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Educating Mathematically Gifted Pupils at the Kolmogorov School

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Abstract
The purpose of this article is to describe the discovery and development of mathematically gifted students at the Kolmogorov School—Special Educational and Scientific Centre of Moscow State University. This residential school, which was founded by Academician Andrey Nikolaevich Kolmogorov, a member of both the Academy of Science and the Academy of Education in the U.S.S.R., in December, 1963, currently fosters the mathematical development of approximately 200 pupils in grades 10 and 11 who are selected through written and oral examinations after participation in regional olympiades. The article illustrates the curricular principles used at the Kolmogorov School.

Everything that the greatest minds of all times have accomplished toward the comprehension of forms by means of concepts is gathered in one great science, mathematics.
J.F. Herbart

The purpose of this article is to provide a brief account of the discovery and development of mathematically gifted pupils at the Kolmogorov School—Special Educational And Scientific Centre of Moscow State University. This school, which opened in 1963, was the first of four residential schools for pupils gifted in mathematics and physics (*fiziko-matematicheskaja shkola-internat*) in the Union of Soviet Socialist Republics. Other residential schools are located in Kiev, Leningrad, and Novosibirsk.

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History
The Kolmogorov School was initiated through the vision and efforts of Academician Andrey Nikolaevich Kolmogorov (1903–1987). Kolmogorov was concerned by the variability in educational experiences favoring students in Moscow. He wanted to ensure educational opportunities for gifted pupils in rural regions as well. In the summer of 1963 Kolmogorov, a member of the Academy of Science and the Academy of Education, invited 50 pupils who had just completed the tenth grade to participate in a one-month summer program in modern mathematics. Most of the invitees were winners of regional Mathematical Olympiades. Nineteen of the participants in the summer program were later invited to study at the Kolmogorov School of Moscow State University, which officially opened in December, 1963.

Selection of Pupils
From its humble beginnings, the Kolmogorov School now instructs about 200 pupils in grades ten and eleven each year from Russia, Byelorussia, Tatarija, Baschkirija, Osetija/ North Osetija, Checheno-Ingushetija and Kabordino-Balkarija. Students are regional winners of olympiades in subjects such as mathematics, physics, chemistry and biology. After winning an olympiad, students are invited to take written examinations consisting of mathematics and physics problems. Students also participate in an oral examination conducted in their home villages by faculty in mathematics and physics from the Kolmogorov School. After the oral examinations, the best students are invited to spend one or two years at the residential school, which commences with a mandatory one-month summer school for tenth graders.

Educational Framework
The educational curriculum in terms of subjects taught parallels the curriculum taught in typical high schools (mathematics, physics, physical education, biology, history, chemistry, literature, and foreign languages). A unique feature of the Kolmogorov School is the fact that all subjects are taught by faculty of Moscow State University. Pupils attend additional lectures and seminars each week in mathematics (two hours of lecture and six hours of seminar) and physics (one hour of lecture and four hours of seminars) than in the typical high school. Curriculum is based on knowledge of the pupil's previous educational experiences and the intuitive judgement of the faculty of the Kolmogorov School. The curriculum framework of the Kolmogorov School focuses on the importance of mathematics in the history of science. The school encourages the development of a clear understanding of the basic concepts of mathematics; for example, the concept of the *limit* in calculus. Such understanding forms the basis for logical and intuitive thinking in mathematics and makes possible further coordination and simplification of practical applications of mathematics in the sciences, for example, wave equations. Students are encouraged to pursue independent study projects in their preferred areas of interest, such as finite geometry, complex numbers, mathematical computing, algebraic theory of equations, and theory of groups. In addition to independent study, students are encouraged to become producers of knowledge by investigating unsolved problems in mathematics. Such work extends the interest of the pupils and models the activity of professional mathematicians. (Sample problems

highlighting the transition form acquiring basic concepts to investigating mathematical problems are shown in Appendix A). In addition, students also receive instruction in computer languages, given the importance of computer applications and technology in the development of science.

Follow-up

After completing their tenure at the Kolmogorov School, students are awarded a high school diploma. Almost all continue to a university or institute for physics or mathematics. To date, approximately 4,000 pupils have studied at the Kolmogorov School. About 10% have earned a doctorate in mathematics or science. About 1% have earned two doctorates in mathematics and science. Many of the alumni are currently faculty members at universities throughout the Soviet Union. The staff of the school look forward to the future contributions and recognition of alumni as they fulfill Kolmogorov's vision.

Appendix A
Basic Limit Problems

1. To prove these identities
   a) $\lim_{n \to \infty} n/a^n = 0$ if $a > 1$;
   b) $\lim_{n \to \infty} a^{1/n} = 1$ for $a > 0$;
   c) $\lim_{n \to \infty} n^{1/n} = 1$;
   d) $\lim_{n \to \infty} \log_a{}^n/n = 0$

Solution

1a. We have $a = 1 + b$, where $b > 0$. By virtue of the Newton binomial theorem
$a^n = (1 + b)^n > 0.5\, n(n-1)b^2$ for $n \geq 2$

Moscow State University *Photo courtesy of Dr Judy Lupart, University of Calgary*

Hence, we obtain $0 < n/a^n < 2/(n-1)b^2$). Passing to the limit in these inequalities at $n \to +\infty$, we get the desired identity.

1b. Without limiting the generality, we can assume that $a > 1$. We have $a^{1/n} + 1 + b_n > 0$.
It is equivalent to $a = (1 + b_n)^n$.
Then by virtue of the Newton binomial theorem
$a > nb_n$ for $n \geq 1$, i.e., $0 < b_n < a/n$.

Passing to the limit in these inequalities at $n \to \infty$
We obtain
$\lim_{n \to \infty} b_n = 0$, i.e., $\lim_{n \to \infty} a^{1/n} = 1$, *q.e.d.*

1c. We have $n^{1/n} = 1 + c_n$, $c_n > 0$. Then by virtue of the Newton binomial theorem
$n = (1 + c_n)^n > 0.5\, n(n-1)\, c_n^2$ for $n \geq 2$,
i.e., $0 < c_n < 2/(n-1)$.
Passing to the limit in last expression, we obtain
$\lim_{n \to \infty} c_n = 0$, i.e. $\lim_{n \to \infty} n^{1/n} = 1$

1d. By continuity of the logarithmic function for $a > 0$, $a = 1$, we have
$\lim_{n \to \infty} \log_a n^{1/n} = \log_a (\lim_{n \to \infty} n^{1/n}) = \log_a 1 = 0$
Then we learn properties of number e (Neper's number), of the solution of Kepler's equation (the existence, continuity, differentiability) for moving two bodies.

Problem Investigation

To find the value S_n of the function f(x) with conditions

$$| S_n - f(x) | < 10^{-n}$$

Solution

The general construction of this is such. We find the function F(x,y) with following properties:

a) $F(x, f(x)) = f(x)$,

b) $F(x, S_n) = S_{2n}$

c) $F'y(x,y)\, |_y = S_n = 0$

For this function we have

$f(x) = F(x, f(x)) = F(x, S_n) + (f(x) - S_n)$
$F'y(x,y)\, |_y = S_n + (S_n - f(x))^2$
$F''yy(x,y)|_y = \xi$,

where $\xi = f(x) + \theta(S_n - f(x)), 0 < \theta < 1$.

If $|F''yy(x,y)\, |_y = \xi| \leq 1$, then we obtain

$f(x) = F(x, S_n) + \theta_1 (S_n - f(x))^2$,

where $| \theta_1| \leq 1$,

i.e., $|f(x) - F(x, S_n)| \leq | S_n - f(x) |^2 \leq 10^{-2n}$,

hence, $S_{2n} = F(x,S_n)$.

For example, $F(x,y) = 0.5\,(y + x/y)$,
$S_{2n} = 0.5\,(S_n + x/S_n)$.

How to Use Gipsy Children's Vernacular in the Kindergarten

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Abstract

This article describes a project based in Hungary which was intended to develop education that was relevant to the various indigenous cultures known as gipsy cultures. The writer emphasises the importance of understanding the culture and then basing education on the strength within the culture giving them recognition and worth.

As our results and difficulties can be explained exactly only in the mirror of the educational policy and social changes, I would first like to report on these.

The works on educational reforms leading Hungary closer to the European standards, were started in 1985. The *Act of Education* worked out then is still in force. This Act opened the way to the alternative schools and curricula, and also enabled the introduction of partial professional autonomy in the schools.

In 1992, a new Act on Education will be presented to the Parliament. This Act is to accomplish the proc-