David is eight years old and has an I.Q. of 145 on the Wechsler Intelligence Scale for Children-Revised (WISC-R). He is capable of grasping anything of interest to him and has lately been "begging" his parents to get geometry books for him. Just as David's parents wonder whether or not he is ready for geometry—or even beginning algebra—the regular classroom teacher may be uncertain of how to design an appropriate mathematics curriculum for him.

As a first step in developing a specialized plan for students with advanced abilities in mathematics, parents and teachers often request an intelligence test as part of an evaluation. Although an I.Q. score can be a useful initial indicator of general academic talent, it does not provide information specific enough for evaluating or planning an educational program based upon a student's strengths. One option for obtaining specific information and meeting the learning needs of a youngster such as David is the diagnostic/prescriptive approach described in this article.

The authors wish to thank Drs. Linda Brody, Mark Saul, and Stephen Willoughby for their helpful comments on an earlier version of this paper, and Dr. Carol Mills for providing information about the CTY Young Students Classes.

by Ann E. Lupkowski, Susan G. Assouline, and Julian C. Stanley
High and high school students who were ready to learn algebra at a faster rate and more advanced level than is typical. The DT-PI method can be adapted for effective use with elementary students who are extraordinarily talented in mathematics. Figure 1 depicts the five steps of the DT-PI model.

Children who have earned a total math score at the 97th percentile, or above, on a standardized, group-administered achievement test such as the Iowa Test of Basic Skills at their own grade level would be suitable candidates for initial screening. A five-step procedure of standardized testing followed by instruction based on test results is employed. (See Figure 1.)

Step 1: Aptitude Test
After initial grade-level screening, the student takes a beyond-level test such as the School and College Ability Test (SCAT). The SCAT is a scholastic aptitude test developed by the Educational Testing Service (ETS) and marketed by CTB/McGraw-Hill, 2500 Garden Road, Monterey, CA 93940. The SCAT was designed for students in the third grade through the college freshman year and therefore is useful as a beyond-level test for talented elementary students. It is highly important to use a difficult enough level of the SCAT. The staff of SMPY uses the following rule of thumb for determining an appropriate starting point: exceptionally able students in grades one through three should be given the level of the test that is two grades beyond their grade placement, and students in grades four through six should be given the test that is three grade levels beyond their grade placement (Cohn, 1988).

SMPY recommends that students earn a score at the 75th percentile or higher for the grade level of the SCAT administered (e.g., for a third grader, use fifth grade norms) before moving on to Step 2. The authors recognize the need for curricular adjustments such as enrichment and problem-solving activities for those talented students scoring below the 75th percentile, but the focus of this article is programming for the extraordinarily talented student.

Using the DT-PI Method With Elementary Students
Diagnostic Testing Prescriptive Instruction (DT-PI) is a beyond-level model (Stanley, 1978, 1979, 1984) developed for use with talented junior high and high school students who were identified by their performance on a mathematical aptitude test that was designed for able students several years older than the youngsters of SMPY. Use of a beyond-level (also referred to as above-level) test is critical to the accurate measurement of aptitude. SMPY’s students are so exceptional in their performance that instruments normed for their age or grade restrict the evaluation of their talents. This type of restriction is known as a ceiling effect, where students figuratively bounce against the ceiling of a test because there are not enough difficult test items and therefore they answer all or most of the items correctly.

From a diagnostic perspective, an educator cannot determine what needs to be learned next when the student has answered virtually all of the items correctly. An above-level test therefore is necessary so that the student is presented with challenging items that can eventually be used diagnostically.

Some of the above-level procedures SMPY has pioneered for junior high and high school students can be adapted for use with younger students who show promise in mathematics. Presented in this article is an adaptation of SMPY’s diagnostic/prescriptive model, which has been used effectively with junior high and high school students. Also included is an outline of how the model can be used with mathematically talented school students.

FIGURE 1

<table>
<thead>
<tr>
<th>1. APTITUDE TEST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student scores ≥ 75th percentile of national norms on out-of-level SCAT.</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>2. ACHIEVEMENT PRE-TEST</th>
</tr>
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<tbody>
<tr>
<td>Student scores ≥ 50th percentile of national norms.</td>
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</table>

<table>
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<tr>
<th>3. READMINISTER MISSED ITEMS</th>
</tr>
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<tbody>
<tr>
<td>Mentor analyzes missed items.</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>4. PRESCRIPTIVE INSTRUCTION</th>
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</thead>
<tbody>
<tr>
<td>(Based upon topics not yet understood.)</td>
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<tr>
<td>Student completes topics.</td>
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<table>
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<tr>
<th>5. POST-TEST</th>
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<tbody>
<tr>
<td>(Use parallel form of test given in step 2.)</td>
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</table>
Step 2: Achievement Pre-Testing

The examiner administers standardized mathematics achievement tests such as those of the Sequential Tests of Educational Progress (STEP). Where- as the primary goal in Step 1 was to determine mathematical aptitude, the purpose of Step 2 is to measure mathematical achievement. There are several levels of the mathematics portion of the STEP tests, and each level includes a Basic Concepts and a Computation test. When originally developed, there were two parallel forms of the STEP mathematics tests, which are still used by SMPY. At present, only one form is available through CTB/McGraw-Hill. However, having at least two forms of a test is critical to the DT-PI process, because the second form will be used in a later step. Obtaining two parallel forms of the STEP test may be accomplished by checking with the support personnel responsible for testing in your school district to see if both forms are on file, or contacting the senior author at the University of North Texas.

The STEP or other mathematics achievement test is administered under standardized conditions. Students are encouraged to answer all questions, but are instructed to indicate those items of which they are unsure by writing a question mark to the left of the item number on the answer sheet. When the testing time expires, the examiner retrieves the test materials from the student, scores the test, and determines the percentile rank of the student’s score. The percentile rank is based upon the norms for the highest grade group available, as this represents the most rigorous standard. For example, a second grader who correctly answers 29 items on Form 4A of the STEP Basic Concepts test would rank at the 50th percentile when compared to first semester 6th graders. If the same score of 29 were compared to first semester 4th graders, the student would rank at the 92nd percentile. Thus, the 6th grade norms provide the most rigorous standard.

Progress to Step 3 in the DT-PI model is determined by the student’s score on this test. SMPY recommends that students score at least at the 50th percentile of the relevant national norms for the grade level of the test administered (Stanley, 1979). Students earning a score at the 50th percentile are likely to learn the rest of that subject quickly. Students scoring below the 50th percentile should be given the preceding level of that test.

Step 3: Readministering Missed Items

If the student scores at least at the 50th percentile under standardized conditions, the examiner returns the test booklet to the student along with a list of the items he or she marked wrong. The examiner does not return the answer sheet to the student or tell the student how those items were missed. The student is asked to rework those items on a separate piece of paper, taking as much time as needed and showing all work systematically. The items are rescored, and instruction is determined based upon the student’s responses.

Step 4: Prescriptive Instruction

The fourth step of the DT-PI procedure is prescriptive instruction based on the testing. Prescriptive instruction is conducted by an individual skilled in mathematics, known as a mentor. Although he or she may be the student’s classroom teacher, it is not necessary for the mentor to be a trained teacher. In Step 4, the mentor examines the test items by which the student has placed a question mark, those missed during the initial administration, and the readministered test items. The mentor places the student’s responses into one of four categories: (1) items answered correctly, but noted with a question mark, indicating that the student may have guessed; (2) items left blank in the initial administration but answered correctly in the readministration, indicating that time may have been a factor; (3) items missed in the initial administration but answered correctly in the readministration, indicating mastery of the material; and (4) items missed in both administrations, indicating that the student did not understand the underlying concept. For items missed twice, the mentor needs to determine if they were missed the same way both times, or not, and how the examinee approached the problem.

It is critical to ascertain which items (and their underlying principles) are correctly understood and therefore which points warrant further instruction. The mentor queries the student about problem-solving procedures in an attempt to clarify further which underlying principles are not understood. An item profile chart (usually available in the test’s manual) may aid this process as well as assist in record-keeping.

For those items answered correctly, but marked by the student with a question mark, the examiner should inform the student that the items were correctly answered, but ask him or her to show how the answer was obtained. In this way, the mentor guards against guessing and “good luck.” (E.g., the student may have eliminated two options and then guessed correctly between the remaining two.)

The mentor then works with the student on the principles (not the items!) he or she does not understand. SMPY recommends using mentor-made problems to help the student study a topic. Textbooks may be useful resources for practice problems. The DT-PI process requires mastering one topic before moving on to the next, but it is imperative that the mentor not require the student to work through every page of a textbook. An advantage of the DT-PI procedure is that students can study new concepts in greater depth than would typically be possible and will not needlessly repeat topics already learned. The purpose of the DT-PI model is to help the student learn mathematics well and at a pace that is challenging to him or her. The goal is not to race through the elementary mathematics curriculum so that the child might start doing algebra as quickly as possible (Stanley, Lupkowski, & Assouline, submitted).

Step 5: Post-Testing

In the final step of the DT-PI mentor model, the student is given the parallel form of the achievement test as a post-test. The goal is for the student to score at least at the 85th percentile of the most rigorous norms for the test, thus indicating mastery of the material. Students who score lower than the 85th percentile require additional instruction and practice with the material. Those who score at the 85th percentile or above, but earn less than a perfect score on the test, require work on the topics they do not yet understand. When the mentor is satisfied that the student has adequately “cleaned up” his or her
knowledge of the topic under study, the mentor and student re-enter the model at Step 2, using achievement tests and materials for the next level or topic. Thus, the student studies the mathematics topics in a linear fashion, demonstrating mastery before moving on.

The Role of the Mentor

For the "prescriptive instruction," one needs a skilled mentor. He or she should be intellectually able, fast-minded, and well-versed in mathematics considerably beyond the subject to be learned by the "mentee(s)." This mentor must not function didactically as an instructor, pre-digesting the course material for the mentee. Instead, he or she must be a pacer, stimulator, clarifier, and extender. (Stanley, 1979, p. 1)

It is not necessary for the mentor to be a trained mathematics teacher. Engineers, college professors, and mathematics majors, as well as high school math teachers have been successful mentors. If the classroom teacher is not the mentor, it is important for mentor and teacher to communicate so that the teacher is aware of the level of mathematics instruction presented to the student.

SMPY recommends that a mentor and a student meet once weekly for two or three hours. In between meetings, it is helpful if the mentor is accessible to the student via telephone to answer questions or make clarifications. Although the DT→PI process involves mentor-paced, rather than student-paced instruction, it is critical for the mentee to take responsibility for his or her learning, especially by doing extensive homework carefully, completely, and well. Especially for younger students, parents may need to supervise homework to ensure that it is completed each week. It is crucial, however, that the student be essentially self-motivated and interested in participating in the DT→PI process, and is not doing it simply to please the parents. Mentor and parents should try to make the work interesting and stimulating. Youths should find it much more fun and more ego-enhancing than regular math classes in school.

The DT→PI process is facilitated if the mentor has an overall plan. Setting particular goals and outlining a linear progression of topics to be covered are part of developing a systematic plan for the student. The item profile chart completed in Step 4 is one device for organizing the plan of study.

It is not necessary for mentors to work with only one student at a time. It is possible for skilled mentors to group as many as five students together for mentoring sessions. Students who are at about the same level in their understanding of mathematics, even if they are not the same age, may be grouped for mentoring sessions. For example, the Center for the Advancement of Academically Talented Youth (CTY) at Johns Hopkins University offers classes based on the DT→PI model. Students are placed in small groups of two to five students according to their performance on the diagnostic tests (Moore & Wood, 1988). Students in these groups benefit from appropriate placement within the mathematics curriculum as well as from the opportunity to interact with their intellectual peers. Nevertheless, a one-to-one setting may be preferable for many students and mentors. The successful interaction between David and his mentor is described in the sidebar to illustrate the DT→PI approach.

Appropriate Instruction at an Appropriate Rate

Although diagnostic testing is an important component of the DT→PI model and has been a focus of this paper, the purpose of this model is not to test the students, but to provide appropriate instruction based upon their needs. A much larger proportion of time is spent in instruction, learning, and practicing compared to the amount of time spent in testing. The model goes beyond testing by using the diagnostic test results to plan instruction.

In the DT→PI model, the study of mathematics is individually paced, so each student covers the material at an appropriate rate. Because students do not have to study topics they already know well, they have more time to study challenging topics, with which they are not familiar, in greater depth than would be possible in the regular classroom. The model is self-correcting because students do not move on to the next topic before mastery is attained. It avoids the pitfall of the "gaps" that may occur with grade-skipping, while ensuring that the student is challenged by the pace at which material is presented and stimulated by the more advanced material. Because extremely talented students who participate in the DT→PI process are consistently presented with challenging material, they avoid developing bad habits such as not writing out or checking their work when solving problems or doing complicated computations. Another advantage is that students learn that testing can be used for reasons other than evaluation. (E.g., testing in the DT→PI model is used to determine appropriate topics and levels of instruction.)

Before starting the DT→PI procedure, the student, parents, and mentors should work in concert with the classroom teacher and other school personnel. All of the people involved must consider the ramifications of participation in the DT→PI procedure. For example, important issues to consider are how the student earns school credit for work completed with the mentor and how the mentoring experience will fit in with the school's curriculum and grading procedures. A pedagogically sound approach is to permit the student to work on the mentor's homework assignments during the regular mathematics periods and give credit for work completed with the mentor, as evaluated by appropriate mathematics teachers in the school.

Although the DT→PI approach to educating mathematically talented...
An Example of the DT-PI Procedure

Step 1: Aptitude Test
At 8 years of age, David earned a total mathematics score at the 98th percentile on the Iowa Test of Basic Skills. Thus, he was eligible to try one of the recommended aptitude tests. David, who was in the third grade, took the quantitative portion of the elementary level of the SCAT test, which was designed for grades three through six. This beyond-level test was appropriately selected following the rule of thumb described in Step 1 of the DT-PI Model for elementary students. He scored above the 75th percentile and therefore was ready to proceed to Step 2.

Step 2: Achievement Pre-testing
The STEP Mathematics Basic Concepts and Computation tests, Level 4, Form A (representing a difficulty level appropriate for grades 4, 5, and 6) were administered under standardized conditions. David was instructed to place question marks on the answer sheet next to those items of which he was uncertain. On the Basic Concepts test, David correctly answered 38 of the 50 items, yielding a score at the 83rd percentile (when compared to the items, yielding a score at the 83rd percentile when compared to 6th graders). The mentor worked with David on Saturdays for two hours per session. David was permitted to read, work on the computer, or do other homework during large-group mathematics instruction. David worked on his mentor-assigned homework while his other classmates worked on their homework. In the evening, his parents checked that the homework was completed. The teacher was very supportive of David and the completion of the mentor-assigned homework and encouraged David to ask any time he had questions concerning the homework.

Step 3: Readministering Missed Items
The examiner kept David’s answer sheet, but returned the test booklet to him. David was asked to retry the 12 incorrectly answered items without a time limit and show all work. He correctly answered seven of the twelve items on the second try.

Step 4: Prescriptive Instruction
Even though David’s score at the 83rd percentile comes close to demonstrating mastery of the topics tested, the DT-PI process requires that he thoroughly understand all points before moving on. Based upon a thorough analysis of David’s performance on the achievement test, the mentor determined what topics needed to be covered. The analysis included an examination of (a) those items by which David placed a question mark; (b) the items he marked incorrectly; and (c) his written solution to the missed problems. These three aspects to the analysis served as the guide to the interview that followed. All of this information was used to determine which principles David did not understand and consequently what content needed to be covered. Steps 2 through 4 were repeated for the Computation test, Form 4A.

The mentor worked with David on Saturdays for two hours per session. David was permitted to read, work on the computer, or do other homework during large-group mathematics instruction. David worked on his mentor-assigned homework while his other classmates worked on their homework. In the evening, his parents checked that the homework was completed. The teacher was very supportive of David and the completion of the mentor-assigned homework and encouraged David to ask any time he had questions concerning the homework.

Step 5: Post-testing
David took Form 4B of the STEP Mathematics Basic Concepts and Computation tests and scored at the 98th percentile on each test when compared to 6th graders. The mentor worked with him on the few items he missed or was unsure of to guarantee mastery of those concepts. Then, David reentered the DT-PI process at Step 2 by taking the next level of the Mathematics Basic Concepts and Computation tests.