THE PRINTING OF MATHEMATICS

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AIDS FOR
AUTHORS AND EDITORS
AND RULES FOR
COMPOSITORS AND READERS AT THE
UNIVERSITY PRESS, OXFORD

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PREFACE

ALTHOUGH mechanical composition had become firmly established in printing-houses long before 1930, no significant attempt had been made before that time to develop the resources of the machine, or adapt the technique of the machine compositor, to the exacting demands of mathematical printing. In that year the first serious approach to the problem was made at the University Press in Oxford.

The early experiments were made in collaboration with Professor G. H. Hardy and Professor R. H. Fowler, and the editors of the *Quarterly Journal of Mathematics* (for which these first essays were designed) and with the Monotype Corporation. Much adaptation and recutting of type faces was necessary before the new system could be brought into use.

These joint preparations included the drafting of an entirely new code of 'Rules for the Composition of Mathematics' which has been reserved hitherto for the use of compositors at the Press and those authors and editors whose work was produced under the Press imprints. It is now felt that these rules should have a wider circulation since, in the twenty years which have intervened, they have acquired a greater significance.

The reasons for this are well known. The end of the War not only released many scientific papers for publication, but found us with much research in progress and many mathematicians and scientists offering the results of their work to a larger scientific public anxious to read it. These demands were made upon a printing industry already struggling to meet greatly increased calls upon its services when it was itself suffering from the reduction of its mechanical capacity by air raids, from the paper scarcity, and, most important of all, from the loss of skilled craftsmen. The learned societies and the printing-trade organizations have sought a remedy for this situation by persuading more printers to undertake mathematical printing. These printers may have produced little work of the kind before, and almost certainly nothing in this field, which is known in the trade as 'higher mathematical printing'.

The original 'Rules', themselves amended by continuous trial and rich experience, are here preceded by two new chapters. The first chapter is a simple explanation of the technique of printing and is addressed to those authors who are curious to know how their writings are transformed to the orderliness of the printed page; the second chapter, begun as the offering of a mathematical author and editor to his fellow-workers in this field, culled from notes gathered over many years, has ended in closest collaboration with the reader who for as many years has reconciled the demands of author, editor, and printer; the third chapter is the aforesaid collection of 'Rules' and is intended for compositors, readers, authors, and editors. Appendixes follow on Handwriting, Types available, and Abbreviations.

It is not expected that anyone will read this book from cover to cover, but it is hoped that both author and printer will find it an acceptable and ready work of reference. The authors wish to emphasize that the rules of style set out here reflect the Oxford practice. We know of other authorities, both in this country and abroad, and, where we differ from them, we do so deliberately for reasons which we believe to be good reasons.

We acknowledge gratefully the help of many authors and editors whose advice and criticism are reflected here. We also thank the Monotype Corporation for their assistance in providing illustrations.

It is hoped that this book will not only help the printer, but will assist the enlarged company of authors to understand the technical problems which are peculiar to the composition of mathematics, so that they can ease the printer's task and their own. We hope, above all, that the publication of these 'Rules' may help to standardize the techniques of both writer and printer. The authors would welcome any criticisms or suggestions calculated to increase the usefulness of this work.

T. W. C.
P. R. B.
December 1953 C. B.

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The letterpress method of printing

THERE are (broadly) three methods of printing: letterpress, lithographic, and gravure; we are concerned here only with the first-named. The letterpress method is based on type, which provides a printing surface cast in substantial relief; hence it is also known as the 'relief' process of printing. Type is cast in moulds: for hand-composition each character is cast separately in quantity and placed in trays (known as 'cases') which are subdivided to provide a little box for each letter, figure, or symbol. In mechanical composition each character may be selected and cast separately and assembled into words and sentences by machine, as in the 'Monotype' method; or the separate matrices may themselves be assembled in a line of complete words with spaces and the whole cast as a thin and solid block of metal (known as a 'slug'), as in the Linotype system, used mostly in newspapers and ordinary bookwork. The complications of mathematical printing demand the greater flexibility of movable types, and so this work is normally set either by hand or, increasingly, on a 'Monotype' machine.

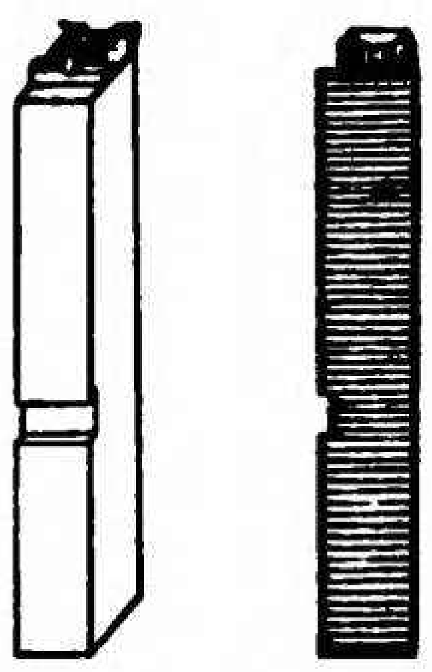
The nature of type

Let us look at the type shown in Fig. 1 (*a*). This drawing represents a single type of the capital M standing upright upon its feet as it would when being printed alongside its fellows. The distance from the feet to the 'face' (which is the printing surface) is a fixed dimension of 0·918 inch and is known as the 'height-to-paper' or just 'type height'.† All the types of every fount used in a printing-house, and all the blocks, must be provided in this uniform height if they are to be assembled and printed together. The bed of the printing machine which carries the type, and the cylinder which is to apply the printing impression, are placed just this distance from one another.

The distance from the front of the type to the back is known as the

† This has become the standard in Great Britain, the United States, and some other countries. Continental heights vary. Oxford has retained one of these heights and casts at 0·938 inch, which is a reflection of the early association of the Press with Dutch and French punch-cutters and type-founders.

'body size'. This dimension will be seen in Fig. 1 (*a*), and even more clearly in Fig. 1 (*b*), which is a silhouette of the type size in which this is printed. The printer defines the body size of his type as of so many 'points', a point being one seventy-second of an inch. This type is known as '11 point' because its body measures eleven seventy-seconds of an inch. All 11-point types, whether in roman or italic, Greek or Hebrew, will measure exactly the same.



(*a*) (*b*)

FIG. 1.

The width of type is variable and is determined by the shape of the character represented: x may be said to be of average width, while M is one of the wider characters, and f one of the narrower. It should also be noted that the different characters occupy varying proportions of the body size: thus the small x occupies a space which is approximately in the centre of the body, but the capital M being an ascending letter fills the centre and top, while the italic *f*, being a letter which both ascends and descends, occupies almost all the depth available. The indentation in the front of the type which is clearly shown in Fig. 1 is the 'nick', whose use will appear later.

Composing type by hand

We shall better understand the complexities of machine composition, particularly of mathematics, if we first look over the shoulder of a hand compositor at work. He stands before a high desk which is, in fact, a printer's 'frame'. On top of the frame and sloping towards him are type cases in pairs, and there are most likely two pairs. The bottom and nearer case will contain the small letters of the alphabet (which explains why the printer always refers to these as 'lower case', usually abbreviated to 'l.c.' on proofs), and other material most commonly required, such as spaces, punctuation marks, and figures. The top case is known as the 'upper case' and holds capitals, small capitals, and other types in less frequent demand.

The compositor holds a metal tool in his left hand which he will refer to as his 'stick', so called because the early printer constructed it of wood. This is a piece of thin but rigid steel which may be 6 inches long and about 3 inches broad, having fixed flanges somewhat less than type height on two adjacent sides, and a movable flange on another. The compositor's first care is to adjust this movable flange to the width of the pages he is to compose. The unit of measurement for this purpose is the 12-point em, which was originally the capital M and was selected because it was a square of the body size in some founts (but by no means in all).

The compositor may still use this letter, or the 'square' space known as the 'em', to set his stick. At this stage you should also be introduced to the 'en', which is half an em in width. Both units will be mentioned frequently in this guide: their derivation should now be clear.

The book to be composed may be of 20 or 30 ems or of any other reasonable width; the page you are reading is of 27 ems. This dimension is always referred to as the 'measure'. The compositor, then, assembles the proper and predetermined number of ems in his stick, and fixes the movable flange so that it presses tightly against this line. Having removed the line of ems, he may begin composition.

Here I should explain that the compositor has an auxiliary aid known as the 'setting rule'. This is a thin strip of brass, cut to the exact length of the measure in which he is to set, and of the same height as the type. The setting rule is placed in the stick and presents a smooth surface which facilitates the assembly of the letters.

So with his 'copy' before him, usually propped up on the lower part of the upper case, the compositor gathers type from his case letter by letter. He picks up each type with his right hand and puts it in his stick face outwards and with nick showing (see Fig. 1), which proves it properly disposed to print the right way up, holding it in place with the thumb of his left hand the while (see Plate I). As each word is completed he adds a space, and so proceeds until he can get no other word in the line, after which he adjusts the spaces until the line is tight. Here he has two cares: first the spaces must be evenly distributed to ensure that the complete page as eventually printed has an even pattern with no unsightly gaps; and secondly that all his lines are of an even tightness so that each type in each page in a series of pages will be held firmly against the hazards of transport and proofing, and inking and printing at speed on machine. Having spaced the line to his satisfaction, the compositor removes the setting rule (on which the line of type has been resting) and, placing it over the completed line in his stick, is ready to continue setting.

At this point I should make it clear that the techniques I am describing, though ancient, are not archaic. Books are rarely set by hand, it is true, but all the major corrections and adjustments necessary to single-type machine-composed types, all principal headings, and much else, still pass through the compositor's stick. This simple tool, and the case and the frame, are still essential to the printer, particularly if it is his business to print mathematics.

But to return to our hand compositor: if he is composing mathematics he may be called upon to build up the pieces of a formula, for instance, not in one line only, but in two or three lines simultaneously. To show how he achieves this an illustration is given below (Fig. 2) in which all the spaces have been pushed up so that they print with the type.

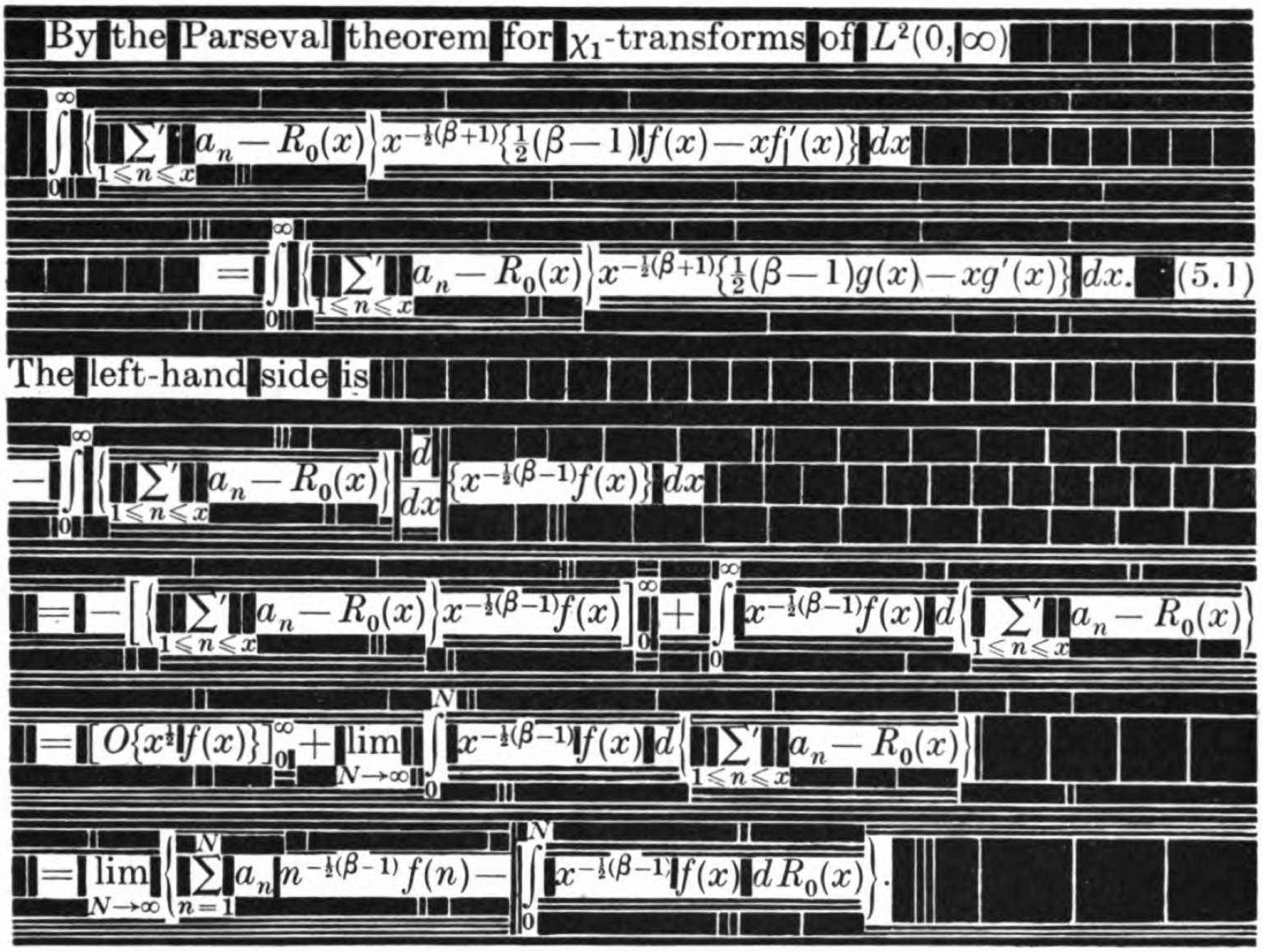


FIG. 2.

THE MACHINE COMPOSITOR

For about 400 years from the invention of printing from movable types all type was set by hand, but in the nineteenth century there was much experiment with methods designed to produce this work with the aid of machines. Of these trials only the solid-line composing machines of the Linotype class, and only the single-type 'Monotype' machine have survived. Both were invented in America, and both were introduced to this country in the closing years of the last century. As I have already observed, it is a 'Monotype' machine which concerns us here and it is this machine which I am now to describe.

A 'Monotype' installation has two separate machines, each of which is independent and complete for the functions it has to perform, and may indeed be operated many miles away from the other. In most printing-offices, and for obvious reasons, they are to be found close together, usually in adjoining rooms and occasionally in the same room, although this is not desirable. The first machine we shall encounter is the keyboard, which may be said to be semi-automatic, and the other is the casting machine, which is fully automatic, and whose function is obvious.

I have occasionally been asked why two machines are necessary to do what appears to be one job. Perhaps I can suggest an answer to the question by mentioning some of the advantages of the present separation. The two independent units are able to produce at their own most economic speeds: the keyboard output is lower than that of the caster on difficult work but is very much higher on simple work; the caster output, on the other hand, is not regulated by the complexity of the matter being cast, but by body size; thus 6-point type may be run a good deal more rapidly than 16 point. So neither machine need retard the progress of the other. Moreover, the work of the keyboard can be put on the shelf and need not be cast until required; the very noisy caster can be isolated, which permits the keyboard operator to work in the quieter conditions the nature of his work demands; and lastly the objection that two men are needed where one should suffice (one-and-a-half really, because one man runs two casters) lacks conviction because the two specialists are free to concentrate on their own jobs, which is of special significance in the case of the keyboard operator, who is one of the most highly skilled people in any printing office.

The 'Monotype' Keyboard

The keys and the perforations

The 'Monotype' keyboard (see Plate III) may look like nothing more than an overgrown typewriter: it is, in fact, very much more. The familiar keyboard is there, it is true, and seems to dominate the machine; there are indeed no fewer than 286 keys. (The average typewriter has 45.) The purpose of the keys is to set in motion a mechanism which releases compressed air to pistons which, in turn, cause punches to rise making perforations in a continuous roll of paper $4\frac{3}{8}$ inches wide. Each key (see Plate V) is marked with the letter or character it represents (as in a typewriter), and as each key is depressed two holes are punched in the paper, one in the left half of the roll and the other in the right half and, of course, in line with one another. There are thirty-one punches, all in a single row, and they punch a combination of holes, differently positioned, for each character. I should add here that the compressed air, which is piped from a compressor, is the only energy supplied to the machine, apart from the manipulations of the operator.

The unit of measurement

As we have seen, the hand compositor is in physical contact with his material; his type grows under his hand and he selects and inserts the spaces he judges to be necessary by empirical methods. But the machine compositor (the 'Monotype' operator) has no such advantages; he handles no type: yet, somehow or other, he and the machine together must so arrange things that the type is ultimately cast to the correct length of line, with spaces of the correct width, properly disposed. Some provision must therefore be made to measure the characters (still represented by mere perforations) and, when the line is nearing completion, to calculate the additional space required between each word to make the line perfect. So it is essential to have some standard unit of measurement, and a calculating machine to do the measuring. This is incorporated in the keyboard.

You and I know that the letters in any one fount vary in width or, in printing terminology, 'set', a term I shall use frequently from now on. The lower-case i and l and the common marks of punctuation are narrow; the lower-case a, d, n, and similar characters are of medium width; the capitals such as M and W are wide. The characters in any one fount may, in fact, be conveniently formed into groups, and all the characters in each group may be so designed as to be of exactly the same set. This has been done; and, so that there may be a system of measurement within the fount, one of the letters of the widest group, let us say the capital W, has been divided into eighteen parts or units. The letters of a fount are, in practice, classified in twelve groups from the smallest which are 5 units to the largest which are 18 units. This unit nomenclature is applied to all founts whether the type be large or small: thus the letter i is, normally, 5 units in set whether in a fount of 6 point or 14 point.

Other complications beset us but must be passed over in this simple account. I think I may now safely determine the unit of measurement and pass on. For this purpose we return to the 12-point em which I have already mentioned: this may be described as being twelve seventy-seconds, or one-sixth, of an inch. I should explain, or confess, that this is an approximation: the 12-point em of the 'Monotype' system is 0·166 inch. This we divide by 12 to discover the dimension of 1 point, and the result by 18 to determine the unit. The quotient is sufficiently 0·0007685 inch, and this is the measurement of a 'Monotype' Base Unit.

The calculating machine

We can now return to the 'Monotype' keyboard, which our description has left bare of everything but key-buttons, and which I deserted after brief reference to its function as a calculating machine. First of all the machine compositor, like the hand compositor, must establish his measure before he can begin work, and a device for doing this, and a great deal more, is mounted just above the keyboard. This is a celluloid scale, known as the 'em scale', marked out to represent ems and ens of set up to 65 ems with zero on the right, and having a moving pointer on its upper face, travelling from left to right. So he sets the pointer to the measure in which he is to compose. Then as he taps each key in turn the pointer will record on the scale, progressively, the unit value of each character composed. This function is performed by a whole mystery of racks and ratchets, and pinions and pawls, all lying behind the em scale and the pointer.

Spacing the line, or justification

In this manner the set value of each character and space is measured and recorded as the operator proceeds steadily towards the end of the line, and short of this point a bell rings to warn him that he must attend to his 'justification', that is the filling of the line with the proper spaces. He has already made a perforation representing a space, after each word. These spaces are accorded a value of four units each, but this value must be increased. It is necessary to know by how much. For this purpose there is another little calculating system.

Mounted over the keyboard is a cylindrical scale, known familiarly as the 'drum', and on its face a pointer. The scale has 72 divisions on its circumference and 20 vertical positions; each of the small squares so created embraces a pair of figures. Each time the operator depresses the space-bar (placed at the bottom of the keyboard as in a typewriter) the pointer moves up one space while the cylinder remains stationary. When the composition is within 4 ems of the end of the line the cylinder begins to revolve and from this point will indicate to the operator the additional space required. But, considering the matter, the operator will probably decide that he can still add another word, or part of a word. He will then glance at the scale to read the figures at which the pointer has stopped. Let us assume that the reading is $\frac{3}{8}$.

Now this justification is based on a system which adds 0·0005 inch, or multiples of this amount, to the spaces which are already in the line. On the keyboard there are two rows of fifteen justification keys, each row being numbered 1 to 15. The keys of the bottom row add progressive multiples of 0·0005 inch to each space; the top row begins where the bottom row leaves off, the keys adding progressive multiples of 0·0075 inch to each space. And so the operator, looking at his justifying scale, sees that the machine has not only calculated what addition has to be made to every space already in the line, but has indicated that all he need do is to touch number 3 key in the top row and number 8 key in the top and bottom row together, to make his work perfect.

The keyboard operator

There is of course a great deal more, and I confess that this account is a simplification, so written for your comfort and mine. But before we turn away from the keyboard to examine the caster, let us take a last look at the operator. He sits before his keyboard with both hands over the keys and glances from time to time at his copy, which is supported on a holder at the left of his machine. He is probably unconscious of our presence because his work demands not only great skill but intense concentration. For not only are he and his machine doing many things at the same time; they are doing these things very rapidly. Indeed, the operator is probably covering anything from five to ten times as much ground as the hand compositor.

Moreover, as I have already said, he is working blind, apart from his own instinct and the intuition which much experience has brought him. The natural difficulties of a manuscript or typescript will not trouble him unduly: he will accept and enjoy them as a challenge to his skill. But badly written or badly prepared copy which is capable of different interpretations will worry him and slow his progress: he will judge such copy unfair and its author inconsiderate (if no worse), particularly if the work is in an unfamiliar language or notation such as Greek, Hebrew, Arabic, or mathematics. Editors, and the printer's own staff, will do their best to tidy a bad manuscript before it reaches the operator, but emergency surgery of this kind is never a complete cure, and, if unnecessary and expensive corrections in the proof stage are to be avoided, it is essential to begin with a healthy body. And is it not just?

THE 'MONOTYPE' CASTING MACHINE

The product of the keyboard now passes to the caster: to the uninitiated it is an insignificant spool of paper, apparently ruined for any useful purpose because it is peppered throughout its length with small wormholes indiscriminately placed. Yet, when this record is 'played back' on the caster, all the painstaking work of the keyboard operator will emerge in bright, newly cast type, line by line, and in due order.

The 'Monotype' casting machine (Plate IV) is a very fine piece of precision engineering and, because it is producing something, more

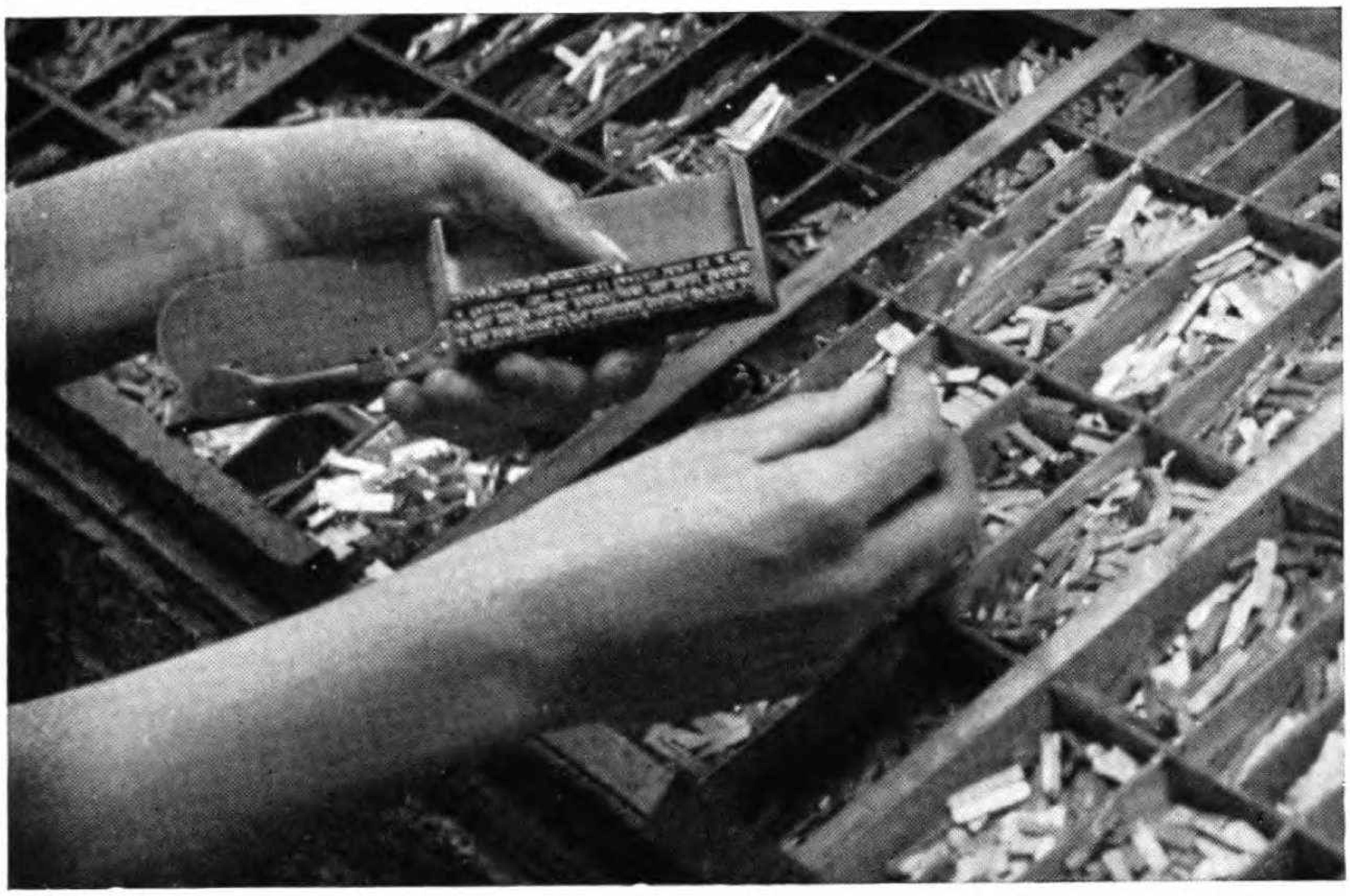


PLATE I. A COMPOSITOR AT WORK

PLATE II. THE MATRIX-CASE

PLATE III. A 'MONOTYPE' KEYBOARD

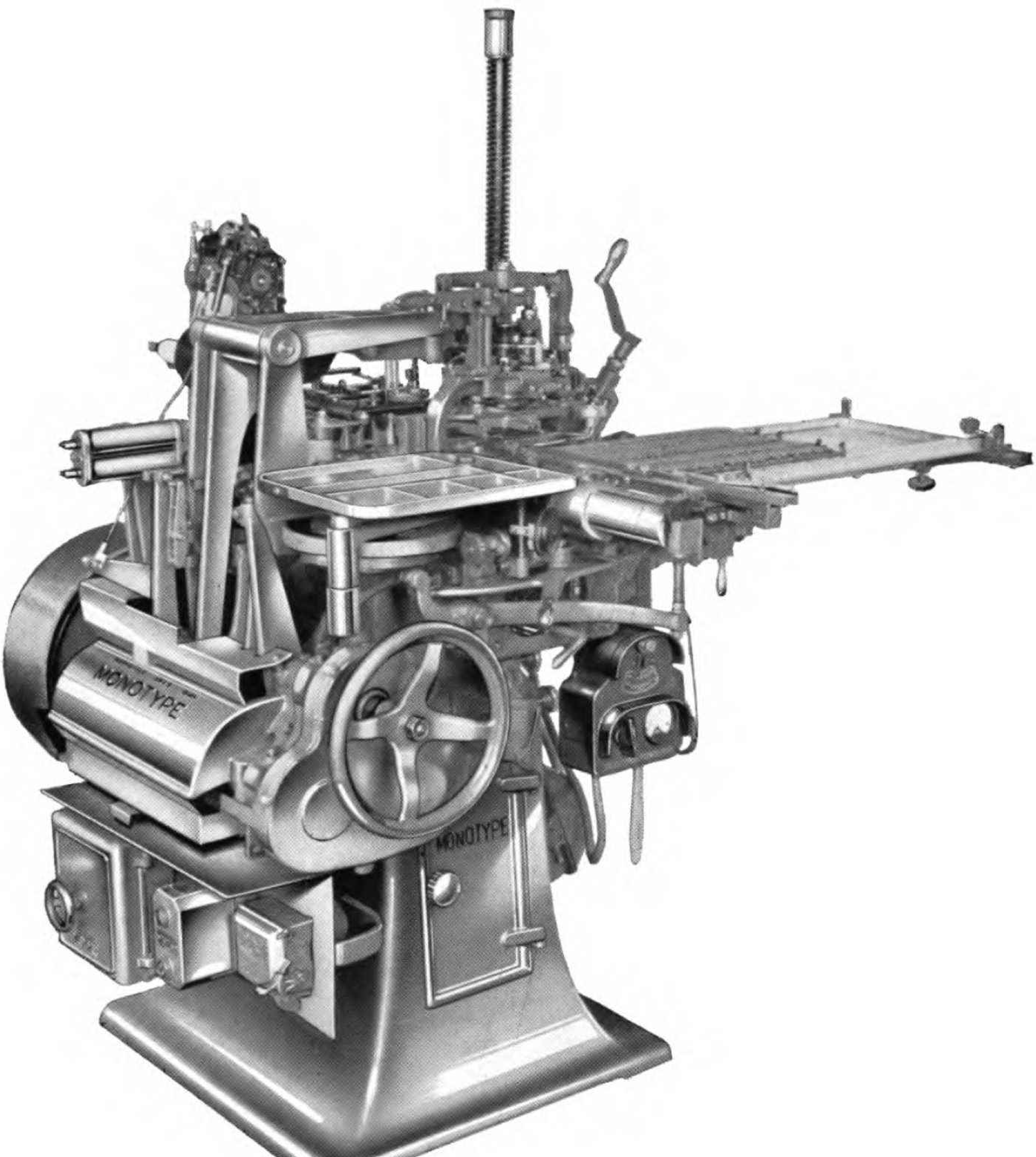


PLATE IV. A 'MONOTYPE' CASTING MACHINE

PLATE V. THE KEYBOARD ARRANGEMENT FOR MATHEMATICAL COMPOSITION interesting to watch than the keyboard. And it is both easy to describe and to understand because most of its working parts, the pieces which do things, are on the top, and visible. I shall mention first the two changeable parts, the die-case (or matrix-case) and the mould.

The die-case or matrix-case

The matrix-case (Plate II) is a steel frame, of a size which allows it to be held comfortably over the palm of the hand, containing the 225, or more, dies or matrices, from which the face of the type or other character is to be cast. Each matrix (Fig. 3) is one-fifth of an inch square in section and a little under half an inch in length. At one end is the face of a character in intaglio, and at the other a 'cone hole'. The matrices may be arranged in fifteen rows of fifteen (or fifteen rows of seventeen, or sixteen rows of seventeen) characters, and the matrix-case fits into a compound slide on the casting machine with the matrices face downwards.

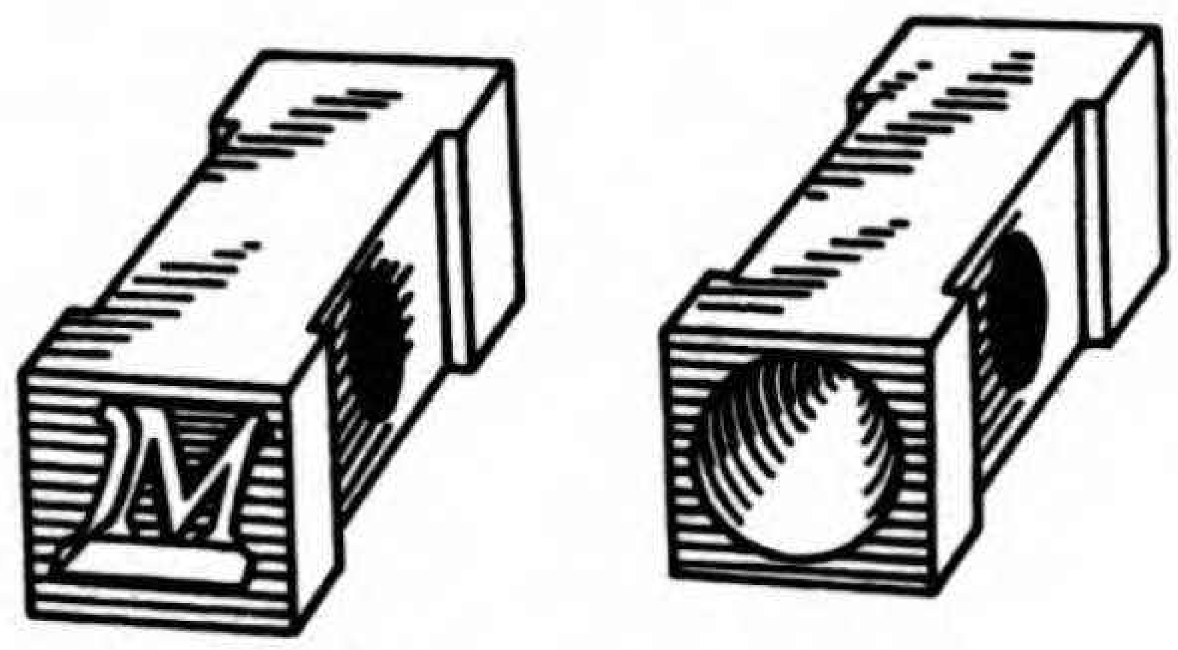


FIG. 3.

The moulds

Separate moulds are needed for each body-size. The mould (Fig. 4) is a comparatively simple but very carefully constructed mechanism, about

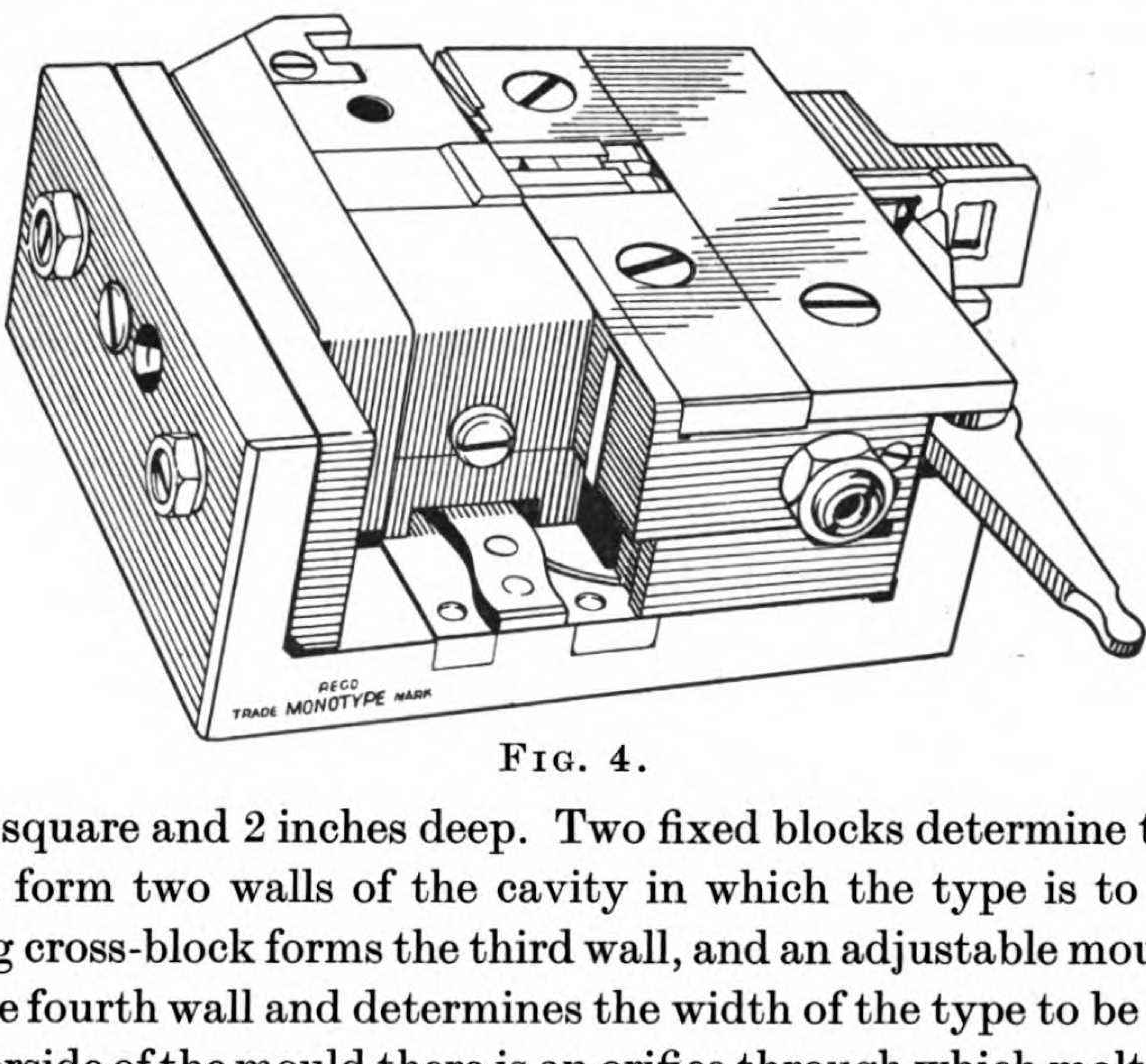


FIG. 4.

4 inches square and 2 inches deep. Two fixed blocks determine the body size and form two walls of the cavity in which the type is to be cast; a moving cross-block forms the third wall, and an adjustable mould blade forms the fourth wall and determines the width of the type to be cast. On the underside of the mould there is an orifice through which molten metal

is pumped: at the moment of casting the open top is covered and closed by a matrix. During casting a small trickle of water is passed through the mould to accelerate cooling, which is practically instantaneous.

Preparing the machine

Let us assume that the caster operator is preparing his machine for running. He will have a number of adjustments to make (and must check very carefully the dimensions and alignment, and sample the quality of his type) before he can begin; we will confine ourselves to those which are the more obvious. He will fix his mould in position on the nearside right centre of the machine, with the orifice downwards; he will insert his matrix-case within the bridge mechanism just over the mould; and he will bring his metal pot under the mould. The metal is an alloy of tin, antimony, and lead, perhaps in the proportions $7\frac{1}{2}:15:77\frac{1}{2}$, which is maintained at a temperature of about 680° Fahrenheit, and is pumped mechanically into the mould. Then he fixes the perforated paper into the paper tower. Now the spool, unwinding on the paper tower, passes over a curved surface having a row of thirty-one holes running across it and, leading from the holes, a number of pipes. The holes are covered by a narrow leather pad which has a groove through which compressed air is conducted and, as the paper passes between the curved surface and the pad, the holes are uncovered wherever there are perforations, and air is admitted to the pipes. The air under pressure is carried by the pipes to the pin blocks which control the movement of the matrix-case.

The moment of casting

This introduces us to the matrix-case positioning mechanism by which the character selected for casting is brought over the mould. As you will remember, there may be fifteen rows of matrices, and fifteen matrices in each row. It is necessary to decide which row is required and then which matrix in that row. The matrix-case is caused to move first in one direction and then in a direction at right angles to the first, and each of these two movements is arrested by a stop. The two pin blocks are on two sides of the matrix-case and at right angles to each other, and each has fifteen stops. The impulses of compressed air, released through perforations originally made by the keyboard operator, pass down the pipes and blow up the stops, so halting the movement of the matrix-case, first on one side and then on the other, thus selecting the proper matrix. Thereafter the matrix-case is clamped down on the mould, exactly positioned by a coned pin which descends into the cone hole at the back of the required matrix, the mould blade being moved to determine the proper set of the character the while; metal is then pumped into the mould. Each letter and space is ejected from the mould as cast and assembled in a line which, when complete, is carried on to a galley, or tray. All this is automatic and produces something of the order of 160 types each minute.

Justification

There is one thing more which I should explain. As you will recall, the last act of the keyboard operator before parting with his line was to make the perforations which would correct, or justify, all the spaces he had already put into the line. I have not hitherto mentioned something which is important: the spool was put into the caster exactly as it came from the keyboard, and was worked through the reverse way. A used pianola roll must be re-wound before it can be played again; a film must be re-reeled; a bandage re-rolled. The 'Monotype' spool undergoes no intervening change of direction before it goes to the caster, but is there *worked backwards*. So the last perforation made on the keyboard was the first to influence the caster, and the last signal of each line decided the spaces between the words throughout the line. Moreover, the last perforation representing a space or a character in the keyboard operator's spool was the first to be cast, and so with every line, from the end, back to the beginning.

The set of all the characters and spaces, that is the width of the opening of the mould blade in the mould, is determined by the movement of wedges in the machine. The wedge affecting the width of type is known as the 'normal' wedge and the two affecting spaces the 'justification' wedges. It is a very simple device, and, as near as may be, infinitely variable.

COMPOSING MATHEMATICS ON THE 'MONOTYPE' MACHINE

We have seen the hand compositor at work and a good deal has been said about the operation of the 'Monotype' keyboard and casting machines. We may now look more closely into our proper business and explore the problems which attend those who set out to compose mathematics by machine.

Earlier in this chapter we saw how the hand compositor was able to build up in his stick, laboriously but surely, characters of diverse shape and size. But the 'Monotype' machine, for all its versatility, can do nothing like that: it is able to produce only one body-size at a time. If, for instance, the machine is producing 11-point *body*, as this is, nothing larger or smaller *in body* will come out of it. But a great deal

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6	1	2	3	4	5	6	7	8	9	0	-	:	;	j	f	i	∫	5
7	a	k	x	y	a	k	x	y	z	s	r	c	e	r	t	s	ħ	6
8	n	p	n	p	α	α	q	v	b	g	o	!	I	z	c	e	$\bar{c}$	7
9	+	−	1	2	3	4	5	k	y	d	h	a	g	a	o	ω	α	8
9	+	−	6	7	8	9	0	x	−	□	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\bar{a}$	ħ	$\dot{x}$	$\bar{\alpha}$	9
10	fl	fi	u	n	S	x	q	k	v	y	b	p	h	d	u	n	η	10
10	F	P	Q	R	A	m	μ	τ	ϕ	□	p	$\bar{\rho}$	fl	fi	J	I	χ	11
12	H	Θ	Ω	Φ	ff	w	J	S	?	ff	Z	C	P	L	F	T	ω	12
13	V	Z	Q	G	O	L	C	Q	V	B	G	O	E	A	w	Γ	Σ	13
14	P	R	B	F	E	T	D	A	ffl	ffi	m	Y	U	R	N	D	𝔄	14
15	→	←	∞	X	Y	K	U	N	H	ffl	ffi	X	K	M	H	m	ℜ	15
16	±	+	−	×	=	⩽	⩾	W	M	∼	$>$	$<$	≠	—	≡	W	□	16
	NI	NL	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	16th Row only
	K		KM										N	KM	K	M	KMN	

of flexibility remains, so long as we respect the limitation the machine imposes upon us at this stage of our work. The machine will produce $\sum$ and $\int$ or ^{2} and $_{4}$ or anything in between; but not $^{2}_{4}$, which are two separate types, $5\frac{1}{2}$ point in size, placed together. Nor will it mix 48-point type such as

$$\sum \quad \text{or} \quad \int \quad \text{or} \quad \surd$$

in an 11-point casting: these must be added later, by hand.

The matrix-case used in mathematics is specially enlarged for this work and offers a wide range of characters, as may be seen in the plan of a matrix-case shown opposite. Moreover, one character may be substituted for another of the same set or width, and in practice those more commonly demanded by a particular manuscript are included in the matrix-case.

The keyboard operator must therefore select from the manuscript only those characters in a formula, for instance, which the machine will produce at one time. If he is composing in 11-point size he will select only those characters which he knows are available in the matrix-case from which the type will eventually be cast. He will move across the folio of manuscript, picking out those characters and leaving spaces for the others. The type will emerge from the caster just as he has recorded his perforations in the spool.

MAKING-UP

The newly cast type, assembled in lines on a long steel tray, called a 'galley', is now passed to the 'maker-up', a compositor possessing special skill and experience. The maker-up stands at a frame on which cases are mounted, just like the hand compositor; indeed he is the modern hand compositor. In his cases will be found the type and spaces required for correction or adjustment; and, either in the cases or near by, the additional characters, large and small, which have yet to be added because, for one reason or another, they could not be included in the matrix-case. These 'sorts', as they are called, are produced by the caster separately, and in quantity, and are stored for use as required.

It is the business of the maker-up to add to the type which has come from the caster whatever is necessary, in material or by adjustment, to make it conform to the author's manuscript, which he has before him. To do this it is likely that he will have to 'break up' some of the type, add new type here and there, and reassemble the whole.

A mathematical working may appear in a manuscript like this:

We consider a variation $\check{d}q^\phi$ of a world-line between $\underset{0}{q}^\phi$ and q^ϕ. Then from (3.21, 23) we have

$$\check{d}\mathring{R} = \check{d}\int_{\tau_0}^{\tau} \mathfrak{L}\,d\tau = \int_{\tau_0}^{\tau}\left(\frac{\partial \mathfrak{L}}{\partial q^\phi}\,\check{d}q^\phi + \frac{\partial \mathfrak{L}}{\partial \overset{*}{q}^\phi}\,\check{d}\overset{*}{q}^\phi\right)d\tau$$

$$= \int_{\tau_0}^{\tau}\left\{\left(\frac{d}{d\tau}\frac{\partial \mathfrak{L}}{\partial \overset{*}{q}^\phi}\right)\check{d}q^\phi + p_\phi\,\check{d}\overset{*}{q}^\phi\right\}d\tau$$

$$= \int_{\tau_0}^{\tau}\left(\overset{*}{p}_\phi\,\check{d}q^\phi + p_\phi\,\check{d}\overset{*}{q}^\phi\right)d\tau \qquad (4.2)$$

$$= \int_{\tau_0}^{\tau} d\left(p_\phi\,\check{d}q^\phi\right)$$

$$= (p_\phi)_{\tau=\tau}\,\check{d}q^\phi - (p_\phi)_{\tau=\tau_0}\,\check{d}\underset{0}{q}^\phi$$

$$(\phi = 0, 1, \ldots, n),$$

from which we see once more that $\mathring{R}$ takes an extreme value. Now, on the other hand, we have

$$\check{d}\mathring{R} = \frac{\partial \mathring{R}}{\partial q^\phi}\,\check{d}q^\phi + \frac{\partial \mathring{R}}{\partial \underset{0}{q}^\phi}\,\check{d}\underset{0}{q}^\phi \quad (\phi = 0, 1, \ldots, n), \qquad (4.3)$$

hence

$$\text{(a)}\ \frac{\partial \mathring{R}}{\partial q^\phi} = p_\phi, \quad \text{(b)}\ \frac{\partial \mathring{R}}{\partial \underset{0}{q}^\phi} = -\underset{0}{p}_\phi \quad (\phi = 0, 1, \ldots, n). \qquad (4.4)$$

and come from the caster like this:

We consider a variation dq^ϕ of a world-line between q^ϕ and q^ϕ. Then from (3.21, 23) we have

$$dR = d \text{ ff } \mathfrak{L}\; d\tau = \text{ff g}\partial\mathfrak{L}\; dq^\phi + \partial\mathfrak{L}\; dq^\phi \text{g } d\tau \quad \partial q^\phi \quad \partial q^\phi$$
$$= \text{ff Kg } d\; \partial\mathfrak{L} \text{ g } dq^\phi + p_\phi\, dq^\phi \text{g } d\tau \quad d\tau \quad \partial q^\phi \qquad (4.2)$$
$$= \text{ff } (p_\phi\, dq^\phi + p_\phi\, dq^\phi)\, d\tau = \text{ff } d(p_\phi\, dq^\phi)$$
$$= (p_\phi)_{\tau=\tau}\, dq^\phi - (p_\phi)_{\tau=\tau}\text{i } dq^\phi \quad (\phi = 0, 1, \ldots, n),$$

from which we see once more that R takes an extreme value. Now, on the other hand, we have

$$dR = \partial R\, dq^\phi + \partial R\, dq^\phi \quad (\phi = 0, 1, \ldots, n), \quad \partial q^\phi \quad \partial q^\phi \qquad (4.3)$$

hence

$$(a) \quad \partial R = p_\phi, \qquad (b) \quad \partial R = -p_\phi \quad (\phi = 0, 1, \ldots, n). \; \partial q^\phi \; \partial q^\phi \qquad (4.4)$$

When the maker-up has completed his work it will look like this:

We consider a variation $\overset{v}{d}q^\phi$ of a world-line between $\underset{0}{q}^\phi$ and q^ϕ. Then from (3.21, 23) we have

$$\overset{v}{d}\overset{0}{R} = \overset{v}{d}\int_{\tau_0}^{\tau} \mathfrak{L}\, d\tau = \int_{\tau_0}^{\tau} \left(\frac{\partial\mathfrak{L}}{\partial q^\phi}\overset{v}{d}q^\phi + \frac{\partial\mathfrak{L}}{\partial \overset{*}{q}^\phi}\overset{v*}{d}q^\phi\right) d\tau$$

$$= \int_{\tau_0}^{\tau} \left\{\left(\frac{d}{d\tau}\frac{\partial\mathfrak{L}}{\partial \overset{*}{q}^\phi}\right)\overset{v}{d}q^\phi + p_\phi\, \overset{v*}{d}q^\phi\right\} d\tau \qquad (4.2)$$

$$= \int_{\tau_0}^{\tau} (\overset{*}{p}_\phi\, \overset{v}{d}q^\phi + p_\phi\, \overset{v*}{d}q^\phi)\, d\tau = \int_{\tau_0}^{\tau} d(p_\phi\, \overset{v}{d}q^\phi)$$

$$= (p_\phi)_{\tau=\tau}\, \overset{v}{d}q^\phi - (p_\phi)_{\tau=\tau_0}\, \overset{v}{d}\underset{0}{q}^\phi \quad (\phi = 0, 1, \ldots, n),$$

from which we see once more that $\overset{0}{R}$ takes an extreme value. Now, on the other hand, we have

$$\overset{v}{d}\overset{0}{R} = \frac{\partial \overset{0}{R}}{\partial q^\phi}\overset{v}{d}q^\phi + \frac{\partial \overset{0}{R}}{\partial \underset{0}{q}^\phi}\overset{v}{d}\underset{0}{q}^\phi \quad (\phi = 0, 1, \ldots, n), \qquad (4.3)$$

hence

$$(a) \quad \frac{\partial \overset{0}{R}}{\partial q^\phi} = p_\phi, \qquad (b) \quad \frac{\partial \overset{0}{R}}{\partial \underset{0}{q}^\phi} = -\underset{0}{p}_\phi \quad (\phi = 0, 1, \ldots, n). \qquad (4.4)$$

It is also the business of the maker-up to add any diagrams which are to appear on text pages, to separate the type into pages, and to add headlines, page numbers, diagrams, and whatever is necessary to complete the pages.

MAKING NEW MATRICES

Every now and then a mathematical author considers it necessary to invent some entirely new symbol to express an idea, and he is always much surprised, and hurt, when he finds editors, publishers, and printers in unanimous and powerful opposition to what he regards as a simple, reasonable, and essential demand. (It is not surprising that the printer includes these esoteric characters among the special sorts which he classifies as 'arbitraries' or 'peculiars'.) There are, indeed, many reasons for this resistance.

In the first place most book printers already possess many special characters which might be made available, and for this reason alone they are unwilling to increase a cataloguing and storage problem which may well be acute already. The mathematical editor (who is well able to speak for himself in this matter) may object that he would be doing no service to science by accepting additions to a notation already overloaded. Editor and publisher will look askance at the expense (20*s*. or more for every sort); and the whole triumvirate will disapprove on grounds of delay (anything up to twelve months) and will assuredly be joined sooner or later by a disappointed and impatient author. So, if a new symbol must be provided, let the author first consult his editor, and the editor his printer, to discover whether anything lies ready to hand which can be used as it stands, or readily adapted.

But if a new matrix must be made it is well to know what this implies. First a drawing must be provided which will be projected and enlarged on to a sheet of paper through a lantern, and traced. From this the draughtsman will make a carefully measured working drawing which will be used to produce a copper-faced pattern in relief about 3 inches square. The pattern is then used in the punch-cutting machine which is operated on the pantograph principle, the follower moving round the outline of the pattern and guiding the rapidly revolving cutter as it shapes a piece of steel into the form of the punch required. When it has been hardened and adjusted the punch is fitted into the matrix stamping machine where the actual matrices are produced for the printer. (It is unnecessary to remind you that each type is a reverse, or negative, of the character it has impressed upon the printed page; a matrix is the reverse of the type it moulds, and therefore matches the printed character; the punch is the reverse of the matrix it is used to produce; and, to complete the cycle, the type is a duplicate of the punch, in another metal.) I should explain, lest you ask the question, that matrices *can* be engraved, and that punch-cutting *can* be eliminated. But a matrix may be required by several, and more often by a great many, printers; and it may wear out or become damaged. So it is much safer, and, in the long run, much cheaper, to have in reserve the comparatively indestructible punch. This is a bare and quite inadequate account of a most exacting procedure: and all for a single character which can very often be done without.

You may remark that, even so, this does not account for the twelve months' delay which I have warned you to expect. Unfortunately ravished Europe is crying out for replacements, and the peoples of the Middle East, India, and Africa for new founts in their own native scripts, which must all come from the same source. It is impossible to exaggerate the magnitude of this problem.

IMPOSITION

A book, to the uninitiated, is a collection of single leaves and most of us discover, in our early and destructive years, that these are, somehow or other, sewn together. And even if we have never seen a book being printed we assume, from economic considerations alone, that several pages must be printed together. The practice of so disposing the pages of a book so that when printed and bound they will appear in proper sequence is called 'imposition'.

Books may be printed sixteen pages at a time, or in thirty-twos, sixty-fours, or even higher multiples: oddments may be printed as eight, four, or two pages, and if need be, as a single page. When the sheets have been printed the first act of the binder is to cut them up and fold them as units of eight, sixteen, or thirty-two pages: these units are known as sections. The most common multiple in which we print is thirty-two pages, and the most common section the sixteen. The disposition of the pages for printing determines these matters.

The pages (still on a galley) may be imposed at any time after the maker-up has completed his work on them. The compositor who is now in charge of affairs lays down the pages, in their proper order, on a large and perfectly flat steel surface called a 'stone' (because in early days it *was* a stone). He then places pieces of wood or metal, known as 'furniture', between the pages ,which are thus separated, to create the margins in the finished book. When this has been done the compositor encloses the whole in a steel frame (known as a chase), and finally he makes all secure with expanding pieces of metal called 'quoins'. The completed imposition is now ready for printing, provided that all corrections have been made. The set of pages, thus assembled and secured in the chase, will henceforward be known as a 'forme'.

On the page opposite is shown a 32-page imposition. This forme, printed first on one side of a sheet of paper and then on the other, will produce 64 pages in all, that is two sheets of 32 pages, or four sixteens.

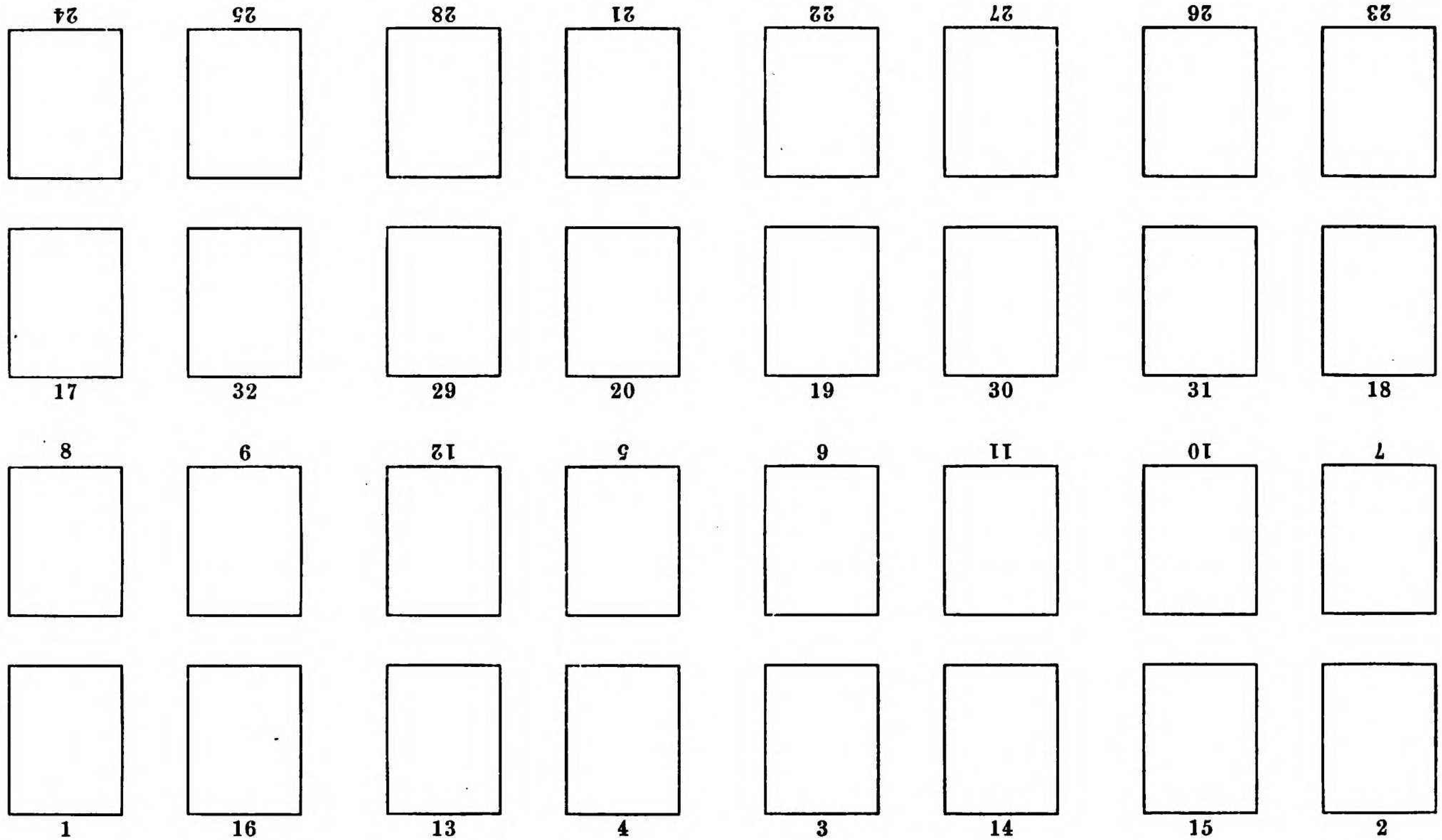
Corrections

In this place I should sound a note of warning. The correction of type is always a troublesome and expensive operation and should never be demanded lightly. The proper place to make final adjustments is in the manuscript. But if corrections must be made, let them be made in the slip proofs where the compositor's task, if difficult, is tolerable. Once the type has been made up into pages any but the smallest adjustment becomes a major operation which may disturb a whole paragraph, or a whole page, or several pages, breeding risks of further errors in proportion to the extent of the disturbance. And after the pages have been imposed, that is the stage immediately preceding the submission of the reader's final 'query' proofs, further correction must be avoided like the plague. For, in the rigidity of the imposed 'forme' the potential risk of error, and the potential enlargement of cost, are multiplied many times.

A NOTE ON OFFPRINTS

Authors who contribute to periodicals are sometimes irritated because the offprints to which they are entitled, or have ordered, are so long in reaching them. This delay is not due to some dark plot of the printer who seeks to deny the author his rights, but is the interval the printer requires to retrace his steps in a somewhat fussy commission which, if truth be told, he would much rather be without. For offprints, in the best practice, mean separation of the articles, reimposition of each article, and the reprinting of contributions in varying numbers, it may be from 25 to 100, or more.

For this purpose the formes are brought back from the printing machine; the pages are then released, rearranged, and imposed again as



Imposition for a folding machine.

so many oddments, and carefully checked. The new formes are then returned to the machine-room and are printed again as a series of small jobs. Sometimes special covers are printed for each contribution, which is another complication. Perhaps the interval which must elapse between the appearance of the journal and the arrival of the offprint is not so unreasonable after all?

C. B.

II. RECOMMENDATIONS TO MATHEMATICAL AUTHORS

1. Introduction

THE production of a piece of printed mathematics requires for its successful completion the co-operation of three parties—the writer, the printer, and the reader. Because of the abstract nature of mathematical thought and of its essential use of symbolism, the writer has always a difficult task in carrying over his meaning to the reader quickly and without uncertainty. The printer, as a middleman, has the double duty of catching the writer's meaning and of passing it on unimpaired to the reader.

The setting of mathematics is a rare and expensive skill not readily acquired, and existing facilities often have difficulty in keeping pace with the rapidly growing output of mathematical writing. For this reason there may be a long and vexatious delay between the writing and the publication of a mathematical work.

A greater and more abiding anxiety is that of the steeply rising costs of mathematical printing. Learned bodies charged with the maintenance of mathematical periodicals are faced with difficult economic problems: are they to raise prices at each increase of costs or must they steadily cut down their acceptance of material? Similar questions confront the publishers of mathematical books.

For these reasons a body of detailed recommendations is presented here for the consideration of mathematicians showing how they can ease the printer's task. They are based, on the one part, on some twenty years' experience as an editor of the *Quarterly Journal of Mathematics*, and, on the other part, on a service, equally long, as Mathematical Reader at the University Press, Oxford. The Editor, while taking full responsibility for what he has written, owes much to his fellow editors for helping to shape many of the recommendations given here: and indeed to the contributors themselves of the *Quarterly Journal* who have already accepted many of them with very good grace.

The Reader, himself trained as a mathematician, has handled more printed mathematics than any single editor can hope (or wish) to have read and has been careful to see that these recommendations are fitted to the practical, day-to-day problems of a mathematical press.

The 'warning examples' that appear in this chapter have sometimes been taken from actually published mathematics; sometimes very elementary examples have been constructed to make their point unencumbered by excessive mathematical detail.

Much in this chapter will be commonplace to the mathematician of experience: it is addressed, in the first instance, to the young mathematician beginning to write for publication. In our editorial experience he is anxious to be shown how he should prepare his manuscript for press and is very willing to conform to suggestions when they are made to him.

The reader, before reaching this chapter, is presumed to have digested the Printer's simplified account of the mechanics of printing. This should be supplemented, wherever opportunity offers, by a visit to a press printing mathematics. It is a sobering experience for an author to contemplate the chain of processes that he sets in motion when he sends a manuscript for publication.

The constraints of type

Under G. H. Hardy's inspiration the London Mathematical Society published in 1932 a pamphlet, 'Notes on the Preparation of Mathematical Papers'. This considered the problems of mathematical printing from the point of view of hand composition. It is hoped (humbly) that the present publication may be regarded as its successor extending its general principles to the conditions of mechanical composition. Many of the problems are the same. Composed type is essentially a mass of hard metal painstakingly built from many small components into a solid whole. Where the printed page shows the blankness of space (or what the printer calls 'white') there is none the less solid metal, invisible only because it does not reach up to the printing face. This is forcefully shown in Fig. 2 of the preceding chapter. Moreover the basis of composition, and especially of mechanical composition, is the formation of lines of type of uniform depth. The printer, like the determinist in the philosophical limerick, is

'A creature that moves
In determinate grooves:
In fact not a bus but a tram.'

In fact, in 'Monotype', with its precise units of type-width as well as the point units of body-depth, we can regard composition as if it were set out on graph paper. On the other hand the mathematician's pen is free to roam, enlarging or compressing his script, filling blank space with symbols, constructing marvels of formulae—even, perhaps, ringing a character or inserting one character within another in a way that cannot easily be directly imitated with metal type.

Text and formulae

It is convenient both here and afterwards, in writings that contain mathematics, to distinguish the *text* from the *formulae* or what printers (and some mathematicians) call 'workings'. The text comprises those portions of the copy that, as here, are written in continuous prose and printed in evenly spaced lines. These can be set up completely and at once from the keyboard at a speed limited only by the legibility of the copy and the operator's dexterity. Nothing is left for the hand compositor at the 'assembly' stage except to 'lead' the completed lines to the desired interspacing.

This can still be true of a text containing mathematical symbols and even of a full-length formula such as

$$(\theta_{12})^2 = \{\exp(a_1 x+b_1 y+c_1 z)\}/(a_2 x+b_2 y+c_2 z).$$

All the characters used here are available on 11-point bodies, and such a 'working', with the spaces filling out the measure, can be set completely from the keyboard. It will be convenient to speak of such a working as being 'linear' in the sense that, like ordinary text, it is composed entirely of a sequence of 11-point types. On the other hand, when written in the mathematically equivalent form,

$$\theta_{12}^2 = \frac{e^{a_1x+b_1y+c_1z}}{a_2 x+b_2 y+c_2 z},$$

the formula departs from 'linearity' in three respects. Clearly the numerator and denominator of the fraction and the 'rule' separating them have been put together by hand. Less obviously, the inferiors and superiors in θ_{12}^2 are separate $5\frac{1}{2}$-point characters built into the symbol by hand; and the 'inferior to superior' in the a_1, b_1, c_1 of the exponent are also inserted and 'justified' on assembly. It might, of course, be suggested that all likely combinations of inferiors with superiors and again all likely 'inferior to superior', 'inferior to inferior', 'superior to inferior', and 'superior to superior' should be separately cast on 11-point bodies, such castings being directly effected from the keyboard.

But here we are met by the limitations of the matrix-case. This, in its enlarged mathematical form, has space for no more than 272 separate matrices; these must include, of course, spacings, marks of punctuation, the complete roman and italic alphabets in both lower case and capital, much of the Greek alphabet, numerals, current mathematical symbols, many of these too in both inferior and superior. All 'sorts' beyond the

272 selected for the particular work in hand must still be inserted at the assembly stage, and it is not of great consequence whether these are inserted as 11-point or composed from separate $5\frac{1}{2}$-point. Moreover all unusual matrices have to be catalogued and stored for immediate access. At the time of writing the University Press, Oxford, has in use or on its shelves some half-million distinct matrices.

By 'linear workings', then, we can effect savings in both time and cost, which, though infinitesimal in a single instance, by accumulation become of considerable importance in the completed volume.

Embellished characters

It is essential to mathematical notation to be able to show association of symbols by symbols suitably 'embellished': thus with x we can associate x', x_1, $\bar{x}$, $\dot{x}$. Here again lateral additions as in x', x_1 are much to be preferred to vertical additions as in $\dot{x}$, $\bar{x}$. The prime $'$ and the suffix $_1$ are put in directly from the keyboard, being separately cast on 11-point bodies and taking their places in the 11-point line alongside the 11-point x. Vertical additions, on the other hand, must be inserted on assembly as pieces of type that have to be supported on each side by lines of blanks; if this occurs in the text, it will certainly disturb the uniform interlinear spacing.

It is true that such familiar symbols as $\dot{x}$, $\bar{x}$ are available as single 11-point types. But, if much of the alphabet is to be required in some embellished form, we may again be confronted with the limitations of the matrix-case, so that the types may still have to be inserted by hand. The author who is accustomed to produce the text of his work on a typewriter, afterwards inserting symbols and formulae with the pen, will be aware of the care and delicacy needed to avoid errors and omissions at this second stage.

There is, of course, no sort of suggestion that mathematical formulae ought always to be linear. It is a matter of professional pride to the compositor and a challenge to his skill to set up anything the author may have written, whatever it may cost in time or effort. But it would obviously be pointless to make these demands where an easy alternative is available; and it is especially heart-breaking to the printer when a complex and really difficult formula is discarded in the corrected proof.

Sufficient has been said in the preceding chapter to discourage the demand for new symbols except where these seem inevitable. In the sentence often attributed to William of Ockham: *entia non sunt multiplicanda praeter necessitatem.*

Consideration for the reader

Many of the detailed recommendations set out below are designed to embody these principles and to assist the printer. But others are more concerned with the convenience of the reader who, as the ultimate recipient, is entitled to the chief consideration; he may often be tired, inexpert, or of foreign speech and may need considerable help. The author therefore has the obligation to see not only that the presentation of his mathematics is complete and impeccable but that his prose is terse and unambiguous and his symbolism well-planned and such as to strike the eye effectively. Those authors are at an advantage who can visualize how their formulae are likely to appear on the printed page.

The art of presenting printed mathematics has much in common with those of display advertising and window-dressing. Crowding is to be avoided; contrast can be used whether of formula against formula or of words against symbols; essential information ought not too often to be hidden away in the small type of inferiors and superiors.

Mr. S. Morison† has the phrase 'assisting the reader's immediacy of comprehension', and a good mathematical presentation is one in which the essential information admits of being 'immediately apprehended'; it should not be sufficient merely to say that it is 'all there' for anyone who has the patience and skill to disengage it.

Aesthetic considerations have not been neglected in these notes. To the pure mathematician (as harshly distinguished from the scientist using mathematics for his own purposes) the aesthetic is part of his technique, and the underlying pattern of a problem is more likely to reveal itself in the printed page than in an author's manuscript or on a blackboard.

Much that is written below is part of the accepted framework of mathematics; some is frankly new and experimental; some even provocative; sometimes decisions have been made merely to secure uniformity; there are repetitions for clearness. There may be conflict with what is recommended in other guides. And certainly what may be convenient in fount or notation for the sciences is not necessarily as desirable in mathematics, whose very life-blood is symbolism. The tradition of mathematical printing is still in the making, and it would be a pity to clamp down too soon on every convention. None the less it is already a

† *First Principles of Typography* (Cambridge, 1951) [Cambridge Authors' and Printers' Guides (I)] p. 15.

great tradition and deserves every support from working mathematicians in its development.

At this time there is strong advocacy to displace the printing of mathematics by photographic methods applied to typescript or manuscript. There is no doubt a place for them in (say) the duplication of lecture notes or in reproducing a doctorate thesis *in extenso*. But it is difficult to believe that these processes can ever satisfy the present demands of mathematicians for the rich arabesques of symbolism they are driven to employ. If increasing complexity of mathematical notation sends up costs so much that publishers are driven more and more to photographic methods, the prospect for mathematicians is bleak. To help postpone this day is one of the purposes of this book.

Finally it may be said that what is recommended here—to authors and through them to editors and printers elsewhere—will represent the Oxford practice in printing mathematics. Authors whose mathematics is being printed at the Oxford University Press will thus know what to expect in the interpretation of their manuscripts.

In conclusion it may be permissible to quote from the preface of a recent Oxford mathematical book. 'Continental authors are often accustomed to begin the real writing of the book when the first proofs come in. From the first the Press made the strict condition that this method should not be followed and accordingly we did our very best to deliver the manuscript in such a form that no avoidable corrections had to be made in the proofs. As a matter of fact the more we worked with this method the more we appreciated it, and we should like to advise other continental authors to give it full consideration.'

This consideration need not be limited to continental authors.

2. Fractions

The solidus. The purpose of the solidus is to keep a fraction 'linear', i.e. within the 11-point limits. For instance a/b is 11-point, but $\frac{a}{b}$ is 24-point.

The solidus should therefore always be used when fractions appear in the text or as part of a formula. Otherwise the fraction oversteps the line of text, which must then be more widely spaced both above and below (as can be seen in the preceding paragraph), disturbing the eye. Thus in the text we print $(x+a)/(y+b)$ and dy/dx, $\partial^2 z/\partial x^2$, etc.

Further, when fractions occur simply in a displayed formula that is otherwise linear, the solidus is conveniently used, for then the formula can be set at once from the keyboard: thus

$$0 < \epsilon < 1/(h+2) \quad \text{for} \quad 0 < \epsilon < \frac{1}{h+2}$$

is both quicker and cheaper.

The solidus is also useful in compound fractions:

$$\frac{\dfrac{x+a}{y+b}+\dfrac{x+a'}{y+b'}}{x+y+1} \tag{1}$$

prints (and reads) better as

$$\frac{(x+a)/(y+b)+(x+a')/(y+b')}{x+y+1} \quad \text{or} \quad \left(\frac{x+a}{y+b}+\frac{x+a'}{y+b'}\right)\bigg/(x+y+1). \tag{2), (3}$$

For reciprocals the index -1 may be more convenient, so that

$$\frac{\dfrac{1}{x+a}+\dfrac{1}{y+b}}{\dfrac{1}{x+a'}+\dfrac{1}{y+b'}} \tag{4}$$

can be printed more compactly as

$$\frac{(x+a)^{-1}+(y+b)^{-1}}{(x+a')^{-1}+(y+b')^{-1}}, \tag{5}$$

as well as

$$\frac{1/(x+a)+1/(y+b)}{1/(x+a')+1/(y+b')} \tag{6}$$

or

$$\left(\frac{1}{x+a}+\frac{1}{y+b}\right)\bigg/\left(\frac{1}{x+a'}+\frac{1}{y+b'}\right). \tag{7}$$

Of these three (5) is probably the clearest.

On the other hand, the solidus should not be used except where it gives some clear advantage. The printer is prepared for displayed blocks of 24-point formulae, which he regards as natural to mathematical expression. Consequently, when a formula is already extended beyond 11-point by some other lengthy symbol, as for instance $\int$ or $\sum$ with limits, the solidus achieves nothing in clarity or convenience, and becomes, in fact, rather inelegant. Thus it is right to print

$$\int_0^\alpha \frac{|f(x)|}{x^2}\,dx, \qquad \sqrt{\left(\frac{2}{\pi}\right)}\int_0^\infty \cos xu\,du, \qquad \sum_0^\infty \frac{(-\frac{1}{2}x^2)^n}{n!}.$$

Bracketing with the solidus. In compound expressions the solidus inherits the priority of $\times$ and $\div$ over $+$ and $-$. Thus $x/y+z$ means $(x/y)+z$, though to the unaccustomed eye it may be clearer to postpone the fraction (when this is possible) and write $z+x/y$. But, when the writer intends $x/(y+z)$ or $(z+x)/y$, he must so bracket them. A sum of fractions such as $x_1/a_1+x_2/a_2+x_3/a_3+\dots$ should, however, be quite unambiguous.

With functional abbreviations like log, sin, etc., careful bracketing is desirable. Thus, unless the context is otherwise decisive, $\log x/a$ may be read as either $\log(x/a)$ or $(\log x)/a$. It is safer to use brackets in each case.

Numerical fractions. Certain numerical fractions of common occurrence appear on the mathematical keyboard as 'small fractions'. These are $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{1}{6}$ $\frac{1}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$.

Such fractions are set and cast as single 11-point units, and their use renders the solidus unnecessary; by comparison the solidus gives a clumsier symbol. Thus we write

$$\tfrac{1}{2}\pi, \quad \tfrac{2}{3}\log x, \quad \sin\tfrac{1}{4}\alpha, \quad \cos\tfrac{1}{2}(x+y)$$

not $$\pi/2, \quad (2\log x)/3, \quad \sin\alpha/4, \quad \cos(x+y)/2,$$

nor, of course,

$$\frac{\pi}{2}, \quad \frac{2\log x}{3}, \quad \sin\frac{\alpha}{4}, \quad \cos\frac{x+y}{2}.$$

With functional symbols where the argument is conventionally enclosed in brackets it may be convenient to repeat a small fraction rather than to double the brackets. Thus

$$\Gamma(\tfrac{1}{2}x+\tfrac{1}{2}y), \quad \Gamma(\tfrac{1}{3}x+\tfrac{2}{3})$$

are better than $$\Gamma\{\tfrac{1}{2}(x+y)\}, \quad \Gamma\{\tfrac{1}{3}(x+2)\},$$

whereas $\Gamma\frac{1}{2}(x+y)$, $\Gamma\frac{1}{3}(x+2)$, being unconventional, may be misread.

In proof corrections and elsewhere small fractions are called for as 'sm. fr.'

Products of fractions. In manuscript it is natural to express a product of separate fractions by simple juxtaposition: thus

$$\frac{1}{2}\frac{a}{b}\frac{x+y}{z}\frac{\sin\theta}{1+\cos\theta}, \quad \frac{y}{x}\frac{dy}{dx}.$$

In print this is inelegant and may be confusing. There the products should be joined up as

$$\frac{a(x+y)\sin\theta}{2bz(1+\cos\theta)}, \quad \frac{y\,dy}{x\,dx}.$$

Where, exceptionally, it is desired to emphasize some aspect of the structure of a product, the full point may be used as a grouping symbol, so that we print (say)

$$\frac{a(x+y)}{2bz}\cdot\frac{\sin\theta}{1+\cos\theta}.$$

The use of fractions and the solidus in indices, etc., is referred to below in § 4.

3. Surds

For practical reasons the printer dislikes the use of the 'vinculum' or bar.† This is therefore to be avoided in the notation of surds, its place being taken where necessary by brackets. With simple forms (unless they are to occur in complex formulae) $\surd$ should be sufficient. Thus we print

$$\surd 2 \text{ (and not } \sqrt{2}\text{)},\quad \surd 311,\quad \surd\tfrac{2}{3},\quad {}^3\surd\pi,\quad \surd x^3,\quad \surd\sin(\alpha+\beta).$$

Compound surds are bracketed thus

$$\surd(2x),\qquad \surd(x+y),\qquad \surd\{(x+a)(x+b)\},$$

and so too $\surd(-311)$, although $i\surd 311$ is usually better.

In numerical surds we treat $\surd 5$ (say) as a single character and print $\surd 5(x+y)$ meaning precisely $(\surd 5)(x+y)$. With algebraic surds $\surd x(y+z)$ is permissible but $(y+z)\surd x$ is less likely to be misread. Such a form as $\surd 2\surd(1-x^2)$ is ugly and should be written $\surd(2-2x^2)$. In any example where confusion seems likely the insertion of sufficient brackets is advised.

Similar principles extend to the use of the 24-point root sign. We print

$$\sqrt{\frac{x}{a}},\qquad \sqrt{\frac{x+a}{y+b}}\quad\text{or}\quad \sqrt{\left(\frac{x+a}{y+b}\right)},\qquad \sqrt{\frac{\sin(\theta+\alpha)}{\cos(\theta+\beta)}}.$$

The index notation sometimes provides a convenient alternative, especially with the higher surds—thus

$$(x+a)^{\frac{1}{2}},\qquad (x^3+a^3)^{\frac{1}{3}},\qquad x^{1/n}+y^{1/n}.$$

The advantage of the index notation will be more fully appreciated when it is explained that such a symbol as ${}^n\surd$ is not one piece of type, but consists of a $\surd$ cast on a specially narrow body into which a small ^{n}

† The 'bar' is actually a piece of metal rule which has to be cut exactly to length and fitted precisely in position over the symbol (or symbols) to which it refers. Two other 'leads' then have to be cut exactly to length to fill up the line so formed and hold the rule securely in place. This is a time-consuming business and, if more than one vinculum is used in the same line, becomes troublesome.

has to be fitted to give the familiar appearance. In the case of a larger $\surd$ sign this, of course, involves justification as well.

The notation $\surd x+a\backslash$ has been used in place of $\sqrt{(x+a)}$, making the surd into one of its own brackets. This conveniently saves a pair of brackets and may be suggested to authors with a taste for experiment.

4. Superiors and inferiors

'Superiors' and 'inferiors' are the printer's terms for indices (or superscripts) and suffixes (or subscripts). Such usual forms as x_1, a^x, $e^{i\theta}$, ${}_2F_1$ give the compositor no difficulty. The superiors and inferiors are each cast separately on the full 11-point body and are in effect independent 11-point characters. Thus $\log_a(1+x^2\sin^2\theta_1)$ is a 'linear' form in the sense defined above and is set directly from the keyboard.

As will be seen by reference to the table on p. 95 the range of superiors and inferiors available at Oxford includes the full italic alphabets in both capitals and lower-case and the lower-case Greek alphabet. A few Greek capitals and other frequently required characters are also in the range.

It must be pointed out that not every 'sort' available in text-size 11-point type is also available as a superior or inferior, although the stock of matrices is being continually increased. The use of a character not listed as superior or inferior may involve the cutting of a special matrix or justification by a hand compositor.

Difficulties of another sort arise when superiors and inferiors have to be set one above the other as in A_1^2, b'_m, θ_0^{n-1}. These superiors and inferiors must each be on a $5\frac{1}{2}$-point body so that, when paired, superior and inferior will build up into a composite 11-point symbol. This, of course, has to be done by hand and requires 'justification' to make a proper fit. Thus, when the context does not forbid, it is much easier for the compositor to set $x_{m,n}$, or even ${}_mx_n$, rather than x_n^m; but ${}_mx_n$ can be confusing in formulae: for example $x_{12}x_{34}$ is clearly better than ${}_1x_2\,{}_3x_4$. With 'double' superiors or inferiors, i.e. superior to superior, superior to inferior, and so on, as in e^{n^2}, x^{n_1}, a_{n^2}, a_{m_n}, the 'second-order' superiors and inferiors need to be specially cast and to be inserted by hand. Moreover, the results are not always easy to read.

When double superiors or inferiors have to be set one over the other in a compound symbol, the task is one of much difficulty. Such symbols as $A_{\kappa_1}^{\kappa'_1}$ or $P^{\kappa'_1\ldots\kappa'_p}{}_{\lambda'_1\ldots\lambda'_q}$ can be built up only by filing down the body size of minute pieces of type so that they may be fitted in the correct relative positions with respect to $^\kappa$ and $_\lambda$ to give the second-order superior and inferior, the whole being carefully fitted together by hand. Such notation should be avoided wherever possible, the more since such composite indices and suffixes do not make easy reading. Some possibilities of avoidance or simplification are suggested below.

With exponents the use of exp for e is now general. It should be employed wherever the exponent is at all elaborate and especially when it contains superiors or inferiors. For such exponents the index notation is not only difficult to print but also tiresome to read. Thus

$$\exp(ax^2+2hxy+by^2), \qquad \exp(\sin\theta_1+\sin\theta_2)$$

are better than $\qquad e^{ax^2+2hxy+by^2}, \qquad e^{\sin\theta_1+\sin\theta_2}.$

It is suggested that this notation could be extended to bases other than e by writing $\exp_a x$ for a^x: this is certainly a logical extension of the notation $\log_a y$. Thus $2^{\frac{1}{2}n(n+1)}$, $\rho^{m^2+mn+n^2}$ could be written and printed

$$\exp_2 \tfrac{1}{2}n(n+1), \qquad \exp_\rho(m^2+mn+n^2).$$

For some purposes we might adapt the notation of the hypergeometric function $F(a,b;c;x)$ in which the semicolons conveniently separate upper and lower parameters and the variable. Thus we could replace

$$I^k_{p,\sigma,\nu_1,\ldots,\nu_k}(a_1,\ldots,a_k)$$

by $$I(k;p,\sigma,\nu_1,\ldots,\nu_k;a_1,\ldots,a_\kappa)$$

or $$I_k(p,\sigma,\nu_1,\ldots,\nu_k;a_1,\ldots,a_k).$$

This principle could be applied to indicial notation by writing

$$\exp\{-\tfrac{1}{4};\lambda-q(x)\} \quad \text{for} \quad \{\lambda-q(x)\}^{-\frac{1}{4}}$$

in cases where the indices need to be specially legible.

The figure 1 is unusual as an exponent, and may easily be mistaken for a prime (accent) in this position. It ought therefore to be specially indicated at its first appearance. When, exceptionally, primes and exponents unity are to occur in the same piece of writing, special means of distinguishing them should be adopted.

$\sum$, $\prod$, $\int$. It is the practice of the University Press to print the limits of a sum, a product, or an integral above and below the symbol $\sum$, $\prod$, $\int$, and not to the side: the gain in legibility is held to justify the extra work of setting. This takes the composite symbol beyond the 11-point range, so that $\sum\limits_1^n$, $\int\limits_0^\infty$ are hand-set. Thus, when $\sum$, $\prod$, $\int$ are used in the text, the limits should be given only when they are essential since their presence, as here, deepens the line beyond the 11-point span.

In dealing with complicated sums or products it is sometimes possible to reduce the amount of work called for both in displayed workings and in the current text by some such simplification of the notation as the following.

Define

$$\Pi_R\begin{bmatrix} a_1, a_2,\ldots, a_M \\ b_1, b_2,\ldots, b_N \end{bmatrix} = \prod_{n=0}^{R} \frac{(1-q^{a_1+n})(1-q^{a_2+n})\ldots(1-q^{a_M+n})}{(1-q^{b_1+n})(1-q^{b_2+n})\ldots(1-q^{b_N+n})},$$

where the left-hand side is much simpler to print than the right. In current text this may be further contracted to $\Pi_R[(a);(b)]$, stating the number of each kind of parameter where necessary. This gives a form which can be set straight from the machine.

The custom has grown up, for example in the theory of numbers, of packing much essential information in small type under $\sum$ or perhaps 'lim'. This is awkward for both printer and reader. It is suggested, for instance, that

$$\sum_{\substack{n\leqslant x \\ n+\kappa_1=a_1^r b \\ a_1>x^\alpha}} 1 = \sum_{\substack{a_1\leqslant(x+\kappa_1)^{1/r} \\ a_1>x^\alpha}} \quad \sum_{\substack{n\leqslant x \\ n+\kappa_1\equiv 0 (\mathrm{mod}\, a_1^r)}} 1$$

is better written and printed as

$$\textstyle\sum_1 1 = \sum_2(\sum_3 1)$$

where

$$n \leqslant x, \quad n+\kappa_1 = a_1^r b, \quad a_1 > x^\alpha \quad \text{in } \textstyle\sum_1,$$

$$x^\alpha < a_1 < (x+\kappa_1)^{1/r} \quad \text{in } \textstyle\sum_2,$$

$$n \leqslant x, \quad n+\kappa_1 \equiv 0 \,(\mathrm{mod}\, a_1^r) \quad \text{in } \textstyle\sum_3,$$

or we could write more formally

$$\textstyle\sum_1 = \sum_1[n \leqslant x;\, n+\kappa_1 = a_1^r b_1;\, a_1 > x^\alpha], \text{ etc.}$$

Commas. In double suffixes and so on, commas are inserted only when needed to avoid confusion. Thus

$$a_{mn}, \qquad a_{m,n-1}, \qquad a_{11}, \qquad a_{9,10}.$$

Use of solidus. Although the solidus is advised for fractional exponents with literal denominators as in $x^{(a+b)/(c+d)}$, simple numerical fractions are generally better expressed with 'small' fractions: thus $x^{\frac{3}{4}}$, $\int^{\frac{1}{2}\pi}$ look better than $x^{3/4}$, $\int^{\pi/2}$.

Correction signs. The customary signs for indicating superiors and inferiors and double superiors and inferiors are explained in § 15.

5. Brackets

Brackets are a great stand-by to the mathematician. They can be inserted for clarity at any point in a formula; they can be added to symbols to create associated symbols as in $[x]$, $\{x\}$ the integer and fractional 'parts' of x. For these reasons it is worth while taking a little care about brackets and their use.

The doubling of brackets within themselves, as in $(x-(a+b))^2$ may trick the eye in a long formula and is foreign to the Oxford practice. The endeavour is therefore to provide a wide variety of brackets. In regular use we have (in printer's language) the three sorts: parentheses ( ), braces { }, and brackets [ ]; and their normal order is [{ (...) }]. When necessary they can be extended by 'full face' brackets and parentheses [ ], ( ). These are cut to the full height of the 11-point body and are consequently slightly longer than the ordinary bracket and parenthesis. They can be called for by 'full' against them in the margin. We then have the extended set of [([{ (...) }])].

Two further sorts of bracket can be made available on the keyboard: 'double' brackets $[\![\ ]\!]$ and 'angular' brackets $\langle\ \rangle$. Double brackets can be placed outside the bracket sequence as

$$[\![[([\{ (\ldots) \}])]]\!].$$

Angular brackets have already been conventionalized in Physics, in connexion with Dirac's 'bra' and 'ket' vectors, and elsewhere. Otherwise they are conveniently retained for the occasion when, in some complicated formula, an inner term, say that within ( ), needs further bracketing. The use of $\langle\ \rangle$ avoids a rearrangement of brackets throughout the formula. Thus we write

$$[1+\{2(a^2+b^2)(x^2+y^2)-(ab+xy)^2\}]^2$$

$$= [1+\{(\langle a+b\rangle^2+\langle a-b\rangle^2)(x^2+y^2)-(ab+xy)^2\}]^2$$

rather than

$$= [\![1+[\{(a+b)^2+(a-b)^2\}(x^2+y^2)-(ab+xy)^2]]\!]$$

where the complete change in the brackets is a little disturbing.

The purpose of such variety is to leave an author free to use some of them symbolically without running short of brackets for ordinary 'grouping' purposes.

Sometimes a little care can save a pair of brackets. Thus we can write $\Gamma(\frac{1}{2}x+\frac{1}{2}y)$ and $\Gamma(-x-y)$ for $\Gamma[\frac{1}{2}(x+y)]$ and $\Gamma[-(x+y)]$. Again, since $\sum x+a$ could be read hastily as $\sum (x+a)$, it is better written as $\sum (x)+a$. But we can avoid this bracketing if we may write it $a+\sum x$.

Symbolic uses. The use of particular brackets in a notational sense, unassisted by a functional symbol, as in $[x]$, $\{x\}$ already briefly mentioned, has its drawbacks. It, of course, reduces the supply of brackets available for their more usual purpose and cannot therefore be pursued very far; and the eye, accustomed to regard brackets as part of the unemphatic punctuation of a mathematical formula, has to be retrained to give them functional significance. On the other hand, in a symbol such as the matrix element a_{ij} or C_{mn} the coefficient in $\sum\sum C_{mn}\,x^m y^n$, the symbols a, C are unimportant as against the parameters i, j or m, n. It would often be tempting to omit a, C and write merely $\langle i,j\rangle$ or $\{m,n\}$, say, bringing these essential symbols into their rightful prominence. This, however, can grow tiring with much use.

Perhaps the right conclusion is that the permanent stereotyping (and therefore sterilization) of a type of bracket is a hindrance, but that temporarily in an article or book it may be convenient to give a nonce value to certain brackets. But in every such case the author should make clear to editors and others in a covering page that such-and-such brackets have a special meaning and therefore must not be tampered with and presumably may not be used elsewhere for current purposes.

A departure from the recognized sequence of brackets should similarly be noted.

To distinguish open and closed intervals the convention has sometimes been made of reversing the bracket at an open end. Thus (a,b) is the completely closed interval, $)a,b)$ is open at a, and $)a,b($ is the interval open at both ends. The printer's attention should be drawn to this use so that he will not ascribe these reversed brackets to a mere error.

Full parentheses ( ) should be used round the upright verticals | | because of their greater height. These are also convenient when bracketed symbols have to be enclosed as coordinates in the usual coordinate parentheses as

$$(n(x_1+x_2),\ n(y_1+y_2)).$$

Where brackets have to spread over two or more lines of print, [ ] are to be preferred to ( ) or { }, which grow bulgy with extension. Thus we print

$${}_2F_1(a,b;c;x) \quad \text{but} \quad {}_2F_1\left[\begin{matrix} a,b; \\ c \end{matrix}\ x\right].$$

Special uses. Brackets, preferably ( ), should be used in 'parenthetic definitions'. Thus in enumerations we write

$$u_r \quad (r=1,\ldots,n), \qquad u_{mn} \quad (m=1,2;\ n=1,3,5),$$

where in the second example the semicolon is used for better separation.

Where such parenthetic definition occurs we do not punctuate u_r, $(r=1,\ldots,n)$,; the added commas are redundant with the brackets ( ).

The 'broken definition' of a function is conveniently printed with a brace as

$$\iint_D \psi_m \psi_n \, dxdy = \begin{cases} 1 & (m = n), \\ 0 & (m \neq n), \end{cases}$$

where the punctuation should be noted.

In limits it is sometimes handy to write $(x \to \infty)$ in place of 'as $x \to \infty$'. We write $x\ (> 1)$ not $x\ (x > 1)$, but, of course, $\sin x\ (x < \pi)$ in full.

The use of ( ) after a functional symbol that is not one of the usual $f, F, \phi,...$ can occasionally lead to ambiguity. Thus, if $x(t)$ is in use, we could read $x(a+b)$ as either the product $(a+b)\times x$ or the value of $x(t)$ at $t = a+b$. We could avoid such a difficulty by using $\langle\ \rangle$ as the functional brackets.

As already mentioned, the 'vinculum' is much disliked by printers and ought to be regarded as obsolete in print, whether with surds or elsewhere. It should always be replaced by brackets.

In the text ( ) carry clauses for which commas are inadequate, or, where they would enclose a bracketed symbol, dashes may be preferred. Interpolations in the text that stand outside the argument, as for instance references, should be given [ ]: 'as has been proved by Cayley [(6) 23, Theorem 1].'

When a mathematical expression occurs in the text, it does not require additional brackets for that reason except when hyphened on to another word. Thus we print '$n+1$ dimensions' not '$(n+1)$ dimensions', but '$(n+1)$-dimensional'.

6. Embellished characters

Some general remarks have already been made about the use and abuse of embellishments over or under mathematical symbols: briefly that these additions are much disliked by printers except when the symbol as embellished is otherwise available as a single character and so can be set at once from the keyboard. The full set of such composite characters available at Oxford is given in Appendix B. It will be seen that 'barred' letters are available for the entire alphabet of italic capitals and for nearly all lower-case italics and lower-case Greek. The dotted alphabets are much less complete, although there is a full lower-case Greek alphabet dotted *below*. For the rest, linguistic study has furnished us with an assortment of letters, English and Greek, variously modified. These are preponderatingly vowels and mostly much too haphazard for mathematical purposes. When, therefore, an author introduces a character, modified above or below, that is not in this list, he is, in effect, asking for a new symbol, and such a symbol should be considered on its own merits for legibility and convenience, as should any new character. Thus it may be questioned whether $\dot{+}$ is sufficiently clear and distinct, especially when seen in proof; some more outstanding plus such as $\boldsymbol{+}$ would have been preferable. In fact dotted symbols are in general to be discouraged except perhaps in traditional devotion to Newton. We must further remember that such composite characters, whether already available or specially cut, mean so many more characters to be accommodated on the keyboard; when its capacity is exceeded, these extra characters must be inserted later by hand.

Bounds and limits. Some authors are accustomed to use $\underline{\lim}$, $\overline{\mathrm{bd}}$, and so on for least and greatest limits and bounds. These notations are being superseded by 'sup', 'inf' for the greatest and least bounds, and correspondingly 'lim sup' and 'lim inf' for the limits of these bounds. These forms can be adapted to subtler limits and bounds, and it is suggested that they should render unnecessary the use of barred notations.

A bar over a symbol is now in use in topology and elsewhere to denote the 'closure' of a set. This is associated with 'fr' and 'int' for the 'frontier' and 'interior' of a set. It would be much more convenient typographically to print 'cl' or 'clo' for closure: the latter is preferable as being pronounceable; 'fro' would be preferable to 'fr' on the same grounds. Over some letters the bar is not easily read, as in $\bar{\delta}$; so too the tilde as in $\tilde{\delta}$. This can be worse in the denominator of a fraction as in $\frac{\alpha\beta}{\bar{\delta}}$.

7. Displayed formulae

The term 'displayed formulae' is used in contradistinction to those formulae or combinations of symbols that are embodied in the lines of text. Most of the formulae of printed mathematics occur as displayed formulae. These are set centrally on the page, i.e. leaving equal amounts of 'white' on each side (though an exceptionally long formula may need the full 'measure'—i.e. length of each line of type on the page). They are generally spaced vertically from the body of the text, the spacings or 'blanks' being obtained by means of leads, i.e. shallower strips of metal that do not reach to the printing surface.

Alternatively a simple formula or expression may be embedded in the text. Such a formula must be short enough not to be broken between two successive lines of text: it is a rule of composition that formulae that cannot be completed in a single line of text are to be displayed even though not so shown in the author's copy. This may happen to a text-formula because, when set up in type, it chances to begin towards the end of a line, which cannot, of course, have been foreseen by the author. It is therefore a safe rule to keep only the shortest formulae in the text; there is then generally sufficient play in the spacing to allow the compositor to follow the author's wishes.

Text formulae, as explained earlier, should be within the 11-point limits of height to avoid interference with the regularity of the linear interspacing, though occasionally, as with $\prod\limits_{0}^{N}$, such interference must be accepted unless, as suggested in § 4, Π_N will serve.

On the other hand, very short expressions or sets of symbols will look awkward and isolated if displayed, and these therefore should be retained in the text.

Broken formulae. When a formula is too long for the page-width and has to be broken into successive lines (and we are now, of course, speaking of displayed formulae), it should be broken, if possible, at the end of a natural 'phrase'; if, for example, it is a much-bracketed formula, it should be broken at the end of one of the major brackets and not at an inner symbol. This natural phrasing (as in music or speech) makes for intelligibility between writer and reader and should not be left to the compositor. An author, when he finds himself writing a longish formula, should indicate a convenient point of fracture in case of need. The compositor will not make use of it if he can possibly get the formula into a single line.

If the over-long expression has, let us say, the general form $\{...\}+\{...\}$, so that it is conveniently broken at the $+$, then, by the Oxford custom, this sign is repeated so that the broken formula appears as

$$\{...\}+$$
$$+\{...\}.$$

This repeated $+$ serves to connect the two parts of the formula, and the first $+$ reminds the reader that there is more of the expression to come—in the way that a speaker's voice retains its pitch at a pause to show that a spoken sentence is incomplete.

The convention includes, of course, the minus sign. A product $\{...\}\{...\}$ is thought of as if written $\{...\}\times\{...\}$, and the $\times$ is repeated when the formula is broken at this point. The principle is conveniently extended to an operator product so that we print

$$(\partial^2/\partial x^2+\partial^2/\partial y^2)\times$$
$$\times f(x,y),$$

where the $\times$ can be read as 'on' or 'operating on'.

A ratio that needs to be broken is better written as a fraction if numerator and denominator do not exceed the measure of the page. Otherwise the division sign $\div$ is to be preferred to the solidus, which is not very distinctive in long expressions. Thus

$$\begin{matrix}\{\ldots\}\div \\ \div\{\ldots\}\end{matrix} \quad \text{is better than} \quad \begin{matrix}\{\ldots\}/ \\ /\{\ldots\}\end{matrix} \quad \text{but} \quad \frac{\{\ldots\}}{\{\ldots\}} \quad \text{is best.}$$

Broken 'assertions'. Symbols of assertion such as $=$, $>$, $\sim$ and the rest are not repeated in this way. Thus we print

$$\begin{aligned}&(A)\\ &= (B)\end{aligned}$$

so long as each member (A), (B) of the equality is complete in itself. This fits accepted practice when, often, an expression (A) is displayed in isolation to be followed by an assertion written out in the text and not stated symbolically with $=$, etc.

Equations are best broken at '$=$' rather than at $+$, $-$, $\times$ if this is possible, and so for other 'assertions'.

A succession of assertions is printed

$$\begin{aligned}(A) &> (B)\\ &= (C)\\ &\geqslant (D)\end{aligned} \qquad \text{or} \qquad \begin{aligned}&(A)\\ &> (B)\\ &= (C)\\ &\geqslant (D)\end{aligned}$$

with the symbols of assertion 'ranged', i.e. set vertically beneath one another. It should be noted that a connecting sign applies strictly to the two expressions immediately connected by it: that is to say, the above formulae imply that

$$(A) > (B), \qquad (B) = (C), \qquad (C) \geqslant (D)$$

and therefore that

$$(A) > (C), \qquad (A) > (D).$$

When some of the steps of the succession do not need the full width of the page, we may be able to print

$$\begin{aligned}(A) > (B) &= (C)\\ &\geqslant (D)\end{aligned} \qquad \text{or} \qquad \begin{aligned}(A) > (B) = (C)&\\ \geqslant (D),&\end{aligned}$$

according to the length of (D). We purposely avoid

$$\begin{aligned}(A) > (B) = (C)&\\ \geqslant (D)&\end{aligned}$$

since, in an actual example, the line of $>$, $\geqslant$ could mask the presence of $=$. We are careful therefore to 'derange' the $>$, $\geqslant$. The compositor has instructions about this, but the author should look for this 'ranging' and 'deranging' both in copy and in proof.

Simultaneous equations are similarly ranged by the $=$ and not by the first or last symbol in each line. Thus

$$\begin{aligned} x &= a, \\ y+z &= b+c, \\ x-y+z &= d. \end{aligned}$$

A 'broken definition' is best displayed in some such form as

$$f(x) = \begin{cases} \sin x & (x \leqslant 0), \\ 0 & (x = 0), \\ \tan x & (x \geqslant 0). \end{cases}$$

'Broken inequalities', which can occur in obtaining upper and lower bounds, can be set thus:

$$\begin{aligned} f(x) &= (x^2+1)/(x^2+2) \\ &\geqslant \tfrac{1}{2} \\ \text{and} \quad &\leqslant 1. \end{aligned}$$

Economy. Since much of the burden of analysis is carried by displayed formulae, it is worth while to give some attention to their economical use. A complicated formula may be straightforward enough for the printer, but may still demand effort from the reader. Heavy formulae can often be lightened by the temporary introduction of a symbol to denote some recurrent element of them. Formulae are helped also by a happy choice or invention of notation.

Analysis must often rely on a long successive reduction or simplification of formulae: on what is, in fact, symbolic computation. How much of this should be written out and how much left to the reader involves decisions many of us have to learn by experience. Ideally the argument should give just sufficient stepping-stones to enable a competent mathematician to stride in safety from each one to the next. Tricks and turns in the computations, especially those crucial twists due to the author's own ingenuity, should be clearly indicated. Straightforward and well-known procedure can be omitted.

Subsidiary results or results of secondary importance may be just enunciated and left with the remark 'it can be proved that' or 'I have been able to show that'.

The writer should often ask himself 'Is this worth several pounds a page?'

8. Notation (miscellaneous)

A few miscellaneous points that can arise in mathematical setting are arranged here.

Abbreviations, where used with a mathematical symbol, become part of mathematical notation and are printed without the full point (and in roman) like mod 2, $\sin x$, $\det |a_{rs}|$. The abbreviation is normally written without a capital: $\max(x, y)$ not $\mathrm{Max}(x, y)$; but we could, if we wished, define Max in some special sense. Other abbreviations, such as 'const' for 'constant', are less elegant and should be avoided. This objection does not apply to 'e.g.', 'i.e.', 'etc.', which pass as current English. Of these 'e.g.' can often be equally well written out as 'for instance' or 'for example', and we can in this way avoid the ugly combination of 'i.e., e.g., ...'. On the other hand, 'i.e.' has no handy alternative and is essential in mathematical narrative linking equivalent statements in much the same way as '$=$' links equivalent forms. When it introduces a displayed formula, it is better placed on the left of this formula, rather than on the right of the preceding line.

The abbreviation 'etc.' is preferred to '&c.' since the ampersand '&' rather suggests a mathematical symbol. This unwanted '&' may, in fact, be offered to writers who are looking for still another symbol for some sort of addition.

Binomial coefficients are now usually written with the notation $\binom{n}{r}$, though this, as here, breaks the line awkwardly when it appears in the text, as it may well do in enumerating the number of members in a set. If this symbol is printed with $5\frac{1}{2}$-point letters as $\tbinom{n}{r}$, it is less quickly read, especially with a more elaborate form such as $\tbinom{mn+p-1}{n}$. The older forms ${}^nC_r$, C^n_r, which still survive, also have this disability and wastefully add the intrusive C. It is suggested that the symbol $(n!r)$, which has been used, might be tried more frequently.

Definitions. The usual form of introducing some symbol as a temporary shorthand for a longer expression is along these lines: 'we write

$$s = x+y+z\text{'},$$

putting the new symbol first and, where possible, displaying the formula of definition (as here), so that it is readily seen on reference back. Sometimes the definition will come naturally at the end of a reduction by successive $=$'s. The convenient form for this is

$$\ldots = \int_0^1 f(x)\,dx + \int_1^\infty f(x)\,dx = I_1 + I_2, \text{ say,}$$

where 'say' is best brought into the line of definition if possible. It is always kind to the reader to let him know without hesitation that the perplexing symbols have been newly introduced and are not some symbols already met whose significance he has foolishly forgotten. For this purpose some authors use $\equiv$ for the identity of definition; others, if they work in the theory of numbers, detest this, since for them $\equiv$ is the congruence symbol. This symbol is also sometimes written for a casual identity that is established in the course of an argument: 'we thus see that $f(x) \equiv ax+b$'. Here we generally get a better emphasis in the form 'we see that, identically in x, $f(x) = ax+b$'.

Limits. It is usual to write $x \to a+0$, $x \to a-0$ to distinguish the limit at $x = a$ approached respectively from above and below. It is suggested as an economy that these could be written $x \to a+$, $x \to a-$ without ambiguity.

The accepted notation for a limit is $\lim\limits_{x\to\infty}$ and so on. This has replaced $\underset{x\to\infty}{\text{Lt}}$ or $\underset{x\to\infty}{\text{lt}}$.

Matrices and determinants. The full 'two-dimensional' notation of matrices and determinants such as

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

is expensive to print and unwieldy to read if there is much of it. Every opportunity should therefore be taken of simplicities of pattern through the absence of elements or otherwise to reduce the notation to a (horizontal) linear form. For the completely general matrix, the form printed above can be replaced by $[a_{rs}]$ or (a_{rs}) alone. Either bracket is permissible here, but [ ] has the better appearance in the elongated form needed in the full notation.

The matrix of a single column can be written $\{x_1,\ldots, x_n\}$ or $\text{col}(x_1,\ldots, x_n)$. The 'diagonal' matrix, in which all the elements are zero except $a_{11},\ldots, a_{nn}$ of the principal diagonal, is written $\text{diag}(a_{11},\ldots, a_{nn})$, and this is extended to $\text{diag}(A_1,\ldots, A_n)$, where $A_1,\ldots, A_r$ are sub-matrices diagonally placed in the matrix, which is completed with zero elements.

For the determinant sufficiently definable in terms of a typical element a_{rs} we can write $\det|a_{rs}|$ or simply $|a_{rs}|$. But, when this latter could be mistaken for a modulus, we can use $||a_{rs}||$.

The normal alignment of a determinant is by its first row, so that we print

$$\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = a^3+b^3+c^3-3abc.$$

There may, however, be occasions on which it is preferable to align on the centre line of the determinant.

Maxima and minima. While max and min are available for the greatest and least values of an expression, these can become awkward in formulae: for instance if we had to write

$$\min\mu^{(r+1)}-\min\mu > \frac{1}{nM^2}(\max\mu^{(r)}-\min\mu)-\epsilon.$$

It is suggested that, unless musicians object, we could make use of $\sharp$, $\flat$ for this purpose, and write instead

$$\mu_\flat^{(r+1)}-\mu_\flat > \frac{1}{nN^2}(\mu_\sharp^{(r)}-\mu_\flat)-\epsilon.$$

Products. In simple products the traditional order is: numerical coefficients, literal coefficients, variable factors as in $2ax$. We help the eye in more elaborate products by putting factorials and symbols such as $(a)_n$ near the beginning; bracketed factors later, and exponentials at the end. Thus $mn!$ could be mistaken for $(mn)!$, and $e^{i\pi x}y$ reads less easily than $ye^{i\pi x}$. We write $2\,n!$ or sometimes, for safety, $2(n!)$ to distinguish it from $(2n)!$, but $n!\,2^n$. We write $f(x)e^{iy}$ but $e^{i\pi}f(x)$.

'Abbreviated' functional symbols like sin, cos should come at the end of a product whenever possible. Thus we use $a\sin\alpha+b\sin\beta$ and not $\sin\alpha\, a+\sin\beta\, b$, which could be read as $\sin(\alpha a)+\sin(\beta b)$. This principle should be maintained in more elaborate examples than need be given here; where it cannot be applied, brackets should be inserted.

Numerical factors are separated by a full point, as $2.4.6....$, and we conveniently extend this to negative numbers without added brackets, as in $-3.-1.1.3.5$.

In *unfinished series* we are reminded that there is more to come by a last + or − at the break, whichever is appropriate:

$$1+2+3+..., \qquad x-\tfrac{1}{2}x^2+\tfrac{1}{3}x^3-... .$$

For an infinite series 'to ∞' may be preferred to 'ad inf.' as

$$1-\tfrac{1}{2}+\tfrac{1}{3}-\dots \text{ to } \infty.$$

Vectors. We print scalar product $\mathbf{a}.\mathbf{b}$, vector product $\mathbf{a}\wedge\mathbf{b}$; or, when it is clear that a, b are vectors, we may print simply $a.b$ and $a\wedge b$. The heavy type used here is known as 'bold-face'. Some authorities, notably the Royal Society, recommend the use of 'sans serif bold' type for vectors ($\mathsf{\mathbf{a}}.\mathsf{\mathbf{b}}$, $\mathsf{\mathbf{A}}.\mathsf{\mathbf{B}}$), as also the use of roman i, j where these denote $\surd(-1)$.

$>$, $=$, $<$. These symbols have currently the force of verbs: 'exceeds', 'equals', etc., and it is thought inelegant to use them in the text as adjectives or unsupported by mathematical symbols. Thus instead of 'a number > 0' we write 'a positive number' or it may be 'a number x (> 0)'; instead of 'the number of solutions > 2' we write 'the number of solutions which are greater than 2' or 'the number of solutions is more than two', whichever is meant.

In the Oxford practice the symbol $\geqslant$ (in this form) is used alike for 'is greater than or equal to' and 'is not less than'. Exceptionally we may wish to distinguish between '$f(x) \geqslant 0$' as meaning $f(x)$ has zero among its values and '$f(x) \nless 0$' as meaning 'I know only that $f(x)$ cannot be negative'.

$(-)^n$ is essentially a symbol of 'sign' and not of number. Thus it is not proper to write $(-)^n/n!$ meaning $(-1)^n/n!$; we do not write $+/n!$ for $1/n!$ On the other hand, $(-)^n x^n$, $(-1)^n x^n$, $(-x)^n$ are all admissible and all mean the same thing.

Sign of substitution. The sign $\Big|_{x=a}$ sometimes used for substitutions is puzzling to compositors, who are used to seeing vertical rules in pairs: the sign is not used often enough to become established in the compositor's mind as a symbol in its own right. It is better to write all substitutions in the form $[\dots]_{x=a}$, where the brackets have the advantage of delimiting the subject of substitution. If, for some reason, these particular brackets are not available, the symbolism $\{\dots\}_{x=0}$ or $(\dots)_{x=a}$ is unlikely to mislead.

Some mathematicians (including the writers) will maintain that symbolism can be overdone; that a remorselessly symbolic mathematics need not be the more intelligible. The passage from mind to mind must be made through the reader's eye, and a microscopic notation, all 'jots and tittles', indices and subscripts, may be as illegible as a macroscopic exposition relying largely on words and phrases. The ideal lies between these, in which an occasional word punctuates the symbolism and a formula or a little knot of symbols breaks the flow of words.

9. Headings and numbering

Here it is convenient to treat together the arrangement of an article for a journal, or a chapter of a book, since the problem is roughly the same for each. In a book, the chapter number will be printed in roman capitals centred on the page, the word 'Chapter' being suppressed. The title of the article or chapter is set in large capitals (indicated where necessary by triple underlining); for a run of articles under the same title (I), (II), etc., are added after the title. This title, or a shortened version of it, may appear as the headline of alternate pages. The author should provide for this purpose a form of title shortened to not more than 40 letters including spaces (that is, for the usual Oxford mathematical *format*). Otherwise the editor or compositor must produce such a variant, which may not give satisfaction. It is often helpful in a book for shortened chapter titles to be used as headlines for left-hand pages, with section titles (shortened where necessary) as headlines for right-hand pages. The right-hand headline will refer to the section containing the top line on this page.

In every case long titles, running maybe into several lines of capitals, are to be deprecated, for appearance's sake and because they have also to be quoted in full in 'Contents' and indexes, and perhaps subsequently in references. It is, of course, recognized that in much specialized mathematical writing it is no easy matter to compose titles that are at once concise yet explicit.

Titles (for elegance) should avoid footnote signs and also, where possible, symbols, since titles will be quoted in indexes, and perhaps elsewhere, in smaller founts.

Sections. An article or chapter is divided into sections introduced by serial numbers and headings in bold type (indicated by a wavy line drawn under the words) set 'full out', i.e. beginning at the left-hand margin. A full point follows the serial number but not the heading. The text commences, indented, in the following line:

> **10. Conclusion**
>
> These methods can be extended . . . .

If section titles are not being used as headlines and the author's invention fails him for a heading, the text runs on after the serial number:

> **10.** These methods can be extended . . . .

Sections are quoted with the symbol § and serial numbers in ordinary type, as § 1, §§ 2, 3, §§ 3–7, etc.

The continuity of an argument will sometimes lead to an overlong section. This may well be broken up into sub-sections with decimal numberings in ordinary type: thus we should subdivide § 4 into 4.1, 4.2, ... These numbers are inserted (with or without italic headings) at the appropriate 'paragraphs' as described below. Such sub-sections are quoted as § 4.1, §§ 4.2–4.5, and so on.

For the reader's understanding an article should generally begin with an introductory section explaining at what point the subject is being taken up and, if possible, indicating the main results achieved. In much mathematical work this 'introduction' will differ little from a 'summary' and may sufficiently take its place.

Occasionally the sections themselves are grouped under PART I, PART II,... or other more informative superior headings. For contrast with the section headings these are conveniently centred and, as here, set in capitals and small capitals with roman numerals.

Paragraphs. The sections themselves will be 'paragraphed' (that is, indented) at convenient intervals. Where there are few displayed formulae, the text should be lightened by frequent indention; but displayed formulae contribute a sufficient 'aeration' of the page, and paragraphing can then follow the broad phrases of the argument. Use of a typewriter often encourages too frequent indention, which then gives an irritating left-hand margin to the page.

Subordinate headings within a section are set in italics, the text running on after a full point. Thus always '*Proof.*', '*Case* (i): $x > 0$.', etc. The principles outlined above will be found exemplified in these pages.

Exceptionally a break within a section, superior to a paragraph, can be made by a 'line of white', i.e. a line of space, built up by setting a line of quads or insertion of leads. The author asks for this by noting marginally 'space' or 'line of space'.

Enunciations. Sections will also be broken from time to time by the enunciations of theorems, lemmas, and the like. These are preceded by the appropriate title THEOREM, LEMMA, COROLLARY, ... in capitals and small capitals (indicated by an appropriate underlining (see § 14) or by 'c. & s.c.' written in the margin).

The enunciations themselves are set in italic. This is indicated by underlining, or a vertical line may be drawn in the left-hand margin running the length of the enunciation and the gloss 'ital.' added. The author should so indicate the exact extent of his enunciation; otherwise this must be guessed at by an editor or compositor.

Theorems, lemmas, etc., are separately paragraphed: that is, they are indented and the line of text immediately following is also indented.

The italicizing of enunciations should be extended to other less precise enunciations such as the statement of a problem, the formulation of an hypothesis, an elaborate definition, or intermediate results that may occur in the course of an argument and which it is desired the reader should note. Such less formal enunciations may be paragraphed or not according to their length or importance. In a sense we may regard this italicizing as the mathematician's quotation marks, taking statements outside and above the level of the ordinary text.

It should be noted that 'lemma', 'theorem', etc., are common nouns spelt with lower-case first letters, but, followed by a number, they become proper nouns and are given capitals. Thus we write: 'we prove these theorems by use of Lemma 1 . . .'. So, too, 'Feuerbach's theorem', 'Fig. 1', 'by reference to the figure'.

Numbering. For most purposes of serial numbering arabic numerals are best. Letters, whether English or Greek, may be confused, in the text, with such letters used symbolically. Roman numerals soon grow cumbersome and I may be misread as a pronoun (as perhaps here).†

Exceptionally, as has been said above, roman capitals in parentheses ( ) are used with titles in a periodical to indicate successive parts (or afterthoughts) of the same study with repeated title; and they are conveniently used for chapter-numbers in a book. Lower-case roman numerals are recommended in the text for enumerating separate cases or where a number of parallel conditions or statements need easy separation: thus

> Let us suppose that (i) $x > 0$, (ii) n is an even integer, (iii) C is some constant, (iv) α, β, γ depend only on x, ...

The advantage of the roman numerals here is that, having no symbolic significance, they are not likely to be confused with any of the symbols x, n, C, α, β, γ.

The numbering of sections, of theorems, etc., and of equations and formulae is always in arabic numerals. Various systems of enumeration, quasi-decimal and otherwise, have been used and have their points of advantage; but over-elaboration is to be avoided. In a series of articles in a periodical the enumeration should be complete for each article and not run on consecutively through the series.

The organization of sections and smaller divisions in a book presents

† This is avoided by writing the numeral always in parentheses as (I).

individual difficulties. Where the sections are relatively few it is often sufficient to number them in one series through the book, ignoring the incidence of chapters. Where large numbers would result from this or where each chapter is more self-contained it is better to start each with section 1. Occasionally it may be helpful to incorporate the chapter number in the section number, so that, e.g., Chapter VI begins with section 6.1. In either case subsections can follow the scheme suggested above, their heading, if any, being in italic set at the beginning of the line. Sub-subsections can be inset and labelled (*a*), (*b*)...; sub-sub-subsections can be distinguished by (i), (ii).... It is essential to retain consistency of order in labelling the smaller divisions. Even more complicated schemes can be devised to satisfy individual whim: discussion at an early stage with the publishers is advised.

Equation numbers. In the Oxford practice equations and formulae are numbered on the *right*, the number being in parentheses ( ). A very few distinguished mathematicians have numbered their equations on the left: this is exceptional. What is so numbered must be displayed; a number cannot, of course, be set against a line of text. The number will normally refer to a single mathematical line, i.e. normally a single printed line: exceptionally a broken formula extending over two or more lines of type, in which case it is set opposite the lowest line. Where several mathematical lines are to share a number, e.g. a set of simultaneous equations, these are braced on the right by a }. The use of a brace and an equation number considerably reduces the effective width of the page, and where the equations involved are long, this means breaking them across two lines. This is frequently avoided by suppressing the brace and printing the equation number below the whole group.

If it should later become necessary to refer to an individual one of these braced lines, a subscript number may be used: thus (7_2) could denote the second equation of the system (7).

Where it is desired to link two separated formulae by 'subdividing' a single number, the forms (7 a), (7 b),... or perhaps (7), (7 a),... are recommended. The latter may be useful when oversight has left two equations with the same number or has left a needed equation unnumbered.

Normally, the numbering of equations runs on consecutively, since it is always new work that is being numbered. But, of course, it is sometimes convenient or necessary to repeat an earlier equation; it may be misleading to omit its earlier number or to give it the new number of its new position. To repeat its earlier number can then give such a deceptive sequence as (4.7), (4.9), (4.10), (4.7), (4.11). In such a case the innovation is suggested of adding brackets [ ] to the repeated number so that the sequence above would appear as [(4.7)], (4.9), (4.10), [(4.7)], (4.11).

In a book the repetition of the section number in the equation number can be justified, particularly if there is much reference from one section to another. The suppression of the full point between these two numbers is less useful. If the chapter number is already incorporated in the section number this makes a 3-figure reference, which is in general only necessary when there is much cross-reference from one chapter to another.

Ideally one should number only those equations and formulae that will be needed in the subsequent course of the analysis or that may be quoted by other writers. Admittedly this presupposes foresight that few of us lay claim to, and one has often been embarrassed in referring to some unnumbered formula. But, on the other hand, excess of numbering may defeat its purpose by making it difficult to pick out the particular numbers that are actually needed.

Subsidiary results. Sometimes a piece of work may lead in passing to a number of results that do not seem to deserve the title of theorem or lemma. These, having been enunciated in italics and paragraphed, may be numbered serially on the *left* in brackets [ ].

10. Footnotes and references

In mathematical printing the custom of the University Press is to indicate footnotes not by small figures but by the signs †, ‡, §, ‖ used in this order and followed, if necessary, by the same symbols duplicated: that is by ††, etc. The asterisk is now avoided since it frequently occurs in mathematical notation. It may be hoped that further reference marks will not be borrowed in this way, since their use sends the eye automatically to the foot of the page to look for what is not there.

Reference marks are preferably attached to words and not to symbols, for then they could be supposed part of the notation. It is better, too, to keep them away from titles and headings. When reference marks fall at the end of a clause or sentence, they are printed after the sign of punctuation as ',†' or '.‡'.

Since pages of the copy will not correspond with the printed pages, an author should write a footnote immediately under the line to which it refers, dividing it from the main text by lines drawn across the page. It is less convenient to have footnotes collected at the end of a chapter or article.

It is good practice in mathematical work to keep footnotes as few as possible; massed footnotes distract a reader. All important information should be worked into the text: this can sometimes be done by insertions in brackets [ ]. References to authorities should be treated as suggested below. Footnotes then remain for interpretations, corrections, irrelevancies, and other such 'asides'. In particular, footnotes, which of course are set in a smaller fount, ought not as a rule to contain formulae —especially when these involve indices, subscripts, or other symbols already in small type.

It must be remembered that the operator, presented with a sheaf of copy, is accustomed to go through the pages picking out all footnotes, and to set these first in their smaller fount; some work and care is then needed in reallocating these to their proper places.

References. Cross-references within a chapter or an article are conveniently dealt with as suggested in § 9. In the case of a book, references to other chapters should include the chapter number, (VI § 2) or (Chap. VI § 2). The chapter number should be given in roman numerals. It is not necessary to include the word chapter or any abbreviated form of it: where this is done, however, the form used (Chap., Ch.) should be consistent throughout. Where the chapter number has been included in the 'decimal' notation (cf. § 9) the reference is more conveniently given as (§ 6.2). References to sections should be given rather than to pages, since these can be written once for all into the copy without waiting for page proofs. Also they are unaltered by any possible subsequent disturbance of the type.

References to other work can be given piecemeal in footnotes as they are needed in the text. If they are at all numerous, it is, however, much better not to give them as footnotes but to collect them together at the end of the article or chapter. There they may appear under the heading 'References' and may be referred to in the text as indicated below. In the case of a book it is sometimes preferable to collect the whole of the references at the end, where they appear as a 'Bibliography'. Authors should consider which method best serves their interests, but are recommended to avoid footnotes unless references are very few.

References should be presented in a standard form. For an article give the author's initials and name, the title of the article (if desired) in quotation marks, the name of the periodical in italics followed by the number of the series (if any) in brackets, the volume-number, the year in brackets, and, finally, the first and last page-numbers of the article. For a book give the author's initials and name, the title of the book in italics, the place and year of publication (or the edition) in brackets. The punctuation is shown in the example below.

'Volume' and 'page' and their abbreviations are omitted. There are recognized abbreviations for the titles of journals. The Royal Society in their *Notes on the Preparation of Papers* (July 1951) give a list of the more frequently cited scientific journals, while a list more suited to the needs of the pure mathematician can be found in *Mathematical Reviews*.

These references are arranged preferably in the alphabetical order of the authors' names, repetition of an author being indicated by a rule, and of a periodical by 'ibid.' An author in collaboration is distinguished from the same author writing alone. The references are then numbered in succession in bold-face type. The following example shows how these detailed rules are observed in practice:

1. D. E. Littlewood, *The Theory of Group Characters and Matrix Representations of Groups* (2nd ed. Oxford, 1950).

2. D. E. Littlewood and A. R. Richardson, 'Immanants of some special matrices', *Quart. J. of Math.* (Oxford) 5 (1934) 269–82.

3. —— 'Some special S-functions and q-series', ibid. 6 (1935) 184–98.

The foregoing scheme is that which appeals generally to the 'pure' mathematician and is followed in detail by such journals as the *Quarterly Journal of Mathematics* (Oxford) and the *Proceedings of the London Mathematical Society*. Other systems differing from this very slightly are in use: the most common is that in which the only variation is that the volume number is printed in bold face, all other details being followed exactly. This is used, e.g., in the *Quarterly Journal of Mechanics and Applied Mathematics*. The Royal Society has standardized a slightly different form for their publications and many authors have followed the lead there given.

Authors should be warned that each journal and, generally, each series of monographs has its own style, and that when writing for a particular journal or series they should familiarize themselves with its style and follow it.

When writing a book to be published by the Oxford University Press they are advised to follow the style here given, or the variant in which the volume number is given in bold face. The Press, however, does not wish to dictate in this matter, and any scheme used consistently throughout the book will as a rule be followed.

The references are, as a rule, to the complete article or book in question, which is then quoted in the text by its serial bold-face type number in parentheses ( ): thus

'The problem has also been discussed by A. N. Other (**1**).'

It is unnecessary and tiresome to print for this

'The problem has been discussed by A. N. Other†',

with a footnote † A. N. Other (**1**), or even † (**1**). When details of page, equation number, etc., become necessary, these are given in brackets thus:

'The proof has been given by A. N. Other [(**1**) 27 (3); see also (**2**) § 6].'

Less conveniently this information is given in a footnote.

This use of (**1**), etc., is much to be preferred to the *ad hoc* reference by initials and so forth, such as A.N.O. (I). And, of course, numbering by means of superscript figures should be avoided if clarity and lack of confusion with mathematical symbols are sought.

It is convenient practice to give an author's initials in the collected 'References', omitting them in the text or in footnotes, unless they are necessary for distinction, as between F. Riesz and M. Riesz. Titles are given only when an individual has been brought into personal relations with the writer: 'My thanks are due to Professor X', 'Dr. Y points out to me that . . .'.

The humane author will be careful to give such a sufficiency of references to earlier work whether by himself or by others that the interested reader can take up the argument without difficulty. It is perhaps a mild form of arrogance to suppose that everyone must be acquainted with the subject in hand. Where the topic is enshrined in an abundant literature, it may be possible to give a useful reference to a wider bibliography elsewhere, but precise references ought to be given to all books and articles from which formulae, theorems, or other details have been quoted.

11. Varieties of type

In this section we are concerned with the type faces used in setting the text of a mathematical book—these are bold-face, italic, large and small roman capitals. We make mention also of special founts, such as script, and the Greek and German alphabets, which can be drawn upon for use as mathematical symbols.

Bold-face type. The use of this type for numbering and titling of sections has already been mentioned, as also its special use for the numbering of references. In mathematical notation heavy type has been conventionalized to denote vectors, although, as their novelty diminishes and their frequency in mathematics increases, the reservation of a specialized type may become less necessary. All publications do not make use of the same fount of heavy type. Oxford has standardized the 'bold face' type shown in the heading of this section, but the Royal Society favours 'sans serif' bold (cf. § 8). In any case bold-face type should be used with economy and discretion in mathematical symbolism because of its strident quality; much of it on a page upsets the evenness of appearance and makes for less easy reading. Certainly such a notation as **(1)** in a mathematical sense would be undesirable since it could be confused with the notation for references.

The sign for bold type is a wavy line beneath the words or symbols in question; for security the word 'bold' may be added in the margin.

Italics, as has been said, are used for enunciation and for subordinate headings. Elsewhere in the text they may be used *very sparingly* (as here) to emphasize a word or phrase. When emphasis is essential in an enunciation or other block of italic type, this is obtained by setting in small capitals.

Formal definitions in the text are conveniently given in italics: 'we call $|f|$ the *modulus* of f'. Looser terminology can be put in quotation marks: 'we apply the method of "maximum descent" at this point'.

In addition to these uses in the text, the bulk of mathematical notation is, of course, carried in italic type. This the printer knows, and it is therefore unnecessary to indicate as italic any recognizable English letter used with a mathematical symbol or occurring in a mathematical formula however short: for example the n in $n+1$ or $n = 2$. On the other hand, when such letter appears in isolation in the text (as does the first n in the foregoing sentence), it comforts the compositor to have it underlined. So too underline the italic letters in x-derivative, z-plane, n-dimensional, nth, adding the hyphen in all except the last. When, however, nth occurs in italic text, as, for example, in an enunciation, we print *n-th* as clearer than *nth*. Also we print $(n-1)$th, $(n-2)$th, $(n-3)$th, not $(n-1)$st, $(n-2)$nd, $(n-3)$rd.

The author must not underline Greek letters, merely because they are used as symbols, nor indeed for any reason whatever. Greek letters are not italic and to write (say)

$$\underline{\alpha},\quad \underline{\gamma},\quad \underline{\kappa},\quad \underline{\mu},\quad \underline{\nu},\quad \underline{\rho}$$

is to run a good risk of getting

$$d, \quad y, \quad k, \quad u, \quad v, \quad p.$$

Standard abbreviations (not single letters) used as symbols are printed in roman and without the full point:

sin, tan, cot, cosec, log, ln, exp, sinh, cosh,

tanh, sn, cn, dn, lim, mod, max, inf, re, im.

The last two have been adopted in the Oxford practice out of the numerous forms for 'real' and 'imaginary' parts.

It may be repeated that italics are indicated by underlining, reinforced for safety by 'ital.' in the margin. Italic is restored to normal (i.e. roman) type by 'rom.' in the margin.

Capitals. Capitals and small capitals are used, as has been said, for the titles of theorems, lemmas, etc., and also, rarely, for emphasis within italic type. By a printing custom the first word or words of an article and of each chapter of a book are set in large and small capitals.

The typographical sign for small capitals is a double underlining and, if necessary 'sm. caps.' in the margin.

Ordinary (roman) capitals appear so frequently in their normal English usage in the text that to invest them with other conventional meanings risks confusion and ambiguity. As has been mentioned above, they are used, always in parentheses ( ), for the serial numbers of titles. But capitals used symbolically, whether in geometry, analysis, or elsewhere, are set in italic. Exceptionally, and somewhat oddly, certain Greek capitals are identified with similar roman capitals. Thus the capitals corresponding to α, β, ϵ, η, ρ are A, B, E, H, P, even when used as mathematical symbols; this clearly renders their frequent use undesirable.

The mark for ordinary roman capitals is a triple underlining and 'cap.' or 'caps.' in the margin. For an italic capital this triad of lines is put *under* the single underlining for the italic; for safety 'ital. cap.' is written in the margin.

Special founts. For use as mathematical symbols, apart from the founts of type already mentioned (roman, italic, and bold face), there is also available a fount of bold-face italic type, which has the same strident quality as its 'roman' counterpart and calls for the same economy and discretion in its use, and also a fount of Script type. Greek and German alphabets are very largely drawn upon, and it should be noted that in each case there is a heavier fount which can be utilized if Greek or

German letters are needed to denote vectors. Difficulty will only arise here if 'embellishments' (dots, bars, etc.) are added: these will involve hand work or the ordering of special matrices.

12. Punctuation

Punctuation can afford to be more lavish in mathematical work than would be thought necessary or even tolerable in ordinary writing. The concentration and complexity of a mathematical argument can be eased by a rather strict adherence to the conventions of English punctuation. These are recapitulated below with adaptations to the special needs of the mathematical sentence. It should be remembered that normal punctuation runs on through the sentence although it may include symbols or whole blocks of displayed formulae. These are essential grammatical components and must have their share in the punctuation. Thus, when a sentence ends with a displayed formula, this must be followed by a full point just as much as if the formula could have been written in words; and consecutive formulae, like consecutive symbols, need to be separated by commas or other punctuating signs.

The *comma* is usefully regarded as a kind of grammatical bracket that gently partitions off words, phrases, and clauses that are slightly out of place in the sentence. Like brackets they normally work in pairs unless the beginning or end of the sentence itself completes the partitioning. A subordinate clause somehow seems more in place when it follows its principal sentence, as here, and for that reason no comma is put before the 'when'. On the other hand, when, as so often in a mathematical argument, the subordinate clause, usually a condition introduced by 'if' or 'when', precedes the principal sentence, then it is important to mark its termination with a comma, often reinforced, as here, with 'then'.

In the preceding sentence the complete conditional clause 'when . . . sentence' is cut off by commas, 'on the other hand' being part of the main sentence. Thus, always, we punctuate 'For, if $x > 0$, $e^x > 1$', not 'For if $x > 0$, $e^x > 1$', since 'for' belongs to the main sentence and, had the order been inverted, we should have written 'For $e^x > 1$ if $x > 0$'.

Because of its frequent technical occurrence in mathematics, it is convenient to regard 'if and only if' as a single conjunction 'if-and-only-if', omitting the commas round 'and only if'. Accordingly we punctuate '$e^x > 1$ if and only if $x > 0$'.

This 'insulation' of interpolated clauses by commas is perhaps the best way of inserting reasons in a mathematical reduction that is being effected in a string of equals. Thus we punctuate

$$\int_0^{\frac{1}{2}\pi} \sin x \log \operatorname{cosec} x \, dx$$

$$= \int_0^{\frac{1}{2}\pi} \frac{(1-\cos x)\cos x}{\sin x} \, dx, \text{ on integration by parts,}$$

$$= \int_0^1 \frac{(1-c)c}{1-c^2} \, dc, \text{ say,}$$

$$= \int_0^1 \left(1 - \frac{1}{1+c}\right) dc, \text{ on reduction,}$$

$$= \log \tfrac{1}{2}e, \text{ by integration.}$$

The successive formulae should be arranged in column as above.

There is a rather subtle use of the comma with relative clauses, which can either assert or define. Those which assert are preceded by a comma, as in the preceding sentence; those which define, as in these two sentences, omit the comma. Briefly, assertive 'which' applies to the whole of a class, definitive 'which' selects certain members of it. Thus the omission of the comma before the first 'which' in this paragraph would imply that there are still other relative clauses that neither assert nor define. The practical test is that assertive 'which' can be replaced by 'and this', 'for these', or some equivalent; definitive 'which' can be replaced by 'that'. This is often preferred in ordinary prose, but, in mathematics, 'that' is already too frequent as a conjunction.

Confusion will not often arise from this source since the experienced reader will be sufficiently warned by the context. The false assertion in 'Mersenne considered numbers of the form 2^p-1, which are prime when p is prime' will deceive only a beginner.

The above distinction applies also to clauses introduced by 'where' or 'when'. The omission of the comma converts the clause into an adverb of place or time: 'This is where we came in'. Thus in definitions we strictly punctuate '$\sin \alpha$, where α denotes . . .'.

A comma may also separate principal sentences connected by 'and', 'but', etc. Here it is a useful convention to omit the comma when the sentences are joined by a common subject, but to insert it when the subject changes with the sentence. But in a succession of 'if'-clauses and the like, the comma should be more readily omitted so that it can more easily separate the conditions from the subsequent assertion.

On the other hand, a comma is insufficient preparation to a sentence opening with 'hence', 'thus', and the rest. These words introduce new principal sentences, and the previous sentence must therefore be closed either with a full stop or at least with a semicolon.

An exception to this rule is the conjunction 'for' which somehow lies between the 'subordinating' and the 'coordinating' conjunctions. It has some slight danger in mathematics from possible confusion with the preposition 'for', for it is frequently used as an ellipsis for 'for the case in which', as in $e^x > 1$ for $x > 0$. After this intentional over-use of 'for', it is suggested that the elliptic 'for' can often be replaced by 'if' or 'when' and the conjunction 'for' by 'since' or 'because'.

Again, a comma is usefully inserted, as here, after words such as 'Again', 'Moreover', 'Finally', 'Above all', introducing a sentence.

It is modern English usage to insert a comma before 'and' with sets of three or more as in 'Tom, Dick, and Harry'. In mathematics with symbols or enumerations the 'and' is best omitted, and we write 'x, y, z are real' or 'We see from (3), (5), (7) . . .', unless the last member has to be specially distinguished. With pairs we may choose between 'x, y' or 'x and y' and so on.

A comma should separate each of a run of equations (not a semicolon unless for a special purpose). Thus

$$x = a, \qquad y = b, \qquad z = c.$$

This holds equally when the equations are in successive lines.

The comma is omitted from a set of equations held by a brace except between equations in the same line of the brace. Thus

$$\left.\begin{aligned} x &= a \\ y &= b \\ z &= c \end{aligned}\right\}, \qquad \left.\begin{aligned} x = a, \quad y = b \\ z = c, \quad u = d \end{aligned}\right\},$$

the group as a whole being punctuated, as here, beyond the brace. Exceptionally with a very long brace the equations may be punctuated as if the brace were absent.

We have learnt to use the commas between substantives 'in apposition' as in 'I, John, take thee . . .'. This practice should not be extended to mathematical symbols. Thus we write free of commas 'the function $f(x)$ is real', 'the point P lies . . .'.

Finally, commas should be slipped in (under the excuse of a sense-pause) to separate numbers from symbols. Thus 'by § 4, $x = 0$', 'in Theorem 1, n is real'.

The *full point* or 'full stop' marks the end of a sentence or an abbreviation. It is not repeated after an abbreviation, such as 'etc.', that ends a sentence, and it is omitted after ! of the factorial symbol. We print, however, '$x_1+x_2+\dots$.' where the full point is distinguishable from the triad of dots preceding it. As has been said, the full point is not used with such abbreviations as 'sin', 'log' that have become established as forms of mathematical notation.

Words and phrases used absolutely as titles or headings are cut from what follows by full points:

'THEOREM 1. *Every solution* . . .',

'*The case of x real.* Here we . . .'.

The *semicolon* can be used both as a weak full stop and as a strong comma. It can be used, with bracketing effect, in a run of equations to break them up into desired groups. Thus we should write the equations of three straight lines in space as

$$x = a, \quad y = z; \qquad y = b, \quad z = x; \qquad z = c, \quad x = y.$$

Likewise in a succession of parallel sentences separated by a comma—as for example in a long statement of conditions—we can take breath at a suitable point by writing there a semicolon in place of the expected comma.

The semicolon can replace the full point when the following sentence has to begin with a symbol that is not a capital letter: to some eyes, indeed, it is repugnant to begin a new sentence (after a full point) with any symbol. In enumerations with (i), (ii), (iii), . . . semicolons are preferred to full points since the roman numerals themselves are lower-case.

The utility of the semicolon in mathematical notation because of its bracketing quality has been referred to elsewhere.

The *colon* is rather precious in its use. It indicates some sort of attachment of the new sentence to its predecessor by way of explanation or expansion. It has been usefully said that a colon is appropriate where a letter-writer would have put a dash.

The punctuation 'as follows:—' is traditional. In mathematical work we omit the dash and print only 'as follows:'. But it is not necessary to make this a rigid rule, since a long exposition may best be introduced by a full stop. We can use the colon in this way before theorems, enunciations, and the like. Thus 'I prove the theorem: THEOREM 1. . . .', 'we have the rule: . . .'. On the other hand, formulae, equations, and so forth are introduced without any mark of punctuation. It is irregular to print

'Solving the equation, $x^5+5x+...$'

'We have the formula: $f(x) = ...$'

unless there is some special reason for requiring the comma or the colon.

The colon is convenient before a restatement preceded by 'that is'. Thus we may punctuate '. . . $f'(x) > 0$: that is, $f(x)$ is monotonic increasing . . .'.

Dashes are sometimes useful in a mathematical text as alternatives to brackets when these may cause confusion with brackets used mathematically elsewhere in the text. Dashes, in their turn, should be used circumspectly to avoid confusion with a minus sign.

Quotation marks show that a word is being employed in some unusual sense. For example, they enclose a word that is being 'talked about' and is not a normal part of the sentence: 'nowadays we write "convergence" rather than "convergency" '. The classical example is perhaps: 'I did not say "and" but "or" ', which is less intelligible printed as 'I did not say and but or'.

Words or phrases used in a special sense, colloquially or in rather bold metaphor, have quotation marks. Thus we might wish to say 'Using the method of "least squares" we "sieve" the solutions'.

As will be seen in the preceding paragraphs, single quotation marks are the normal usage, double quotation marks serving for a quotation within a quotation.

The *apostrophe* presents few points of difficulty. There is the long-standing contention between 'St. James's Park' and 'St. James' Park'. The mathematician will probably prefer to spell as he pronounces and to write 'Gauss's solution', but 'for goodness' sake'.

It is English idiom to write 'a theorem of Ramanujan's, and not 'a theorem of Ramanujan'. Exemplars are 'a friend of mine'—not of course, 'a friend of me'—or Browning's title, 'A Toccata of Galuppi's'. We could say that much English mathematical writing was a result of Ramanujan, but, naturally, not that it was a result of Ramanujan's. We may be able to avoid the strict idiom, where it would be cumbrous, by writing 'a result due to Ramanujan' or 'given by Ramanujan'.

We can sometimes use an apostrophe to save a hyphen and, for example, write 'nine points' circle' on the analogy of Shakespeare's 'The nine men's morris is filled up with mud'.

Symbolic plurals with an apostrophe, as 'the x_r's are positive', can read awkwardly. It is suggested that we should accustom ourselves to regarding the plural of x_r as x_r itself in the same way that a sportsman shoots elephant and, in the garden, sows cabbage and plants coreopsis. We have also the analogy of 'for all x'.

Hyphens. There is a thought-provoking analysis of the use of the hyphen in H. W. Fowler's *Modern English Usage*. For mathematical purposes it may be summarized by saying that the first purpose of the hyphen is to link two or more words into a single part of speech, as in the previous sentence, where the hyphen combines two words into a single adjective 'thought-provoking'. It is also used to show that two words have not their precise everyday sense as in 'the inequality is best-possible', as distinguished from 'the best of all possible worlds'. This represents a stage in evolution from 'the inequality is "best possible"', when the technical phrase was less usual. The final and improbable form would be 'the inequality is bestpossible'.

In speech this linking of words is shown by an assimilation of stress: the strong accent goes forward to the first word of the group, and it is this shift of stress which is translated into print by the hyphen. Thus we distinguish by accent and hyphen 'a lást-minute contribution' from the very different 'a last mínute contribution'; or perhaps, in mathematics 'a fírst-order term' from a first órder-term'.

Where words have their normal grammatical sense the hyphen is out of place. Thus we write 'theory of the zeta function' without the hyphen, but of course 'zeta-function theory' with it: in the first phrase we have the customary English usage of a noun acting as an adjective. So we write 'the theorem is well known' with the adverb modifying the particicipial adjective in the regular way, but we should write 'a well-known theorem' because the compound adjective is usually so stressed.

We write 'a positive-definite form', but 'the form is positive definite'. We hyphen 'sub-sequence', 'co-cycle' if we wish to avoid the accentuation of 'subsequent', 'bicycle'.

13. Wording

So far in these notes the printer, as much as the reader, has been our first consideration. The suggestions that follow deal with points of phraseology in the text, where, of course, there are few, if any, printing difficulties. Here the primary concern will be to help the reader.

In the first place the completeness and exactitude of mathematical argument often engender long and involved sentences with repeated 'since', 'if', 'when'. It is generally kinder to break these up into simpler forms. 'Therefore' has many synonyms: 'then', 'thus', 'hence', 'accordingly', 'in consequence', 'it follows that'. Sometimes alternative cases or successive conditions can be broken up with (i), (ii), etc.

Besides prolixity we need also to avoid ambiguity. A doubtful statement may mean either that the author's intention is actually mistaken, or more probably, that the reader must suspend judgement till he has read further or must return and re-read. In either case there is an irritating pause that breaks the continuity of the argument and, therewith, the reader's assimilation of it.

There are a number of words or forms of expression frequent in mathematical writing that are not immediately clear, and precautions should be taken in their use.

Assertions. The body of a mathematical argument necessarily contains many assertions of mathematical fact (or belief). These will often differ much in their quality and origin. Let us consider, as it were, a diagrammatic assertion: 'all λ-functions are bounded'. Its *raison d'être* in an argument may be one of several:

(i) it may be immediately obvious (to any competent mathematician) from the definition of a λ-function;
(ii) it may be a well-known theorem, dating, say, from Euler;
(iii) it may be a theorem hidden somewhere in the literature of the subject;
(iv) it may follow, with or without effort, from what the author has just established;
(v) it may enunciate a result that the author is about to prove;
(vi) it may be a statement (without proof) of a less important consequence of the author's argument, which he has printed elsewhere or even has not yet published.

It clearly aids the reader when the assertion can be ushered in by a phrase that distinguishes between the above possibilities. Thus one could say:

(i) 'it follows at once from the definition that';
(ii) 'by Euler's well-known theorem';
(iii) 'K. N. Owall has proved', adding the reference;
(iv) 'we can (readily) deduce', omitting 'readily' when that would be insincere;
(v) 'I shall now prove that';
(vi) 'it can be shown that'.

Proofs. In the same way it should always be made clear to the reader where a proof begins and ends, so that he may know when to give that closer attention that a proof demands and when to relax that attention. A formal proof is frequently headed '*Proof*'. Less formally we can commence with the conjunction 'For'. We notify the end of the argument by such a phrase as 'This completes the proof'.

Definitions. We should always emphasize when we are introducing a new symbol or term; otherwise the reader may blame his own negligence for forgetting its meaning. It is helpful, in the same way, to give some reminder when a symbol reappears after lying dormant for several pages. To the new symbol we attach a word of definition immediately before or immediately after its first occurrence. As 'I write $X = x^2$' or 'cuts XY in a point P (say)'.

For a reappearance we can say something like 'which equals ξ, as defined in (17)', or whatever may be appropriate.

'*Assume*'. This word is sometimes used in several senses that are not equivalent. The author may mean that he is *quoting* or *relying on* an established result. It may refer to a justifiable classification of cases or a preliminary clarification. We could then replace it by 'suppose' and say: '(i) We first suppose x real', or again 'Renaming the suffixes, if necessary, we sufficiently suppose that $x_1 > x_2 > x_3 > \ldots$'.

This leaves 'assume' free for what is truly hypothetical, as in an inductive proof, 'Assuming the theorem true in n variables'; or when the author can prove his theorem only under restrictive conditions, 'Assuming that all λ-functions are differentiable'.

A temporary hypothesis set up only to be demolished by deducing (say) a contradiction can be introduced by 'Suppose, if possible, that'.

We often say 'Suppose this is true', but we should *write* in full 'Suppose that this is true' or 'Suppose this to be true' or just 'Suppose this true'; and similarly 'provided that $x > 0$' and not 'provided $x > 0$', still less 'providing $x > 0$'.

'*Arbitrary*' in its mathematical sense, can be given two opposite meanings. If we sometimes fancifully picture a mathematical argument as a two-handed game played against an inexorable adversary, then it matters much whether 'arbitrary' means at the author's choice or the adversary's choice. Thus the notorious arbitrary ϵ of convergence-theory is at the adversary's disposal and therefore restricts the author's choice of $n(\epsilon)$. On the other hand, the arbitrary constants in the solution $y = \sum c_n \eta_n$, of a linear differential equation are at the author's disposal, and enable him to satisfy conditions imposed on the solution. We can distinguish these senses of 'arbitrary' by speaking of a 'given' or 'prescribed' ϵ and, if necessary, referring to the constants as 'disposable'.

'Or' in the sense of 'or equivalently' for a statement rephrased or, more often, for a formula simplified or recast can be confused with 'or' implying a true alternative. To read 'the equation is satisfied by $x = \sin 30°$ or $x = \frac{1}{2}$' will not give pause to a mathematician, but in a more elaborate example the reader may have to read back, or read on, or to make a mental computation before deciding which sort of 'or' is meant.

It is suggested that the 'or' of equivalence should always be replaced by 'i.e.' or sometimes by 'thus' or 'that is'; sometimes even by 'or equivalently'. It is regrettable that we have no better conjunction than this abbreviation of the Latin.

'As'. To write 'as' in the sense of 'since' is discouraged by some authorities, unless perhaps the 'as'-clause comes first as in the following sentence. As 'as' has a sufficiency of other uses—as for instance, as equivalent to 'while' as in 'as $x \to 0$'—we might well exclude this particular use of 'as' in mathematical work. In exceptional cases it could even be confusing.

'Only'. For clarity 'only' in mathematics ought to be attached closely to the word or phrase it qualifies even though this would be thought pedantic in common speech. Thus we should say that (at $x = 0$) $x^n \sin(x^{-1})$ is *continuous only* when $n > 0$ but is *only continuous* (and not differentiable, say) when $0 < n \leqslant 1$. The principle extends, of course, to such equivalent words as 'solely', 'merely', 'simply', and 'alone'.

So far we have been concerned with wording that could be ambiguous or deceptive or might in some way trip up and delay the reader. Certain subtleties and refinements are now mentioned for such as may be interested.

'I', *'we'*. It is no longer *de rigueur* for the single author to disguise himself as 'we' or as 'the author of the present paper'. 'I' is perfectly correct wherever it is appropriate. The natural distinction between 'I' and 'we' is that which would be made by a lecturer to his audience who might say 'I shall now prove' and (hopefully) 'as we have seen'. There are many intermediate statements in which either pronoun could be used.

More generally it is helpful to retain 'we' for accepted practice or terminology and to distinguish with 'I' the author's departures from orthodoxy or his contributions to knowledge or notation. Sometimes 'my' is better replaced by 'the' or 'this'.†

† G. H. Hardy once rebuked me, when I had written 'my inequalities', with the comment 'Megalomania! they are God's inequalities'. [T. W. C.]

An occasional change to the passive or the imperative helps to lighten the monotony of perpetual 'we'. Thus one could write 'it can be seen', 'differentiate and proceed to the limit'.

'Can', *'may'*. There is a distinction between 'can' and 'may' that is sometimes worth preserving. Whereas 'can' denotes ability, 'may' refers either to what is legitimate or to what could possibly happen. Thus we can generally differentiate a series term by term. But *may* we? We *may* solve a differential equation, by trial and error perhaps, and it is certainly permissible. But the hard question is 'can we?' It is perhaps a pity to obliterate this distinction by using 'may' as a gentler form of 'can'.

'Will', *'shall'*. The normal conjugation is 'I shall, (thou wilt), he will, we shall, you will, they will', and any departure from this is purposeful and carries emphasis.

Evidently 'will' suggests free-will, and it is polite to attribute this gift to second and third persons; for ourselves we retain 'shall' which is unthinking and automatic. Applied to others it implies 'compulsion'; it is the verb of commandments and by-laws: 'Thou shalt not steal', 'Five shall be a quorum'. This use has appeared in mathematics, though it sounds harsh to older ears.

In the first person 'will' shows effort and intention: 'If I shall be in town, I will gladly dine with you', and the emphatic 'I will' of marriage.

The use of 'should' in the sense of 'ought', of course, stands outside these distinctions.

Mathematicians who are writing in English are asked not to forget the dignity and traditions of the language. What they write purports to be English prose, even though symbols have replaced many of its words; it should be both readable and speakable as well as printable. Thus symbols such as '∴', '∵' or end-tags like 'q.e.d.', 'q.e.f.' are best left behind in the schoolroom. What they say can be as well said in plain English.

When 'with respect to' grows tedious by repetition, it need not be cut to 'w.r.t.', which is not current English. The preposition 'in' will generally serve. Thus 'continuity in x' leads on to 'differentiability in x' and 'differentiation in x'; 'reflection in a line' gives 'inversion in a circle', 'conjugate and pole and polar in a conic', and so on.

A hybrid formation like 'number-theoretic' can generally be avoided by saying 'of number theory', unless we can boldly use 'arithmetic' for the theory and therefore 'arithmetical' for the adjective. Alternatively the compound noun, like any other noun, may properly be used as an adjective: 'in function-theory language' or 'in the language of function theory' are equally admissible.

Subjunctives. The subjunctive survives with difficulty in current English speech, but it has perhaps more virtue in mathematics than elsewhere. It is appropriate in hypotheses and conditions. Formerly one wrote 'if P be a point', but now it comes naturally enough to put 'if P is a point', retaining the subjunctive for the less likely hypotheses 'if I were you', or 'if x were positive' leading to a contradiction.

The subjunctive is retained in conditional clauses either as the composite 'should be' or the starker 'be': 'the condition that the roots be real requires that Δ should be positive'.

Split infinitives are a contentious topic. But any splitting of the verb reads awkwardly, e.g. the separation of an auxiliary from its participle or infinitive, as in 'we may, if the series is uniformly convergent, integrate term by term'. Clearly the if-clause is better first or last.

Unattached participles are discouraged by the purist. Thus we ought not to write 'eating my supper, a friend called'. By analogy it is an error (though a very convenient error) to write 'differentiating in x, the series can be summed'. This is avoided if we write 'on differentiation' or perhaps 'by differentiation', or say 'differentiating in x we can sum the series'.

'Combined operations' seem to prefer a singular verb. Thus we write 'Differentiation, cancellation, and summation gives' not 'give'.

In changing practice some mathematical terms have shortened. Thus we say 'convergence' rather than 'convergency'; 'algebraic', 'geometric' generally replace 'algebraical', 'geometrical', and even 'arithmétic' (so accented), can be an adjective as in 'arithmetic mean'. These curtailments ease the rhythm of the sentence.

There are some handy words for picking out pieces of a mathematical formula. We speak of a *term* of a summation (finite or infinite) and a *factor* of a product. We can refer to the *right-hand side* of an equation, or (borrowing from the French) speak of the *first member*, *second member*, etc. This is convenient when there is a 'run' of equations $A = B = C = \dots$. We call a_{rs} an *element* of the matrix (a_{rs}) or of the determinant $|a_{rs}|$. We can term x the *argument* of $f(x)$ and it may be useful to distinguish the *parameters* a, b, c from the *variable* x in such a function as the hypergeometric $F(a, b; c; x)$.

In a product we may wish to distinguish a *numerical coefficient* or a *numerical multiplier* from an *algebraic factor*.

14. Preparing Copy

Authors can save time, vexation, and expense both to themselves and to the printer by careful preparation of copy. The principles and points of detail that have been set out in these pages represent the Oxford practice in the printing of mathematics. These will be followed in setting an author's manuscript unless sound and sufficient reasons have been established for some particular departure from them. It will accordingly be supposed that an author has already acquainted himself with them. With these as background some additional points need to be emphasized.

Confusable characters. Mathematical notation has of necessity called into service such a wide variety of alphabets and symbols that it is not surprising that editors and compositors are often perplexed to interpret characters written by hands schooled in many different traditions.

In Appendix B is shown a full normal range of alphabets and mathematical symbols either as contained in the standard keyboard layout or as available in the matrix case. By its use any desired letter or symbol can be indicated by its name or its number. This is an extreme precaution needed only in an occasional emergency, e.g. in proof correction. In Appendix A roman and greek alphabets and arabic numerals are shown, written in a hand in which care has been taken to give distinguishable form to characters often confused. These are recommended for imitation when an author fears that his normal script may lead to confusion or ambiguity.

Much of the ambiguity arises between elements from different 'alphabets', and this is at once avoided by use of the conventional modes of distinguishing these 'alphabets'. Thus italics, small capitals, and capitals are indicated by single, double, and triple underlining respectively, and where necessary, by adding 'ital.', 's.c.', and 'cap.' respectively in the margin. For an italic capital we combine the single and triple underlining and put 'ital. cap.' in the margin. When necessary, lower-case letters are distinguished from capitals and roman letters from italic by the respective marginal glosses 'l.c.' and 'rom.'. Bold-face type is indicated by a wavy line beneath and 'bold' in the margin. 'Bold-face' is the correct name and not 'Clarendon' which is a different sort of heavy type now largely discarded for this purpose. Arabic numerals are noted by 'fig.' in the margin. In symbols $\underline{a}$ (ital.), $\underline{\underline{\text{A}}}$ (sm. cap.), $\underline{\underline{\underline{\text{A}}}}$ (cap.), $\underline{\underline{\underline{\underline{A}}}}$ (ital. cap.), a (l.c., rom.), $\utilde{\text{a}}$ (bold), 1 (fig.).

These commonly recognized symbols will be extended in the Oxford use by $\times$ written under a Greek letter, 'cap.' being added in the margin for a Greek capital. Thus

$$\underset{\times}{\alpha}, \qquad \underset{\underset{\equiv}{\times}}{\Lambda}.$$

These conventions avoid the inconvenience of coloured inks or pencils, which are better kept for other purposes.

When, as is generally the case, an author's handwriting is uniform and distinctive, the indications mentioned above need only be given at the first few appearances of a doubtful character to establish its meaning in the compositor's mind—and, perhaps, as a reminder at a later re-entry if it has been unused for a number of pages.

Exceptionally an author who writes two different characters in scarcely distinguishable form should use the markings throughout.

As has been said, it is not necessary to underline italic letters appearing in obvious mathematical formulae. Still less is it right to underline *all* formulae, since figures and Greek letters, for example, are not indicated in this way. But isolated italic letters occurring in the text should always be underlined.

The following are the more notorious cases of confusion that authors are asked to guard against.

Greek and italic: α, a, d; γ, y, Y; ϵ, e; η, y; κ, k, K; μ, u; ν, v, r, τ; σ, o; χ, X; ω, w, W.

These are at once distinguished as

$$\underset{\times}{\omega}, \quad \underset{-}{w}, \quad \underset{\equiv}{W} \text{ (cap.), etc.}$$

Within the Greek alphabet itself we distinguish

$$\xi \text{ (xi)}, \qquad \zeta \text{ (zeta)}$$

by their names as here.

Numeral and italic:

$$0, \ o, \ O; \qquad 1, \ i, \ I, \ \text{typed l}; \qquad 2, \ 3, \ z.$$

We distinguish as

$$0 \text{ (fig.)}, \qquad \underset{-}{o}, \qquad \underset{\equiv}{O} \text{ (cap.), etc.}$$

Capital and lower-case: C, c; O, o; S, s; U, u; V, v; W, w; X, x; Z, z; Θ, θ; Φ, ϕ; Υ, υ.

These can be distinguished by exaggeration of size, addition of serifs, or as $\underset{\equiv}{U}$ (cap.), $\underset{-}{u}$; $\underset{\underset{\equiv}{\times}}{\Theta}$ (cap.), $\underset{\times}{\theta}$; etc. In some hands the pairs h, n; e, l; n, x; n, r; r, s; m, n may be confused. Only careful penmanship will avoid this.

Mathematical notation. The ‘upright rule’ | can be distinguished from the ‘solidus’ / by noting the name in the margin. Superior ^{1} may be mistaken for an accent, inferior $_1$ for a comma , as in x^1, x'; $x_1, x_,$. Distinguish as $\overset{1}{\vee}$ (fig.), $\overset{\prime}{\vee}$ (accent); $\underset{1}{\wedge}$ (fig.), , (comma). To separate F_n from Fn note $\underset{n}{\wedge}$ marginally for the former.

Typescript. Reasonably good manuscript is quite acceptable to the printer as ‘copy’, and has the great advantage of being the work of the author at first-hand, thus eliminating the possibility of errors inadvertently introduced in the process of copying. Copy typed by the author or by a typist with mathematical experience is, of course, less ambiguous than manuscript. Attention is, however, needed to make certain that the symbols and formulae left to be inserted with the pen have, in fact, been inserted. But, whereas the experienced compositor will decipher a difficult or poorly written manuscript of symbols and technical terms more successfully than a lay typist, he will almost certainly be misled by any mistake made in typing, e.g. w for ω. The author is therefore advised to give the work of such a typist the most careful supervision, with special attention to formulae and symbols. Typescript should be double spaced.

Special symbols. The contents of the normal mathematical keyboard (i) are shown in Plate V, the resources of the matrix-case (ii) on p. 12, and a list of available letters embellished with diacritics (iii) on pp. 91–95. It assists editors and printers when an author can indicate in a preliminary sheet which (if any) of the special characters in (iii) he has needed in his work and whether any of (i) do not occur in his work. He should also call attention to any special notational use of brackets.

New symbols ought not to be introduced lightly and unadvisedly. The cutting of the necessary matrices is both slow and expensive and may impose unwanted delays in printing. As an alternative authors should explore the possibility of combining existing symbols for their purpose or perhaps exploit the resources of other alphabets such as Hebrew or Gaelic (even Chinese characters may be permitted where available).

Where a novel symbol will have only a temporary use, it is best, of course, to build it up out of existing material or borrow it from some convenient fount. Where the new symbol promises to become an established element of mathematical notation, then the expense and long delay of getting the new matrix must be faced. In this case it is of no advantage that the new symbol could have been composed by hand; thus $\dot{+}$ is no better than $\boldsymbol{+}$. What now matters is that the new symbol should be both 'see-able' and 'say-able'. Here $\dot{+}$ is quite say-able but the dot is not very see-able. On the other hand $n!$ is a clear symbol and can be spoken either as 'factorial n' or (by others) as 'n shriek!'.

The inversion of symbols to form new symbols is not recommended since the inverted symbol does not conform to the line of type. ∇, which began as an inverted Δ, is now a special sort, and so is $\exists$.

Text-figures. Figures in the text are reproduced photographically from drawings supplied by the author. The linear dimensions are (approximately) halved in the process, so the drawings should be large enough to bear this reduction. This applies also to the thickness of the individual lines, and great care should be taken to ensure that this thickness is uniform. For a good result drawings should be in ink and on good paper. The diagrams are inserted into the text as blocks, which are expensive to make, and very difficult to correct. Accordingly there should be economy in their use, and a final careful check should be made that drawings and text agree before both are submitted to the printer. They should be as simple as will serve their purpose, avoiding excessive detail; in particular they should not carry explanations, equations to curves, and so on. It should be sufficient to identify curves or details of the figure by numbers or other simple symbols, these, and any other necessary lettering being left in pencil, or, even better, placed on a self-locating transparent overlay. The necessary explanations can be given as 'underlines': that is, written in the copy under the drawing together with the number and title of the figure. They will then be set up in type together with the title in the ordinary way. Since captions for figures are set in small type, and therefore on a different machine from the main text, it is an advantage if the author supplies a separate list of underlines which will then be set up as a whole, the distribution to the appropriate figures being the task of the 'maker-up'. It is important to indicate in the manuscript the position at which the figure should appear. The printer will then insert it at the nearest convenient place.

Figures are, of course, numbered consecutively in an article. In a book they can be numbered in one series throughout the book, or by chapters. The former is preferred for the convenience of all except the author. Line blocks should be in a separate series from half-tones. The latter are then printed as separate plates. No figure, however insignificant, should be left unnumbered.

Tables. Most authors who prepare tables of functions have determined ideas about their layout. In tables giving numerical values of a mathematical function, usually to a certain number of significant figures at regular intervals of values of the argument, modern practice is to omit vertical rules. The horizontal lines are usually printed in groups of five separated by a line of white. The argument may be in bold face as may the headings for the dependent variable. Numbers preceding the decimal point are printed only for the first of each group of five lines unless their value changes in this range. If a change takes place, the last unchanged number is repeated; the new number appears on the next line and is repeated at the beginning of the next group of five: change of sign is dealt with in a similar way. When 'differences' have to be printed, first differences are placed on a line half-way between the lines of entry of the table, second differences are placed on the same line as the middle one of the three entries concerned.

Descriptive tables which set out the results of experiments are printed quite differently. Here there is no functional relationship between the values of successive entries. All entries have therefore to be given in full and both horizontal and vertical rules have frequently to be called into play to demonstrate clearly the relationships of the various quantities involved. Even here it is well to keep the number of rules to the minimum, since each rule whether horizontal or vertical has to be cut to its exact length and fitted in by hand.

Numerical results in tabular form present little difficulty so long as they can be got into the page either upright or sideways. Column headings should be concise enough to fit easily into the column; if not, the columns should be lettered, numbered, or otherwise indicated, and the information given below the table against the appropriate letter or number.

Certain corrections are expensive to make. If a correction widens a column, the space has to be taken from other columns, and this means recutting every horizontal rule and much more. Similarly, insertion of a new line necessitates recutting all the vertical rules. Care should therefore be taken with the copy to make such corrections unlikely. In particular, where a line of space is to be used to break up the column of figures, as in tables of logarithms, this should be made quite clear in the copy.

Final precautions. The author should check his numberings of sections, references, and particularly of formulae and equations to be certain that they run consecutively with neither gaps nor repetitions. He should also see that such numbers when quoted in the text are quoted correctly. It is advisable to do this as late as possible since numbering may go astray at last-minute revisions of the text. To avoid much renumbering of formulae a gap can be filled by assigning a number to an unimportant form and repetition corrected by 'twinning' a number with (*a*), (*b*), etc.

When the copy is not in typescript, it is helpful to repeat unusual names and words thought doubtful in block capitals either directly underneath or in the margin. We are most of us conscious of some of the deficiencies of our own handwriting.

15. Correction of Proofs

Proofs are sent out to authors as the work proceeds. Normally one is marked for return, one is included for the author to keep (or for each author if more than one), and one 'shuttle' (for use between authors). If more than this number are needed, e.g. if Dr. X who is helping with the correction needs a set, they should be asked for early enough to be pulled with the others. It will involve delay and additional expense if extra proofs have to be pulled separately. It should be noted that such a request is taken to mean that only *first* proofs are to be seen by Dr. X: revises will not generally be sent.

It is good practice to make all corrections on the kept proof in the first place and then, when the author is finally satisfied that the corrections are to his liking, to transfer them to the clean 'return' proof.

The signs used and the general manner of correcting mathematical proofs are indicated on pp. 88–89. Here are shown a supposed mathematical proof with corrections marked and the same passage printed after correction.†

The English practice‡ as shown here is to mark the error in the text and write the correct form in whichever margin is more convenient. Corrections, separated by upright strokes, are written in the order in which the errors occur in the line of type. It is unnecessary to invent pairs of quaint signs to link error and correction. This puzzles rather than helps the compositor who is used to the English method. Exceptionally, an error may be joined to its correction by a continuous line.

Many of the conventions used in correcting will have been mentioned in the relevant sections above. It should be noted here that full points and colons are ringed for clarity; that superiors are 'raised' by a $\surd$-shaped mark and inferiors 'depressed' by a ⌃ mark or its reflection. For double

† In case the authorship of the extract may be recognizable, it should be explained that the 'proof' was created by inserting the greatest variety of possible errors into a correctly printed passage.

‡ A British Standards publication (B.S. 1219: 1945, *Printers' and Authors' Proof Corrections*) contains a table of standard marks used in the correction of proofs. This table may also be obtained separately on a card.

superiors and double inferiors these marks are doubled; they are suitably combined for inferior in superior or superior in inferior. When a character has been wrongly printed as a superior or inferior and it is desired to restore it to its normal position, this is done by writing the character in the margin without added marks. The effect of the several spacing signs and their normal use are explained in Chapter III.

Only the actual letters or symbols that need to be changed should be marked and their corrections written in the margin (not, of course, in the text itself); exceptionally, in a difficult or involved correction, the complete form desired may be added for safety. Corrections, of course, must themselves be legible. They should be large enough, and, for sharpness of definition, pen-and-ink is preferred to pencil or ball-pointed pen. When it is evidently some trick of the author's hand that has baffled the compositor, the correction needs to be 'bigger and better'; or reference may be made to the form correctly printed elsewhere in the proof, or an erratic character may be indicated by reference to Appendix B as explained in § 14.

A proof may have queries inserted by an editor or a press reader calling attention to some doubtful point in a mathematical argument or apparent discrepancy of notation or numbering. These, of course, the author will deal with as well as correcting misprints or errors that he has overlooked in his copy. 'Second thoughts' by the author are not excluded, but, if they become expensive, they may be charged to him.

Authors should remember too that to destroy an elaborate formula that has cost the compositor time and effort to construct can be very discouraging to him.

Naturally corrections are preferred that make the least disturbance of the printed page; sometimes this can be conveniently done by a slight change in wording or in symbolism. A faulty sequence in numbering of equations or formulae can be most easily righted by numbering a formula needlessly if there has been an omission, or by suppressing an unquoted reference-number if there has been duplication.

When reference-numbers have to be changed, care must always be taken to correct these numbers wherever they are quoted in the text.

Some minor imperfections will appear only in proof. A displayed formula may print awkwardly in a way that could not easily be foreseen. Notation in the text may be broken across two lines: this can sometimes be corrected by a minor change of wording, or it can be displayed if it is long enough. A reference-number in the text falling at the end of a line may confuse the eye with the ordinary reference-numbers. This can generally be moved inwards or into the line below by change of phrase or of spacing.

In every case the constant aim of the printers, with the author's connivance, is impeccable printing.

Authors should be reminded that proofs are only rough indications of the final result, showing what will appear rather than how it will appear. They are pulled without elaborate preparation, not on the best paper; the inking may be uneven and there will be many points about them not to be tolerated in the printed copy.

The tinier pieces of notation such as fractional indices or double subscripts often suffer from this imperfection of the proof, but this need not mean that they will not be perfectly printed in the outcome. Where they are actually invisible or quite illegible, the correct form should be repeated for safety. This apart, the finer points of typography can well be left to a Press jealous of its reputation. The final printing, it should be remembered, will be on good paper, with the type carefully locked into the forme, the machines accurately adjusted, and the inking carefully watched. It may be encouraging, when faced with rough proofs, to recall the appearance of the finished mathematical work of the Press.

Proofs should, of course, always be returned as quickly as possible. The Press is as anxious as the author to proceed to publication. There is no need to return the copy with the proofs. This may be a consideration when, as often, proofs travel by air-mail. In the case of a book the author should return the proofs piecemeal as he finishes with them. The printers will then proceed with the work of revising and imposition, so that an author may be receiving revises of the first sheets before he has received the whole of the first proof. There are, however, considerations which justify delay in returning the early proofs of a book: if, for instance, there are many forward page references, it may be well to wait for further proofs so that page numbers may be inserted, and the proof as returned for revise reasonably complete.

Errata and corrigenda

If, in spite of the care of the author and the Press, errors are discovered in a finally printed article, subsequent 'Errata' or 'Corrigenda' are not, as a rule, published separately, on the ground that they are seldom likely to catch the attention of those reading the faulty original passages. When, however, an article is followed by a sequel on the same topic, this may well notice outstanding errors in the earlier article, whether of detail or substance. In the case of a book, errors discovered at a very late stage may be noted on an 'errata' slip inserted in the book between 'advance copy' stage and publication. This will, of course, delay publication slightly. Every effort should be made to render an 'errata' slip unnecessary.

16. Final queries and offprints

The last piecemeal bits of printing an author is likely to receive from the Press will be 'final queries' if he is writing a book or offprints if he is contributing to a periodical; and it is fitting to end with them.

'Final queries', as has been said in Chapter I, are the last proofs, pulled when the completed type has been locked into the chase. These are read by the mathematical reader himself, and this is normally his first acquaintance with the work whether in copy or in proof; earlier 'house' corrections on proofs sent to the author will have been made at lower levels. The mathematical reader is concerned in removing typographical blemishes, inconsistencies of notation, numberings that have gone astray, and technical faults generally that have escaped previous readings. But his scrutiny may also discover real (or apparent) mathematical inconsistencies of argument or calculation. These and such other presumed errors that require the author's attention are addressed to him for his immediate and final decision as 'final queries'. These are not to be regarded as author's 'proofs' and they are certainly not to be understood as an invitation to the author to begin thinking again. The opportunity for that passed with his last proofs. The delicacy and danger of correction when the type is in chase cannot be overemphasized; putting one thing right may inadvertently put several other things wrong. However a misprint (or a 'miswrite') that happens to be noticed in the 'final queries' should be marked.

In a periodical final queries are sent to an editor and are not seen by an author unless, desperately, he has to be consulted on some obscure point of his notation or terminology that cannot be resolved by the combined wisdom of available experts. Otherwise his last contact with the press is the receipt of offprints. The printer has explained why this must usually occur after what strikes the author as a terrible delay: to set up any more rapid system would entail very considerable expense. In one writer's experience offprints seem seldom to arrive until the author has begun to blame himself for some blunder in filling up the necessary form. Perhaps this admission may comfort others.

T. W. C.
P. R. B.

III. RULES FOR THE COMPOSITION OF MATHEMATICS AT THE UNIVERSITY PRESS, OXFORD

I. GENERAL

1. **General directions for composition** are laid down in *Rules for Compositors and Readers* (36th ed., Oxford, 1952).

2. **Indention** is to follow House style.

3. **Make-up.** In making up into pages avoid where possible breaking italic enunciations, etc., at the end of a page: but correct page-length must be maintained.

4. **Connect main paragraph** and following dependent sub-paragraphs, where necessary, by colon without rule.

5. **Numbering paragraphs.** It is impracticable to lay down definite rules for numbering paragraphs and subdivisions of paragraphs. *Copy must be followed in this respect.*

   Where the letters (*a*), (*b*), (*c*), etc., are required for sub-paragraphs they are to be printed in italic; lower-case numerals (which are to be preferred) always in roman.

6. **Section sign** (§) is to be used when a section is referred to in the text, but is to be *omitted* before section numbers at the head of a section.

7. **Numbered theorems, formulae, and equations.** Print theorem numbers at beginning of paragraphs to left; formula or equation numbers in roman in parentheses full out to right.

8. **References to formula and equation numbers** in the text are to be in roman and enclosed in parentheses, e.g. (1.42). Avoid, wherever possible, putting these at the end of a line in the text.

9. **References to books and articles** should be printed thus:

   J. C. Burkill, 'On Mellin's inversion formula', *Proc. Cambridge Phil. Soc.* 23 (1927) 356–60.

   Proc. London Math. Soc. (2) 33 (1931) 22–31.

   E. W. Hobson, *Theory of Functions of a Real Variable*, 2 (Cambridge, 1927) 57.

In certain journals and certain series of monographs the volume number is printed in bold-face type, all other details of style being as given here. The Royal Society and the Royal Astronomical Society have their own styles, and some authors prefer to adopt these. Where this is consistently done, copy is to be followed.

10. Reference marks. Print †, ‡, §, ||, then, if necessary, ††, ‡‡, etc. The symbols †, ‡, etc., should be attached to words rather than to mathematical symbols. Where attached to the latter in copy the reader should query.

11. Letters used as symbols, unless otherwise marked in copy, are to be always in italic, whether occurring in roman or italic matter, or in bold-face cross-lines.

Example: THEOREM III. *If* $a(z) = \sum a_n z^n$ *and* $b(z) = \sum b_n z^n$ *are uniform in the*, etc.

Where letters used as symbols are specially marked to be bold face, roman, script, etc., the markings should be followed and these letters, too, should be always printed in the same fount, whether occurring in roman or italic matter, or in bold-face cross-lines.

12. Kerned letters. Use kerned sorts of $C\ D\ E\ F\ H\ I\ J\ K\ M\ N\ P\ S\ T\ U\ V\ W\ Y\ \Gamma\ \Psi$ P when followed by inferior suffixes only:

Examples: $P_j \quad P_{l-1}$

but use the ordinary fount capital when followed by a superior over an inferior:

Examples: $P_j^l \quad P_m^{l+1}$

13. Solidus. In order to avoid uneven spacing between lines and to facilitate setting by machine, simple fractions occurring in current text may be replaced by forms using the solidus, e.g. print

$$a/b \text{ not } \frac{a}{b} \qquad dy/dx \text{ not } \frac{dy}{dx}.$$

This applies also to forms such as $\frac{x}{3}$, $\frac{y^2}{4}$, which may be printed $x/3$, $y^2/4$; but in these cases it is neater to print $\frac{1}{3}x$, $\frac{1}{4}y^2$. Always print $\frac{1}{2}\pi$, $\frac{1}{4}\pi$, etc., for $\frac{\pi}{2}$, $\pi/2$, $\frac{\pi}{4}$, $\pi/4$, etc.

The solidus may also be used in cases such as $e^{\frac{a}{b}}$, $e^{\frac{a}{2}+\frac{b}{3}}$; these are best printed $e^{a/b}$, $e^{\frac{1}{2}a+\frac{1}{3}b}$.

14. Brackets. An author's sequence of brackets must be followed by the compositor, even if against that usually adopted, which is [—{—(—)—}—].

15. Abbreviations sinh (not sin h), cosh (not cos h), sin, cos, re, im, exp, max, min, lim, sgn, mod, log, ln, sn, dn, cn, are to be printed in roman and without a full point. This rule applies generally to any functional symbol which consists of more than one letter (usually the abbreviation of a word or the initials of two or more words). (For the spacing of a 'mod-formula' see Rule 35.)

Examples:

$$\sinh t \qquad \cosh t \qquad \sin x \qquad \mathrm{re}\, e^{x+iy} \qquad \max(u, v) \qquad \mathrm{sn}\, u$$

Use roman in the same way for inferiors:

Examples: f_{max} $\quad y_{\mathrm{min}}$ $\quad p_{\mathrm{av}}$ $\quad v_{\mathrm{crit}}$

16. Superior with inferior. Where both superior and inferior follow a letter, the inferior is to be printed immediately under the superior.

Examples: P'_1 $\quad \theta_1^{n-1}$ $\quad \psi_{n-2}^2$

17. The accent (or prime). The superior in x', y' is almost always an accent. It is rarely a figure 1, except in special studies (e.g. Differential Geometry), when it should be so marked on copy.

18. Root sign. Where $\surd$ is followed by a single symbol or number the rule should not be printed. (For size to be used see Rule 21.)

Examples:

$$\surd n \quad \surd 2 \quad \surd\frac{a}{b} \quad \surd\frac{a+b}{b+c} \quad \text{not} \quad \sqrt{n} \quad \sqrt{2} \quad \sqrt{\frac{a}{b}} \quad \sqrt{\left(\frac{a+b}{b+c}\right)}$$

$$\surd 135 \text{ not } \sqrt{135} \qquad \surd(-135) \text{ not } \sqrt{-135} \text{ or } \surd{-135}$$

$$\surd|x-y| \text{ not } \sqrt{|x-y|}$$

If the omission of the rule may introduce ambiguity it must be replaced by brackets (parentheses, braces) to show where the root begins and ends.

Examples:

$$\surd(a+b) \text{ not } \surd a+b \text{ for } \sqrt{a+b}$$

$$\surd\{x+(y+z)/w\} \text{ not } \surd x+(y+z)/w \text{ for } \sqrt{x+(y+z)/w}$$

Sometimes brackets are printed in two-line workings when they are not strictly necessary, e.g. $\surd\left(\frac{a}{b}\right)$, $\surd\left(\frac{a+b}{b+c}\right)$ (cf. above), but never print $\surd(n)$, $\surd(2)$.

19. Fractions. The size of a *numerical* fraction is to be governed usually by that of the symbol immediately following. When fractions are formed with italic or Greek letters, always use the large fractions.

Examples:

$$\tfrac{1}{2}\gamma\frac{\partial\phi_j}{\partial x_k} \qquad \tfrac{1}{2}\log\frac{\phi(-1)}{\phi(1)} \qquad \frac{1}{2}\sum\frac{1}{m+1} \qquad \tfrac{1}{2}\int y\,dx$$

$$\frac{1}{4}\sqrt{\left(\frac{2}{\pi}\right)} \qquad \frac{1}{2}\left(\frac{\partial\phi_j}{\partial x_k}\right) \qquad \frac{1}{2}-\frac{x}{a} \qquad \frac{2}{\pi}\int\limits_0^\infty J\,dt$$

$$\frac{1}{\pi a}\sum_{m=0}^{\infty}\frac{1}{m+1} \qquad \sqrt{\left(\frac{2}{\pi}\right)}\Gamma(\tfrac{1}{2}+it)\cos(\tfrac{1}{4}\pi+\tfrac{1}{2}it)$$

Sequences of fractions occurring in the same line should be of the same size.

Example: $\frac{1}{2}, \frac{1}{3},\ldots, \frac{1}{n}$ not $\tfrac{1}{2}, \tfrac{1}{3},\ldots, \frac{1}{n}$

20. Compound fractions. The rule dividing the main fraction is to be thicker than those separating the lesser.

Example:

$$\frac{\dfrac{a+b}{z}+x}{b+y} \quad \text{but better as} \quad \frac{(a+b)/z+x}{b+y}$$

21. Symbols $\int \sum \prod \surd$ and brackets ( ) { } [ ]. The size of the symbols is to be varied according to the depth of the formula to which they apply.

Use full face $\surd$ before full face letters, brackets, parentheses, symbols, etc.

Use full face ( ) when they are used outside a modulus sign: e.g. $f(\sin|x|)$.

In single-line undisplayed workings in the text use 11-pt. $\int \sum \prod \surd$ and brackets.

In displayed formulae

(*a*) use 11-pt. $\sum$ and 18-pt $\int$ with single-line workings, and generally

18-pt. $\sum$ and 24-pt. $\int$ will be used if preceded or followed by two-line fractions.

Examples:

$$\int_0^T u^{-\frac{1}{2}+it} \qquad \sum_{s=1}^{\infty} s^{-2}Q_s$$

$$\int_0^{\infty} \frac{|f(x)|}{x^2}\,dx \qquad \int f(x)\,dx \int \frac{F(y)}{y^2}\,dy \qquad \sqrt{\left(\frac{2}{\pi}\right)} \int_0^{\infty} \cos xu\,du$$

$$2\sum_0^{\infty} \frac{(-\frac{1}{2}x^2)^n}{n!} \qquad E = \pi x \sum_{\nu=0}^{\infty} p\nu + \sum_{\nu=0}^{\infty} \frac{\nu}{\nu^2+1}$$

(*b*) the largest symbol $\int$ used in any line is to be used *throughout that line*. For this purpose a working broken by the compositor for lack of space is to be treated as a single line.

Example: $$\int u(x)\,dx + 2\int \frac{P(x)}{Q(x)}\,dx = \int R(x)\,dx$$

(*c*) where $\sum$ is followed by other $\sum$'s without intervening +, —, etc., these $\sum$'s must be all of the same size, viz. that of the largest needed.

Example: $$\sum_m mP_m = \sum Q_m \sum \frac{1+m}{1-m} R_m$$

(*d*) Subject to Rule (*b*) the two sides of an equation (or inequality) are regarded as being independent, so that in a case such as the following 18-pt. $\int$ would be used in spite of the 24-point working on the right-hand side.

$$\int_0^x u^{1-s}e^{2\pi i\nu u}\,du = O\left\{x^{1-\sigma}\left(\frac{t}{x^2}\right)^{-\frac{1}{2}}\right\}$$

The size of $\prod$ used in any working follows the same rule as for $\sum$.

[*These examples are in* 11-*pt. Where the matter is in larger or smaller type symbols of appropriate size will be used.*]

22. Limits. Print limits of integration, summation, and product above and below the symbols $\int$ $\sum$ $\prod$ and not at the side.

With integrals the limits above the $\int$ are to be centred above the top 'hook', and those below under the bottom 'hook', and not on the centre of the body.

Examples: $$\int_0^{\infty} \qquad \int_0^{\rho} \frac{n(u)}{u} \qquad \sum_{m=1}^{n} \qquad \prod_{r=1}^{m}$$

When limits occur above and below $\int \sum \prod$ the symbols preceding and following are to stand clear of these limits.

Examples:

$$-\frac{i}{x}\int_{R}^{2R-t} e^{-ix} \qquad \bar{P}(x) = \sum_{0\leqslant n\leqslant N}{}' r(n)-\pi x$$

In 'terms at the limits' represented by $[\,]_a^b$ the limits b, a should be separated vertically by a wider space than in (say) P_a^b; they should extend above and below the brackets.

23. Miscellaneous.

Print etc. (not &c.) ...as in Lemma 3. ...to Theorem 1.

x-axis z-plane nth (but *n-th* when in italic line) zeros

mil mol (both without full point).

Omit commas in the phrase 'if and only if'.

Example: ...is true, if and only if x is...

Use $\geqslant$ $\leqslant$, not $\geq$ $\leq$ or $\geqq$ $\leqq$; $\gtreqless$ not $\gtrless$ or $\gtrless$.

II. FORMULAE

24. Undisplayed formulae (that is, formulae run in as part of the text) must never be broken at the end of a line. Where a break is unavoidable, the formula should be displayed and the text continued full out on the following line. If the formula is short it should be got in or turned over (i.e. *not* displayed). In case of difficulty consult the Mathematical Reader. The word 'formulae' here includes forms like

$$x_n\ (n = 0, 1, 2,...) \quad \text{or} \quad x_1, x_2, x_3,..., x_n$$

which should be got into one line or displayed.

25. Displayed formulae or equations are those which are placed in a separate line from the text

either, (*a*) by the author, in order to stress their importance or because of their complexity,

or, (*b*) by the compositor, in accordance with Rule 24 above.

A single displayed formula should always be centred on the page. Successive lines of displayed matter should as a rule be centred independently of each other.

Example:

$$y_1+1 > 0, \quad \ldots, \quad y_n+1 > 0,$$
$$(y_2+1)\ldots(y_n+1) > 2^n.$$

As a general rule groups of EQUATIONS should be ranged on the equals signs, the type being set to give a generally central effect. This can frequently be done by centring the longest line on the page.

Example:

$$Y+iX = 2\pi\rho\Big\{cA_0+A_0 B_1+i\sum_{n=1}^{\infty} n(n+1)A_n B_{n+1}\Big\},$$
$$M = 2\pi\rho Ri\Big\{cA_1+\sum_{n=1}^{\infty} n^2 A_n B_n\Big\}.$$

In other cases the block of formulae must be centred as a whole as in (1) on p. 82.

In a succession of equations or inequalities with more than one equals or inequality sign in one line the ranging should be broken, otherwise the presence of the second sign is masked.

Example:

$$|z+t|^2 \geqslant |z|^2+t^2-2|z|t\cos\tfrac{1}{2}\delta = (|z|+t)^2-2|z|t(1+\cos\tfrac{1}{2}\delta)$$
$$= (|z|+t)^2-4|z|t\cos^2\tfrac{1}{4}\delta \geqslant (|z|+t)^2\sin^2\tfrac{1}{4}\delta$$

Breaking of formulae. Where a displayed formula or equation is too long to come into the measure, it must be broken and continued on the next line.

As a general rule it is better to break at any of the signs $= > < \geqslant \leqslant$. Avoid where possible breaking bracketed formulae. In cases where such a break cannot be avoided, the formula must be broken at the end of a mathematical 'phrase', i.e. generally at a major bracket if possible.

Example:

$$J = \tfrac{1}{2}P\operatorname{cosech}\gamma\pi[\gamma(\gamma^2+1)\{(\alpha^2+\beta^2)^2-2(\alpha^2-\beta^2)+1\}]\times$$
$$\times\Bigg\{(a+b)\{(\alpha^2+\beta^2)^2+2\gamma^2(\alpha^2-\beta^2)+\gamma^4\}+$$
$$+d\Bigg[\{(\alpha^2+\beta^2)^2-2(\alpha^2-\beta^2)+\gamma^4\}+\frac{\gamma^2+1}{\alpha^2+\beta^2}(3\alpha^2-\beta^2-1)\Bigg]\Bigg\}^{-1}$$

In a displayed formula which is broken it is necessary to give a warning if the formula is not complete where broken.

For instance, suppose $x+y+z$ is to be broken after y. Since $x+y$ might be complete in itself the $+$ must be added at the break on the first line to warn the eye that there is more to follow. The symbol should then be repeated before the continuation in the next line. Suppose, however, the formula to be divided is $x+y = z$, again to be broken at y. Since $x+y$ is now complete there is no need for warning and the $=$ is therefore put only at the beginning of the second line.

For this reason $+ \; - \; \times \; \div$ are to be repeated, while $= \; > \; < \; \geqslant \; \leqslant$ are to be printed only at the beginning of the second portion of the broken formula. A few examples are given:

$$I(l, m, s, \phi) = \frac{\pi^s p^{-s} q^{-s+n/\lambda}}{\Gamma(s)\Delta_\phi^{1/T}} \int_0^\infty \exp[-\pi q t \Phi(l_1 \ldots l_n) -$$

$$-\pi\phi(m_1 \ldots m_n)/tp] t^{-s-1-n/\lambda}\, dt.$$

$$-\frac{1}{\pi}\cos\tfrac{1}{2}(\mu-\nu)\pi K_\mu(x)K_\nu(x)$$

$$= \sin\tfrac{1}{2}(\mu+\nu)\pi \int_0^\infty J_{\mu+\nu}(2x\sinh t)\cosh(\mu-\nu)t\, dt +$$

$$+\cos\tfrac{1}{2}(\mu+\nu)\pi \int_0^\infty Y_{\mu+\nu}(2x\sinh t)\cosh(\mu-\nu)t\, dt.$$

$$I_1 = \frac{e^\xi}{\xi} - \frac{e^\xi}{\xi}\sum_1^{2m-1} e^{-p\xi/m}\left\{\phi\left(1-\frac{\phi-1}{m}\right) - \phi\left(1-\frac{p}{m}\right)\right\} -$$

$$-\phi\left(-1+\frac{1}{m}\right)\frac{e^{-\xi}}{\xi}.$$

$$X^{s-1}f_1(X) - (s-1)X^{s-2}f_2(X) -$$

$$-\delta^{s-1}f_1(\delta) + (s-1)\delta^{s-2}f_2(\delta) +$$

$$+(s-1)(s-2).$$

A formula broken between two bracketed expressions such as

$$(a+b+c)(x+y+z)$$

must be printed with the multiplication sign $\times$ at the end of the first and the beginning of the second lines.

Example:

$$(a+b+c)\times$$

$$\times(x+y+z)$$

Indention of broken formulae should be such as to give where possible a central effect (i.e. a fair overlap in the centre of the page).

Example:

$$\int_0^\infty \{l_r(ax)J_s(bx)+J_r(ax)l_s(bx)\}\,dx$$
$$= \int_0^\infty \{m_r(ax)J_s(bx)+J_r(ax)m_s(bx)\}\,dx = 0$$

is preferred to

$$\int_0^\infty \{l_r(ax)J_s(bx)+J_r(ax)l_s(bx)\}\,dx$$
$$= \int_0^\infty \{m_r(ax)J_s(bx)+J_r(ax)m_s(bx)\}\,dx = 0$$

A broken FORMULA at the end of a page must be got in or turned over (i.e. it must never be split across the two pages).

A broken EQUATION or inequality ($= \; > \; < \; \geqslant \; \leqslant$) should not as a rule be split across two pages.

It is less objectionable (and especially if the two pages face) in a series of equalities such as

$$\begin{aligned} r_1 r_2 \cos\theta &= x_1 x_2+y_1 y_2+z_1 z_2 \\ &= r_1 l_1 r_2 l_2+r_1 m_1 r_2 m_2+r_1 n_1 r_2 n_2 \qquad (1) \\ &= r_1 r_2(l_1 l_2+m_1 m_2+n_1 n_2). \end{aligned}$$

Where the text finishes as a short turn-over line followed by a short displayed formula, the latter is not to be printed as a separate line but centred and let up into the previous half-line.

Example:

If we consider the same direction as positive throughout, it is clear that $$pq+qr+rs = ps.$$

In cases such as the following, where sufficient space intervenes between the letterpress and the displayed working, centre the latter in the same line as the letterpress introducing it.

Hence $$f(x+h) = 0,$$

and $$f(x+h) = f(x)+hf'(x).$$

We have $$\frac{4\sinh^2(\frac{1}{2}z)}{z^2} = f(z).$$

III. SPACING

26. The spaces in use are:

Thin space	=	3 units	indicated thus ∧
Thick space	=	5 ,,	,, ,, ↑
En quad	=	9 ,,	,, ,, ≙
Em quad	=	18 ,,	,, ,, ⇑

Where no guidance as to spacing is offered in the manuscript the following rules are to be observed:

27. No space should be used

(*a*) between a number and the symbol it multiplies.

Examples: $2a$ 4θ

(*b*) between two symbols which together represent a product of the individual symbols.

Examples: ab θy $2xz$

Exception: See Rule 29 (*b*).

(*c*) before or after the symbols $+ \; - \; \pm \; \mp \; \times \; \div$.

Examples: $a+b$ $x-2y$ $ab\times xy$

Exceptions:

(i) In such instances as $\cos x + \cos y$ where the addition is '$\cos x$' + '$\cos y$' (and not $x+\cos$).

(ii) Where this rule becomes subject to a more important spacing rule, such as 30 (*c*).

Examples:

$$\int f(\theta)\, d\theta + C \qquad \int \phi(x)\, dx - \int f(y)\, dy$$

(*d*) in the grouped abbreviations logloglog, logcos, etc.

Examples: $\mathrm{logloglog}\, x$ $\mathrm{logcos}\, \phi$

(*e*) before and after superiors, parentheses (except as 29 (*d*) below), braces, brackets, and vertical rules. (See Rule 29 (*b*) for inferiors.)

Examples:

$$\sin^2 A \qquad \sin(A+B) \qquad y\{M_\nu(bx)\cos\alpha - J_\nu(bx)\sin\alpha\}$$

$$2|\log R(z)|+|\Theta(z)|$$

(*f*) after , in inferiors and superiors.

Examples: $A_{y,p}$ $L_{m.n}$

(*g*) in limits above and below the symbols $\int \sum \prod$.

Examples: $\sum_{m=0}^{\infty}$ $\prod_{r \geqslant 1}$

(*h*) between multiple integrals.

Example: $\int\int\int$

28. Thin space should be used between figure and letter where a, b, etc., are added to formula numbers. Also in the numbering of figures, tables, etc., where a letter is used with a number, whether or not the letter is in parentheses.

Examples:

Equations (7 a), (7 b) Fig. 3 (*a*) Table II (*b*)

29. Thin space should also be used

(*a*) before and after abbreviations used (i) singly as sin, cos, log, exp, max, etc., (ii) in groups as logloglog, logcos, etc.

Examples: $2\,\sin\,\theta$ $2\,\sin^2\theta$

Exception: If preceded or followed by brackets (of any shape) or vertical rules follow Rule 27 (*e*) above, as $\sin(a+b)$.

(*b*) after inferiors, unless followed by brackets (of any shape) or vertical rules, to set off the *apparent* space preceding the inferiors. (See Rule 27 (*e*) for superiors.)

Examples: $\phi_s\,\psi_r$ $\phi_s\,a_r$ $f_x(x,y,z)$

(*c*) before *f*, *F*, ϕ, ψ, etc., when employed as functional symbols, e.g. $f(x)$, $\phi(x,y)$, $\psi(x_1, x_2,...)$ except when figures, parentheses, or indices precede them.

Examples:

$\sin \tfrac{1}{2}s\pi\,f(x)$ $\sin(\tfrac{1}{2}s\pi)f(x)$ $4f(mn)$

(*d*) after *o* when the symbol $o\,(x)$, $o\,(N)$, or such combination of lower-case *o* and another symbol stands alone. This applies to lower-case *o* (and not cap. *O*).

Examples: $o\,(h)$ $O(1/x)$

(*e*) after the , in coordinates of points, etc.

Examples: (x,y) $(1,1)$ (a,b,c)

(*f*) before and after dx, dp, ds, or similar combinations of d with another symbol when used without the sign $\int$.

Example: $y\,dx-x\,dy = 2dS$

(*g*) in such cases as

$x^2\sin x$ $x^m\log x$ $\sin^{-1}\alpha$ $\sin^{-1}2$ $\cos^2\tfrac{1}{2}x$

(*h*) after ! in such expressions as

$$\frac{n!}{r!\,(n-r)!}, \qquad \frac{1}{3!\,6!\,8!\,9!}$$

(*i*) before 'back' inferiors.

Example:

$${}_1\Phi_0(a;rE)\theta_0 = G(pax;px)\,{}_1\Psi_1(a;pax;xQ)\theta_0$$

30. Thick space must always be used

(*a*) before and after $= \neq \equiv \not\equiv \doteqdot \triangleq \sim \approx \simeq \cong \rightarrow < > \leqslant \geqslant \ll \gg \nless \ngtr \prec \succ \propto \barwedge \subset \supset \in \bar{\in} \notin \subseteq \supseteq$.

Examples: $x = 0$ $\alpha \geqslant 1$ $x+y = -z$

(*b*) before and after the symbols of integration, summation, and product $\int \sum \prod$.

Examples:

$$\int kx^2\,dx \qquad \tfrac{1}{2}\int x^a\,dx \qquad \sum a_n b_n \qquad x^m\prod_s(\delta+b_s)z$$

(*c*) before and after ds dp dx, and similar combinations of d with another symbol, in integrals (i.e. following $\int$).

Example: $\int f(x)\,dx$ but print $\iint zr\,d\theta dr$

(*d*) after comma in sequences of fractions such as

$$\frac{x}{a},\ \frac{y}{b},\ \frac{z}{c},\ldots$$

which would take a thin space if height were 11 point instead of 24 point.

(*e*) also in sets of symbols if displayed:

Example: $x_1,\ x_2,\ldots,\ x_n$

31. Em space must be used where possible (and in any case not less than a thick) before a parenthetic definition.

Examples:

$$L_p \quad (1 < p < 2) \qquad F(w) = O(w^\lambda) \quad (w \to \infty)$$

Parentheses should not as a rule be omitted. Query on proof if so omitted.

No comma is needed with parentheses, i.e. not L_p, $(1 < p < 2)$.

Use em space between a formula and a verbal explanation.

Examples: $b \equiv 0 \pmod q \quad$ for some q.

$$E_n(t) \sim e^t/t \quad \text{as } t \to \infty.$$

32. Two-em space or more is to be placed between two separate equations or inequalities in the same line.

Example: $m^2 - 2l^2 = M, \qquad -3l^2 > N$

33. Grouping symbol. Use full point on nut body (with no extra space each side) as a grouping symbol.

Example: $$\frac{dT}{dr} = \mu_1 \rho c.4\pi^2 n^2 r^2$$

34. Enumerations are to be set up in the following manner when displayed:

$$r = 1, 2,..., n \qquad (r = 0, 1,..., m-1)$$

Use ordinary full points with no space before or between.

In current text ordinary line space should be used instead of thin space: compare the last example of Rule 36.

35. Congruences are to be printed with thick space before the parenthesis and thin space after mod.

Example: $x \equiv a \pmod n$

36. Functions of many arguments should be printed thus:

Examples:

$$f(\xi_1, \xi_2,..., \xi_n) \qquad f(a, b, c,..., n) \qquad F(a, b; c; x)$$

If commas are systematically omitted in copy (e.g. in Differential Geometry) they are not to be inserted, but use a thick space instead of a thin.

Example: $$f(x^1\ x^2 \dots x^n)$$

Where the symbols occur as separate variables in the text use commas and the ordinary line space:

Example: Let $f_1, f_2, \dots, f_N$ be the most general polynomials of degrees $n_1, n_2, \dots, n_N$ respectively.

In all cases use ordinary full points with no space before or between.

MARKS USED IN CORRECTION OF PROOFS

On the following two pages are shown a supposed mathematical proof with marks of correction, faced by the page as it would appear after the corrections have been made.

[THE CORRECTED PROOF]

5. *Exceptions to the rule*

The analogy between the error term for $n(\psi)$ and the order of the fourier coefficients does not by *any* means always hold. This is because the behaviour of $\phi(t)$ in the middle of the interval $(-1, 1)$ has little effect on $|f(z)|$ when $|x|$ is large. In Fact we have

Theorem XIII. *If* $\phi(t)$ *is continuous in* $(-1, -1+\eta)$ *and* $(1-\eta, 1)$, then *the results of Theorems X, XI, and XII hold, proided that we replace* $\psi(1/\rho)$ *in each result by*

$$\omega\left(\frac{1}{\rho}\right) = \text{Max}\left\{\omega\left(\frac{1}{\rho}\right); \frac{\log \rho}{\rho}\right\}.$$

For $x > 0$ we write

$$f(z) = \int_{1-\eta}^{-1} e^{zt}\phi(t)\, dt + \int_{1-\eta}^{1} e^{zt}\phi(t) dt = I_1 + I_2. \quad (5.1)$$

As in 4.2

$$I_2 \ne \frac{e^z}{z}\left\{1 + O\left(\frac{r\omega(1/r)}{x}\right)\right\}. \quad (5.3)$$

It follows from Lemma *A* that

$$I_1 = o\{e^{(1-\eta)x}\} = o\left\{\frac{e^x}{r} \cdot re^{-\eta x}\right\} = O\left\{\frac{e^x}{r}\right\},$$

for $x = (\log r)/\eta$, and *a fortiori*

$$I_1 = O\left\{\frac{e^x \log r}{rx}\right\}, \quad (5.3)$$

for $x \geqslant (\log r)/\eta$. Combining 5.2 and (5.3) with (5.1), we have

$$f_1(z) = e_z\left\{1 + \phi\left(\frac{r\omega_1(1/r)}{x}\right)\right].$$

The result follows as before with ω_1 instead of ω. In order to obtain the results of theorems XI and XII we have in addition to use (4.32), *i.e.* we have to show that

$$R(z) < e^{H_{16} r\omega_1(1/r)} \quad (5.4)$$

for $|x| \leqslant 10Kr\omega_1(1/r)$. We only need consider the integral from $-1+\eta$ to $1-\eta$. For the usual arguments show that

$$|zI_2| \mp e^{H_{17} r\omega_1(1/r)}, \quad (5.5)$$

and a similar inequality holds for the integral from -1 to $-1+\eta$

It remains to give an example of a function for which the analogy fails. Consider

$$\psi(t) = \sum_{m=1}^{\infty} \frac{e^{2\pi i t}}{m^2}$$

5. Exceptions to the rule

The analogy between the error term for $n(\rho)$ and the order of the Fourier coefficients does not by any means always hold. This is because the behaviour of $\phi(t)$ in the middle of the interval $(-1, 1)$ has little effect on $|f(z)|$ when $|x|$ is large. In fact we have

THEOREM XIII. *If $\phi(t)$ is continuous in $(-1, -1+\eta)$ and $(1-\eta, 1)$, then the results of Theorems X, XI, and XII hold, provided that we replace $\omega(1/\rho)$ in each result by*

$$\omega_1\left(\frac{1}{\rho}\right) = \max\left\{\omega\left(\frac{1}{\rho}\right); \frac{\log\rho}{\rho}\right\}.$$

For $x > 0$ we write

$$f(z) = \int\limits_{1-\eta}^{-1} e^{zt}\phi(t)\,dt + \int\limits_{1-\eta}^{1} e^{zt}\phi(t)\,dt = I_1 + I_2. \tag{5.1}$$

As in 4.2
$$I_2 = \frac{e^z}{z}\left\{1+O\left(\frac{r\omega(1/r)}{x}\right)\right\}. \tag{5.2}$$

It follows from Lemma A that

$$I_1 = o\{e^{(1-\eta)x}\} = o\left\{\frac{e^x}{r}.re^{-\eta x}\right\} = o\left\{\frac{e^x}{r}\right\},$$

for $x = (\log r)/\eta$, and *a fortiori*

$$I_1 = O\left\{\frac{e^x \log r}{rx}\right\}, \tag{5.3}$$

for $x \geqslant (\log r)/\eta$. Combining (5.2) and (5.3) with (5.1), we have

$$f_1(z) = e^z\left\{1+O\left(\frac{r\omega_1(1/r)}{x}\right)\right\}.$$

The result of Theorem X follows as before with ω_1 instead of ω.

In order to obtain the results of Theorems XI and XII we have in addition to use (4.32), i.e. we have to show that

$$R(z) < e^{H_{16} r\omega_1(1/r)} \tag{5.4}$$

for $|x| \leqslant 10Kr\omega_1(1/r)$. We need only consider the integral from $-1+\eta$ to $1-\eta$. For the usual arguments show that

$$|zI_2| < e^{H_{17} r\omega_1(1/r)}, \tag{5.5}$$

and a similar inequality holds for the integral from -1 to $-1+\eta$.

It remains to give an example of a function for which the analogy fails. Consider

$$\psi(t) = \sum_{m=1}^{\infty} \frac{e^{2^m it}}{m^2}.$$

APPENDIX A

LEGIBLE HANDWRITING

BELOW are given the complete Latin and Greek alphabets, capital and lower-case, and also the arabic numerals, written in a hand which, it is suggested, will minimize the danger of confusion discussed on pp. 65–66. Here differences between 'confusable' characters have been emphasized, and the form given is recommended for imitation by authors, especially when using single letters as symbols or when attempting in marginal correction of proofs to make clear a symbol which has already puzzled the compositor.

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APPENDIX B

TYPE SPECIMENS AND LIST OF SPECIAL SORTS

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Greek Series 106

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αβγδεζηθικλμνξοπρσςτυφχψω β̄γ̄ζ̄κ̄λ̄μ̄ν̄ξ̄ρ̄σ̄ς̄τ̄φ̄ψ̄ ϑ ϖ ῡ

5½ pt.

αβγδεζηθικλμνξοπρσςτυφχψω

Greek Series 90

11 pt.

ΓΔΘΛΞΠΣΥΦΨΩ Γ̄Λ̄Π̄Φ̄Ψ̄Ω̄Δ̇Φ̇Ψ̇Ω̃∇ θφ̇φ

9 pt.

ΓΔΘΛΞΠΣΥΦΨΩ εθηπφ ∇ Γ̄

6 pt.

ΓΔΘΛΞΠΣΥΦΨΩ θ φ

Greek Series 91: 11 pt.

ϱ ϰ

Greek Upright Display Series 92

11 pt.

ΑΒΓΔΕΖΗΘΙΚΛΜΝΞΟΠΡΣΤΥΦΧΨΩ

αβγδεζηθικλμνξοπρσςτυχφψω Π̇ Π̈ ὠ ϖ ε̂ ε̃

9 pt.

Π Ψ αβγδεζηθικλμνξοπρσςτυχφψω ε̂

Kerned Caps 11 pt.

Γ Ψ Ρ

Fraktur Series 28

10 pt.

ABCDEFGHIJKLMNOPQRSTUVWXYZ S̿

abcdefghijklmnopqrstuvwxyz āb̄c̄f̄ḡm̄n̄p̄q̄s̄t̄ âb̂ĉĝŝt̂ äöü ãq̃m̃

9 pt.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

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6 pt.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz āf̄c̄s̄

Fraktur Series 29

10 pt.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz **k̄ōw̄**

9 pt.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz

6 pt.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz

Script 11 pt.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

g i n p t u q *Ê P̄*

Script 11 pt. Superiors and Inferiors

F L M N V *C F R S*

Script Capitals 8/9

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Superiors in 11 pt.

ABCDEFGHIJKLMNOPQRSTUVWXYZ IV c

abcdefghijklmnopqrstuvwxyz . , · *x̄ħ* *ẋẏż*

ΓΔΛΩ *αβγδεζηθικλμνξοπρσςτυφχψω* *ϖ* *ϑ* [ ] ( )

½⅓⅔¼¾⅕⅖⅗⅘⅙⅚⅛⅜⅝⅞ † | ′ − + √ ± ∞ 1234567890

Inferiors in 11 pt.

ABCDEFGHIJKLMNOPQRSTUVWXYZ ABIV

abcdefghijklmnopqrstuvwxyz . , ; · ( ) [ ] + = − ± ∞ , ° / | * 1234567890

Δ *αβγδεζηθικλμνξϙπρσςτφχψω* *ϑ* *ζ̄* ½⅓⅔¼¾⅛

Superiors in 9 pt.

CEH LNRSV IV *abcdefghijklmnopqrstuvwxyz* . ,

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Inferiors in 9 pt.

ABCDEFGHKNRSTVZ *abcdefghijklmnopqrstuvwxyz* I . , ′

1234567890 / ( ) [ ] + − = × ± ∞ ½⅓⅔¼¾⅛ ° *αβγδεζηθικλμνξοπρστυφχψω*

2. SYMBOLS USED IN MATHEMATICAL COMPOSITION

	Symbol and Point Size						Symbol and Point Size				
No.	*11*	*10*	*9*	*8*	*6*	*No.*	*11*	*10*	*9*	*8*	*6*
1	+	+	+	+	+	40	∣	∣	∣	∣	∣
2	±	±	±	±	±	41	′	′	′	′	′
3	=	=	=	=	=	42	″	″	″	″	″
4	∔	..	..	..	..	43	‴	..	..	..	‴
5	+̊	..	..	..	..	44	⁗	..	..	..	..
6	≠	≠	≠	≠	≠	45	°	°	°	°	°
7	≢	..	..	≢	≢	46	∘̣	..	..	∘̣	..
8	≡	..	≡	≡	≡	47	⋮	⋮	⋮	⋮	⋮
9	≣	..	..	..	..	48	:	:	:	:	:
10	÷	÷	÷	÷	÷	49	∞	∞	∞	∞	∞
11	≑	≑	≑	..	..	50	∼	∼	∼	∼	∼
12	×	×	×	×	×	51	≃	..	≃	..	..
13	−	−	−	−	−	52	≈	..	≈	..	..
14	√	√	√	√	√	53	≅	..	..	..	..
15	√	√	√	√	√	54	↓	↓	↓	↓	↓
16	∫	..	..	∫	..	55	↑	↑	↑	↑	↑
17	)	)	)	)	)	56	∧	..	∧	..	..
18	(	(	(	(	(	57	⊼	..	..	..	..
19	]	]	]	]	]	58	⊥	..	..	..	⊥
20	[	[	[	[	[	59	∠	..	∠	∠	∠
21	{	{	{	{	{	60	≍	..	..	..	..
22	}	}	}	}	}	61	⇀	⇀	..	..	..
23	⊁	..	..	..	⊁	62	⥫	..	..	..	..
24	≻	..	≻	..	..	63	⥭	..	..	..	..
25	>	>	>	>	>	64	⥪	..	..	..	..
26	<	<	<	<	<	65	⥬	..	..	..	..
27	⊀	..	..	..	..	66	⇋	⇋	⇋	..	..
28	⩾	..	⩾	⩾	⩾	67	⇌	⇌	⇌	⇌	⇌
29	⩽	..	⩽	⩽	⩽	68	≬	..	..	..	..
30	≮	..	≮	..	..	69	#	..	#	..	..
31	≯	..	≯	..	..	70	∴	∴	∴	..	..
32	≪	..	≪	..	≪	71	≏	≏	..	≏	..
33	≫	..	≫	..	≫	72	♌	♌	..	..	..
34	≷	≷	..	..	..	73	♈	♈	..	..	..
35	≶	≶	..	..	..	74	∝	..	..	..	∝
36	←	←	←	←	←	75	⌖	..	..	..	..
37	→	→	→	→	→	76	∇	..	..	∇	..
38	↣	..	..	↣	..	77	Δ	..	..	Δ	..
39	/	/	/	/	/	78	∇	..	∇	..	..

	Symbol and Point Size						*Symbol and Point Size*				
No.	*11*	*10*	*9*	*8*	*6*	*No.*	*11*	*10*	*9*	*8*	*6*
79	Δ	..	Δ	..	..	99	..	..	..	$\oplus$	$\oplus$
80	$\oint$	..	..	..	..	100	..	..	..	$\ominus$	$\ominus$
81	$\wp$	..	$\wp$	..	$\wp$	101	∂	∂	∂	∂	∂
82	$\subset$	..	$\subset$	..	..	102	$\exists$	..	$\exists$	..	..
83	$\supset$	..	$\supset$	$\supset$	..	103	Ⓢ	Ⓢ	..	..	..
84	$\not\subset$	..	..	..	..	104	Ⓜ	Ⓜ	..	..	..
85	$\subseteq$	..	..	..	..	105	Ⓗ	Ⓗ	..	..	..
86	$\supseteq$	..	..	..	..	106	Ⓟ	..	Ⓟ	..	..
87	$\overset{\supset}{\subset}$	..	..	..	..	107	©	..	©	..	..
88	$\underline{\ni}$	..	..	..	..	108	Ⓚ	Ⓚ	..	..	..
89	$\overline{\in}$	..	..	..	..	109	Ⓑ	Ⓑ	..	..	..
90	$\in$	..	$\in$	..	$\in$	110	Ⓔ	Ⓔ	..	..	..
91	$\notin$	..	..	..	..	111	Ⓛ	Ⓛ	..	..	..
92	$\cup$	..	..	..	..	112	$\odot$	$\odot$	$\odot$	..	..
93	$\cap$	..	..	..	..	113	..	..	..	$\circledast$	..
94	$\#$	..	..	..	..	114	$\sum$	..	..	..	..
95	$\Leftarrow$	..	$\Leftarrow$	..	..	115	$\prod$	..	..	..	..
96	$\Leftrightarrow$	..	$\Leftrightarrow$	..	..	116	$\bigcup$	..	..	..	..
97	⦶	..	..	..	..	117	$\bigcap$	..	..	..	..
98	..	$\otimes$	..	$\otimes$	..						

3. DISPLAY SYMBOLS

Point Size	*Number of Symbol*											
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
48	$\surd$	$\int$	$($	$[$	$\sum$	$\}$	..	..	..	..	..	..
42	..	..	$($	$[$	..	$\}$	..	..	..	..	..	..
36	$\surd$	$\int$	$($	$[$	$\sum$	$\}$	..	..	..	$\mathbf{/}$	$\vert$	$\vdots$
30	$\surd$	$\int$	$($	$[$	$\sum$	$\}$	..	..	..	$\mathbf{/}$	..	..
24	$\surd$	$\int$	$($	$[$	$\sum$	$\}$	..	$\oint$	$/$	$\mathbf{/}$	$\vert$	..
20	..	$\int$	..	..	..	..	..	..	..	..	..	..
18	$\surd$	$\int$	$($	$[$	$\sum$	$\}$	$\prod$	$\oint$	$/$	..	$\vert$	$\vdots$
14	..	$\int$	..	..	..	$\}$	..	..	..	..	..	..

APPENDIX C

ABBREVIATIONS

1. Names of Units

We give here an alphabetical list of the generally accepted abbreviations for the more common units likely to occur in the writings of physicists and others whose work contains an appreciable amount of mathematics. Abbreviations of prefixes to the names of metric units which indicate multiples and sub-multiples are also included. It will be noted that letters of both the Greek and Latin alphabets are used, and it should be pointed out that in the latter case such abbreviations are always printed in roman type as opposed to the use of italics for *symbols*. If the abbreviation stands for a single word the full point is omitted, except where its omission might lead to ambiguity: thus we print l. for litre and in. for inch. The full point is used, however, if the abbreviation stands for more than one word, e.g. ft. lb., e.s.u., the final point being used after 'lb', for example, although it would not appear in this abbreviation standing alone.

ampere	A, amp
Ångström	Å
atmosphere (pressure)	atm
bar	b
British thermal unit	B.T.U.
calorie	cal
centi-	c
centistokes	cS
coulomb	C
cubic inch	in^3, cu. in.
metre	cm^3, cu. cm., c.c.
cycles per second	c/s
Debye unit	D
deci-	d
decibel	dB
degree	deg
Centigrade	° C
Fahrenheit	° F
Kelvin	° K
Rankine	° R
dyne	dyn
electromagnetic unit	e.m.u.
electron-volt	eV
electrostatic unit	e.s.u.
farad	F
foot	ft
foot-pound	ft. lb.
gallon	gal
gauss	G
grain	gr
gramme	g
henry	H
horse-power	h.p.
hour	h
hundredweight	cwt
hydrogen-ion exponent	pH
inch	in.
joule	J

kilo-	k	pico-	p
kilocalorie	kcal	poise	P
kilogramme	kg	pound	lb
		poundal	pdl
litre	l.	revolutions per minute	rev/min
mega-	m	second	s, sec
metre	m	square inch	in^2, sq. in.
micro-	μ	metre	m^2, sq.m.
micron	μ		
milli-	m	volt	V
minute	min	volt-ampere	VA
		volt-coulomb	VC
newton	N	watt	W
		weber	Wb
ohm	Ω		
ounce	oz	yard	yd

2. Names of Periodicals

Apart from the comprehensive *World List of Scientific Periodicals* (3rd revised edition, Butterworth, 1952) there are already in print extensive lists of recognized abbreviations of scientific journals. The Royal Society publishes a list covering scientific periodicals generally (see p. 50); *Mathematical Reviews* has a list that includes all periodicals likely to contain mathematics. These are in substantial agreement, and it does not seem necessary to include a further list here. Below are given some abbreviations of words commonly to be found in titles of mathematical or scientific publications. Pronounceable abbreviations are an advantage and it is, of course, desirable that titles (and places of origin) should be unambiguous. Articles, prepositions, conjunctions, etc., occurring in the full title are normally omitted in the abbreviation, unless there is a special reason for their inclusion: they are sometimes essential, e.g. to distinguish *J. de Math.* from *J. für Math.* A forthcoming British Standard embodies a Code for the abbreviation of titles of periodicals. Where, as has been recommended in these pages, references are kept from footnotes, there is less need to save a few ems of space.

Abhandlungen	Abh.	Allgemeine	Allg.
Abteilung	Abt.	Anales	An.
Academy, Académie	Acad.	Angewandte	Angew.
Accademia	Accad.	Annals, etc.	Ann.
Akademie	Akad.	Applied, applicata, etc.	App.

Archiv	Arch.
Astronomy	Astr.
Astrophysics	Astrophys.
Berichte	Ber.
Bulletin	Bull.
Bureau	Bur.
Circolo	Circ.
College, etc.	Coll.
Comptes Rendus	C.R.
Crystallographica	Cryst.
Economics	Econ.
Electric, -al	Elec.
Elektrische	Elek.
Engineering	Engng.
Engineers	Engnrs.
Faculty	Fac.
Forschungshefte	Forschungsh.
Fysik	Fys.
Gazette	Gaz.
Geofysik	Geofys.
Geophysics, etc.	Geophys.
Gesellschaft	Ges.
Giornale	Giorn.
Industry, etc.	Ind.
Institute, etc.	Inst.
Istituto	Ist.
Jahresbericht	Jber.
Journal	J.
Magazine	Mag.
Matematico, etc.	Mat.
Mathematics, -tische -tical, etc.	Math.
Mechanics	Mech.
Memoirs, etc.	Mem.
Meteorology	Met.
Mitteilungen	Mitt.
Nachrichten	Nachr.
National	Nat.
Nazionale	Naz.
Notices	Not.
Optics, -al	Opt.
Philosophical	Phil.
Physics, Physik, etc.	Phys.
Proceedings	Proc.
Publication	Pub.
Quarterly	Quart.
Recueil	Rec.
Report	Rep.
Research	Res.
Review, Revue, etc.	Rev.
Rivista	Riv.
Royal	Roy.
Schriften	Schr.
Science, scientific, etc.	Sci.
Section	Sect.
Seminar	Sem.
Series	Ser.
Sitzungsberichte	S.B.
Skrifter	Skr.
Society, Société, etc.	Soc.
Statistics, -al	Statist.
Studies	Stud.
Technical	Tech.
Tijdschrift	Tijdschr.
Transactions	Trans.
Travaux	Trav.
University	Univ.
Vereinigung	Verein.
Verhandelingen	Verh.
Wetenschappen	Wetensch.
Wissenschaften	Wiss.
Zeitschrift	Z.

INDEX

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