of interest that 30% of Lubinski et al.'s (2006) SMPY sample had at least one parent who was born outside the USA.

Parents

Occupations were listed for most of the mothers. Fifteen percent were described as home makers. Of the remainder, over one-quarter (27%) were teachers, ten percent were nurses, ten percent were medical practitioners or dentists, six percent were pharmacists, and a further 16% were in a mathematics or science related field, for example, accountant, computer analyst, laboratory pathologist, scientific programmer, and statistician. The other working mothers had diverse occupations including: business manager, editor, librarian, personal assistant, and sales assistant.

Many of the fathers (20%) worked in a mathematics or science related field, for example, computer scientist, biochemist, metallurgist, mathematician, scientist, or statistician. A further 14% were engineers. Other occupations included business/manager (14%), medical practitioner (12%), teacher (8%), and accountant (7%), as well as cook, home trader, minister of religion, and public servant. Overall, it can be inferred, the medallists' parents were tertiary educated.

The mathematicians and able mathematics students investigated respectively by Gustin (1985) and Csikszentmihalyi et al. (1993) similarly had well educated parents. In contrast, 30% of the 70 mathematicians in Burton's (2004) sample "came from a working class family where there was no history of university attendance" (p. 38).

6 If a similar proportion of the 690 medals awarded over the period of interest were to multiple medallists then the 420 letters mailed out represent virtual full coverage of the 1978-2006 medallists. This is consistent with the comprehensive data base maintained by the AMC.

Medallists

Favourite subject at school. Almost two-thirds of those who responded to this item (61%, i.e., 45 out of 74) gave mathematics as their favourite or joint favourite subject. Reasons for the choice of mathematics varied, as can be seen from the representative comments that follow.

Being good at mathematics was a reason given by many: “the ability to obtain 100% marks in tests”; “able to answer the questions and the certainty of the answers”; “I was good at it and enjoyed that”; “I often did well”.

Some focussed on the beauty of mathematics: “mathematics is elegant”; has “inherent beauty”; is “methodical but exquisite” were some of the phrases used.

Others liked the logic of mathematics and wrote: “everything fits together; “logical”; “logic, structure, precision!”; “logical thinking, problem solving; mathematics requires logic and understanding much more than hard work and good memory”; “It was the most intellectually rigorous subject”.

The perceived certainty of mathematics was also given as a reason: “clear cut answers”, “it was unambiguous. You were either right or wrong”.

Many enjoyed the problem solving component and the challenges: “solving difficult problems”, “finding and fixing problems”, “The challenge of problem solving, the satisfaction that came when a new concept was understood”, “intellectually stimulating, great satisfaction from solving problems”, “I was able to solve things by myself”, “the pleasure of figuring something out that was not initially obvious”. The thrust of many of the answers given is neatly captured in the following comment: “The interesting non-routine problems that required extensive exploring and creative, logical thinking”.

Especially noteworthy is the fact that a number of respondents gave being able to do extension work beyond the regular syllabus as their reason for liking mathematics.

Of those who did not select mathematics as their favourite subject, 11 nominated other science subjects. Six selected English or another language, with the subject’s perceived logic and challenge again being the attraction: Latin because “it was the most challenging subject; logical thought processes”; Japanese because of its “blend of science skills and arts skills”. Also nominated was music because it provided “the opportunity to study something that is universal. I enjoyed playing the piano”, and outdoor education because “it consisted mainly of exciting things like rock climbing, kayaking, and rafting … . It was very social. It was something physical that I was good at, so I felt less geeky.”

Careers. As can be seen from the data in Table 1, the majority of the medallists were in, or intended to aim for, mathematics or fields heavily dependent on mathematics.

Table 1

Medallists' Careers

Career: actual or clearly stated as intended	Frequency
Academic/Professor (field not stated)	3
Actuary	8
Artificial intelligence researcher	1
Astrophysicist /Mathematical physicist/scientist (field not stated)	3
Doctor (general practitioner/various specialists)	12
Economist	2
Engineer (various fields)	6
Hedge Fund trader/finance	4
IT/computer science	5
Management	3
Mathematician (academic/researcher)	10
Software engineer/developer	6
Statistician	3
Student (currently & no clear career indication)	8
Other (e.g., music teacher, lawyer, video game developer)	5
Total	**79**

Most of the respondents indicated that mathematics was important in their work. In some cases the reason for this was clearly tied to the demands of their job. For example: "Yes. I am a mathematics professor (!), so I teach AND research it" (Similar answers were given by another nine respondents); "Finance is underpinned by understanding of math and statistics"; "Yes. I am a Statistical Methodologist for [a company] and my role depends on my mathematical comprehension". Other comments included:

> Vital; both in my current work as a statistician and my previous work in biomedical research, I'm essentially an applied mathematician, so it comes into almost every aspect of what I do.

> Yes, being a hedge fund trader required keen understanding of probability, expectation and risk, and most importantly not to be fooled by the randomness of many daily events. People from a mathematical background are much more likely to have an appreciation of these things.

Some responses illustrated how subtly mathematics is integrated into other occupations:

> Yes. Sales forecasting requires trend analysis and pattern detection.

> I am grateful at its practical application in medication dose calculation and working out people's kidney functions. But in a broader sense the statistical analysis of drug trials influence our daily prescribing decisions.

> Yes but less than I thought it would be when I thought I was going to be an engineer. I'm now officially an artist, but I work with computers and it really helps to have a basic knowledge of calculus and a good knowledge of techniques associated with 3 dimensional geometry.

Even those who considered mathematics to have limited applicability to their current job often appreciated the broader skills they had gained from studying mathematics:

> Not directly, but I do think the logical processes and problem solving skills are used all the time in any sort of work, and certainly in teaching and music making. …I think quite mathematically often.

> While doing maths I learnt formal logical skills and ability to learn in a flexible way. These are used regularly at work. However the technical mathematical skills are very rarely used.

Collectively, the medallists' attitudes to mathematics and choice of careers and working preferences reflect the qualities identified by Wieczerkowski et al. (2000) as characteristics of high mathematics achievers: "placing a high value on mathematics", and "seeing oneself as capable of being successful in the mathematics field" (p. 419).

Leisure occupations. The medallists' leisure time pursuits were eclectic. They included sport (e.g., football, golf, hiking, rock climbing, running, soccer, squash, swimming, tennis, volleyball), music (including guitar, piano, singing, violin, and writing music), card games, playing chess, photography, reading, socializing/ spending time with family, and writing. The list of leisure activities closely mirrors those nominated by the students in Csikszentmihalyi et al.'s (1993) study. Stanley et al. (1974) and Lubinski et al. (2001) have also pointed out the diversity of interests of students in the SMPY program, the diverse fields to which they are attracted, and the variety of careers in which they ultimately engage.

Winning a medal. None of the medallists mentioned negative aspects of winning a medal. Many indicated that winning a medal gave them great satisfaction and pride in having their mathematics achievement recognized. Attending the actual award giving ceremony was also prized by many. New or continuing opportunities to attend special courses and do advanced mathematical work with others who liked mathematics and were good at it were seen as particular benefits. Others talked of specific doors being opened, or of longer term benefits.

> A source of pride – we were immensely competitive in a good-natured way at school and there were 3 or 4 students in my year who won AMC medals in various years. We still get together every year to do the Westpac / AMC competition paper over dinner (our 15th year this year).

Working preferences and motivation. Table 2 contains aspects of the medallists' working preferences and motivations.

Table 2

Medallists' Working Preferences and Motivations (in Percentages)

Item	Stornly Agree/ Agree	Neutral	Strongly Disagree/ Disagree
I enjoy working competitively with others	67	14	19
I thrive when I work at something which is challenging and difficult	96	4	-
When working in a group I would rather be in charge	43	43	8
Good relations with my fellow workers are more important than task performance	37	37	26
I worry that my success may cause others to dislike me	25	19	56
It is important for me to perform better than others on a task	48	32	20
I try harder when I am cooperating with others on a task	57	28	15
I sometimes work at less than my best in case others resent my performance	7	10	83
I'd rather work alone if at all possible	40	27	34
Once I undertake a task, I persist	81	14	6
It annoys me when others do better than me	34	28	38
I prefer working in situations that require a high level of skill	93	6	1

Several features stand out. Medallist thrive on doing difficult, challenging, and highly skilled work. Once started, they persist with a task. Their motivation and task commitment is high. Working competitively but also working cooperatively was liked by many. These qualities mirror those of Csikszentmihalyi et al.'s (1993) sample who were described as having "high achievement motivation and endurance" (p. 207).

Being in charge of a group seemed not to be important. Those who preferred to work alone slightly outnumbered those who did not. In general, the medallists wanted to perform well, irrespective of the reactions of their peers. About the same number agreed, as disagreed, that they were annoyed when others did better on a task.

Self descriptions. Many of the medallists (75%) provided a description of themselves. Space constraints allow only a few (representative) comments to be included:

> Very laid back and even-keeled. I like to solve problems, whether they be arguments (or miscommunications) or technical problems about how we can make something in our game look better. I tend to get very absorbed in what I'm doing and spend too much time on it.

> An affable introvert. Keen to please people, scared of confrontation. A good listener but reluctant to open up to others. Able to dig through a lot of dull work provided there's some nugget to excite my imagination. A good teacher, able to communicate concepts simply. Modest. Reasonably self-assured. Tend to fantasise about possible achievements although I lack the ambition to define and pursue my own goals. I could go on ...

> Quietly fun, easily entertained, patient; caring, principled, sensitive to some emotions; proud, perhaps a little arrogant, attention-loving; thinking, logical, analysing; careful, shy, well-meaning.

> Irritable, passive aggressive, a little obsessive compulsive, a bit reserved, mercurial, a little anxious, a little paranoid, worry a little too much, a bit of a hypochondriac, complain a lot, not easily offended or shocked, vulgar at times, easy going, depressive at times, irreverent, not too serious, good sense of humour...

Concluding Comments

Many of the findings of the current study, based on questionnaire responses, are as expected – with respect to family background, the medallists' reasons for liking mathematics, their varied career choices in areas in which they saw mathematics used directly or indirectly, their delight in a challenge, their high level of motivation and persistence, and their wide range of leisure activities. The next stage of the study, that is, interviews with participants involved in the first phase, is an opportunity to probe contextual nuances and individual differences subsumed in the more general findings.

Particularly provocative for those involved in mathematics education is a recurring theme that permeated many of the responses: the most exciting and fulfilling mathematics came from opportunities to do advanced mathematical work with mathematically talented peers outside the regular school curriculum. This finding, too, can be explored in more depth during the interviews.

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