Fast-Paced Precalculus Mathematics for Talented Junior High Students: Two Recent SMPY Programs

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Introduction

For ten years, the Study of Mathematically Precocious Youth (SMPY), based at The Johns Hopkins University, has been concerned with identifying and facilitating the education of youths who can reason extremely well mathematically.[1] The principal means of identification used by SMPY has been the College Board Scholastic Aptitude Test (SAT) (see George, 1979a). Since 1971, SMPY has used the SAT in seven talent searches for individuals of predominantly seventh-grade age. In an effort to develop programs for the mathematically talented students who have been identified, SMPY has sponsored a dozen accelerated and enriched precalculus mathematics classes. A detailed description of the earlier classes is given by Fox (1974), George (1976), Stanley (1976), George and Denham (1976), and George (1979b).

During the summers of 1978 and 1979, mathematics classes sponsored by SMPY were held primarily for seventh-grade-age (12 or 13 years old) students. The objective of both summer programs was that each participant learn well and at a high level of understanding as much precalculus mathematics as was feasible during the eight-week program. The students who were selected to participate were exceptionally able in mathematics relative to national age-grade norms. The 1978 group was the abler, as discussed later in this paper, in terms of their scores on the mathematics section of the SAT, and achievement during the summer program.[2]

The DT→PI Approach

The instructional method of "diagnostic testing followed by prescriptive instruction" (DT→PI) was used during both summer programs. The DT→PI approach has been found to be both an efficient and effective means for teaching precalculus mathematics to very talented students on a one-to-one basis (see Stanley, 1978). A detailed description of the DT→PI approach can be found in Bartkovich and George (1980). In the 1978 and 1979 summer programs, the DT→PI method was applied to a classroom situation. Since the students' aptitude for mathematics was high but their educational backgrounds diverse, the DT→PI approach of individualized teaching was expected to work well.

In brief, the DT→PI classroom approach begins with diagnostic testing. Each student is administered a standardized test in the subject that follows the last precalculus mathematics course for which credit was obtained in school. Testing in succeeding courses is continued to the point where a student does not score well (better than 75th percentile on national norms) on the test. This is the initial course in that individual's program of instruction. SMPY has found that many mathematically brilliant youths know much first-year algebra without formal instruction in that course (Cohn, 1977).

After the level of instruction has been determined for each student, an individualized program of "prescriptive" instruction can be devised. By using an item content classification chart (based on course concepts), the instructor can scrutinize missed test items to determine which mathematical concepts are not already known. The resulting instruction should include only those topics not yet learned. Little or no class time is spent on concepts already acquired well. Review of such topics can be done through homework reinforcement.

Although material is covered at a rate much faster than in the usual class, this does not imply that any topics are left out, or that the student does not master those topics. Rather, the course is streamlined by eliminating unnecessary repetition. Since the class is comprised of very able students, the material can be treated in greater depth than is usual.

Homework and the ability to work independently are crucial elements in the fast-paced and enriched mathematics classroom. Assignments should be used to reinforce newly acquired concepts and skills, and also as a learning tool. These youths are quite able to learn a number of topics largely on their own through homework assignments. Assignments should be designed cleverly to cover in an extensive manner the appropriate topics while not being overly repetitious.

When a student has completed a course (a teacher judgment based on homework and classroom quizzes, a standardized test is administered for certification purposes. The tests, used by SMPY both as a diagnostic tool and as a certification measure, are the Cooperative Mathematics Tests (Educational Testing Service, 1974). There are two comparable forms available for each precalculus subject in the sequence, making this series ideal for pre- and postinstructional testing.

The instructor's goal is for each student to obtain near-perfect scores on these tests, indicating complete mastery of the material. For purposes of certification, however, an individual scoring above the 85th percentile on national high school norms is deemed to have completed satisfactorily the course covered by that test (the stricter eighth-grade national norms are used for Algebra I). If a student achieves a score just below the criterion, he or she may be certified on the basis of completing extra work in the areas of weakness. After a course is completed, the instructor follows, as much as possible, the course sequence appropriate to the student's school district when choosing the next course to begin.

The 1978 Program

During the eight-week period from 20 June through 10 August 1978, SMPY sponsored a program for mathematically talented junior high students. Participants typically had just finished the seventh grade. The students attended class on Wednesday of each week.

To enroll in the summer 1978 program, students first had to be participants in SMPY's 1978 Talent Search. Individuals were eligible for the Talent Search only if they were of seventh grade age, and had previously scored in the upper three percent on national in-grade norms on a standardized test of mathematical aptitude. A total of 2,798 such youths from a five-state region and Washington, DC entered the 1978 Talent Search. Participation involved taking the SAT, which is designed for above average high school juniors and seniors, at the regular national administration in late January of 1978.

Invitations to attend the classes were extended to those individuals scoring at least 600 on the mathematics section of the SAT (SAT-M), and at least 1100 total for the mathematics section plus the verbal section of the SAT (SAT-M + SAT-V). (A score of 600 on SAT-M exceeds 79% of all college-bound male seniors who take the SAT. The average score for college-bound male seniors is 493.) There were 61 students in the 1978 search who met the SAT criteria. Of this group, 33 (30 boys and 3 girls) chose to participate in the program. (One exception to the entrance criteria was made for an 11-year-old girl who scored 590 on SAT-M and 600 on SAT-V.) This group of 33 was extremely select because they were chosen from the top 61 out of 2798, who, in turn, were from the top 3% of their age group. Therefore, these 33 students each constituted a level of selectivity of approximately 1 in 1500 in their age group nationally. The average SAT-M score of the participants was 669 or better than the score of about 92% of college-bound male seniors. The group's average SAT-V score was 528, or better than about 80% of the college-bound male seniors who take the test.

On the basis of the diagnostic testing results, the 33 students were divided into five separate classes which were as homogeneous as possible in relation to knowledge of precalculus. Four of the classes had one instructor each, while the fifth class had two instructors for all but one week. The instructors were highly successful graduates of SMPY's earlier fast-paced mathematics classes. Despite the small class sizes and individualization, it was found that students often interacted in still smaller groups or pairs, aiding each other with particular concepts. As they progressed through various topics, those studying the same material were encouraged to work together. This interaction became a major strength and component of the program.

The first of the eight meetings was used mainly for diagnostic testing, and little time was spent on instruction that day. Classes met for the next seven consecutive Wednesdays. Students who had not quite finished a particular subject by the end of the last session were given an extensive homework assignment covering the remainder of the course. A few weeks later, two tutoring days were offered. The individuals who attended one or both of those sessions had an opportunity to study the coursework further, ask questions of the instructors, and take a standardized test for the purpose of course certification. The total number of instructional hours was no more than 40 for the summer program itself, with a possible 6-8 more hours available during the two extra days of tutoring.

Each student participated at a cost of $25. The remaining support for the program came from the Dodge and Dreyfus Foundations. Each individual agreed to do *at least* four hours of homework each week. In addition, all homework assignments were to be done neatly and thoroughly. Any student who missed a class was still required to complete the corresponding assignment. A variety of textbooks was used during the summer. Among them were *Algebra and the Elementary Functions* (Hart, 1974); *Geometry* (Moise & Downs, 1975); *Modern Trigonometry* (Wooten, Beckenbach, Buchanan, & Dociani, 1973); and *Analytic Geometry* (Protter & Morrey, 1975).

Diagnostic testing showed how diverse the students were in their knowledge of precalculus mathematics. The students' formal backgrounds differed, as did the amount of mathematics that they had acquired on their own. Eight individuals had previously taken Algebra I in school, while the others had taken prealgebra or seventh-grade general mathematics. If a student had learned a mathematics course on his or her own, he or she was certified via testing for having mastered the material. Individuals who were certified for a particular course during the diagnostic testing spent little or no time on that course during the summer program. Table 1 shows the number of students who completed each precalculus course during the summer. Courses completed via instruction are tabulated separately from courses certified as mastered via preinstructional testing. An examination of Table 1 reveals that a total of 77 courses were taught to the 33 students; therefore, each participant learned an average of 2.33 precalculus courses. Seventeen students were certified for 28 courses via the diagnostic testing (an average of 1.65 course each for those 17 students, or an average of 0.85 for the group of 33). When the number of courses learned through instruction is combined with the number of courses certified through preinstructional testing, 105 courses were acquired by the 33 students, for an average of 3.18 each. The courses were covered in the order of the

Table 1
Number of Students Completing the Various Precalculus Mathematics Courses During 1978 Summer Program

Course	Number of students (N = 33)		
	Through Instruction	Through Preinstructional Certification	Total
Algebra I	6	5	11
Algebra II	21	5	26
Algebra III	20	5	25
Plane Geometry	6	11	17
Trigonometry	12	2	14
Analytic Geometry	12	0	12
Total	77	28	105

precalculus sequence: Algebra I, Algebra II, Algebra III, Plane Geometry, Trigonometry, and Analytic Geometry. Twelve students completed Analytic Geometry; they were only eighth-grade (in the fall) age or younger, and yet were well prepared to enroll in a rigorous calculus course.

Table 2 shows a breakdown of the participants by the number of courses that they completed during the summer, through instruction and preinstructional testing certification.

Note that each student completed at least one course during the 1978 summer program. Six of the 33 students finished five courses (Algebra II through Analytic Geometry, having already had Algebra I in school.)

The 1979 Summer Program

Encouraged by the results of the 1978 program, SMPY organized a similar eight-week mathematics program for the summer of 1979. Most of the participants were seventh-grade age youths who were identified through the 1979 Talent Search. Eligibility for the 1979 Talent Search was the same as in 1978. The entrance criteria for the 1979 summer program were somewhat lower than they had been for the 1978 program, in order to allow more students to participate. A student had to score at least 500 on SAT-M, and at least 1000 on SAT-M + SAT-V. Out of 3,675 participants in the 1979 search, 290 achieved such scores. A total of 96 youths elected to participate in the program; four individuals dropped out rather quickly, leaving 92 students (22 girls and 70 boys) to finish the program. The students were chosen from the top 290 out of 3,675, who, in turn, were in the upper 3% of their age group nationally in mathematical ability; therefore, these highly select youths were approximately the top 1 in 500 of their age group nationally. The average SAT-M score for the 92 students was 592. This score exceeds the scores of more than 76% of college-bound male seniors who take the test. The participants' average SAT-V score of 503 was higher than about 72% of the college-bound male seniors score. As in the 1978 program, most of the students who attended the classes came from areas some distance from Baltimore.

The attendance, homework, and study habit requirements were the same as for the 1978 program. Diagnostic testing was conducted prior to the first class. The textbooks were the same ones used in the 1978 program, supplemented by *Intermediate Algebra* (Zuckerman, 1976). The Cooperative Mathematics Tests series was again used for diagnostic testing and certification of course mastery. A fee of $50 was charged to students; the remainder of the costs were absorbed by a National Science Foundation grant (#SPI 78-27896).

The students attended class one day each week from June 26 through August 16, 1979. Each individual chose to attend on either Tuesday, Wednesday, or Thursday, the same day for all eight weeks. The numbers who chose each of the days were approximately equal. There were five hours of classroom instruction on each day, or 40 hours total. Some youths completed courses after the end of the program in the same manner as did some 1978 program participants, and then received up to six hours of additional instruction.

As a result of the diagnostic testing, each of the three classes (T, W, and Th) was divided into three groups of about 10 students each: high, middle, and low, based on the students' current knowledge of precalculus mathematics. The qualitative labels "high, middle, and low" refer only to the amount of precalculus knowledge acquired before the summer program began. Differences in mathematical reasoning ability were not assumed or implied by the grouping.

The instructional staff consisted of three experienced senior instructors and four junior instructors. The three senior instructors were graduates of SMPY's earlier fast-paced mathematics classes (two had been instructors in the 1978 summer program). Three of the junior instructors were each paired with a senior instructor. Each pair was then assigned to the high, middle, or low group. The other junior instructor spent his time among all of the

Table 2
Number of Courses Completed by Students in 1978 Summer Programs

Number of Courses	Number of students (N = 33)		
	Through Instruction	Through Preinstructional Certification	All Courses Completed
0	0	16	0
1	7	8	3
2	12	7	8
3	10	2	8
4	4	0	8
5	0	0	6

Table 3
Number of Students Completing the Various Precalculus Mathematics Courses During 1979 Summer Program

Course	Instructional Class							
	High (N = 30)			Middle (N = 31)			Low (N = 31)	
	Instruction	Testing	Total*	Instruction	Testing	Total*	Instruction	Total*
Algebra I	0	3	3	10	3	13	28	28
Algebra II	14	11	25	29	0	29	25	25
Algebra III	16	3	19	4	0	4	6	6
Plane Geometry	9	2	11	7	0	7	0	0
Trigonometry	14	0	14	3	0	3	0	0
Analytic Geometry	6	0	6	0	0	0	0	0
Totals	59	19	78	53	3	56	59	59

*Total number of courses completed

groups, depending on where his assistance was most needed (in fact, he spent most of his time with the low group). The teaching style was essentially the same as the year before, following the DT→PI approach. The standards for certification were identical with those of the previous summer.

The students in the 1979 program were even more varied in their mathematics backgrounds than the 1978 group. All students in the high group had already completed Algebra I and knew some other precalculus. The middle group either had taken Algebra I in school or learned much of it on their own, but knew little other precalculus. The low group either had no previous instruction in Algebra I or needed review in some topics of that course.

Table 3 summarizes the number of students who completed each precalculus course, either through instruction or preinstructional testing certification. The high, middle, and low groups are treated separately. Students in the high group had been certified before instruction for a total of 19 courses, an average of 0.63 courses each, mostly in Algebra II. Students in the middle group were certified for only three courses before instruction, all in Algebra I. No students in the low group were certified for any courses prior to instruction. An individual was not given a test for certification in a course which had previously been completed in school. A test was used as a diagnostic tool, however, if there appeared to be gaps in the student's knowledge of that course.

Table 4 shows the number of courses completed by students in each group during the summer program, via instruction or preinstructional certification. The 30 students in the high group completed 59 courses through instruction, or 1.97 courses each on the average. When preinstructional certification is included, the total number of courses completed by the high group is 78, or 2.6 courses each. The middle group learned 53 courses through instruction, and a total of 56 when preinstructional courses are included. For these 31 students, the average achievement was 1.71 courses learned, and 1.80 total course completed. The students in the low group

Table 4
Number of Courses Completed by Students in 1979 Summer Program

Number of Courses	Instructional Class							
	High (N = 30)			Middle (N = 31)			Low (N = 31)	
	Instruction	Testing	Total*	Instruction	Testing	Total*	Instruction	Total*
0	1	16	0	1	28	1	1	1
1	8	10	8	11	3	9	6	6
2	14	3	7	15	0	16	19	19
3	5	1	8	4	0	5	5	5
4	2	0	3	0	0	0	0	0
5	0	0	4	0	0	0	0	0

*Total number of courses completed

Table 5
Average Scholastic Aptitude Test (SAT) Scores for 1978 and 1979 Program Participants

		SAT-Mathematics		SAT-Verbal	
		Mean	Standard Deviation	Mean	Standard Deviation
Class	1978 N = 33	669	50	528	64
	1979 high N = 29[a]	622	48	501	61
	1979 middle N = 31	587	49	510	60
	1979 low N = 31	570	42	500	63

[a]There were 30 students in the class, but one student did not have SAT scores and was accepted into the program based on previous achievement in mathematics.

learned 59 courses, or an average of 1.90 courses each.

Role of SAT as predictor of mathematics achievement

The 1978 program participants had higher SAT-M scores than the 1979 students; the 1978 group also completed more mathematics courses through instruction or certification than did students in any of the 1979 classes.

Presented in Table 5 are the mean SAT scores and standard deviations for all groups. Students in the 1978 class had an average SAT-M score of almost 48 points higher than the top 1979 group, and a full 100 points higher than the 1979 low group. The average SAT-V score for students in the 1978 program was appreciably higher than for the three groups in 1979, but all three 1979 groups were approximately equivalent on SAT-V. It is interesting to examine what relationship these facts have with the level of achievement among the groups.

In the first column of the correlation matrix in Table 6, the correlation of SAT-M with the number of courses learned during the program is shown for each group. There is a clear trend here, with the magnitude of the correlation increasing as the average SAT-M score for the group decreases. The reverse trend is observed for the correlation of SAT-M with the number of courses certified through preinstructional testing (see column 2 in Table 6). This latter finding indicates that students with very high SAT-M scores are more likely to have acquired, on their own, a considerable amount of precalculus mathematics knowledge. According to these data, the following rough generalization can be made: students who score over 600 on SAT-M probably know at least some precalculus course material without having had a formal course in school. Students scoring between 500 and 600 on SAT-M are less likely to know much precalculus, unless they have learned it in school. In addition, the data from the 1979 program suggests that, after a certain level, SAT-M does not seem to be a major factor in the amount a student learns in the fast-paced program.

The relationship of SAT-V with number of courses learned is shown in the last three columns of Table 6. Correlations hovered around zero in most cases, and do not indicate that SAT-V plays a role in achievement for these highly select youths. If no combined criteria of SAT-M + SAT-V was set, and SAT-V scores could vary freely, more might be learned about the relative importance of SAT-V. Some fast-paced mathematics program in the future should probably try this, by requiring only a high SAT-M score for enrollment.

Table 6
Correlation Between Scholastic Aptitude Test (SAT) Scores and Mathematics Achievement

	Correlation of SAT-Mathematics with:			*Correlation of SAT-Verbal with:*		
Group	Number of courses learned	Number of courses certified before instruction	Total	Number of courses learned	Number of courses certified before instruction	Total
1978 (N = 33)	.12	.53**	.50**	.19	-.23	-.03
1979 High (N = 29[a])	.20	.41*	.38+	.08	-.16	-.04
1979 Middle (N = 31)	.29	.29	.40+	.07	-.20	-.01
1979 Low (N = 31)	.53**	—[b]	.53**	-.01	—[b]	-.01

[a]One student had no SAT scores and therefore had to be omitted from calculations.
[b] No student in the group was certified before instruction.
+.01 < p < .025 *.005 < p < .01 **p < .005

Counseling

During both summer programs, the students were exposed to topics in applied mathematics through talks given by faculty members of The Johns Hopkins University. The presentations, lasting about one hour each, were designed to illustrate how mathematics is used in problem solving in many different fields. Faculty members from the disciplines of engineering, sociology, biology, computer science, psychology, and statistics participated. Generally, the students enjoyed these informal talks and showed their interest by asking many questions and initiating discussions. SMPY thought it was quite important to emphasize the role of mathematics in many contemporary professions and occupations, not only to underscore its relevancy, but also to encourage the development of career-oriented interests.

Towards the end of each summer program, individual counseling was offered to the students and their parents. Short-range counsel was based on each student's achievement during the summer and his/her instructor's recommendations. An appropriate mathematics course for the coming year was suggested. Also discussed were possible ways of working in cooperation with the local school system in order to design a suitable educational program for the individual. All counseling sessions stressed obtaining appropriate course placement first, and then negotiating for high school credit for coursework completed during the summer and following school year.

The long-range counsel concerned future academic opportunities and career options. Considerations such as grade skipping, early graduation, taking college courses while still in high school, obtaining college credit through the College Board's Advanced Placement Program examinations, and other accelerative and enriching options were emphasized. This counsel focused chiefly on the program of courses in mathematics, but it also took into account the individual's other interests and strengths.

Evaluation

The results of the 1978 and 1979 summer programs demonstrate that, via the DT→PI approach, mathematically talented youths of seventh-grade age were able to complete vast portions of the precalculus sequence, given 40 to 48 hours of instruction. The approximately two courses completed (through instruction) on the average in each of the programs would normally take 1.5 to 2 years (270-380 50-minute class periods) to finish in a typical junior or senior high school.

A major source of success of both programs was the avoidance of several pitfalls that have been observed in other similar programs. First, there was little if any reteaching of concepts that students had already mastered. As previously explained, this is inherent in the DT→PI style of instruction. In addition, the classes were not sidetracked by an overemphasis on course grades. Students are often driven to compete with one another for high grades. This can be especially detrimental in a group of highly talented youths. The grades, therefore, were based on performances on the standardized tests. Thus, the participants were competing against national norms and not one another.

It is also commonly thought that gaps in knowledge result from rapid completion of mathematics. This did not happen in either program, as evidenced by the performance of participants in more advanced mathematics courses. From the 1978 group, 12 students (more than a third of the group) were prepared to take calculus at eighth grade age. Eight of those individuals enrolled that fall in an SMPY-sponsored Sunday afternoon class in college-level calculus. This class met each week during the school year to prepare for the more comprehensive (BC) level of the Advanced Placement calculus test. On a scale of 1 to 5, where 5 is the highest, the results were as follows: four 5s, one 4, and three 3s. A score of 3 or better will gain the examinee two semesters of credit in calculus at many selective colleges. Another boy from the 1978 class taught himself calculus and took the BC level calculus test; he scored a 5 on it. These high scores demonstrate the thoroughness with which those students in the summer class covered the precalculus mathematics. Despite having somewhat less able participants, the 1979 class also compiled an impressive record of achievement.

Questionnaire Results

Students in both summer programs completed questionnaires at the end of the program. They answered a variety of questions regarding the instruction that they had received, whether or not their attitudes towards mathematics had changed, and what improvements should be made for future similar programs. Students were asked to rate how the teaching skills of their instructor compared with the skills of mathematics teachers that they had had in school. The students overwhelmingly agreed that their instructors in the summer program were as good as or better than the teachers that they had in school. The favorable responses may have been in part due to the teaching and classroom *style*, rather than the teacher per se. The students enjoyed being able to move ahead at their own rate, something not common in the regular school setting. This idea is substantiated by a question asked to the 1979 group. When requested to make a comparison, the vast majority of students asserted that the summer program, although it was more difficult, was more fun, more exciting, and more productive than their other experiences in mathematics.

Students in both programs were asked, "How has your inclination to work on your own in mathematics changed since your participation in the summer program?" Of the 30 students responding from the 1978 group, 19 said that they were more inclined to work on their own, 10 reported no change, and 1 said he was less inclined to do so. Of the 86 students answering the same question in 1979, 46 said that they would be more inclined to do independent work, 33 said that there was no change, and 7 said that they would be less inclined.

During the fall of 1978, the students from the 1978 program were requested to complete a follow-up questionnaire (no follow-up was conducted in 1979). The most important question was whether the summer instruction had prepared them adequately for the mathematics course that they were taking in school. Twenty-nine of the 33 students responded to the questionnaire. Of those, 26 said that their summer instruction had prepared them adequately. Two students said it did not prepare them for their present course (geometry), since they had studied algebra during the summer and the subject matter was unrelated. One boy, who was taking calculus (and had *not* studied trigonometry or analytic geometry during the summer; calculus was not recommended by SMPY) replied that he needed further study in those two courses.

A similar question was asked on a questionnaire sent to the parents of the 1979 program participants early in the fall of 1979. Of 79 parents responding, 55 said that they felt their children had thoroughly mastered the material covered during the summer. Six felt that their children had not learned all of the material completely, and 18 were unsure or had no way of knowing how thoroughly the material was learned. Parents were also asked "Are you satisfied with the progress made by your child this summer?" Seventy-three responded "yes," four answered "undecided," one said "no," and one left the item blank.

Another criterion for evaluation is the cooperation of the participants' schools in subject-matter placement and granting of course credit. These data are not formally available for the 1978 summer group. Based on the responses from the parents of students in the 1979 program, only eight stated that the local school was uncooperative or somewhat uncooperative in granting credit for courses certified as completed, and only one student was required by the school to repeat any subject learned during the summer. Eleven students were enrolled in either a high school or college calculus program.

Conclusion

The earlier fast-paced and enriched precalculus mathematics classes sponsored by SMPY had shown that a homogeneous class of mathematically gifted students was able to learn precalculus well, at a rate three or four times as fast as what is considered normal for the standard curriculum. The 1978 and 1979 summer programs demonstrated similar results using an individualized (DT→PI) teaching approach. This is significant because, as the diagnostic testing clearly showed, the mathematics backgrounds of these highly talented youths in the program were quite diverse. The success of the DT→PI approach is also important because many areas, particularly rural localities, do not have a population sufficiently large to set up a homogeneous class for gifted youths. The cost-effectiveness of such a method makes it even more attractive.

The progress of the students in these programs can be perceived adequately only in the context of what happens to most youths who in the seventh grade reason extremely well mathematically. Across the country most of them must wait until at least the eighth grade — usually the ninth — in order to begin the progression from the first-year algebra to wherever they terminate the high school mathematics sequence. Many never get beyond plane geometry because they have to "study" Algebra I for 180-190 45- or 50-minute periods during a whole school year, when they need far less time for it, and they get off to a poor start. After Algebra I they are caught in the same Carnegie unit lockstep for Algebra II, for plane geometry, and so on. Typically, they will complete those three subjects by the end of the tenth, or more likely, the eleventh grade. Contrast that with the mathematics subject level attained by students in SMPY's 1978 and 1979 brief summer programs.

The time saved, boredom and frustration avoided, and increased productivity are immeasurable qualities that illustrate the need for fast-paced and enriched mathematics classes.[3] As concerned educators, we must provide special programs such as those reported here in order to effectively meet the intellectual needs of gifted youths and aid them in their desire to fulfill their tremendous potential.[4]

Footnotes

1. The identification aspect of SMPY's work was assumed in July, 1979, by a new office at The Johns Hopkins University, the Office of Talent Identification and Development (OTID). OTID operates the annual Talent Searches in cooperation with SMPY.
2. The precalculus mathematics sequence refers to the courses beginning with first-year algebra which prepare an individual for the study of calculus. The order of completion used in these summer programs was: Algebra I, Algebra II, Algebra III (college algebra), plane geometry, trigonometry, and analytic geometry.
3. At the time of this writing, a number of fast-paced programs using SMPY's models were underway for the summer and/or fall 1980. The New Jersey State Department of Education will sponsor four or five regional centers; Allegheny Community College (near Pittsburgh) will conduct a program, as will the Delaware State Department of Education; Woodberry Forest School in Woodberry Forest, Virginia; Frederick County, Maryland; and the Association for High Potential Children in Eau Claire, Wisconsin. A program is also underway at Arizona State University in Tempe, Arizona. Kanawha County, West Virginia (Charleston area), is starting a program in the fall of 1980, and the North Carolina School of Science and Mathematics will use the DT→PI model for some of its instruction. Other similar programs are already underway in Illinois, Nebraska, and Charles County, Maryland. Duke University is replicating the SMPY talent search model and program model in a 13 state region of the southeastern United States.
4. The authors thank William C. George and Julian C. Stanley for their comments and suggestions on earlier versions of this paper.

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The following references were inadvertently omitted from the manuscript entitled "Exceptionally Gifted Boys and Their Parents" by Robert S. Albert which appeared in the *Gifted Child Quarterly,* Volume 24, No. 4, 1980.

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