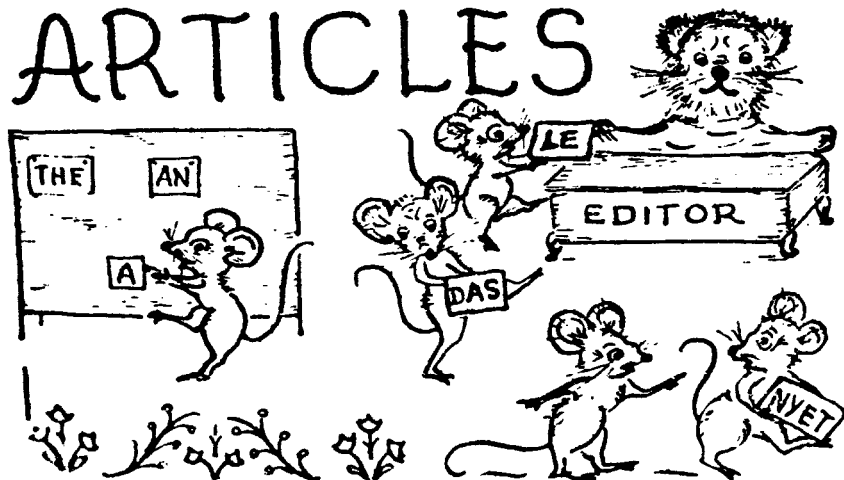


ARTICLES



ACCELERATING MATHEMATICS INSTRUCTION FOR THE MATHEMATICALLY TALENTED

William C. George

*Study Of Mathematically Precocious Youth
The Johns Hopkins University*

Abstract

Fast-paced country-wide mathematics classes meeting outside of regular hours were established to meet the needs of highly talented mathematical reasoners. The results from the original two programs demonstrated that four and one-half years of precalculus mathematics could be taught in approximately 120 hours. These classes show the importance of homogeneous grouping. Class success was based on identification of qualified students through appropriately difficult mathematics tests, voluntary participation by students, and carefully done homework assignments. The programs' success resulted in different school systems adopting the model. This paper concerns the various classes and the implications of fast-paced mathematics.

Education is considered extremely vital to our society. Through the work of psychologists such as Spearman (1923), Terman (1916), Thorndike (1928), Thurstone (1935), and Wechsler (1939),

educators began to recognize via psychometric means that intellectual abilities vary considerably. The resulting effect was an educational system supposedly geared to meet the individual's needs. There has been a great deal of talk on the subject of individual differences, but too little has been accomplished, especially in the area of the gifted student (Williams, 1974). Even with the finest seed, proper care and cultivation are necessary to harvest a plentiful crop. The Study of Mathematically Precocious Youth (SMPY) has been working with highly able mathematical reasoners since 1969. While Terman's longitudinal study¹ made the public more aware of the intellectually gifted child and his/her importance to the society, there have been far too few strong, systematic efforts to help such students learn mathematics and the physical sciences fast enough and well.

Establishing the Fast-Math Concept

In an effort to assist the mathematically talented student, SMPY has developed a smorgasbord of accelerative techniques (Stanley, Keating, and Fox, 1974; Keating, 1976). One of the most successful of these innovations begun by SMPY is its concept of special fast-paced mathematics classes. The original class was established in June of 1972 with the idea of setting up a curriculum that would meet the mathematics needs of junior high school students (chiefly seventh graders) who reason extremely well mathematically. The staff of SMPY (see Fox, 1974) reasoned that if students with interest and ability in mathematics were given the opportunity to learn as fast as they could, they would achieve far better than under the usual school conditions. This was demonstrated in the first program by the fact that 15 of the original 23 persons completed Algebra I and Algebra II. Nine of the sixteen continued through Algebra III, trigonometry, and analytic geometry; eight of these also finished plane geometry (taught quickly last), demonstrating that four and one-half years of precalculus mathematics can be learned well by a sizable percentage of mathematically talented young students studying two hours on Saturday mornings for approximately 12 months (Fox, 1974; and Stanley, 1976, pp. 156-159).

For the five volumes of the Genetic Studies of Genius series thus far see Terman (1925), Cox (1926), Burks, Jensen, and Terman (1930), Terman and Oden (1947), and Terman and Oden (1959). Also see Oden (1968) and Sears and Barbe (volume being prepared).

Verifying The Fast-Math Concept

The success of this program encouraged us to establish during the summer of 1973 a second class, but composed chiefly of ninth graders who were similar in nature. The 31 students (9 girls and 22 boys) electing to take this program came from either Baltimore County or Howard County. Before the age of 14 each participant had scored at least 500 on the mathematics portion (M) of the College Entrance Examination Board's Scholastic Aptitude Test (SAT) and 400 on the SAT-verbal (V) portion. This means that these individuals were already obtaining mathematics and verbal aptitude reasoning scores higher than many college-bound high school juniors and seniors (College Entrance Examination Board, 1974).

In addition, each person had to demonstrate good knowledge of Algebra I as indicated by score on both 40-item forms of the Educational Testing Service's Cooperative Mathematics Test, Algebra I. The class met for two and one-half hour sessions a week during the summer and one two-hour session a week during the school year. It was in lieu of each person's regular mathematics class.

Cognitive Background

In order to learn more about each student so that appropriate counseling could be given, the SMPY staff administered a battery of cognitive tests. The results of this test battery were informative. The girls scored significantly lower on the SAT-M and Form AA of the Bennett Mechanical Comprehension Test (1968) than the boys. From these differences it appears likely that the boys had more mathematical ability and knowledge than the girls from the outset. In addition, the boys' mean score of the initial Algebra I test was higher. Since a greater percentage of girls than boys had already taken Algebra I, and girls are usually more diligent school students than boys are, it seems likely that a considerable number of these boys acquired some of their mathematical skills from sources outside the classroom (Keating, 1974; Astin, 1974).

At the end of the summer portion of the program four girls and one boy chose to leave the program for various reasons. This was not necessarily due to lack of Algebra II proficiency. There was no significant difference in mean SAT-M score between those persons choosing to remain and those deciding to leave. There was little

mean difference between the two groups on Algebra I scores and the non-verbal reasoning Progressive Matrices (Raven, 1960) scores, which seems to indicate that mathematical reasoning ability alone will not predict success or continued interest in this type of program WITHIN a homeogeneously high-level group.

In the fall two more boys joined the class, making the enrollment during the school year 5 girls and 23 boys.

Variations in Learning Styles

Even within a highly select group such as this one there is a great deal of differentiation (Stanley, 1973; and Fox, 1976). At the end of the Algebra III segment it was decided to split the class of 28 students into two sections. The majority had been able to adapt and keep up with the fast, theoretically styled pace. Some of the students, however, needed more detail than the current teaching style was giving them. In addition, this group tended to be more socially interactive in learning approach. Therefore, when plane geometry was started a second class section consisting of three girls and two boys was begun in the hope that these talented youths could keep up with the rest of the class. In the new section they were given more attention and a more detailed approach to the subject matter. Teacher style in the smaller classroom situation was rather different than that of the larger class. Homework and fast pacing were, however, still the crucial factors in both teaching approaches. The need for coordinated student and teacher styles in order to learn mathematics optimally seems realistic (Renzulli, 1975; Ohnmacht, 1960; and Witkin, 1973).

During the rest of the school year both classes covered the same material, but in a different manner. The smaller class was much more personalistic and group-oriented. Math games were used to keep incentive high and make learning mathematics fast a stimulating experience. In the faster-paced group (N= 23) teaching was more individualistic, and new solutions were suggested in a much more theoretical manner. Both teachers, however, insisted on supplemental self-instruction through a properly paced homework process. Before-class preparation was a necessity. Neither class was slowed by those students who chose not to read and complete their homework. By keeping up with their assignments students were expected to cover what they missed in their fast-paced classes as well as study the finer points of the problem.

Results

The two sections with their contrasted learning styles were both successful. Twenty-eight students learned Algebra II and plane geometry at a high level of proficiency as indicated by the Cooperative Mathematics Test (SMT) for each particular curriculum segment. Twenty-three of these completed Algebra III. Fourteen boys successfully completed trigonometry and analytic geometry and were ready to take calculus in the fall of 1974.

In September of 1974 eight of these highly able mathematical reasoners began a college calculus class, taught to them and seven others for two hours each week by the mathematics department chairman of Loyola College in Baltimore. The course was designed to help prepare these young men for the College Entrance Examination Board's Advanced Placement Program (APP) examination in calculus at the more difficult BC level. This course supplemented the regular high school course in calculus each person was taking. By earning a 3, 4 or 5 on the BC level of this test each student would receive college credits in calculus and the opportunity to take more advanced mathematics courses such as advanced calculus or linear algebra. In May 1975 all eight took the APP more difficult level calculus exam. There were five 5's and three 4's. In addition, three other members of the class took the BC level calculus examination at the same time. They earned two 3's and one 4. The remaining three chose to take calculus directly as a part-time college course during the 1974-75 school year. (See Stanley, 1976).

School System Adoption of the Fast-Math Concept

The success of these programs was so great that in the fall of 1974 two Maryland public school systems, those of Charles County and Montgomery County, adopted this model on a county-wide basis. If the SMPY model could be replicated without real difficulty by these two systems the feasibility of the fast-math model would be demonstrated. A strong reason for this impact would be that in many ways the two counties are exact opposites. Montgomery County is a talent-rich school system bordering the Washington, D. C. metropolitan area. It has approximately 60,683 students in its secondary school system, with 57 secondary schools. On the other hand, Charles County is small, rural and located in Southern Maryland. It is at least one hour away from either the Baltimore or Washington metropolitan areas. There are only eight secondary schools. The total secondary school population is 8,720.

Montgomery County Program

With the start of the 1974-75 school year two fast-math classes were established. Students were selected on the basis of the 1974 Maryland Mathematics Talent Search SAT-M scores that SMPY reported to each school system in the spring of 1974. Being talent-rich but having funds for only two classes, Montgomery County was forced to choose a high cutoff score of 600 on the SAT-M for inviting students to participate in their initial program. This was because the special classes could not handle most of the 356 seventh and eighth grade students identified by SMPY as being highly able mathematical reasoners. It is important to remember that these students identified by SMPY from this county as being mathematically talented came on voluntary basis to the 1974 Talent Search. If exhaustive recruitment techniques for the Talent Search had been implemented it is estimated that the number of students identified as mathematically talented might have been doubled. (The criterion score for being identified as mathematically talented was only 420 on the SAT-M, this being the 57th percentile of a random sample of male high school juniors and seniors.)

There were 62 eligible students. Fifty-four of them (43 boys and 11 girls) chose to enroll in this program. During the 1974-75 school year 13 students dropped out for various reasons (10 during the first month), leaving 41 (34 boys and 7 girls) in two classes that met one-two hour period per week. At the end of the 36 weeks (72 hours) of instruction these classes had covered Algebra II and plane geometry well. Contrast this with the approximately 300 hours usually used to cover this subject matter, a savings of 228 hours. No student scored below the 79 percentile of national high school norms as measured by the Cooperative Mathematics Test (CMT) -- Algebra II segment after 34 hours of instruction or below the 95th percentile of national high school norms on the CMT -- plane geometry achievement test. Only 9 out of 38 persons scored below the 99th percentile in the area of plane geometry, while 29 out of the 39 persons tested for knowledge of Algebra II scored above the 95th percentile. Unfortunately, there were several people in the class of 41 who missed the plane geometry and Algebra II testing. The remainder of the pre-calculus segment (i.e., college algebra, trigonometry and analytic geometry) was learned during the 1975-76 school year. Those students who successfully completed this program signed up to take calculus as either 10th or 11th graders during the 1976-77 school year.

The success of the first program encouraged Montgomery County

to expand this opportunity to more highly able reasoners in the area of mathematics during the 1975-76 school year. Four more classes containing a total of 100 students used the fast-paced class concept to learn mathematics well. Those students, identified on the basis of the Educational Testing Service's School and College Ability Test (SCAT), Form 1, which is the level normally used with college freshmen, were end of the year sixth-, seventh-, or eighth-graders in the spring of 1975. In order to take the SCAT a student had to score in the top one percent in both math AND verbal aptitude as measured by the Cognitive Abilities Test (CAT) and in the ninth stanine on mathematics concepts using the Iowa Test of Basic Skills. Cutoff scores on the SCAT were equated to SAT-M scores, with a minimum qualifying score of 600 for 8th graders and 500 for 6th and 7th graders.

Charles County Program

On the basis of SMPY's success, which has been documented by Fox (1974), George and Denham (1976), and Stanley (1976), Charles County became interested in this learning style for mathematically gifted students. With a smaller school population and voluntary participation of students in the 1974 Talent Search, only 26 students were identified as mathematically talented. Twenty-five of these students were still in the public school system in the fall of 1974. In October, 1974, Charles County's first fast-mathematics program was in operation with 23 students. Four of these dropped out during the first year and a half. This is excellent, considering the variability in mathematical aptitude even within this highly select group; the scores on SAT-M ranged from 420 to 640.

The success of the Charles County program cannot, however, be discounted even though that county does not have the talent-rich pool of Montgomery County. At the end of 26 hours of instruction in Algebra I, as opposed to the usual 180 50-minute classroom periods, the class mean on the CMT Algebra I test was at the 95th percentile of national ninth grade norms. Compare this with the second SMPY program, whose class mean after 150 hours of instruction was at the 95th percentile. Remember, the second SMPY class was more rigidly selected, and over 80 percent of its students had just completed one full year of Algebra I when they were tested. This is another strong indication that highly able students need stimulation other than what is offered in the routine classroom.

During the second semester of the 1974-75 school year this

Charles County class of 19 studied plane geometry, again meeting one two-hour period per week at night. After 26 hours of instruction the class mean for knowledge of plane geometry was at the 80th percentile of national high school norms. During the 1975-76 school year this group completed the precalculus mathematics sequence. During the 1976-77 school year most of those students are taking calculus as high school sophomores and juniors. That is quite remarkable, because this is the first time high school calculus has been offered in Charles County. Hitherto, calculus has not been available in this public school system due to its rural nature and small number of mathematically talented youths.

The Charles County program does have one variation from the SMPY model. While covering Algebra I, plane geometry, college algebra, and computer science at night using the fast-math approach, these students learned Algebra II, trigonometry, and analytic geometry in the regular classroom setting, grouped by ability. This was done to insure that all the pre-calculus mathematics sequence would be learned by these bright students, most of whom are on the lower end of the mathematically talented spectrum.

Using this same variation in the style of fast mathematics, the county established a second fast-math program in January 1975. This time the students were screened in October of 1974 via the Preliminary Scholastic Aptitude Test (PSAT) which is normally taken primarily by high school juniors. Twenty seventh and eighth graders who were already in the upper two percent of their age-grade norms in math reasoning ability were selected for the class. The class mean on the PSAT-M, 44.2, was at the 79th percentile of sophomore boys taking the test. In addition, a minimum verbal proficiency of 36.0 as indicated by the PSAT-Verbal was required.

At the end of 26 hours of instruction this able group scored at the 93rd percentile of national ninth grade norms in Algebra I as measured by that CMT segment. During the 1975-76 school year the students learned Algebra II and college algebra, meeting only one-two hour period per week. During the regular school day participants learned plane geometry. This class will be ready for the calculus during the 1977-78 school year.

As a result of the continuing success of these programs, this innovative county expanded its plans to include a third class, which started in January of 1976. The same identification procedures used in the second program were followed for the third. This time regular

quizzes and strictly graded homework were included as one means of improving study habits, which often are poor due to the lack of proper prior competition.

EVALUATING THE FAST-MATH CONCEPT

In reviewing the model classes from SMPY, Charles County and Montgomery county, it appears that all have been successful and accomplished their goals of stimulating and challenging mathematically able students while they learn the precalculus mathematics sequence quickly and thoroughly. The three approaches are similar in principle and yet different in details because of the varying abilities and situational factors in each system.

Identification Procedures

Table 1 shows a variety of aptitude measures used to identify excellent mathematical reasoners. Such measures include the numerical portion of the Academic Promise Test (APT), the mathematical part of the Scholastic Aptitude Test (SAT-M), the Preliminary Scholastic Aptitude Test (PSAT) and the quantitative part of the School and College Ability Test (SCAT-Q). Factors such as cost, size of talent pool, age of student, and selection efficiency have resulted in using the various screening techniques previously mentioned. All these aptitude tests have one thing in common, and that is a high enough test ceiling for the groups tested. Each measure is difficult enough to differentiate among young enough students who are already in the top one out of every 100 persons on an age-grade test.

Unfortunately, the typical age-grade tests used by schools will not succeed in differentiating an already seemingly homogeneous upper 1% or 2% group. Each participant included in the class listed in Table 1 is in the top 2% of his/her age-grade group in mathematical reasoning ability. From this fact alone one might assume that all students should have performed equally well in the precalculus sequence regardless of the program in which they happened to be enrolled. Comparing the mean SAT test scores (see Table 1) for the various programs, we find that the first Montgomery County program has the greatest talent pool. No student enrolled in it had an SAT-M score of less than 600. The SAT-M mean for this program is significantly higher (.001 p .002) than that of the select second SMPY program, which had a minimum cutoff score of 500. Furthermore, the first Charles County class is significantly lower (.001 p .002) than the second SMPY program. This county in its rural location had the widest variation in SAT-M scores of the three

programs. Even within its own program in consecutive years the difference between the Montgomery County classes is significant ($.001 < p < .002$). One possible reason for this difference is that the age range of the second class was lowered by one year to include sixth graders.

With this information in mind an effective comparison of the group accomplishments can be made. Even though our expectations will vary, all six programs should far exceed the typical high school classroom learning situation. Confirmation of this hypothesis will help show the flexibility of the fast-math concept.

Curriculum

Since Charles County and Montgomery County have ongoing programs, to indicate mastery level one can compare only those precalculus segments they have completed and on which they have been tested. The three comparable segments are Algebra I, Algebra II, and plane geometry. As Table 2 indicates, all five groups have learned these segments well as measured by the Cooperative Mathematics Tests. All groups scored high on the most appropriate national norms. Montgomery County's first class, as would be expected, far surpassed the two SMPY classes and the first Charles County class. Using the national high school norms, its class mean for plane geometry was at the 99.7th percentile. Their knowledge as measured by the CMT standardized achievement tests was significantly higher than SMPY class number two ($.001 < p < .002$) in plane geometry and Algebra II ($.025 < p < .05$) SMPY class number two was significantly higher on plane geometry than Charles County ($.001 < p < .002$) and SMPY class number one ($.01 < p < .02$).

These results may make the first Charles County program seem poor until one considers two factors. The first is that, when compared with the national norms, the class mean of 48.2 on plane geometry is at the 80th percentile. Second, after 26 hours of instruction these students knew geometry better than four-fifths of the nation's students who complete plane geometry in the regular 180 45- or 50-minute periods. In addition, by looking closely at the Algebra I results in Table 2 one notes that the two Charles County classes knew Algebra I better than the first SMPY class (which, however, consisted mainly of pre-seventh-graders and studied the subject for only nine two-hour periods), and in one case better than the second SMPY class. Even between the two Charles County classes there is a significant difference ($.025 < p < .05$) in favor of the first class. In this case, a "creaming-off" process may be operating--i.e., the ablest students get into the first class.

This evaluation indicates that most of the students in all three

programs learned parts of the precalculus curriculum quickly and well as measured by the Cooperative Mathematics Tests. This is a crucial point, since many persons find it difficult to believe that quite a few students can learn the entire precalculus mathematics sequence well in one-fifth the time usually taken in the regular classroom situation.

Sex Differences

As indicated in Table 1, boys in the second SMPY class scored significantly higher than the girls in that class ($.005 < p < .01$) on SAT-M and in the second Montgomery County class on SCAT-Q ($.001 < p < .002$). In the other classes conducted by SMPY and Montgomery County the identification scores on the APT-N and SAT-M favored the boys, but not significantly. The opposite was true for the Charles County program. Girls tended to score higher on final screening devices, as indicated in Table 1, though not significantly.

Boys tend to outnumber girls in these classes on the average ratio of 2:1 of 3:1. One reason, as mentioned by Fox (1976) and Astin (1974), may be that girls do not perceive mathematics as a field easily accessible to them. Traditionally, it is generally assumed to be a male-oriented area.

With these factors in mind it seems likely that boys would score significantly higher than girls on the various curriculum segments and that the attrition rate for girls would be greater in the Charles County and Montgomery County programs (George and Denham, 1974; and Fox, 1974.) This was not the case. Girls tended to remain in the county-wide programs and in one instance scored significantly higher ($p < .05$) than the boys (first Charles County class in Algebra I). It was encouraging to see that significant sex differences in performance did not appear here, as they indeed did strongly in the first two SMPY groups. This may be due to the changing attitude toward women and mathematics or perhaps in the Montgomery County special classes to the extremely great mathematical aptitude of both sexes.

Summary

In retrospect the following five items seem needed for a successful class: (1) the identification of qualified, mathematically oriented, and highly apt students through appropriately difficult tests of mathematical and verbal reasoning ability and prerequisite achievement; (2) the selection of a bright, dynamic, assertive teacher who can create an atmosphere of fun and productivity while introducing the mathematical reasoner to challenging materials at a

rapid-fire pace; (3) compatible learning styles between student and teacher; (4) the development of good study habits, learning new materials by doing homework well; and (5) voluntary participation and self-motivation by the students. It appears that once these considerations are met the class will proceed "naturally" in both the academic and social aspects, as evidenced from these various programs. Quite a few students will need a somewhat slower, more detailed pace, especially because some of them will not work sufficiently hard or long (several hours per week) on homework. A socially oriented class, rather than an investigative environment, may be better for some individuals. In addition, the attrition rate seems to decrease in the county-run programs, especially for girls. This may be because the class is perceived as being socially acceptable and credit is assured, since the course is sanctioned by the school system.

In conclusion, the benefits of fast-paced mathematics classes far outweigh the costs, even monetarily. Well-run programs can effect net savings. As indicated by these various programs, the fast-math concept is viable and easily adapted to a wide range of circumstances. Already other programs following this model are in existence. These include in-school programs in Baltimore City, private-school programs in Prince Georges County, and two county-wide classes in Baltimore County (Smith, 1976). Students in these classes are challenged to move at a fast pace, which is appropriate for them. Their time is being used more wisely, without the boredom that so often occurs in the regular classroom situation. The development of good study habits is encouraged by means of fast pacing and suitable competition from other able students before it is too difficult to learn them.

Fast-math programs can easily be justified when one considers the impact that they have on highly apt students. For the ablest and best motivated youths such programs mean a chance to go into better careers in which their fine knowledge of mathematics will be highly useful. Neither the students nor society can afford to keep the best mathematical reasoners proceeding at the usual pace of most regular mathematics classrooms.

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Table 1: Four Mathematical Aptitude Tests Used As Identification Measures for Six Fast-Math Programs

Program	Identification Measure ^a	Total					Boys		Girls		
		N	Mean	S.D.	Rank %ile ^c	N	Mean	S.D.	N	Mean	S.D.
First SMPY	APT-N	19	46.7	6.0	99+	11	48.4	6.9	8	44.4	3.3
Second SMPY	SAT-M	33	580.9	49.5	91	24	593.3*	49.0	9	547.8	34.6
First Montgomery County	SAT-M	41	646.3	43.4	97	34	649.4	45.4	7	631.4	28.0
First Charles County	SAT-M	21	509.5	56.2	81	14	504.3	56.3	7	520.0	54.5
Second Charles County	PSAT-M	24	44.2	4.4	79	17	43.6	4.0	7	45.9	4.6
Second Montgomery County	SCAT-Q ^b	152	582.7	62.7	91	117	591.79**	63.0	35	552.3	51.2

* .005 < p < .01

** .001 < p < .002

^a The identification measures used are as follows: Academic Promise Test-Numerical (Apt-N), a 60-item test of numerical ability used for grades 6-9; the Scholastic Aptitude Test -- Mathematics (SAT-M), a difficult 60-item test of mathematical reasoning ability designed for high school juniors and seniors; the Preliminary Scholastic Aptitude Test -- Mathematics (PSAT-M), an abbreviated form of the SAT-M designed primarily for high school juniors; and the School and College Ability Test -- Quantitative (SCAT-Q), a 50-item test for college freshmen designed to measure numerical skills and mathematical reasoning ability.

^b The SCAT-Q scores discussed in this table have been transformed to equivalent SAT scores.
^c Percentile ranks used are the following: spring sixth grade norms (APT-N); random sample of high school juniors and seniors, whose mean score is 395 and standard deviation 120 (SAT-M); representative sample of sophomore boys tested during a special administration in 1966 (PSAT-M).

Table 2: Comparison of the Cooperative Mathematics Test Results for Three Precalculus Curriculum Segments Taught in the Various Fast-Math Programs

Program	Algebra I				Algebra II				Plane Geometry						
	N	Hrs. of Instruc.	Mean	S.D.	%ile ^f Rank	N	Hrs. of Instruc.	Mean	S.D.	%ile ^g Rank	N	Hrs. of Instruc.	Mean	S.D.	%ile ^h Rank
First SMPY	22	18	28.1	6.6	80	16	36	32.4	7.9	94	8	12	58.3	2.9	97
Second SMPY	31	150 ^d	65.6 ^e	7.7	95	28	34	34.0	3.5	97	28	36	62.9	7.8	99
First Montgomery County ^a	--	---	----	---	--	39	34	35.8	3.5	98	38	38	68.5	5.1	99.7
First Charles County ^b	23	26	33.4	4.0	96	--	--	----	---	--	19	26	48.2	9.8	80
Second Charles County ^c	20	26	30.6	4.0	93	--	--	----	---	--	--	--	----	---	--

^a The Montgomery County Program started with Algebra II, since many individuals had already completed Algebra I the previous year.

^b The Algebra II segment in the first Charles County program was being taught concurrently with Algebra I, but in the regular classroom setting.

^c The plane geometry segment was being taught concurrently with Algebra I, but in the regular classroom setting. There are no Algebra II results, since this segment had not been completed as these data were being analyzed.

^d Many of the students invited to participate had already completed 180 50-minute sessions of Algebra I as part of their regular secondary school curriculum, and this is the time listed.

^e Both 40-item forms of the Algebra I test were administered to all students to indicate mastery of Algebra I concepts. This was necessary before they were allowed to enter the second fast-math program, which started with the Algebra II segment.

^f Percentile ranks are shown for national grade nine norms.