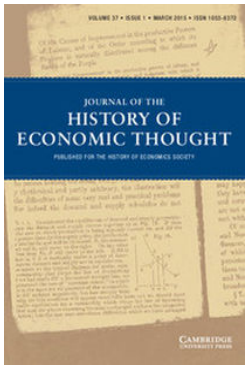

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Journal of the History of Economic Thought / Volume 25 / Issue 04 / December 2003, pp 413 - 435
DOI: 10.1080/1042771032000147498, Published online: 11 June 2009

Journal of the History of Economic Thought,
Volume 25, Number 4, December 2003

Routledge
Taylor & Francis Group

RAMSEY'S THEORY OF NATIONAL SAVING: A MATHEMATICIAN IN CAMBRIDGE

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I. INTRODUCTION

In the December 1928 issue of the *Economic Journal*, Frank Ramsey asked the question "how much of its income should a nation save?" Few of the Cambridge economists of the 1930s were convinced by his highly formalized answer. His contribution sank quickly into oblivion, remaining there for about thirty-five years. In the 1960s, the success of the Hamiltonian formalism and the increasing interest for optimal growth led on the contrary to a quasi "natural" use of Ramsey's former intuitions.[1] These mathematical tools became so widespread that, a few years later, new classical macroeconomics uses a new interpretation of the "*à la* Ramsey" models, within the setting of representative agent models, in order to bypass the Arrow and Sonnenschein-Mantel-Debreu "impossibility results."[2] The "*à la* Ramsey" model is the backbone of modern new classical macroeconomics. It is thus not surprising that these successive moves in macroeconomic theory came to foster a slanted interpretation of Ramsey's 1928 article. In this respect, Roger E. A. Farmer's point of view is representative of the retrospective tribute sometimes paid to Ramsey's article:

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[1] The purpose is to combine Ramsey's model with Phelps's Golden Rule.

[2] We, in fact, interpret the Sonnenschein-Mantel-Debreu results as negative results: any micro functions are consistent with any macro functions; well-defined micro functions (individual demand functions or micro Cobb-Douglas functions) are neither necessary nor sufficient for macro well-defined functions. New classical macroeconomics avoids answering the theoretical challenge in adopting, from the outset, the case of perfect aggregation conditions: the agents are either identical or only differs in their initial endowments.

ISSN 1042-7716 print; ISSN 1469-9656 online/03/040413-23 © 2003 The History of Economics Society
DOI: 10.1080/1042771032000147498

> F. Ramsey was one of the first economists to study how an infinitely lived agent should allocate his resources over time. His work was at the forefront of mathematical economics at the time it was written but his approach has now become a standard part of graduate macroeconomic courses. Many applications of Ramsey's work assume that there is only one agent in the economy and that this representative agent can be thought of as a stand-in for the workings of the market mechanism (Farmer 1993, p. 77).

In the shape of representative agent models, the "*à la* Ramsey model" has in fact become the formal hardcore of growth theory as well as real business cycles theory and is nowadays a general model-building instrument. As a representative agent model, its analytical significance rests on the use of the second "welfare theorem," the intertemporal-Pareto-optimal choice of a representative agent being associated with a decentralized allocation of resources. As a general equilibrium macroeconomic representation, its legitimacy is founded on an instrumentalist methodology. Finally, the calibration of the parameters leads to a widening range of applied economic models. But it should not be doubted that, at the end of the story, the "*à la* Ramsey" model has almost nothing to do with the original contribution Ramsey exposed in 1928. In contrast with this contemporary use of "*à la* Ramsey" models, I propose to come back to Ramsey's original article, both from a theoretical and a methodological perspective.

My intention in this paper is twofold. First, I aim at clarifying what this 1928 isolated contribution was, which ultimately paved the way toward "neoclassical macroeconomics." Ramsey's mathematical theory of saving is entirely directed toward answering the inaugural question mentioned above, what should be the good level of savings for an entire nation. Therefore, it was not a question of microfoundation but a welfarist interrogation that motivated the construction. To tackle this question, Ramsey chooses to model a planned economy, which allows him both to make a perfect information and foresight assumption and to provide a mathematical treatment of the problem. The originality of his model is then based on the use of an aggregate utility function and an aggregate production function, with a view to transposing the marginalist reasoning from the microeconomic level to a macroeconomic entity embodying the nation. Thus, in excluding from the beginning any problem linked with the coordination of individual decisions, this construction focuses primarily on the global inter-temporal allocation of income. From an historical point of view, this constitutes the main contribution of the 1928 article. In doing so, Ramsey achieved a breakthrough in economic theory, providing macroeconomic theory with one of its first "control" theory models, and paving the way for a normative tradition in macroeconomics.

Secondly, I want to provide an historical background to Ramsey's model. In spite of the novelty of Ramsey's theory and methodology, his modeling choices did not come by chance. As John Maynard Keynes puts it in his famous obituary in the *Economic Journal*: "From a very early age, about sixteen I think, his precocious mind was intensely interested in economic problems. Economists living in Cambridge have been accustomed from his undergraduate days to try their theories on the keen edge of his critical and logical faculties" (Keynes 1930, p. 335).

It is not so speculative to assert that Ramsey's "mathematical theory of saving" is an interesting example of how a mathematician can apply his skills to an economic problem. From a Cambridge theoretical and philosophical basis, Ramsey works out an intertemporal allocation model grounded on a systematic but careful use of mathematics. More precisely, I shall argue that his contribution can in fact be fruitfully interpreted in relation to the Cambridge background at the beginning of the twentieth century. Ramsey was to a certain extent a typical Cambridge "product." To put it in a few words, his thought was at the junction of two particularly active circles in Cambridge in the first decades of twentieth century: a philosophical one, which was shaken up by flourishing logicism and pragmatism; and an economic one, which was still imbued with the Marshallian legacy but was also experiencing the first quivers of "the Years of High Theory." On these Cambridge foundations, Ramsey applies a presentation and logical treatment of his own to the latent, but until then not clearly defined question, of optimal saving.

II. RAMSEY'S THEORY OF SAVING

We today have only a limited appreciation of Ramsey's original theory. Before inquiring into the origins of the 1928 article, it seems necessary to linger on the main features of Ramsey's theory. I want to underline three characteristics of this original contribution: It is a "synthetic," a normative and a mathematical theory of saving.

A Synthetic Theory of Saving

Ramsey's contribution to optimal saving theory is the first synthetic representation of the double theoretical function of saving underscored by his predecessors. Classical authors more or less considered savings—taken as a whole—as the source of capital accumulation and growth. On the other hand, the marginalist revolution put the emphasis on saving as the result of an individual allocation problem, and thus as a temporary sacrifice of present satisfaction. The challenge of Ramsey's article is to reconcile these approaches into a unified theoretical framework. Ramsey takes up this challenge through radical and audacious simplifications for the time.

Ramsey sets out to answer the initial question "How much of its income should a nation save?" in three parts. The first part is concerned with the well-known planning problem of a community which lives forever. Under simplifying assumptions, Ramsey demonstrates his famous rule—the first version of the so-called—"Keynes-Ramsey" rule: "The rate of saving multiplied by the marginal utility of money should always be equal to the amount by which the total net rate of enjoyment of utility falls short of the maximum possible rate of enjoyment" (Ramsey 1928, p. 543).

Ramsey then widens the range of application of the rule in a series of theoretical variations: he proposes a graphical determination of the optimal savings rate, which allows him to deal successively with the question of the

income allocation of a selfish individual who lives a finite time and does not care for his children (part II), and that of an altruistic individual who on the contrary takes his descendants into account. In the conclusion of the article (part III), Ramsey then turns to the improvement of the preceding developments for the study of a kind of simple decentralized economy. He sketches the main characteristics of a long-term macroeconomic equilibrium, which he presents as the emergent result of individual saving decisions consistent with Ramsey's rule. It is noteworthy that Ramsey introduces only marginal changes in the rule in each of its applications. These changes are to be understood as necessary adaptations to each particular institutional framework, but Ramsey never recasts the allocation rule. The collective mathematical rule is thus the keystone of the entire article.

To obtain this initial version of the intertemporal allocation rule, Ramsey assumes from the outset a one-good world, whose structure remains unchanged in the course of time. The nation is represented by a collective cardinal utility function and the productive structure is represented by an aggregate production function[3] with the total amounts of labor and capital (available in the economy) as arguments. Remarkably enough, Ramsey does not try to justify the use of such macroeconomic functions, and *a fortiori*, he carefully avoids referring to any kind of aggregating procedure or "representative agent" concept.[4]

In the model, the aggregate production function plays the role of an accumulation constraint allowing solution to the utility maximization program of the community. Savings and investment decisions are thus identified in accordance with the Classical view.[5] In contrast, unlike the Classical representation, capital originates in the both profits and wages of the members of the community.[6] The main advantage of this overall presentation is to exempt Ramsey from analyzing the coordination of individual decentralized decisions and to dispense from the

[3] Mark Blaug finds in John Hicks's "Theory of Wages" (1932) the first occurrence of the idea that "the functional distribution of income may be explained simply by invoking the principles of marginal productivity as enshrined in an aggregate production function for the economy as a whole" (Blaug 1962, p. 487). Although Philip H. Wicksteed (1894) was using a global satisfaction function, he studied the question of factor pricing without appealing to the concept of aggregate production function.

[4] Ramsey deals in the same manner but separately with what he considers to be independent questions: the collective choice (which is the object of his first section) and the individual choice (which then articulates the following sections). This point is explicit when he introduces a utility discount rate to adapt the initial rule to the determination of the individual question: "In assuming the rate of discount constant, I do not mean that it is the same for all individuals, since we are at present only concerned with one individual or community" (Ramsey 1928, p. 553).

[5] Ramsey neglects to make clear the way capital is transferred between generations: private and collective transfers are not distinguished.

[6] This fact becomes clear in the second part of the 1928 article, where Ramsey proposes a graphical determination of the optimal amount of savings. After having specified a particular production function, with constant returns to scale, he distinguishes the global income distribution between "earned income" and "unearned income" (1928, p. 550). He then notices that "not only would the whole unearned income be saved, but part of the earned income also" (1928, p. 550). An interesting analysis by Frank Raymond reminds us that this feature echoes the historical development of capitalism, the emergence of a middle class of workers and savers, and the expansion of a financial intermediaries system that can collect savings funds as a source for investment (Raymond 2000, p. 17).

outset with any kind of adjustment process. This question is solved by definition. For all that, however, Ramsey makes an extensive use of marginalist reasoning at an aggregate level. He particularly mobilizes a result that in neoclassical literature stands for equilibrium of the individual, and that his marginalist predecessors would have balked at transposing to the aggregate level: the production factors are paid their marginal productivity. The rate of interest "earned by waiting" (Ramsey 1928, p. 546) is given by the marginal productivity of capital. This result is used in particular by Ramsey to legitimate the equation (3) presented below—that is, to articulate the optimal intertemporal national income allocation. However, more than an equilibrium condition to follow, the status of this marginalist methodology might be more adequately interpreted as an implicit tautology, expressible by an omniscient planning entity. The economical organization at every instant t is in fact assumed to be totally governed by the marginalist maximization principle, but based on what we could anachronistically call shadow prices.

This theoretical synthesis and analytical integration of two different historical traditions will not offend the present-day reader. On the contrary, standing apart from this synthetic program, another salient feature of Ramsey's model might at first sight appear much more peculiar. I am thinking about a particular property of the intertemporal collective utility function in the model. This utility function is deprived of any rate of discount, and Ramsey would clearly have regarded any other assumption as "ethically indefensible" (Ramsey 1928, p. 543). The analytical consequence of this ethical position is to introduce from the outset what he calls bliss (B), that is, a satiation point, or the "maximum obtainable rate of enjoyment or utility" (Ramsey 1928, p. 545).[7] It is the result of a normative (prescriptive) choice made by Ramsey.

A Normative Approach

Ramsey is not searching for a new statement of the economic determinants of saving. The initial question of the paper is clearly normative. So is the answer Ramsey sets forth, and so is the theory he aims at implementing: a "control theory." The purpose of theorizing here is to determine rules one could impose on reality in order to shape this reality according to some predefined objectives. More than that, this aim is sustained from the outset by the properties of the intertemporal collective utility function.

The intertemporal collective utility function—that is the objective function to be maximized under constraint—is the sum in continuous time of instantaneous collective satisfactions; it is a collective intertemporal additive utility function. The adopted framework does not make clear the achievement of instantaneous collective utilities. The aggregation procedure of individual utilities is not described, and we cannot determine whether Ramsey applies a weighted sum of individual satisfactions. This appears to be secondary since the mission allotted

[7] The first part of the article also excludes population growth, technical progress, or unforeseeable events like wars or earthquakes.

to the planning entity is neither a distribution nor a coordination one. On the contrary, the planning entity has to orchestrate the intergenerational income transfer in order to define an optimal path of consumption. At this moment of the construction, Ramsey refuses to introduce a discount rate of future utilities, thus adopting what is conventionally called an intertemporal "utilitarian" criterion.[8] Indeed, there is no reason why the planning entity should privilege one generation against another. The optimal situation looked for by the community is double-sided. On the one hand, the planner's objective is to achieve bliss (*B*), an ideal stationary state, in which capital accumulation and saving become useless and in which all national income is devoted to consumption. This stationary state is nearly postulated by Ramsey, who appeals to common sense to rally his readers to the assumption.[9] The enjoyment increases as long as the (decreasing) marginal productivity of capital is positive. On the other hand, the way to get close to bliss within an infinite time requires a rule: the optimal allocation rule. According to it, the course of consumption is such that along the entire path, it is not possible to increase by one unit the utility level of a generation, without deteriorating the utility level of the next generation, by more than one unit.

The normative character of Ramsey's proposals in other respects goes far beyond the ethical question. To answer the inaugural question of the 1928 article, Ramsey, in fact, assumes that the community has what we can call a substantive rational behavior.[10] More than a descriptive- or reality-oriented standpoint, Ramsey gives himself a kind of "model-able" behavior, likely to provide the analysis with an abstract and general foundation. This model-able behavior is an ideal behavior. He therefore constructs the armature of a general prescriptive decision model, opened to the generalization proposed in the subsequent sections of the article. The institutional setting is never explicitly defined. But in the case

[8] This expression refers to the transposition of the Benthamite static criterion to intertemporal aggregation. Notwithstanding, this choice does not echo any hedonist principle. In wrongly identifying the Benthamite utilitarianism with such a philosophy, Ramsey rejects "the psychology of the Utilitarians, in which pleasure has a dominating position. The theory I propose to adopt is that we seek things which we want, which may be our own or other people's pleasure, or anything else whatever, and our actions are such that we think most likely to realize these goods" (Ramsey 1926, p. 69).

[9] After having noticed that the global net enjoyment increases with the amount of capital, Ramsey (1928, p. 545) successively contemplates two possibilities. The first one consists in supposing the existence of a saturation level of net enjoyment, due either to a saturation of capital or to an intrinsic saturation of enjoyment to a level called the "maximum *conceivable* rate of enjoyment." The second one is to suppose that net enjoyment never stops increasing. Two logical cases are again conceivable: either the rate of enjoyment will increase to infinity, a possibility *a priori* ruled out by Ramsey, or it will approach asymptotically to the "maximum *obtainable* rate of enjoyment" (bliss).

[10] We here use the sense Simon has given to the notion of "substantive rationality." Ramsey does not clearly use such an expression. However, this conception of rationality was still the foundation of the theory of subjective probability he proposed in "Truth and Probability" (1926). On this occasion, Ramsey opposed to Keynes's appeal of a bounded rationality a representation of individual decision founded on three main features, which one finds again in 1928: (1) The agent aims at getting the maximum of satisfaction, but the theory is silent about the content of goals and value. (2) The condition for the validity of the theory is the global consistency of behavior. (3) the theory postulates "one world": "behavior is objectively rational in relation to its total environment, including both present and future environment as the actor moves through time" (Simon 1986, p. 26).

of the community, as well as for the individual decision, Ramsey always deals with a perfect information framework. In these conditions, the choice of every agent results from a maximization behavior that orders the behavior one should adopt because it is the one of a perfectly informed agent. Ramsey's commentaries on the hypothesis of rational behavior in "Truth and Probability" (1926) enhance the methodologically normative status of this representation. When he proposes adopting the hypothesis of expected utility maximization,[11] he then specifies:

> It is a simple theory and one which many psychologists would obviously like to preserve by introducing unconscious desires and unconscious opinions in order to bring it more in harmony with the facts. How far such fictions can achieve the required result I do not attempt to judge: I only claim for what follows approximate truth, or truth in relation to this artificial system of psychology, which like Newtonian mechanics can, I think, still be profitably used even though it is known to be false (Ramsey 1926, p. 69).

This approximation is useful because it allows us to establish general rules[12] to guide human behavior. In the limiting case where "[the] subject has no doubts about anything, but certain opinions about all propositions, we can say that he will always choose the course of action which will lead to his opinion of the greatest sum of good" (1926, p. 70). What appeared like a limiting case in 1926 becomes the central approximation of the 1928 paper, within the context of an implicitly perfect information framework. In this context, the choice to reject discount rates of future utility can, besides the ethical aspect, be associated with a logical requirement of consistency. Indeed, to give greater importance to present satisfaction would reveal a defective expectation faculty, which can characterize an isolated individual but which cannot affect an omniscient planner.

To conclude with the normative construction of the paper, the rational behavior conditioning the establishment of the allocation rule appears as an obviously false but nevertheless fruitful representation. The validity of this will afterwards be evaluated in the light of its practical improvements, according to Ramsey's pragmatist epistemology. As will be seen later in detail, Ramsey aims at defining a general mathematical rule able to guide collective, as well as individual, decisions and actions. The set of extreme simplifying assumptions described above indeed allows him to set a relatively simple system of mathematical reasoning.

A Mathematical Theory

Ramsey is often regarded as the first figure to have achieved an idea Edgeworth (1881, p. 109) had first expressed: to exploit, for economic questions—the calculus of variations, a mathematical tool that was for a long time a standard tool of physical sciences. The entry devoted to the Ramsey model in the *New Palgrave Dictionary of* Economics is in this respect symptomatic: "The main contribution of the paper was to pose a fruitful question—what should the rate

[11] "The theory that we act in the way we think most likely to realize the objects of our desires, so that a person's actions are completely determined by his desires and opinions" (Ramsey 1926, p. 69).
[12] Such rules are justified by a logical or mathematical reasoning as we will see in the last section of this paper.

of savings be—and propose a method of analysis—that of intertemporal welfare maximization using the techniques of dynamic optimization, in this case, the calculus of variations'' (Newbery 1987, p. 47).

Nevertheless, it appears that Ramsey seems quite reticent to use this method explicitly to tackle his dynamic optimization problem. In fact, Ramsey demonstrates his allocation rule in two steps. At first sight, one would try to solve an optimization program (the objective being to get close to bliss) in which the constraint calls out the aggregate production function. This way would imply the direct use of the calculus of variations. Instead of taking this way directly, Ramsey provides two demonstrations. Both are only partial demonstrations, but they complete each other. Hereafter, I will refer to the first one as the "economic demonstration'' and to the second one as the "mathematical demonstration.''[13] Although both make an extensive use of mathematics, mathematics is not on same footing in the one and in the other.

In the "economic demonstration,'' mathematics frames the reasoning. It shows the logical improvements of three fundamental equations summing up the assumptions mentioned above. These three initial equations are supposed to be sufficiently evident to evade any further justification. Their validity rests on a transposition to the macroeconomic level of the standard marginalist results. Mathematics then logically expresses and makes explicit the intrinsic content of the equations.

The first equation is an accounting identity. Labor (a) and a stock of capital (c) are used jointly to produce a flow of output $f(a,c)$. This output is then used either for consumption (x) or for saving, which is identified with capital accumulation ($\dot{c}$):

$$\dot{c} + x = f(a, c) \tag{1}$$

The second equation equates the marginal satisfaction drawn from consumption and the marginal effort due to work. With $v(a)$ standing for the marginal disutility of labor and $u(x)$ standing for the marginal utility of consumption,[14] we obtain:

$$v(a) = f_a(a, c) \cdot u(x) \tag{2}$$

The third equation expresses the intertemporal balance of the "rational community:''

$$\frac{d}{dt}u(x) = -f_c(a, c) \cdot u(x)^{15} \tag{3}$$

It in fact sums up a double equilibrium condition. On the one hand, it expresses that the utility of the last present consumption unit must be equal to the utility

[13] I borrow these terms from Newbery (1987).

[14] In Ramsey's paper, $U(x)$ denotes the total utility of consumption; $V(a)$ stands for the total disutility of labor. We have $u(x) = U'(x)$ and $v(a) = V'(a)$.

[15] This equation results from the arbitrage relationship equating the marginal utility of consuming a unit now with the marginal utility of consuming the product or investing the unit until the next instant of time: $u\{x(t)\} = \{1 + f_c(a, c) \cdot \Delta t\} u\{x(t + \Delta t)\}$. Ramsey then identifies the marginal productivity of capital with the rate of interest.

drawn tomorrow from the last invested unit. On the other hand, it stems implicitly from the identification of the interest rate with the marginal productivity of capital, which appears to be an equilibrium condition for the society envisaged like a giant firm. However, considered at the level of the planned system, this result expresses more a tautology than a real optimal allocation condition. Indeed, the nation cannot be looked upon as a price-taking firm that would have to apply the standard profit maximization condition; it is to be understood as the choice of a perfectly advised planner on the basis of "shadow prices," with the interest rate being one of the "shadow prices."[16]

On this triple departure point, Ramsey attempts to present his allocation rule as a purely logical conclusion of a few operations of derivation, integral calculus and substitution of variables: "Equations (1) (2) and (3) are sufficient to solve our problem provided we know c_0, the given capital with the nation starts at $t = 0$, the other initial condition being supplied by considerations as to the behavior of the function as $t \to \infty$" (Ramsey 1928, p. 546). These operations yield, in fact, almost the allocation rule announced at the beginning of the article:

$$\dot{c} = f(a, c) - x = \frac{K - \{U(x) - V(a)\}}{u(x)} \tag{4}$$

where K is an unspecified integration constant. However, Ramsey comes up against the problem of the economic meaning of the constant K. His intuition is to identify K with B, the level of utility the community would reached in the state of bliss, but he does not succeed immediately in this ultimate step.

He then has to replace the problem from a mere mathematical standpoint, which consists in minimizing the integral $\int_0^\infty [B - (U(x) - V(a))]dt$, that is to minimize the sum of the gap between the net current satisfaction (which purports to be the total utility of consumption $U(x)$ minus the total disutility of labor $V(a)$) and bliss, under the technical constraint given by equation (1). Here we come to the second demonstration, the "mathematical demonstration" of the allocation rule. That is also the famous moment where Ramsey alludes to the usefulness of the calculus of variations to deal with his problem. However, this way is nevertheless rapidly set apart: "If we apply the calculus of variations straight away, using equation (1), we get equations (2) and (3) again; but if instead of this, we first change the independent variable [t] to c, we get a great simplification" (Ramsey 1928, p. 547), a simplification that amounts to a first order minimization condition. He finally establishes the mathematical expression of his income allocation rule:

$$\dot{c} = \frac{B - (U(x) - V(a))}{u(x)} \tag{5}$$

Thus, the income allocation rule results in fact from a parallel drawn between equation (4) and (5). In fact, in this way Ramsey avoids using explicitly the calculus of variations.

[16] Let us note that this third equation describes what we incorrectly call the "Keynes-Ramsey's condition."

The two previous demonstrations admittedly complete each other, but they join intuitively and not formally, thanks to an implicit mathematical definition of bliss. Although B is presented by Ramsey as an intuitive concept—appealing to the usual culture of economists when they evoke the "stationary state" of the classics—it is in fact given a precise mathematical definition.[17] It is then amazing that Ramsey chose to insist on the economic demonstration, whereas the problem was sufficiently well defined to allow a more direct mathematical demonstration of the allocation rule.[18] To understand the reasons for this roundabout demonstration of the mathematical rule of income allocation, as well as to throw light on the main features underlined in this first section, I think that a historical inquiry into the general context Ramsey was working may be helpful.

III. AN ORIGINAL EXPLOITATION OF CAMBRIDGE TOPICS

Ramsey's economic papers are sometimes regarded as a kind of pastime for a young open-minded philosopher. Ramsey's philosophical work has been the subject of many historical investigations: his role in the widening of Russell's logicist program, his close relations to Wittgenstein's philosophy, the originality of his pragmatist epistemology are now relatively well-known facets of his intellectual contribution. On the other hand, Ramsey's pioneering economic work, and in particular the 1928 article, passed mainly unnoticed in the 1930s and was for a long time regarded as only a marginal chapter in the history of economic thought. Both his construction and his main results opposed the tide of history: whereas Ramsey advocated a comparatively high level of saving, the Depression originated in excessive saving; whereas he was toughening the use of perfect information frameworks, Cambridge was progressing toward disequilib-

[17] Ramsey's intuitive considerations about the bliss situation have indeed a precise mathematical counterpart. They include veiled affirmation that there necessarily exists a superior bound to the definition set of the net utility function (the greatest rate of enjoyment *conceivable*) and therefore that this function admits a finite limit B, the maximum rate of enjoyment *obtainable*. Thus, the saturation of satisfaction imputed to the saturation of consumption implies the existence of a finite limit of consumption $\hat{x}$, such that

$$U(\hat{x}) - V(\hat{a}) = \sup_{x \geqslant 0} [U(x) - V(a)].$$

This alternative is then associated with the second possibility, in which

$$B = \lim_{t \to \infty} [U(x(t)) - V(a(t))].$$

This mathematical limit eventually gives the terminal condition required for every definition of an optimal path of a state variable.

[18] It is, however, surprising that Ramsey so swiftly passed over such an opportunity. As he himself observes, this method would indeed have led to the equations (2) and (3), central to the economical interpretation of the reasoning. The complete calculus would have given the mathematical expression of the rule in the form of the equation (5). In fact, the mathematical translation of the intuitive definition of bliss was sufficiently precise to give the missing initial condition and to solve the differential equations stemming from Euler's equations. I give a demonstration of this affirmation in Gaspard (2001).

rium or imperfect competition theories. All this gives an historical rationale for the delayed interest the historians took in Ramsey's economic work.

To trace the genealogy of the 1928 paper is, moreover, a bit hazardous. Few clues are available in Ramsey's article about a possible theoretical influence. There is no indicating footnote and no bibliography in the 1928 article. The first impulse would be to present Ramsey the economist as a kind of "meteor" in the sky of economic theory.[19] Accordingly, the theoretical jumps in the 1928 paper mentioned above are mainly unprecedented and they appear all the more innovative as they failed to find a receptive public. However, such an interpretation would be quite frustrating. If we take a wider glance at Cambridge topics and examine Ramsey's notes, lectures, or philosophical articles carefully, it is my view that some more constructive elements can be found that will shed new light on the origins and the construction of the 1928 article.

Ramsey and Cambridge Economics: An Informal Integration

In my opinion, the 1928 article is a compromise between a very personal methodology and a general Cambridge set of themes. Connections between Ramsey's peculiar theoretical and methodological choices and the Cambridge community may be drawn, if not directly through explicit parallels between Ramsey's theoretical arguments and other contemporary theoretical constructions, then at least through the insertion of Ramsey's contribution in a common Cambridge culture.

Ramsey's integration in the economists' community passes, rather, through indirect, if not informal, channels. It owes a lot to the open intellectual Cambridge community. Ramsey was first of all a mathematician. Yet, on the fringe of his mathematician's degree course, the assiduous frequenting of pluri-disciplinary discussion circles gave him occasions to express his growing interest in analytical philosophy, and also in the social sciences. He was particularly well received in the *Moral Sciences Club*, where he was close to George Edward Moore. He was as well received in the famous *Apostles Society*, then frequented by Bertrand Russell, Moore, and Keynes. There, he delivered a series of eclectic lectures between 1921 and 1925, which made great impression on the "brothers" of the *Society*.[20] He then became famous in every field distinguishing Cambridge at

[19] It is admittedly possible to find in the history of economic ideas the intuitions that Ramsey uses in the 1928 paper. Raymond (2000) for instance, draws a rather complete picture of the previous writings and representations of saving that, from Adam Smith to Knut Wicksell, might have influenced Ramsey's conceptions. However, this kind of work is soon confronted with the lack of explicit material or references and so necessarily ends up in general conjectures. If Ramsey had certainly acquired a pretty good knowledge of political economy, mainly through the Cambridge economic lectures, it seems rash to speculate on whether he had read Irving Fisher, John Bates Clark, or Wicksell. Given the synthetic character of Ramsey's theory of saving, many authors could figure in good position as might-be precursors.

[20] Ramsey's interventions for the *Apostles Society* may be found in the *Notes on Philosophy* (Ramsey 1991, pp. 290ff). He here deals with different questions such as the social responsibility of the mathematicians, the problem of induction, the interest of psychoanalysis, or the recent evolution of women's position in society.

the beginning of the twentieth century. This training period allowed him to familiarize himself with the logicism of Whitehead and Russell on the one hand, and with Ludwig Wittgenstein's philosophical thought on the other. He especially carried on the program of logical foundations of mathematics of Whitehead and Russell in an immediately famous paper (Ramsey 1925) and translated the *Tractatus logico philosophicus* of Wittgenstein in 1922.

Ramsey's philosophical successes opened to him the doors toward the world of economic theory. The fact that economics was still conceived as a "branch of philosophical analysis" and as a "moral science" by Keynes[21] facilitated Ramsey's penetration of this world.[22] Keynes and Ramsey were introduced by Richard Braithewaite, and their friendship and mutual admiration strengthened both thanks to the *Apostles* meetings and as a result of their long debate on Keynes' *Treatise on Probability*. Keynes was then largely responsible for the institutional legitimacy of the young mathematician, having found him a fellowship (1924) and a lectureship (1926) in King's College, and subsequently publishing his articles in the *Economic Journal*.[23] Ramsey then quickly attained the status of the brilliant young mathematician, whose critical skills could represent a precious help for economists. Ramsey thus worked behind the scene with important figures like Pierro Sraffa,[24] Roy Harrod,[25] and Arthur Cecil Pigou. This work gave him a regular contact with, as well as a certain practice in, economic theory.

Cambridge Economic Topics

In addition to these underground contributions close to Cambridge economists, a few forerunners of Ramsey's own interest in economic questions can be found

[21] See, for instance, Roy O'Donnell (1982) or John Davis (1991) for analysis of this affirmation and for some precise quotations of Keynes.

[22] Keynes suggested in his obituary of Ramsey that the young philosopher could have soon "exchanged the tormenting exercises of the foundations of thought and of psychology, where the mind tries to catch his own tail, for the delightful paths of our own most agreeable branch of the moral sciences, in which theory and fact, intuitive imagination and practical judgement, are blended in a manner comfortable to the human intellect" (Keynes 1930, p. 335).

[23] It seems that their discussions do not suffice to put their theoretical contributions together. Common philosophical background (their shared admiration for Moore, Russell, and Wittgenstein) led them toward different theoretical and methodological positions, as seen in Ramsey's severe criticism of the methodological foundations of the *Treatise on Probability*. Ramsey adopts in his own essay on probability what Keynes vigorously rejected: mathematical expectations to represent the psychology of individuals that were deprived from any social links.

[24] The work accomplished by Ramsey with Sraffa is revealing. In his preface to *Production of Commodities by Means of Commodities* (1960), Sraffa expresses his "greatest debt" to three mathematicians: S. Besicovitch, A. Watson, and F. Ramsey for their "invaluable mathematical help" (Sraffa 1960, pp. vi–vii). Sraffa's Cambridge collections reveal that Ramsey collaborated on the working out of the system of equations that finally found their way into the 1960 book. Sraffa's diary shows the existence of many conversations between the two men, taking place between June 1928, and November 1929. For the details of Ramsey's mathematical corrections by Sraffa, see Kurz and Salvadori (2001).

[25] We learn from Harrod's biography of Keynes (1951), that Keynes asked Ramsey to report on Harrod's article proposed for the *Economic Journal* in 1928 ("On supply"). Ramsey suggested many substantial modifications to the paper.

in his lectures to the *Apostles*. His very first intervention, "The Rights and Wrongs of Doing Nothing to Improve the Lot of Fellow Men" (Ramsey 1991, pp. 291–94) displays his wish to see the mathematicians use their abstract knowledge in order to "share the burden imposed others by accidents" and "to give something for the rest of the community in return for their bread and butter." Still more interesting is a contribution of February 1923, entitled "Socialism and Equality of Income" (Ramsey 1923, p. 313). Here, Ramsey enthusiastically lists the advantages of a controlled economy over a liberal system. He answers a pessimistic conclusion of Keynes, according to which there could be no satisfactory solution to the lasting underemployment.[26] Ramsey aims at solving this question through a socialist economic organization. The question of unemployment, however, appears to be the pretext for a long study of "whether if the state were to own and control industry, more or less wealth would be produced and on the proportions in which this is distributed among the population" (1923, p. 313). The paper reveals the young philosopher's mastery of many economic arguments as well as showing the up to date character of his preoccupations. Enumerating the "advantages and disadvantages which would arise from the unification of industry and the substitution of public benefits for private gain as the object of production" (1923, p. 313), he at first alludes to the "savings of time, energy, disutilities or money" that this kind of conversion would permit. Costs linked with advertising and commercial travels could be eliminated. The State would choose the optimal size of production in every industry, avoiding many diseases of liberalism: monopolies and the linked unreasonable profits; the waste of natural and human resources (costs linked with the bad health of unemployed people); economical fluctuations stemming from "defects in our industrial and financial system and in the wisdom of our business men" (1923, p. 315). In the same manner, human energy and ingenuity would not be lost in financial speculation, strikes or trade unionism.

Secondly, Ramsey deals with the urgency of income redistribution and so of taxation policies that could increase the global utility of the community in accordance with the "law of diminishing marginal utility" and avoid the existence of hereditary classes, "a fact that is injurious to the community" (1923, p. 318).

Besides its anecdotal character, this youthful contribution reveals both what makes the Cambridge anchorage of Ramsey, through the set of themes often tackled there, and the economic questions likely to be of interest for the young mathematician. Each proposition of the program echoes with the economic and moral questions strongly discussed at that time.[27] Faced with the problems of unemployment, financial speculation, or recurrent social claims, many Cambridge economists questioned themselves about the appropriateness of planning, not only from the theoretical point of view (as in the debate on the general equilibrium *vs*. central controlled economy) but also from a moral slant.

[26] Ramsey seems to refer to a 1923 contribution at the *Apostles*, in which Keynes identifies three evils afflicting the modern society: "unearned income, uncertainty, and unemployment" (*Collected Writings*, *Vol*. *XIX*, p. 824). See O'Donnell (1982, p. 170) for more information.

[27] Each of these themes is the object of long discussions in Keynes's Papers for the *Apostles*, and in many articles in the 1930s. They are still present in prospect in the *General Theory*. See the detailed analysis O'Donnell (1982) proposes in chapter 13, "Political Philosophy."

Ramsey's plea for the scientists' or philosophers' commitment to improve the life of their contemporaries is recurrent in the Cambridge moral tradition.[28] It extends naturally to economics, viewed as "the servant of ethics" (O'Donnell 1982, p. 172)—that is, as the instrument of a moral and intellectual blossoming of individuals *via* an improvement of their material conditions.[29] This conception can be found in Alfred Marshall[30] and Pigou,[31] and imbues the whole of Keynes' philosophy. If these authors are certainly not known for their enthusiastic defense of the nationalization of the British economy, each of them relies on economic progress as the origin of social progress. Pigou's welfare theory aims, in particular, at conceiving economic welfare as an important portion of subjective happiness, of which objective measurement lies in the national income. The very notion of national dividend or national income seems to acquire in Cambridge the status of central theoretical concept. Henry Spiegel finds in Marshall the first occurrence of the word, and the concept becomes the hard core of Pigovian as well as of Keynesian analysis. Cambridge thus resuscitates the classical preoccupation with the size, the growth and the sharing of the domestic product—preoccupations that had mainly disappeared from the dominant tradition of the end of the nineteenth century. At that time, the reintegration of these questions gives a new breath to dominant British economic theory. The synthetic theory that Ramsey published in 1928 thus proves to be one of the possible ways to re-integrate macroeconomic preoccupations in the neoclassical framework of the allocation of scarce resources. It is an original way of doing so, which differs in the manner but perhaps not in the ambition, from the two theoretical systems of Keynes and Pigou.

Pigou's work is in this respect important. *Economics and Welfare* (1920) actualized the debate in proposing three criteria of economic welfare: the size, the distribution, and the stability of national income. It also paved the way toward State intervention, underlining the private defective actions that could counter the achievement of optimal social welfare. The Pigovian reflections on the necessity to correct the divergence between the "social marginal net product" and the "marginal private net product" seemed to have directly influenced Ramsey's economic papers. In "A Contribution to the Theory of Taxation" (1927b) Ramsey tackles a problem Pigou had suggested to him: "Given R (the infinitesimal amount of government revenue to be raised), how should the linear commodity taxes be chosen in order that the values of the consumption levels (chosen by consumers) shall make the level of utility a maximum?" (Ramsey 1927b, p. 49). With this paper, Ramsey has the first occasion to come back to one of the central questions evoked in the enormous research program he was

[28] Moore and Wittgenstein, for instance, describe philosophers as "therapists."

[29] See, for instance, Davis (1991).

[30] Pigou argues then that "Economics for [Marshall] was the handmaid to ethics, not an end in itself, but a means to a further end: an instrument by the perfecting of what it might be possible to better the conditions of human life" (1925, p. 82).

[31] "Economics and Ethics are mutually dependent. The practical art of social service requires them both. The first is handmaid of the second" (Pigou 1908, pp. 13–14). In *The Economics of Welfare*, Pigou argues that the economic theories are *instruments* for the bettering of human life, and that they therefore may help social improvement (Pigou 1920, p. 4).

sketching in "Socialism and Equality of Income." In doing so, he makes his mark in the statement of the question of optimal taxation. Indeed, he shifts the emphasis to an economy in which the social marginal net product and the individual marginal net product are made equal by definition. He already then is working in a perfect information framework—where every kind of dysfunction is ruled out—and adopts a highly formalized treatment of the question.

Cambridge was also interested in the welfare of future generations. In a well-argued paper, David Collard (1996) shows the central place of saving in many Cambridge analyses. Focusing on the measurement of annual economic income, Marshall and Pigou included savings as the center of intertemporal income transfer: savings are "the contribution to this year's economic activity, to future consumption" and "to maximize the national income is to optimize the consumption of present and futures generations taken together" (Collard 1996, p. 587). Ramsey was thus systematizing a still wholly theoretical idea and was explicitly inserting his contribution into this debate. In this respect, I think one of the most essential moments in Ramsey's 1928 argument is the mentioning of the intuitive reasoning that Keynes would have suggested to him as an illustration for the mathematical expression of the rule.

Furthermore, Collard puts forward a stimulating analysis showing that "there was a strong Cambridge tradition (Sidgwick-Marshall-Pigou-Ramsey)[32] against discounting future utilities" (Collard 1996, p. 585). I would also suggest that in addition to the egalitarian utilitarian tradition inherited from Mill and Sidgwick, Ramsey's paper contains the implicit and recurrent idea that discounting takes its roots in a "defective telescopic faculty"—according to Pigou's expression—which leads to an underestimation of future satisfaction. The ethical and logical requirements are thus united in the dismissal of discount rates in the first part of the 1928 article: a community behaving like a rational and omniscient entity could not logically be accused of such a misrepresentation.

The question of intertemporal welfare is already briefly raised in the 1927 paper on optimal taxation. In the fourth part of this paper, Ramsey incidentally poses the problem of exempting savings from an income tax: "We must suppose the taxes imposed only for a very short time and that they raised no expectation of similar taxation in the future; since otherwise we require a mathematical theory considerably more difficult than anything in this paper" (Ramsey 1927b, p. 59).

In fact, overcoming the stage of static analyses means introducing, from an optimization perspective, the mathematical processing of the temporal paths of economical variables. This is a forward step taken in the 1928 article. However, if we speak about the Cambridge influence on Ramsey's work, we cannot avoid studying the fields where this influence is the most extreme: his philosophy of mathematics and his general epistemology.

IV. STATING A RULE FOR ECONOMIC JUDGEMENT

A kind of contradiction could be underlined when confronting the preceding developments of this paper. One should in fact note that the 1928 article does

[32] Collard adds John Stuart Mill to this tradition.

not *a priori* correspond to the hallmarks sometimes associated with Cambridge economics. Ramsey publishes a highly formalized paper, giving greater place to an abstract economic reasoning in which the analysis of every economic adjustment process is ruled out. Each of these features could be opposed to the main characteristics commonly attributed to the Cambridge tradition: a tradition which, from Marshall to Keynes, preferred well-defined and realistic assumptions, the aims of which were to provide practical recommendations rather than abstract or idealized economic laws, and which appeared relatively reluctant to employ excessive formalization. Moreover, Cambridge economics used to represent economic phenomena using organic topics, which contrasts with the deterministic point of view inherent in Ramsey's theory.

The contradiction mentioned above could be qualified if we consider the philosophical Cambridge background that profoundly influences the nature of Ramsey's theorizing. This philosophical background was the one of the expansion of analytic philosophy in Cambridge. As we have seen, the starting point for Ramsey's work is Russell's logicism (Ramsey 1925). Although he finally rejected the logicist project,[33] he always stressed the importance of formal logic and mathematics as the essential tool for clarifying our thoughts. However, another Cambridge event that was decisive in Ramsey's affirmation of his own epistemology was the publication of Keynes's *Treatise on Probability* (1921). Ramsey in fact proposed in "Truth and Probability" (1926) a stern criticism of Keynes's logical theory of subjective probability. In this essay, he set out an alternative theory of subjective probability—a psychological theory of human decision making under uncertainty—as well as establishing foundations for his original philosophy of science. Two axes regulate his epistemology from "Truth and Probability" onwards. The first is the constituent and regulating role of mathematical analysis as a logical instrument allowing the identification of consistent and meaningful propositions. The second one is a pragmatist criterion for the valuation of philosophical as well as of scientific propositions.

The Clarification of Thoughts

In accordance with a vision borrowed from the logicist philosophy of Whitehead and Russell, Ramsey expresses a constant trust in logic and mathematics to clarify human discourse. Mathematical language, as a part of logic or more precisely as one of the methods of logic (Ramsey 1926, p. 87), is the "par excellence" clear and universal language, the essential tool for every rationalization. Scientific propositions on the one hand, logic and mathematics on the other hand, are linked in Ramsey's thought by the agency of analytical philosophy: logic is the hard core of philosophy, viewed as a critical activity aiming at the clarification of thought. This representation echoes with, among others, Wittgenstein's affirmations in the *Tractatus*: "The object of philosophy is the logical clarification of thoughts" (1922, 4.112, p. 77).

This object turns out to be the gist of philosophical activity in Ramsey's

[33] See, for example, Sahlin (1990).

conception. In the note 57 taken from the *Notes on Philosophy, Probability and Mathematics* he says, "Philosophy is Analysis of Possibilities." Ramsey so sums up science's dependency on logic: "Philosophy is useful in all sciences because it clarifies them ... [Now] Philosophy needs logic, because logic gives the schemes of analysis" (Ramsey 1991, p. 226).

But more than the single tool of operational transcription of "reality"—and besides its role of knowledge rationalizing—logic acquires for the young Cambridge philosopher the status of a true epistemological norm: the norm of well-ordered and meaningful discourses and theories. Logic and mathematics not only transcribe the common or scientific language in rigorous terms, they show the possible forms of the propositions and their relations and so show us what we ought to think. They have a prescriptive part: "logic is concerned not with what men actually believe but what they ought to believe, or what it would be reasonable to believe" (Ramsey 1926, p. 89). In the 1926 essay on subjective probability, formal logic is the condition of a valid theory of individual rational behavior under uncertainty.[34] Under certain axiomatic preconditions regarding the preference relation of the individual, and given a psychological law of maximization of expected utility, he shows that a well-defined measurement procedure of subjective utilities and partial beliefs allows the application of the mathematical laws of probability. The possibility of the use of mathematical laws is therefore a condition of rationality, rationality being defined as the global consistency of the choices emerging from the subjective data. In that case, no Dutch book could be prevented against the individual (Ramsey 1926, p. 78). The validity of the theory is then defined by the possibility of applying mathematical laws—that is, of applying some of the rules of formal logic, what Ramsey also calls "the logic of consistency" (Ramsey 1926, p. 87).

More generally, formal logic and mathematics have the power to delimit what could be said (or believed) from what we are not authorized to say. Scientific theories purport to be sets of assumptions, axioms and definitions, in which logic is determining the propositions or combinations of propositions that are neither contradictory nor absurd.

Logic and mathematics order propositions. Nevertheless, they do not order facts. As in Wittgenstein's *Tractatus*, where "mathematical propositions express no thoughts" (1922, 6.21, p. 169), mathematical propositions in Ramsey's philosophy express nothing about reality, nothing that is not known before: "Logic issues in tautologies, mathematics in identities, philosophy in definitions; all trivial but all part of the vital work of clarifying and organizing our thought" (Ramsey 1990, p. 2).[35]

Their contribution lies entirely in calculus but they represent a precious tool, which allows getting over "the obstacle of complexity": "Complexity is an

[34] In "Truth and Probability," Ramsey identifies two kinds of subjective data underlying individual choice: the psychological value the individual attaches to the possible states of the world (what we would call "utilities"), on the one hand, and the subject's degree of belief in a proposition p that affirms the occurrence of a possible event, on the other. Ramsey tries to establish a method for revealing and measuring the individual utilities and degrees of belief, thanks to their betting behavior.

[35] "Formal logic can be interpreted objectively as a body of tautology and subjectively as the laws of consistent thought" (Ramsey 1926, p. 84).

obstacle because we may understand symbols separately but not in complicated combinations [. . .] we get over this obstacle by calculation, logical or mathematical'' (Ramsey 1991, p. 42).

This activity is necessary since it offers true understanding of every assertion's improvements. Now, understanding is the essential contribution of philosophical and logical analyses, because to understand propositions means ''being able to proceed from them to appropriate other statements or actions'' (Ramsey 1991, p. 42). According to Ramsey, the expected result of the use of mathematics is to provide action-oriented theories.

A Pragmatist Epistemology

In showing the limits of what we can express, mathematics as an extension of logic shows us the limits of the reality as we can live it. Models and theories do not pretend to represent the world as it is. They represent the world in such a way that we could act in it. The validity criterion of such constructions is in fact their ability to successfully guide our future actions.

The first traces of pragmatism can again be found in ''Truth and Probability'' (1926), wherein Ramsey explicitly refers to Charles Sanders Peirce's work (1926 pp. 82, 90).[36] Alongside of formal logic, which guarantees the consistency of the subjective data, Ramsey introduces what he calls ''human logic'' (or the logic of truth), which evaluates the suitability of the individual beliefs according to the actions they imply. In fact, ''we want our beliefs to be consistent not merely with one another but also with the facts'' (Ramsey 1926, p. 87). Like formal logic, human logic says us to what we should think: it directs us to entertain the beliefs that yield successful actions. In the 1926 theory, the individual degrees of belief could be evaluated thanks to the human logic: the best belief to entertain is the belief that leads to the more often successful actions.

It is in Ramsey's subsequent writings that pragmatism, as a knowledge philosophy, overrides the logical atomism[37] he was defending in his youthful days. Ramsey gives up the idea that general propositions could be seen as the conjunction of atomic propositions. If atomic propositions may be said to be true or false, general propositions have no ''truth-value.'' Scientific theories, as general propositions, are notwithstanding, useful because, ''Causal laws form the system with which the speaker meets the future'' (Ramsey 1929b, p. 149).

They indeed stay in daily life as the foundation of our expectations and beliefs and therefore rule each of our actions. Thus, in Ramsey's terms, what we call

[36] Ramsey's main reference is always the work of Peirce (quoted in Ramsey 1926, 1927a, 1929). Ramsey also refers to William James's definition of true beliefs as useful beliefs but contests the antirealism of James's view (Ramsey 1991). On Ramsey's version of pragmatism, see Sahlin (1990) and Mellor (1980).

[37] A complete study of Ramsey's philosophical thought and conversions can be found in N. E. Sahlin (1990). The author notes a radical change in Ramsey's epistemology, as well as in his presentation of logic between ''Facts and Propositions'' (1927a) and ''General Propositions and Causality'' (1929). He shows that the logical position Ramsey adopts with the question of the ''axiom of infinity'' strongly determines his epistemological distinction between scientific laws and single accidental generalizations.

general propositions or laws are not "judgements," susceptible to being right or wrong, but are "rules for judging" (Ramsey, 1929b, p. 149), in which we place more or less confidence. The object of science is then the formulating of these rules. Scientific theories so become "plans" in the double sense of the word: they are at the same time kinds of topological maps and definitions of future actions (planning). They thus draw the cognitive plans that guide our future actions. Formalization aims at systematizing knowledge in a "planning" perspective: it gives rules for judging and acting, rules of an as-large-as-possible reach.

However, a noteworthy point must be added. When dealing more precisely with economic decision-making (in the 1927b as well as in the 1928 articles), Ramsey gives up the formalization of decision-making under uncertainty and aims at determining rules for human decision-making in a perfect information framework. Yet, whereas beliefs play a prevailing part in Ramsey's philosophy, they completely vanish from his theoretical work in economics. The necessity to consider human logic disappears, and pure theory becomes the realm of formal logic, the logic of consistency. As in 1926, in 1928 Ramsey expresses his interest for the study of a pure decision theory. However, he deals in 1928 with the decision of an omniscient planner[38] and without uncertainty, the only rules to follow are then the rules of consistency: the consistency of the choices at every moment in the time, according to the utility function of the community. The pragmatist test of the theory, which was crucial in the theory of individual decision under uncertainty, is then ruled outside the boundaries of pure economic theory. Ramsey does not discuss the possible application of his theory of saving. Nevertheless, if the pragmatist inflection of Ramsey's thought does not clearly appear in his theoretical choices in 1928, we can say that to state a "mathematical theory of saving" was meaning to state a "mathematical rule for economic judgement": a rule that could give a guideline for macroeconomic policy.

These epistemological reflections give further information on Ramsey's ambitions through his normative and mathematical theory of saving. In conjunction with the methodological recommendations of Cambridge economists, they also may help us to better understand Ramsey's articulation of economic and mathematical demonstrations mentioned above in my first section.

The Roundabout Use of Variations Calculus

Ramsey could not propose anything but a mathematical investigation. The value of the "economic demonstration" of the allocation rule rests precisely on its logico-deductive shape, which gives the guarantee of a well-ordered discourse and the preservation of the economical sense of the derived result. Mathematics is in this first instance used to translate the corpus of assumptions and already-established laws of economics in a consistent set, and to sum up the arbitrage

[38] We could also notice that in his 1926 response to Keynes' *Treatise on Probability*, Ramsey had cautiously avoided dealing with the social component of uncertainty, which was crucial in Keynes's conception of uncertainty. Ramsey, in fact, proposed a theory of individuals who were alone against the nature. Once more he (1928) avoids any consideration of the coordination of individual decisions and prefers to concentrate on a pure decision problem.

reasoning followed by the community. The equation (4) appears then as the logical improvement of the three initial equations. Mathematics' "vital" contribution to clarification therefore allows the statement of general rules: Ramsey bases the remainder of the article on marginal changes brought about in the initial rule. These successive extensions are conditioned by the mathematical expression of the rule. Ramsey in fact rejects Keynes' intuitive reasoning in the name of the general spread of his mathematical rule:

> Mr. Keynes, to whom I am indebted for several suggestions, has shown me that this result can also be obtained by the following simple reasoning ... Unfortunately this simple reasoning cannot be applied when we take account of time-discounting, and I have therefore retained my equations (1)-(4), which can easily be extended to deal with more difficult problems (Ramsey 1928, pp. 547–48).[39]

However, the introduction of the calculus of variations seems to have been secondary in Ramsey's eyes. He is anxious to expose the "economic demonstration" of the rule although it does not suffice to completely establish the income allocation rule. On the contrary, he does not linger on the judiciousness of the calculus of variations. For instance, in the second section of the paper, Ramsey proposes to adapt the allocation rule to a second object: the individual allocation rule.[40] But although the individual problem would have been susceptible of a much easier mathematical treatment than the collective question—the integral to minimize would have been finite—Ramsey prefers marginally modifying the equation (3) rather than using a traditional result of variations calculus.

Ramsey thus seems to narrow his use of mathematics to already well-diffused methods in the economics literature. This may be interpreted as the sign of pedagogical preoccupations. But it is also a revealing sign of the Cambridge stamp in Ramsey's text. Though Ramsey was one of the most brilliant mathematicians in Cambridge, his contribution presents no imperialist ambition. Mathematics is indispensable, but its scope is limited to the elaboration of a new theory. The 1928 article consists more in casting new light on a recurrent economical problem than in importing new mathematical tools. Ramsey does not transpose by analogy an equations system, which would be *a priori* available.[41] In the context in which Ramsey was writing, such processes would have required a rigorous justification of the legitimacy of such tools. The Cambridge apprehension for excessive mathematical analysis within economics is often underlined. It led Roy Weintraub (1970, p. 167) to interpret it as a "maths-phoebia" of Cambridge economists. Marshall and Keynes, in spite of their solid mathematical formation, were both anxious to avoid the elevation of mathematics to the highest form of economic reasoning.

[39] Keynes's intuitionism was still the first object of his criticism in 1926.

[40] The transposition implies two modifications: it is now relevant to introduce a discount rate of future satisfactions, and it is necessary to take account of the fact that the individual lives a finite time.

[41] When dealing with an infinite planning horizon, Ramsey had to compromise with a technical constraint: suitable general mathematical results did not yet exist at that time (see, for example, Blot and Michel 1996). Physics had not yet asked the question of optimal trajectories on infinite horizons. Ramsey then could not tack on by analogy a general mathematical package to his analysis. However, the problem was solvable thanks to the careful mathematical definition of bliss, a work that is well done by Ramsey.

Ramsey was however first of all a mathematician and a logician. In each of his contribution to economics, he takes a theoretical question and gives an answer based on pure mathematics or logic, what presupposes rash simplifications and assumptions. He never was interested in the study of true economic mechanisms. What mattered for Ramsey was the legislative power of mathematics applied to economics and the guarantee of consistency it implies. The roundabout way Ramsey is introducing the calculus of variations—the adding of a graphical representation of the rule in the second part of the article—all those are nevertheless the signs of the desire to give an economic anchorage to the mathematical reasoning. The often-quoted admiration Keynes (1930) expressed for the 1928 contribution reveals the admittedly relative success of Ramsey's construction.[42]

V. CONCLUSION

As a conclusion, it might be worthwhile to underline two general results drawn from the preceding study, which in hindsight appear quite interesting. The first one is closely related to the methodological point just considered above. In stating his mathematical rule of economic judgement, Ramsey was providing, in 1928, one of the first economic models in the history of macroeconomic theory. The use of the calculus of variations is imposed by the economic problem, but it does not represent in itself the main purpose of the article. Ramsey only implicitly uses the calculus of variations, but he is aiming to state it within a true economic reasoning. Ramsey does not tack on by analogy a general "mathematical package" to his economic analysis. This fact distinguishes Ramsey's original contribution from the later restatement of the "Ramsey model" in the 1960s.[43]

The account of the Cambridge legacy in Ramsey's 1928 conclusion leads to a second, more ironic result. The "*à la* Ramsey" models are now in such a general use that their origins are largely forgotten. From all my analysis, it follows that Cambridge was the cradle of the two dominant macroeconomic "paradigms" that clashed throughout the second half of the twentieth century: Keynesian and neoclassical—then new-classical—macroeconomics. Although Ramsey's theoretical choices were not picked up in Cambridge at the time, the gap between these two traditions originates in the audacious simplifications Ramsey (1928) applied to Cambridge questions.

[42] According to Keynes, the 1928 article is "one of the most remarkable contributions to mathematical economics ever made." Nevertheless, Keynes continues, "The article is terribly difficult reading for an economist, but it is not difficult to appreciate how scientific and aesthetic qualities are combined in it together" (Keynes 1930, pp. 335–36).

[43] Bellman's dynamic linear programming and the discovery of Pontryagin's work in the anglophone world from 1962 yield then a sudden widening of dynamic optimization tools in economic theory. The results of general operational research were in fact sometimes explicitly transposed by analogy in some optimal growth models in the 1960. This fact is quite clear for David Cass's (1965) or Tjalling Koopman's (1965) attempts. The argument of Nancy Wulwick (1995) is quite convincing in this respect. I believe this kind of analysis cannot, however, characterize Ramsey's original contribution.

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