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# FACTORIAL INVARIANCE AND ITS RELATION TO RACE, SEX, AND IQ 

## by

## PHILIP HART RAMSEY

## Dissertation Committee:

Julia R. Vane, Ph. D., Chairman Harold E. Yuker, Ph. D. Allan Easton, Ph. D.


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## ABSTRACT

A factor analytic study was conducted on the 18 items In age levels $V$, VI, and VII of the 1960 revision of the Stanford-Binet Intelligence Scale. A total of 827 test results were collected from seven school districts, two nursery schools and two day care centers in Nassau County. The only restriction on the selection of these subjects was that their mental ages be between 4 years, 6 months and 7 years, 6 months. This restriction insured the applicability of the items under investigation. A special computer program was written to check the scoring accuracy of the test results. Of the tests determined to be accurately scored 600 were randomly selected In such a manner as to form a normal distribution of $I Q$ scores. This sample had a mean IQ of 99.4 and a standard deviation of 16.2. Chronological age ranged from 3 years, 1 month to 11 years, 2 months.

The purpose of the study was to investigate changes in factors under various conditions. Both Principal Components Analysis (PCA) and Principal Factor Analysis (PFA) were used. Rotation to Kaiser's varimax criterion was employed. Guttman's unit eigenvalue rule indicated the existence of $s i x$ factors. These factors were identified as: Visual Judgment, Verbal Abstract Ability, Definitions, Numeric Memory, Difficulty Level, and Verbal. The coefficient of congruence and Cattell's salient variable similarity index (s-index) showed high and significant agreement between factors extracted by PCA and those extracted by PFA. Six factors were extracted in all
subsample analyses.
Split halves reliability was determined for the six factors by dividing the 600 subjects randomly into two subsamples of 300 and analyzing each subsample separately. The factor solutions were rotated to maximum agreement with the solution for the total sample by Cliff's least square procedure. Factors from one subsample were compared to corresponding factors from the other subsample both by the coefficient of congruence and the s-index. All six pairs from the two samples were found to be significantly related by the s-index and were therefore considered reliable. The same procedure was used to determine reliability for samples of size 150 and 100. Although the number of reliable factors appeared to be less with smaller sample size, McNemar's exact test of correlated proportions indicated that the differences were not significant. Therefore sample size could not be demonstrated to show significant differences in the number of reliable factors. Reliability of factors in both high and low IQ groups was also investigated and no significant differences were found between these groups in number of reliable factors. These investigations were done for both PCA and PFA and no significant differences were found between the two methods as to the number of reliable factors.

Additional subsamples were selected from the total sample so that they differed systematically on various subject variables. A male and a female sample were selected, as well as black and white samples, and samples differing only in the
standard deviation of the IQ scores. PCA only was employed on these samples. The s-index was again used to determine the significance of the relationship between factors from these samples which differed in race, sex, $I Q$ level or $S D$ of IQ scores. The number of factors which were significantly related when each of these variables were investigated was not found to be significantly different from the number of reliable factors from samples of comparable size. Therefore no differences in factors could be attributed to differences in race, sex, or IQ. This was in general agreement with previous studies.

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## CHAPTER I

## INTRODUCTION

Factor analysis in psychology seems to have been used primarily either for theory construction (Horn and Cattell, 1967) or for estimating the number of separate "abilities" measured by one or more tests (Guilford, 1956). Although a great deal of work has been done on the factor analysis of psychological tests, little has been done to demonstrate factor reliability or invariance. Only a few studies have investigated such variables as race or sex and these have lacked tests of significance.

The present study was designed to investigate factor reliability of the Stanford-Binet Intelligence Scale with children aged four through seven years, using both a Principal Components Analysis (PCA) and a Principal Factor Analysis (PFA). These two methods have been widely used since the introduction of the electronic computer.

In view of the fact that factor analysis has been a popular technique in fields such as psychology, business and blology, one might expect substantial concern with simple reliability and validity. This has not been the case, however. Armstrong and Soelberg (1968) report a survey of 46 recent articles in which two-thirds provided no measure of reliability. The authors then dramatized what can happen when there is no measure of rellability. They used random numbers to create a matrix of intercorrelations or arbitrary traits and demonstrated that they were able to "1dentify" the factors which emerged.

They pointed out that a simple measure of reliability would have demonstrated the worthlessness of their results. The authors then suggested three methods which would permit reliability estimation, namely: a priori models, Monte Carlo simulation, and split samples.

In the a priori method the researcher works out, in as much detail as possible, the structure of the solution that he expects to find. He postulates the number of factors he expects to appear, which variables should load together, relationships which should exist among factors, and what variables he expects will dominate which factors. Predictions are based on behavioral models, previous findings reported in the literature, or merely on "well-educated" hunches. The results may, of course, agree with an a priori model as the result of luck or chance, but the as priori specification of a model provides a much more objective benchmark than is provided by a posteriori appeal.

Monte Carlo simulation is used when prior information about the underlying behavioral processes is too weak for the use of a priori models and the sample sizes are too small to split the sample. In such cases researchers simulate their results by factor analyzing suitable samples of random data, chosen to conform to the actual data in terms of sample size, number of variables, and assumed underlying distributions. The analysis of random data is replicated many times in order to obtain distributions of the various factor statistics. By comparing the results based on actual data with the distributions
from Monte Carlo simulations one can judge whether the former appeared to be "significantly" different from the latter.

The split samples method is the most practical reliability method for use in studies of empirical variables, and will be used in this study. In the split sample method the sample to be factor analyzed must be large enough to be divided in half randomly, and each subsample factor analyzed. The factors obtained from the analysis of each subsample are compared statistically. If the factors are found to be significantly related they may then be considered reliable.

## Literature Review

The problem of obtaining factor reliability, or factorial invariance, is complicated by the lack of statistical tests in factor analysis. Although psychologists continue to compare results of separate factor analyses by inspection (Zachert and Friedman, 1953; Tiliman, 1966), only investigations making quantitative comparisons have been reviewed here.

Peterson (1965) reviewed a number of studies of personality factors and concluded that two broad personality factors should be retained rather than a number of narrower factors since only the broad factors showed reasonable invariance over studies. He based his conclusions upon studies using the correlation coefficient to correlate factor loadings.

Another study in which factor loadings were correlated was done by Rosenblatt (1966). Using Monte Carlo simulation he constructed 20 mathematical factor models which ranged from

2 to 5 factors and from ten to fifty percent theoretical error varlance. These models were then used to generate test scores through a computerized random number generation. Three samples of 100 subjects were generated for each factor model. Sixty principal axis factor analyses with squared multiple correlations in the main diagonals were performed and were followed by varimax rotations. Rosenblatt extracted twice as many factors as were built into the model. Using the interclass correlation to compare the various factors in the principal axis solutions he found the built-in factors had higher correlations (. 45 to . 79) than the additional factors (. 14 to .21). For the varimax solutions he averaged the correlations between the three samples and also found higher correlations for the built-in factors (. 86 to .92) than for additional factors (. 31 to .46). From these figures it can be seen that by using this method the factors built in on the basis of mathematical formulas were more rellable than random factors extracted afterwards. It can be seen also that the varimax solutions give consistently higher correlations than the principal axis solutions.

The only real limitation to Rosenblatt's study other than the fact that the data is artificial is that he measured factorial invariance by the correlation coefficient which involves correlating factor loadings. This same weakness applies to Peterson's study, since he too used the correlation of factor loadings. A number of investigators (Barlow and Burt, 1954; Leyden, 1953; Burt, 1964; and Pinneau and Newhouse, 1964) have objected to correlating factor loadings. These objections center around the fact that loadings of .9, .8, . 7 on
one factor would correlate -1.0 with loadings of .7, . $8, .9$ on another factor. In other words even though all three items have large positive loadings on both factors the difference. in order of the loadings leads to a correlation which implies that the two factors are opposites.

In attempting to overcome the deficiencies of the correlation coefficient several authors independentiy derived a measure known as the "unadjusted correlation" (Burt, 1949) or coefficient of congruence (Tucker, 1951) or coefficient of similarity (Wrigley and Neuhaus, 1955). This coefficient's simplicity has undoubtedly led to much of its popularity. For orthogonal factors it is calculated by simple summation of the cross products of all item loadings on any two factors being compared and then this sum is divided by the square root of the product of the sums of squared loadings of the two factors. If two factors, $A$ and $B$, have loadings:

A

$$
\begin{aligned}
& a_{1}, a_{2}, \ldots, a_{n} \\
& b_{1}, b_{2}, \ldots, b_{n}
\end{aligned}
$$

The coefficient of congruence, $\phi$, is calculated by:

$$
\phi=\frac{\sum a_{1} b_{1}}{\sqrt{\left(\sum a_{1}{ }^{2}\right) \sum\left(b_{1}{ }^{2}\right)}}
$$

This coefficient ranges from -l to -1 with the same interpretation being given to values of -1 , and -1 as is given to the correlation coefficient. Unfortunately no test of significance seems to have been reported by any of the authors of the coefficient of congruence.

Both Harmon (1967, p. 271) and Vandenberg (1959, p.263) present accounts of a study by Tucker (1951) using the coefficient of congruence. (The present author was unable to obtain this study from the U. S. Army.) Tucker reportedly analyzed two studies -- one involving 18 variables for a sample of Naval Recruits and the other involving 44 variables for a sample of Airmen and Soldiers -- in which 10 variables were common and the six factors of the smaller study were matched with six of the twelve factors of the larger. Values for the coefficient of congruence were: . 999883 on Factor A, verbal relations; . 999984 on Factor B, perceptual speed; . 939811 on Factor C, a numerical factor; . 999875 on Factor D, tentatively identified as a reasoning factor; . 999670 on Factor E, technical information; and . 459917 on Factor F, a spatial visualization factor weakly represented by test items about electric circuits and automotive mechanics. These coefficients were calculated after rotating the two factor structures into maximum congruence. According to Harmon (1967), Tucker "... accepts coefficients ranging from . 999984 down to . 939811 as defining congruent factors, but rejects a value of . 459717 as, 'definitely low so that this factor will not be considered as a congruent factor (p.19)' (p.271)." This rather arbitrary decision as to what values of the coefficient of congruence are acceptable highlights the need for a test of significance. Another example of the use of the coefficient of congruence applied to factor analysis is in a study by Vandenberg (1959). He performed a factor analysis on the results of a number of Thurstone's tests given to Chinese
students studying in the $U$. $S$. He then rotated to maximum congruence by Tucker's technique and calculated coefficients of congruence between the Chinese data and Thurstone's data (1938). Values of the coefficient of congruence were: .873 for Factor S (Spatial), . 910 for Factor $V$ (verbal), . 855 for Factor $N$ (number), . 830 for Factor M (memory), and . 730 for Factor $P$ (perceptual speed). In order to evaluate his results Vandenberg cited examples given by Ahmavaara, who perfected a mathematical technique for comparing factors.

Ahmavaara (1954) applied his technique twice: first, to the results of the 60 -test and 21 -test study of 14 -year-old children reported by Thurstone (1951). Ahmavaara reports the following values for his technique after the factors in both studies had been made orthogonal:

| W | .979 |
| :--- | :--- |
| S | .968 |
| V | .967 |
| M | .929 |


| R | .848 |
| :--- | :--- |
| N | .744 |
| P | .689 |

Then he applied his transformation analysis to the results of Thurstone's 57 -test PMA study (1938a) and the results of the 27 -test study of the perceptual factor (1938b). For both these studies the subjects were college students and the factors were orthogonal. Ahmavaara reported values as follows:

| W | .617 | I | .609 |
| :--- | :--- | :--- | :--- |
| S | .782 | N | .891 |
| V | .591 | P | .698 |
| M | .774 |  |  |

Vandenberg then stated, "Unless the difference between his (Ahmavaara's) and Tucker's techniques results in marked differences in the values of the respective invariance
coefficients, the results of the Chinese students - United States students comparison shows an agreement between factors that is as close as or even closer than the agreement between factors found for two groups of United States students (p.300)." Since Vandenberg made no comparison between the coefficient of congruence and Ahmavaara's technique, this is at best a highly questionable conclusion. Tests of significance would simplify the problem of comparing factors.

Saunders (1959) attempted to apply a test of slgnificance to the coefficient of congruence. He performed a factor analysis of the Wechsler Adult Intelligence Scale in which he divided many of the subtests into parts so as to produce 19 variables. He performed this type of analysis on two samples, high school males and college males. Although he did not specify the type of factor analysis the results appear to be from a principal axis or possibly a centroid. Successive iterations were used to determine 9 factors for sample A in 6 iterations and 10 factors for Sample $B$ in 8 iterations. Varimax rotations were used for interpretation. After rotation to maximum congruence by Tucker's technique (1951) there were 7 factors with coefficients of congruence equal to one (within . 003 the accuracy of the computations). Two more factors were significant (. 05 level) by an F-test (received by private communication from Tucker) in which the numerator degrees of freedom were unknown and were assumed to be of reasonable value. Since the numerator df's were considered conservative, Saunders concluded that at least nine factors must be significantly related in the two samples. As can be seen an estimate was necessary
in order to perform the test of significance.
Quersh1 (1967a) collected data on the Illinois Test of Psycholinguistic Abilities (ITPA) and Stanford-Binet, MA, on 700 children ranging in age from 2 years, 6 months to 9 years, 0 months and with IQ's ranging from 80 to 120. Correlation matrices of the 10 ITPA items and Stanford-Binet, MA, were later compared factorially across 7 different age ranges (1967b). This study is the first reviewed in which factors in one sample were compared with factors in another sample, when the samples differed systematically, i.e. each sample represented a different age group. The matrices were factored by the square root method with unities in the diagonals and coefficients of congruence were calculated between the first four factors in all seven groups. The results are given in the following tables:

Table 1
Coefficients of Congruence across Seven Age Group Samples for Factor A (above the Diagonal) and Factor $B$ (below the Diagonal) (decimals omitted)

| Samples (Age Groups) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Samples | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 |  | 979 | 923 | 959 | 970 | 944 | 955 |
| 2 | 981 |  | 959 | 974 | 964 | 950 | 967 |
| 3 | 982 | 986 |  | 976 | 966 | 917 | 939 |
| 4 | 984 | 982 | 994 |  | 980 | 960 | 965 |
| 5 | 948 | 940 | 982 | 981 |  | 962 | 956 |
| 6 | 943 | 948 | 977 | 968 | 979 |  | 957 |
| 7 | 896 | 915 | 931 | 908 | 910 | 939 |  |

Table 2
Coefficients of Congruence across Seven Age Group Samples for Factor C (above the Diagonal) and Factor D (below the Diagonal) (decimals omitted)

## Samples (Age Groups)

| Samples | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | 943 | 915 | 930 | 970 | 935 | 789 |
| 2 | 940 |  | 970 | 951 | 962 | 967 | 787 |
| 3 | 932 | 954 |  | 947 | 961 | 975 | 749 |
| 4 | 981 | 957 | 969 |  | 959 | 980 | 761 |
| 5 | 962 | 926 | 963 | 985 |  | 959 | 733 |
| 6 | 955 | 923 | 957 | 983 | 981 |  | 792 |
| 7 | 956 | 905 | 921 | 976 | 981 | 968 |  |

From these tables it can be seen that the coefficients range as follows: . 917 to . 980 for A, . 896 to . 994 for B, . 733 to .980 for C, and . 921 to . 985 for factor D. The medians of these coefficients for factors $A, B, C$ and $D$ are $.960, .968, .947$, and .957 respectively. Quershi interprets these as representing, "a high degree of stability...(p.809)", which is probably true but again one wonders what the results of a statistical test or comparisons across other parameters such as sex, IQ, or race would show.

Lindsey (1966) investigated some of these questions when he performed a principal axis analysis on the Wechsler Intelligence Scale for Children comparing samples differing in sex and race. The subscales of the Wechsler scale were divided so as to form 26 variables. Four additional variables were provided by: the Peabody Picture Vocabulary, the Ammons Full-Range Picture Vocabulary Test, the Manipulation of Areas Test, and the Number Concept Test. Since Lindsey refers to
this as Hotelling's method he presumably used ones in the main diagonal. He extracted all factors whose eigenvalues were greater than one. The results were then rotated by Kaiser's varimax method. The tests were administered three times: (1) at the beginning of the first grade in 1961 , (2) at the end of the first grade in 1962, and (3) at the end of the third grade in 1964. Originally there were 163 white children with a mean age of 6 years, 2 months and lll black children with a mean age of 6 years, 1 month. The mean full scale IQ was 103 for the white group and 84 for the black group. Distribution by sex was essentially equal for the original groups. In the Spring of 1964 there were 217 of the original 274 subjects available for testing, 125 whites and 92 blacks. The mean IQ for whites was then 108 and for blacks 91. Six separate samples were selected for the analysis: (1) 1961 white sample, (2) 1962 white sample, (3) 1964 white sample, (4) 1964 black sample, (5) 1964 male sample, (6) 1964 female sample. Coefficients of congruence were calculated for comparisons of the flve factors common to all groups. Coefficients between the 1961 white sample and the 1964 white sample were:

| Factor A: | Expressive Psycholinguistics | .8285 |
| :--- | :--- | :--- |
| Factor B: | Perceptual Organization | .7800 |
| Factor D: | Perception of Incongruity | .5754 |
| Factor E: | Numerical Ability | .6152 |
| Factor F: | Education of Conceptual Relations | .7035 |

For comparison of the 1964 white sample and the 1964 black sample and the comparison between sexes the coefficients were respectively:

| Whites vs. Blacks | Males vs. Females |  |
| :--- | :--- | :--- |
| Factor A: | .8247 |  |
| Factor B: | .8660 | Factor A: |
| Factor D: | .72267 |  |
| Factor E: | .5579 | Factor B: |
| Factor F: | .68825 |  |
|  | Factor D: | .7835 |
|  | Factor F: | .8633 |

Lindsey, while admitting that these results can only be tentative, suggested that on the basis of factor structure the black sample is somewhat more similar to the white sample than the white sample is to its own previous factor structure. This occurred even though these two racial groups were considerably different in IQ. Although it appears that there were no differences due to race, $I Q$, or sex, caution is warranted because the absence of statistical tests leaves one without any frame of reference. For example we might have expected the greatest agreement to be between the 1961 and 1964 white samples since the same individuals were tested on the same tests. The problem is that we do not know whether these coefficients were high enough to insure a significant relationship but we are tempted to guess that they were because correlation coefficients of this size certainiy would be. On a logical basis, however, one can argue that there could be very different factors in the 1961 and 1964 samples. For example, even if the same tests were used in both cases the particular items which determined most of the differences between individuals taking the tests might be different. Test designers attempt to keep the items the same over various age levels but this is generally accomplished by intuitive ideas of what items are similar. If the item changes are different enough one might expect different factor patterns at the two age
levels. In fact, one might even argue that there was good evidence for just such a change in the present case because the two samples of different sexes are consistently more similar in factor organization than two samples of the same individuals collected two years apart.

Rather than use the coefficient of congruence. some experimenters such as Hamilton (1968) have preferred Ahmavaara's technique because it is "mathematically elegant." Hamilton reported a principal components analysis of 17-1tem rating scales administered to 152 men and 120 women suffering from primary depressive illness. A correlation matrix was generated for each sex and after factorization was rotated by Kaiser's varimax method. The two matrices were then compared by Ahmavaara's method for both the principal components solutions and the varimax solutions. Coefficients for the corresponding 6 factors were $1.00, .93,-.70, .68,-.51$ and -.03 while the coefficients for the non-corresponding factors ranged from -.66 to . 65. For the varimax solution the corresponding factor coefficients were $.97, .93,-.66, .92,-.74$, and .62 with non-corresponding factor coefficients ranging from -. 53 to .59. Clearly the varimax solution indicated an agreement not evident in the principal components solution. Since there was no statistical test, Hamilton divided the 120 women into two groups of 50 and 70 each, factored, rotated and compared the two varimax solutions. He found the coefficients for the 6 factors were $.76,-.86,-.70,-.81, .03$, and .63 while the non-corresponding factor coefficients ranged from -. 58 to .70 . Hamilton then stated, "It would seem, on the evidence, that the

Varimax method of rotation cannot guarantee the appearance of invariant factors." (p.109). He then questioned the restriction to orthogonality. Hamilton did not question the sample size which is strange since factor analysts routinely require more than 100 subjects in a sample for the explicit purpose of insuring stability.

Werdin (1962a) developed a transformational method similar to Ahmavaara's which he demonstrated on previous data of his own (1962b). Corresponding coefficients for the 5 common factors are: .94,.90,.99,.88, and .995. Noncorresponding factor coefficients range from -.23 to .35. Again no statistical test was used.

Several additional methods have been developed for comparing factors all of which show great promise (Pinneau and Newhouse, 1964; Nanda, 1967; and Levine, 1968). Unfortunately, none of these have provided statistical tests nor have they been widely used.

Guilford's study (1966) in which he attempted to provide an empirical base for his structure-of-intellect model is an example of the ambiguity that results when no test of significance is provided. Guilford performed a factor analysis using Cliff's rotational methods (1966) in order to determine how closely his data could be brought into agreement with his model. After a principal axis solution with communality estimates in the diagonal he proceeded to rotate to the appropriate target matrix by Cliff's method, "Successive target matrices were tried, in an effort to approach maximally the criteria of simple structure, positive manif'old, and psychological
meaningfulness (p. 54)." Two slight graphical rotations were made from the analytic solution in order to clarify four factors. Later during the discussion of his results Guilford went on to say, "A number of the 'miscarriages' stubbornly remained, indicating that the advantages taken in a pattern type of rotation cannot take full liberty with data. (p.64)." Since no statistical test is available to check the closeness of the data to his target matrix one is left in doubt as to just how much "liberty" can be taken with data.

In another article (Guilford and Hoepfner, 1969) comparisons were made between data which were both rotated to simple structure by the varimax method and rotated to targets based upon the Structure-of-Intellect (SI) factors as opposed to a 67 percent identification in the target-rotation factors. This statement was then made: "In view of the expected advantage of the target method in this respect, a comparison here is not completely fair, but the target method yields about three times as many identifications.... If we had depended upon the varimax method to arrive at a general theory of intelligence, it is doubtful whether the SI theory, or any other theory, could have been generated from the factor-analytical results. (Pp9-10)" As can be seen, Guilford did not use a test of significance so his results can be questioned.

Perhaps the most promising treatment for interbattery comparison, at least from a statistical point of view, is the method of cannonical correlations. Unfortunately it cannot be used in such studies as the present because it requires the use of different tests with the same individuals. Although
this method has been available for over 30 years (Hotelling, 1935), it has been little used because the technique is complex and presentations of the method generally have not shown how to obtain loadings for factors which would allow for interpretations as in factor analysis. Also it requires the use of one's in the main diagonal which supposedly reduces the agreement between samples. Ragland (1967) has removed the first two difficulties by presenting a relatively simple explanation of the system as well as procedures for deriving factor loadings for the purpose of interpretation. The first canonical correlation gives the maximum possible correlation between two sets of test results and corresponds to a measure of the agreement between the first principal components of the two batteries. In like manner the following correlations compare each successive principal component. Canonical correlations unlike most methods of interbattery comparison do have tests of statistical significance.

Ragland quoted Jones (1964) as using canonical correlations to predict talented behavior in students. Jones gathered test data on 450 students in the seventh grade and used it to predict their performance as high school seniors. Test measures were grade average, aptitude and achievement test scores, teacher and peer nominations for various kinds of talents, and awards received for talented achievements. Factor analysis led to 21 criterion factors and 7 predictor factors, each set orthogonal. Canonical correlations for the first 5 matched factors were significant and ranged from . 78 to .29. Although Jones was able to use the canonical weights
for interpretive purposes due to the orthogonality of the factors, Ragland warns that this is not possible, in general, but shows how loadings may be obtained which can be interpreted.

Burt (1947) used artificial data to show the comparison between canonical correlation analysis and factor analysis and seemed to indicate that canonical correlation analysis (CCA) was best for comparison purposes and factor analysis (FA) for interpretive. Ragland insists that the two purposes can be accomplished at the same time. However, Das (1965), following the procedures outlined by Burt, analyzed the scores of 223 Indian college students on 5 "experimental non-verbal reasoning tests (p.61)" and 12 "reference tests for reasoning (p.61)." A principal axis FA with communality estimates was performed on the sets of 5 and 12 . The canonical correlations found were: .629,.016, 041, .035, and .022. However, no significance levels were reported. Since no rotation was performed Das had some trouble making interpretations. The first factor was identified as 'g'. The remaining factors had many positive and negative loadings as is characteristic of unrotated factors. Since the second factor had verbal tests with negative loadings and non-verbal tests with positive loadings, Das stated that it, "emphasizes the non-verbal nature of the five tests. (p.65)" Das might not have struggled with the interpretation of these factors if he had checked on their significance.

King, Bowman, and Moreland (1961) seeking factors common to biochemical levels and intelilgence, misinterpreted

Burt as suggesting that factors can be identified by their weight in a canonical correlational analysis. They performed the CCA between 7 amino acid factors derived from 21 amino acid variables and 7 intelligence variables from French (1954). The sample size was rather small with only 58 subjects and only one canonical correlation was mentioned. It had a value of .599 and was significant beyond the . 025 level.

Osborne, Anderson, and Bashaw (1967) applied canonical correlation analysis to the data presented by Lindsey (see page 10 above). They organized the data into only three groups: (1) Pre-school, 1961; (2) Grade 1, 1962; and (3) Grade 3, 1964. Only two significant correlations were found between 1961 and 1964; . 850 and .523 . However, there were four significant canonical correlations between 1961 and 1962; .867, .545, .530, and .468. Three significant correlations were found between 1962 and 1964; .870, .542, and .472. Since the factors had already been interpreted on the basis of the varimax rotation solutions, the canonical correlations were not used for interpretive purposes. The first canonical correlation was taken as a measure of the reliability from one group to the next and an overall rellability of .86 was reported. The authors seem to have considered this a reliability estimate of the whole battery rather than only the first principal component as it should be.

There still remains one major objection to the practical use of canonical correlations, that is, the use of one's in the main diagonal rather than the more popular communality estimates. Meredith (1964a) offered a possible
solution. He developed a technique for correcting the canonical correlations based upon the reliabilities of the tests involved. The technique was demonstrated on a set of data from Wechsler (1949). The intercorrelations on the Wechsler Intelligence Scale for Children subtests for 100 boys and 100 girls seven years of age were presented as the two sets to be compared. Six canonical correlations were found: .68, .20, 16, .12, .11, and . 05. Applying Bartlett's significance test (1941, 1947) the first canonical correlation was significant well beyond the .01 level but none of the others reached significance. When corrected for attenuation these became .97, .47, . 35, . 30, . 24, and .10. Applying the same statistical tests, the first 4 correlations were significant beyond the .001 level and the fifth at the . Ol level.

One investigator who developed a technique for factor comparison which included a test of significance was Cattell (1949). He proposed the basic idea for the salient variable similarity index which essentially used Fisher's exact probability to test the chance expectation that two factors to be matched both have the same items as salient variables (1.e., loadings above some minimum value such as .30). He later used it to compare the general ability factor which turned up in personality tests in 7 separate studies (1957). Cattell found that the salient variable similarity index, (Cattell and Baggaley, 1960) was more conservative than the correlation coefficient which sometimes gave significance when it was unjustified. This is not surprising in light of the many objections already raised to the use of the
correlation coefficient with factor loadings. The latest modification of Cattell's s-index (Cattell, et. al., 1969) was used in the present study.

The first factor analysis of the Binet Scale was performed by Ruth Wright on the 1916 version and her results were published in 1939. In it 456 ten year olds were selected so that a sufficient number of items within a specified range were included. The centroid method was employed and upon rotation two explanatory hypotheses were offered for the common factor which remained even after rotation. The first was In favor of a general factor of intelligence. The second and "more tenable" was an effect due to maturation. The seven f'actors which were found and labeled were:

Ages VII - XII

1) General or Maturation
$2)$ Number
3 Space
4 Verbal Relations
5 Induction (tentatively)
"Apparentiy involves a reasoning ability" Cannot be interpreted but a possibility that the method or ability is more common to children at a lower level of development"

McNemar performed the second published analysis using the standardization population of the 1937 revision of the Binet. He employed Thurstone's centroid method and extracted only three factors. He pointed out that since the reliabilities and therefore communalities were on the average . 65, this meant that 35 percent of the variance would be due to unreliability. Since the first factor accounted for 40 percent of the variance in most cases, there seemed little justification to extract more than one factor but to be on
the safe side he extracted three factors, which he did not label. He performed a total of 14 analyses and there was overlap between analyses at each adjacent level. For example, all the items at age level II and II-6 appeared in the first analysis, all the items at II-6 and III in the second analysis, and so forth. In general Mc Nemar seemed to feel that enough of the variance was due to the first factor to justify employing the Binet as a measure of general intelligence. In other words, roughly equal IQ's of two individuals could be considered to be measuring the same kind of intelligence. Although admitting the possibility of isolating meaningful factors by means of rotating centroid axes, Mc Nemar stated that, "these small 'group factors' could not contribute sufficiently to $I Q$ variance to invalidate the comparability of IQ's of the same magnitude for individuals of approximately the same life age." He admitted, however, that at age levels II, II-6, V, VI, and XVIII the 'group factors' (all after the lst) contributed too much to the test variance to equate comparable IQ's for individuals at those levels (p.116).

In comparing overlapping tests used in adjacent analyses of 136 pairs, only 12 showed differences large enough to attract attention. Of these 12 , only 3 seemed to be significant. (Analysis on different samples.) Loadings of similar items or the same item at repeated levels tended to be the same. There are, however, a few exceptions which forced Mc Nemar to, "belleve that some differences do exist in the common factor called for at various age levels." (p.122). Factor structure shows patterns of factors in a set. Although
he worked with both forms $L$ and $M$, Mc Nemar found no observable differences as to factorial structure.

The third analysis was by Cyril Burt and Enid
John in 1942 in which two methods of factor analysis were employed on a group of 483 boys and girls with MA's between 10 and $11 \frac{1}{2}$ and CA's between 10 and $14 \frac{1}{2}$ and mental ratios between 80 and 110. Using bipolar and group method they showed that the data could be adequately interpreted in either of these two methods. First, the bipolar method led to a large first factor accounting for 40 percent of the variance, as well as a number of bipolar factors (both positive and negative loadings) which were explained as a kind of special ability leading to deviations above or below the general intelligence represented by the first factor. The group method on the other hand leads to a number of distinct factors each having all positive loadings and representing separate abilities.

Agreement was found with Ruth Wright on the general factor which remained after rotation and may be Spearman's 'g' or more probably a factor of maturation. "With increasing age, the influence of the specific functions becomes more and more conspicuous and that of the general factor less and less predominant (p.119)". The eight factors found by Burt and John were:

Ages X and XII

| 1 |  |  |
| :--- | :--- | :--- |
| 2 | General | 5 |
| Age | 6 | Memory |
| Verbal | 7 |  |
| 4 | Vomprehension |  |
| Vocabulary | 8 | Numerical |
| Spatial |  |  |

A fourth analysis was conducted by Lyle Jones in 1949 on Mc Nemar's data at ages VII, IX, XI, and XIII. Rotation procedures applied to the centroid solutions indicated the correlations could be explained completely by group factors. Factors at the respective age levels were:

Age VII
I) Verbal
2) Reasoning

3 Memory
4 ) Number (rather indistinct factor found only at this age)

Age XI

1) Verbal
2) Memory

3 Spatial
4) Residual

Age IX

## 1) Verbal

2) Reasoning

3 Memory
4 Spatial
5) Residual

## Age XIII

1) Verbal
2) Reasoning I
3. Reasoning II

4 Memory
5 Visualization
6) Spatial

These four age levels represented four separate factor analyses, but as can be seen from the results, the factors were largely the same. The factors at age XIII were more clear-cut for interpretation than at lower levels according to Jones. He felt that the trend agreed with the general literature on this subject.

The fif'th study was carried out by Douglas Dean in 1950 and performed on 145 children ( 60 girls and 8.5 boys) in the first grade. Age levels VI, VII and VIII were used. In addition to the Stanford-Binet, the SRA Primary Mental Ability Battery was factor analyzed by Thurstone's group centroid method. Dean found that both these tests had an equally important verbal influence but the PMA stressed perceptual
values, not on the Binet, while the Binet put more stress on individual memory. Although it was possible to interpret the first factor (unrotated) as "g", Dean considered it more "psychologically meaningful" in terms of group factors. He also pointed out that IQ's were not strictly comparable. The same claim was made by Jones as has been already pointed out. Since several of Dean's factors turned out to be oblique with one another, he concluded that this could be interpreted as support for Garrett's hypothesis of less differentiated intelligence at earlier age levels. Again this is in general agreement with Jones' findings. Dean identified six factors:

Ages VI - VIII

1) Verbal

2 Perceptual Speed
3 Spatial
4 Reasoning
5 Memory
6) Spatial in nature but other elements involved In the sixth analysis, Lyle Jones, in 1954, refactored his previous data at age XIII with an oblique rotation and extracted 10 factors: 3 verbal, 2 memory, space, reasoning, closure, carefulness, and residual. There was little difference except to clarify the psychological meaning of some of the factors.

The seventh analysis was a rather elaborate one by George Edward Stormer in 1966 in which the 1960 revision of ${ }^{\prime}$ the Stanford-Binet and a $5 \frac{1}{2}$ hour battery of reference tests chosen as stable measures of specific intellectual abilities were given to a random sample of typical 15 -year-old students. The sample was taken from all over the State of Illinois and was selected to fit the ten-point socio-economic scale
devised by Warner. The group was stratified into three age ranges; Low Range - XI to XIV, Middle Range - XIII to SAII, Upper Range - AA to SAIII. Since all the children were the same age, this meant the stratification was essentially into three IQ groups. The sizes of these groups were as high as 428 and never lower than 100. Ten factors were extracted at each age and identified as:

Low Range Middle Range

| 1) | Verbal |
| :---: | :---: |
| $2)$ | Memory |
| 3 | Spatial |
| 4 | Divergent Production |
| 5 | Sentence Use |
| 6 | Sentence Production |
| 7. | Attention |
| 8 | Anxiety |
| 9 | Space Orientation |
| $10)$ | Minkus |

1) Verbal

2 Spatial
3 Memory
4 Divergent Production
5 Orientation in Space
6 Sentence Word Production
7 Attention
8 Verbal Induction
9 ) Intuitive Reasoning
10) Concrete Reasoning

Upper Range

1) Reasoning-Memory

2 Reasoning-Spatial
3 Verbal Production
4 Divergent Production
5 Verbal-Precision
6 Dimensional Reasoning
7. Verbal Reasoning

8 Unidentified
9 Abstract Thinking
10) Spatial

One of the factors found with loadings from the reference tests but not from the Stanford-Binet was divergent production indicating that this factor is not measured by the Stanford Binet. This would not be surprising for anyone familiar with the Stanford-Binet. Memory and spatial aptitudes were measured only minimally by the Binet. The major portion of the variance in the Binet seemed to be attributed to the verbal factors of fluency, reasoning, and production.
"This would imply that typical identification procedures based directly or indirectly on the Binet, measure primarily the ability to achieve grades and academic recognition in the typical school program (p.110)." It might be pointed out that predicting academic success was exactly what the Binet was constructed to do. Perhaps the most important conclusion of Stormer's study is that children of the same CA but dirferent MA have very different patterns of intellectual function1ng.

Ramsey (1968) worked with preschool children and found seven factors which he called verbal fluency, visual motor ability, visual judgment, control, persistence, general knowledge, and visualization. Conclusions from this study were tentative, however, because the size of the sample which numbered only 152 children, was considered too small for definitive conclusions, particularly in view of the fact that no rellabllity test was used.

On the basis of the articles reviewed it is apparent that psychologists have become concerned about the use of factor analysis because of the omission of tests of reliability, (Humphreys, 1962; McNemar, 1964; Maxwell, 1961). Prior to the introduction of the electronic computer, however, the repeated analyses required to establish reliability represented a monumental task and it is understandable why such analyses were not done.

## CHAPTER II

## PROBLEM AND METHODOLOGY

The present study was designed to investigate the factor reliability of the 1960 revision of the Stanford-Binet Intelligence Scale at age levels V, VI, and VII. StanfordBinet tests of 827 children were collected from public schools, nursery schools, and day care centers in eleven different communities in Nassau County. The results of 600 tests were used for the first factor analysis. The sample of 600 was then divided into two subsamples of 300 each to permit a check of reliability using the split sample method. The total sample was then redivided into four smaller groups to provide reliability estimates for samples of smaller size.

The total sample was also divided on the basis of IQ into groups containing high IQs and low IQs. Subsamples of males and females, whites and blacks, and subsamples selected on the basis of large and small standard deviations were considered.

All the items from age levels V, VI, and VII of the 1960 revision of the Stanford Binet Intelligence Scale, totaling 18 in all, were used in the factor analyses. The structure of the Binet is such that all items correlate well with the test as a whole because this was a major criterion for selection of the items by the test authors. This high item intercorrelation is a desirable feature for items in any factor analysis because it ensures a good deal of shared variance.

## Types of Factor Analyses

Two types of analyses are used in this study, namely, Principal Components Analysis (PCA) and Principal Factor Analysis (PFA). Both types of analysis are based upon Hoetelling's Principal Axis method. Principal Components Analysis requires the placing of ones in the diagonal of the correlation matrix. Principal Factor Analysis on the other hand calls for some estimate of shared variance (communality) to be placed in the diagonal. The present PFA employed the largest, absolute, off-diagonal element as the communality estimate. This means the largest correlation which any item had was used as its diagonal entry.

## Rotation

One of the complications of factor analysis is that the particular configuration of factors in an analysis is arbitrary. It is analogous to an algebra equation in which a simple curve can be placed on a graph at a peculiar angle so that its algebraic equation is very complex. By moving the curve to a new position on the graph, it is possible to simplify the algebraic equation. The curve itself is unchanged. Only its relation to the axes has been changed.

This characteristic of analytic geometry led to the use of graphical rotation procedures to clarify factor relationships. By graphing the factors it was often possible to see simpler ways of expressing the same results. Thurstone (1935) specified characteristics of simplified factor relationships which he called, "simple structure." Simple structure
was supposed to make factors easier to identify and to lead to better factor stability. Factor analysts have been accustomed to using graphical rotations before attempting to name and interpret a set of factors. Recently Kaiser (1958) has provided the varimax criterion which defines mathematically a factor relationship similar to simple structure. In the present study all solutions were rotated to the varimax criterion of simple structure.

In order to compare separate analyses of different samples it is necessary to rotate them to a common position. The PCA solution for the total sample was used as a common position for all other PCA solutions and the PFA solution for the total sample was used as a common position for all PFA solutions. In order to rotate the other samples to this common position represented by the total sample, Cliff's procedure (1966) was used. Cliff's procedure involves rotating an analysis to a least squares fit to a "target" solution. A target solution is a solution which the experimenter tries to match. In the present study the solution obtained for the total sample was used as a target for the later solutions.

## Comparisons Between Factors

## Statistical Significance

Once the factors were rotated to a common position
a factor in one sample could be compared to a factor in another sample. The significance of the relationship between factors could be determined by Cattell, et. al.'s (1969) salient variable similarity index (s-index). When using the Index some level of sallency must be set for the factors.

Item loadings on a single factor can be grouped into three categories: (1) positively salient (loading above some value such as . 30); (2) hyperplane loadings (loadings between plus and minus . 30); and (3) negatively salient (loadings below -. 30). Two factors which are to be considered the same should have the same items in each of these three groups. For example, an item which is above .30 on one factor should be above . 30 on the other factor. Figure 1, from Cattell, et. al. (1969, p.784), gives the possible combinations of item categories for two factors being compared:

Factor 2

|  |  | PS | H | NS |
| :---: | :---: | :---: | :---: | :---: |
|  | PS | $\mathrm{f}_{11}$ | $\mathrm{f}_{12}$ | $\mathrm{f}^{\prime} 13$ |
| Factor 1 | H | $\mathrm{f}_{21}$ | $\mathrm{f}_{22}$ | $\mathrm{f}_{23}$ |
|  | NS | $\mathrm{f}_{31}$ | $\mathrm{f}_{32}$ | ${ }^{\text {f }} 33$ |

PS - positive salient variables (loading above . 30 )
H - hyperplane variables (loading between . 30 and -. 30)
NS - negative salient variables (loading below -. 30)
$\mathrm{f}_{1 j^{-}}$a joint frequency
Figure 1. Schematic Representation of Cross-classification of the Variables of Two Factors.

The s-index is calculated from the frequencies in Figure 1 by the formula:
$s=\frac{f_{11}-f_{33}-f_{13}-f_{31}}{\left(f_{11}-f_{33}-f_{13}-f_{31}-f_{12}-f_{21}-f_{23}-f_{32}\right) / 2}$
The possible values of the s-index ranged from -l for perfectly opposite factors to +1 for perfectly related factors.

As with the correlation coefficient, a value of zero represents no relation between the factors.

The value of the s-index is determined completely by the relative freguencies in the categories mentioned above. An item with a loading of .30 would be placed in the same category (positively salient) as an item with a loading of .90. Therefore when two factors are compared by means of the s-index, an item which loads . 30 on one factor and .90 on the other factor is considered to be in perfect agreement across the two factors. This type of agreement suggests that the value obtained for the s-index may not be a good estimate of the shared variance of the two factors, although it does give a level of significance.

In the present study some of the values calculated using the criterion of .30 saliency were not to be found on the table, because when a factor has more than $40 \%$ of the items salient, the s-index value is not given, therefore, using the .30 sallency level it was necessary to estimate 10 of the 30 comparisons shown in Table 4 (p.46). Since this procedure was not considered satisfactory, another method of calculating the s-values was used. When the s-index is used, the experimenter must choose a level of saliency and any level applies equally well. It was decided, therefore, to use the level that would permit an approximately equal number of items to be included in each comparison as salient items. The number of salient items was limited to $40 \%$ of the total number of items so that an s-value would be obtained whose significance
could be found in the table. Under this new procedure one of four different sallency values was chosen for each comparison, either $.20, .30, .40$, or .50 . Significance values for the present study are presented in Table 27.

## Percent of Shared Variance

As was noted in the review of the literature the coefficient of congruence is a widely used measure of factor agreement. Although it has no formal statistical test of significance it is considered to be a better measure of shared variance than the s-index. The square of the coefficient of congruence, therefore, was used in all comparisons as a measure of the shared variance.

## Sample

The testing of all subjects for the present study had been conducted previously by schools or testing agencies. This permitted the analysis of a "real life" sample and avoided the bias that could have been introduced by a single examiner testing all the children. In order to avoid excessive sampling of emotionally disturbed children, no hospital or clinic test results were used. IQ, mental age (MA), chronological age (CA), race, sex, and socio-economic status as measured by father's occupation were recorded for each child. The only restriction on the selection of subjects was that their MA be between 4 years 6 months and 7 years 6 months. This restriction was made in order to insure the applicability of the items in the analysis to the subjects being used. The Binet employs different items at different age levels so this restriction in MA eliminated subjects who had not been given
the items used in the present study.
A total of 827 tests were collected from seven school districts, two nursery schools and two day care centers In Nassau County. A measure of the accuracy of the data was provided by a computer program written to calculate the MA from all the item scores and compare it to the MA recorded on the test.

Of the original 827 test results collected, 116 or $14 \%$ were in error. By checking the coding of the cards from the forms on which the test data had been recorded, 39 coding errors were found. This was about $4.7 \%$ of the total or almost one third of the errors.

Three of the agencies providing test results were revisited in order to trace the errors. These 3 had furnished 358 of the 827 test results. Six of the errors or $.7 \%$ of the original 827 tests were found to be errors in transcribing information from the agency records. Errors in scoring were found on another 20 test results in the agencies. Altogether 65 of the 116 errors were accounted for. The remaining error rate of 51 in 827 was $6.2 \%$.

Since it was not possible to trace all errors to their original sources it was decided to eliminate all test results with excessive errors. Of the 51 remaining errors 34 were found to have errors of more than 2 months in MA and were eliminated from the sample. On the remaining 17 test results the recorded $M A$ was altered to agree with the calculated MA and the IQ's were re-evaluated.

Of the 65 corrected errors 31 were errors in the item scores and 34 were errors in the recorded MA scorc. If this same split can be assumed to hold for the remaining 17 errors, then the half which had inaccurate MA's could be considered to be correct. This would leave an error rate of no more than about 9 in 793 or $1.1 \%$.

The 793 subjects remaining after the error analysis were used to form the sample of 600. This sample conformed to the normal distribution as established by the authors of the Stanford-Binet scale (Table 28). The 193 excess cases were eliminated randomly with the aid of a random number table.

The actual mean of the total sample of 600 was 99.4 with a standard deviation of 16.2. The standardized mean IQ for the Binet is 100 and the standard deviation is 16 . The range of IQ's for the total sample was from 43 to 157 . There were 499 whites; 332 boys and 167 girls.

The fact that there were twice as many boys as girls is not surprising in that the usual ratio of boys to girls for testing in the schools, even for routine intellectual examinations, is in the neighborhood of 3 to 1 . The fact that the present ratio is only 2 to 1 is the result of using a large number of tests from schools where whole classes had been tested.

Of the 101 black children 52 were boys and 49 were girls. The mean MA for all children was 5.8 years with a standard deviation of .9 years. Chronological age ranged from 3 years 1 month to 11 years 2 months with a mean of 5.9 years and a standard deviation of 1.2 years.

## Procedure

Various combinations of the total sample were used to form different groups for analysis, as shown below:

| Group | Sample | No | Method of Selection |
| :---: | :---: | :---: | :---: |
| 1 | A | 300 | Randomly selected from total sample |
|  | B | 300 | Remaining after sample A selected |
| 2 | C | 150 | Randomly selected from total sample |
|  | D | 150 | Randomly selected after sample $C$ selected |
| 3 | C-2 | 100 | Randomly selected from 300 cases which remained after samples $C$ and $D$ selected |
|  | D-2 | 100 | Randomly selected after sample $C$ selected |
| 4 | A-10w | 150 | Lower half of IQ range of Sample A-Group 1 |
|  | B-low | 150 | Lower half of IQ range of Sample B-Group 1 |
| 5 | A-high | 150 | Upper half of IQ range of Sample A-Group 1 |
|  | B-high | 150 | Upper half of IQ range of Sample B-Group 1 |
| 6 | E-very low F-very low | 100 | Random division into two groups of lowest 200 of $I Q$ range of total sample of 600 |
| 7 | E-very high F-very high | 100 | Random division into two groups of highest 200 of $I Q$ range of total sample of 600 |
| 8 | Male-10w | 100 | Random selection of males below 100 IQ |
|  | Female-low | 100 | Random selection of females below 100 IQ |
| 9 | Male-high | 100 | Random selection of males above 100 IQ |
|  | Female-high | 100 | Random selection of females above 100 IQ |
| 10 | Black | 101 | IQ scores of whites in sample matched to scores of total blacks in sample |
|  | White | 101 | within a 10 point distribution range |
| 11 | G | 300 | Randomly selected from total of 793 cases to fit a normal distribution with a mean of 99.5 and a SD of 12 . |
|  | H | 300 | Randomly selected from the remaining 493 cases to fit a normal distribution with a mean of 99.5 and a $S D$ of 21. |

Randomization was accomplished by the use of an IBM random number generator subroutine, RANDU.

The samples used in Groups 1 to 11 represent all
the samples used in the present study. Additional comparisons were made by regrouping the samples as follows:

| Group | Samples |
| :---: | :---: |
| 12 | $\begin{aligned} & \text { A-1ow IQ } \\ & \text { A-high IQ } \end{aligned}$ |
| 13 | $\begin{aligned} & \mathrm{B}-10 \mathrm{IQ} \\ & \mathrm{~B}-\mathrm{high} \mathrm{IQ} \end{aligned}$ |
| 14 | $\begin{aligned} & \text { A low IQ } \\ & \text { B-high IQ } \end{aligned}$ |
| 15 | $\begin{aligned} & \mathrm{B}-10 \mathrm{IQ} \\ & \text { A-high IQ } \end{aligned}$ |
| 16 | E-very low IQ <br> E-very high IQ |
| 17 | F- very low IQ <br> F- very high IQ |
| 18 | E-very low IQ <br> F-very high IQ |
| 19 | F-very low IQ <br> E-very high IQ |
| 20 | $\begin{aligned} & \text { Male-low IQ } \\ & \text { Male-high IQ } \end{aligned}$ |
| 21 | $\begin{aligned} & \text { Female-low IQ } \\ & \text { Female-high IQ } \end{aligned}$ |

Since most of the items on the Binet are dichotomous with only pass or fail scoring, most correlational techniques are inappropriate. Previous investigators have used the tetrachoric correlation because it gives a good estimate of the relationship between normally distributed dichotomous variables. Hayes (1943), however, has pointed out that the tetrachoric correlation is quite unstable and may overestimate
the actual correlation. The phi coefficient was preferred in the present study because it does not require the assumption of normality and gives a conservative estimate of the correlation between dichotomous variables.

The total sample was first analyzed by both Principal Components Analysis and Principal Factor Analysis. The number of factors was determined by Guttman's unit eigenvalue rule as described by Kaiser (1960). Once the number of factors was determined that same number was used in all analyses. The PCA and PFA solutions for the total samples were used as standard solutions for all later analyses of the same type as explained in the section on rotation.

Reliability as a function of sample size was deter-mined by comparing samples within Group l sample size 300 , Group 2 sample size 150, and Group 3 sample size 100. This was done for both PCA and PFA. Reliability was further investigated for both types of analysis by comparing the samples within Group 4 low-IQ and Group 5 high-IQ. For the remaining comparisons only the principal components analysis was used.

The reliability for low and high IQ levels was examined by comparing the samples within groups $4,5,6$, and 7 . Comparisons were made both on the same IQ level and across IQ levels. Cross-IQ comparisons were the comparisons in groups 12 to 21.

Differences with respect to sex on the same IQ
level were investigated by comparisons within groups 8 and 9. The effect of race was considered by comparing the white and black samples in group 10. Final comparisons were made
between small and large standard deviations as represented by the samples in group 11.

## CHAPTER III

## RESULTS

In the present study 22 subsamples were used in addition to the single total sample of 600 children. Table 3 gives mean IQ's, standard deviations, and ranges for the 23 samples studied. The significance of the differences between means and variances of samples in the same groups are also presented in this table. The t-test of the differences between means was taken from Ostle (1954). This test makes no assumption about the equality or lack of equality of sample variances. It does require that the sample sizes be the same. Only two groups, 3 and 7, had significantly different means. Even with random division such differences can occur. The greatest difference was only 7.2 IQ points (Group 3). An F-test was used to test the differences between variances.

## Identification of Factors

The results of a Principal Components Analysis are called components. The term factor is reserved for the results of a factor analysis such as the Principal Factor Analysis. Because of the wide popularity of the term factor deriving from factor analysis both components and factors are often referred to under the general term f'actor. In some cases the same usage was necessary here.

Table 3

Means, Standard Deviations and Tests of Significance for all Basic Samples

| Group | No. | Mean | $\underline{t}^{\text {a }}$ | SD | F | Range |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total sample | 600 | 99.4 |  | 16.2 |  | 43-157 |
| 1 A Large sample | $\begin{aligned} & 300 \\ & 300 \end{aligned}$ | $\begin{aligned} & 99.1 \\ & 99.7 \end{aligned}$ | . 72 | 15.6 16.8 | 1.16 | $59-143$ $43-157$ |
| $2 \begin{aligned} & \text { C Medium sample } \\ & \text { D Medium sample }\end{aligned}$ | 150 150 | 99.5 100.8 | 1.22 | 15.4 14.7 | 1.10 | $50-136$ $50-136$ |
| 3 C2 Small sample | 100 100 | 103.4 96.2 | -3.38** | 18.2 16.5 | 1.22 | $\begin{aligned} & 43-143 \\ & 59-157 \end{aligned}$ |
| $4 \begin{aligned} & \text { A Low IQ } \\ & \text { B Low IQ }\end{aligned}$ | 150 150 | 86.5 86.5 | . 00 | 9.2 10.2 | 1.23 | $59-100$ $43-100$ |
| $5 \begin{gathered}\text { A H1gh IQ } \\ \text { B High IQ }\end{gathered}$ | 150 150 | 111.7 112.9 | 1.20 | 9.2 20.6 | 1.33 | $\begin{aligned} & 100-143 \\ & 100-157 \end{aligned}$ |
| $6 \begin{gathered}\text { E Very low } \\ \text { F Very low } \\ \text { IQ }\end{gathered}$ | 100 100 | 81.2 82.2 | . 94 | 9.0 7.6 | 1.40* | $\begin{aligned} & 43-93 \\ & 61-93 \end{aligned}$ |
| $7 \underset{\text { F Very high IQ }}{\text { E }}$ ( ${ }^{\text {E }}$ | 100 100 | 115.8 118.4 | 2.00* | 7.1 10.0 | 1.98* | $107-145$ $107-145$ |
| $8 \begin{aligned} & \text { Male low IQ } \\ & \text { Female low IQ }\end{aligned}$ | 100 100 | $\begin{aligned} & 87.5 \\ & 86.7 \end{aligned}$ | -. 64 | $\begin{aligned} & 8.8 \\ & 9.6 \end{aligned}$ | 1.19 | $\begin{aligned} & 58-99 \\ & 59-99 \end{aligned}$ |
| $9 \begin{aligned} & \text { Male high IQ } \\ & \text { Female high IQ }\end{aligned}$ | 100 100 | 113.5 110.9 | -1.98 | 9.4 11.2 | 1.42* | $\begin{aligned} & 100-140 \\ & 100-157 \end{aligned}$ |
| $10 \begin{aligned} & \text { White } \\ & \text { Black }\end{aligned}$ | $\begin{aligned} & 101 \\ & 101 \end{aligned}$ | $\begin{aligned} & 93.2 \\ & 93.2 \end{aligned}$ | . 07 | $\begin{aligned} & 14.0 \\ & 13.8 \end{aligned}$ | 1.03 | $\begin{aligned} & 66-123 \\ & 6 I-126 \end{aligned}$ |
| 11 Small SD | $\begin{aligned} & 300 \\ & 300 \end{aligned}$ | $\begin{aligned} & 99.4 \\ & 99.3 \end{aligned}$ | -. 09 | $\begin{aligned} & 13.2 \\ & 20.9 \end{aligned}$ | 2.50*** | $\begin{aligned} & 50-144 \\ & 39-160 \end{aligned}$ |

a Degrees of freedom in all groups are one less than the common sample size of the 2 samples in each.

$$
\begin{aligned}
* \mathrm{p} & <.05 \\
* * \mathrm{p} & <.01 \\
* * * \mathrm{p} & <.001
\end{aligned}
$$

Six factors were extracted in 21122 analyses using the unit eigenvalue rule which has been established as the criterion in the present study. When the total sample was analyzed by PCA , the six components extracted were matched with the six factors extracted in the PFA, although the factors and components came out in different orders. Listings of the loadings for the factors and components from all analyses can be found in Tables 36 to 69. The factors have been numbered 1, 2, 5, 4, 6, and 3. This ordering gives identical numbers to corresponding factors and components.

For each of the factors extracted a tentative name was given based upon the items which loaded above .30 on the factor. The following are the factors and loadings which appeared when the total sample at 600 was analyzed by both the PCA and PFA methods.

## Factor 1: Visual Judgment

Only six items loaded above .30 on this factor for both the PCA and PFA. These items and loadings were:

Item
V-4
Copying a Square Loading
.68 .67 .62 . 42 .39 . 3138V-1 Picture Completion: ManVI-4 Number Concepts 31 34

All these items involve visual ability as well as judgment. Four of the items had motor components, but success with pictorial similarities and differences and number concepts
is dependent upon visual judgment but not upon motor ability. Since exactly the same items were found to load above .30 for both PCA and PFA, the s-index between the component and factor had a value of 1.00 . This was significant well beyond the . 01 level. The coefficient of congruence had a value of .967 . Squaring the coefficient of congruence gave the estimated per cent of shared variance of $93 \%$. Factor 2: Verbal Abstract Ab11ity

Only two items loaded above . 30 for this factor. Again the same two were found in both PCA and PFA.

PCA Loading

## Item

VII-2 Similarities: Two Things
VII-5 Opposite Analogies III
.85
.57

PFA
Loading
.53
.45

The s-index again had a highly significant value of 1.00 . The coefficient of congruence had a value of .927 which gives an estimated per cent of shared variance of $86 \%$.

## Factor 3: Definitions

The only item loading above . 30 for either PCA or
PFA was Definitions. This item is apparently a factorially pure item at these age levels. That is, passing Definitions appears to depend on no other ability required by the 17 remaining items.

| Item | PCA <br> Loading | PFA <br> Loading |
| :--- | :---: | :---: |
| Definitions | .93 | .46 |

The occurrence of this single item as the only item above .30 in either PCA or PFA gives an s-index value of 1.00 . This
value is significant beyond the . Ol level. The coefficient of congruence was . 867 which when squared gave $75 \%$ as the estimated per cent of shared variance between the factor and component.

Factor 4: Numeric Memory
The same two items are involved in the two types of analysis for this factor.

| Item | PCA <br> Loading | PFA <br> Loading |
| :--- | :---: | :---: |
| Repeating 5 Digits | .88 | .53 |
| Number Concepts | .37 | .35 |

Again an s-value of 1.00 was significant beyond the . Ol level. The coefficient of congruence of .939 gives an estimated per cent of shared variance of $88 \%$.

Factor 5: Difficulty Level
The s-value relating this component and factor was .92 which was significant beyond the . 01 level. The coefficient of congruence was . 974 which squared gives $95 \%$ as the estimated per cent of shared variance.

PCA PFA
Item Loading

Loading

VII-1 Picture Absurdities 1
VII-4 Comprehension IV . 68
.61
VII-3 Copying a Diamond . 60
V-б Patience: Rectangles . 39
VI-4 Number Concepts . 38
VI-3 Mutilated Pictures . 32
.31
V-2 Paper Folding: Triangle . 31
. 51
.45

Factor 5 presents the most complex configuration of items of any of the factors, and the items do not seem logically related to one another. A large number of diverse items are included. Several items which correlate highly with the test as a whole such as Vocabulary, Similarities: Two Things, and Opposite Analogies II and III are missing so it cannot be considered a g-factor. The three top loadings in both analyses seem to account for most of the variance of this factor. These items are quite diverse but are all on the 7-year level. Since the Binet items are arranged in order of difficulty with each succeeding age level being more difficult than the one preceding it, there is a possibility of the occurrence of a factor relating to difficulty. Factor 5 seems to be such a factor.

## Factor 6: Verbal

This factor contained nine salient loadings (above .30) in the PCA and eight in the PFA.

## Item

VI-5 Opposite Analogies II
VI-2 Differences
VI-1 Vocabulary
V-1 Picture Completion: Man
VI-3 Mutilated Pictures . 51
VII-5 Opposite Analogies III
.47

.43
VI-4 Number Concepts .

.41
VI-6 Maze Tracing ..... 30

.37 ..... 37
V-6 Patience: Rectangles67
43V-1 Picture Completion: Man60

PFA Loading4963 . 4541 3231
36
.51

The s-index relating this factor and component was .94 which was significant beyond the .01 level. The coefficient of congruence was . 997 which gives an estimated $99 \%$ of shared variance between the factor and component.

Since the Bine was constructed as a test of general intelligence one must consider the possibility of a g-factor in these rotated solutions. Factor 6 contains many verbal items and is the best candidate for a g-factor. Factor 6 was not considered to be a g-factor however, because several items which correlate highly with the test as a whole, including Similarities and Definitions, do not appear in Factor 6.

Both PCA and PFA were used on a number of subsamples In addition to the total sample. Samples $A$ and $B$ represented random division of the total sample of 600 into two samples of 300 each. These two samples formed Group 1. Comparison of the components in sample $A$ with those in sample $B$ gave a measure of reliability for the six components when the sample size was 300. Group 1 in Table 4 gives the values of the s-index when the components in samples $A$ and $B$ were compared using .30 as the saliency value. Groups 2 and 3 give s-index values for the comparison of components from samples of smaller sizes.

Group 4 represents comparisons of components in sample A low IQ with sample $B$ low IQ. Group 5 gives the comparisons between sample $A$ high $I Q$ and sample $B$ high IQ. All of these five groups were also analyzed by PFA. Results of comparsons of the factors are given in Table 5 .

Table 4
Salient variable similarity index for the Principal Components Analysis: A sallency value at .30 was used. The percents refer to hyperplane percents. That is the percent of items with loadings within .30 of zero.

| Group | Components |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| $N=\frac{1}{300}$ | $.67^{* *}$ | $\begin{aligned} & .50^{* *} \\ & 78 \% \end{aligned}$ | $\begin{aligned} & .67 * * \\ & 92 \% \end{aligned}$ | $\begin{aligned} & .67^{* *} \\ & 83 \% \end{aligned}$ | $\begin{aligned} & .86 * * \\ & 61 \% \end{aligned}$ | $.82(* *)$ |
| $\stackrel{2}{N=150}$ | . $33 * *$ | $.75 * *$ $78 \%$ | $.67 * *$ $83 \%$ | $\begin{aligned} & .86 * * \\ & 81 \% \end{aligned}$ | $.36 * *$ 59\% | $.84(* *)$ |
| $\stackrel{3}{\mathrm{~N}=100}$ | $.80\left(^{* *}\right)$ $58 \%$ | . $578 *$ | .00 <br> $81 \%$ | . 22 | ${ }_{56 \%}^{63(* *)}$ | $\begin{aligned} & .74(* *) \\ & 47 \% \end{aligned}$ |
| $\mathrm{N}=\stackrel{4}{150}$ | . 63 (**) | $.50 * *$ $78 \%$ | $.67 * *$ $78 \%$ | $.55 * *$ $69 \%$ | 63(**) $56 \%$ | $.82\left({ }^{* *)}\right.$ |
| $\begin{gathered} 5 \\ N=150 \end{gathered}$ | $\begin{aligned} & .62^{* *} \\ & 64 \% \end{aligned}$ | $\begin{aligned} & .67 * * \\ & 83 \% \end{aligned}$ | $.40 \%$ $86 \%$ | $\begin{aligned} & .40 * \\ & 86 \% \end{aligned}$ | $.80(* *)$ | $.90(* *)$ |

Table 5
Salient variable similarity index for the Principal Factor Analysis: A saliency value at .30 was used. The percents refer to hyperplane percents. That is the percent of items with loadings within .30 of zero.

| Group | Factors |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| $\mathrm{N}=\frac{1}{300}$ | . 83 \% ${ }^{\text {6\% }}$ | $.67 * *$ $75 \%$ | . $50 \% *$ | $.67 * *$ $83 \%$ | $.67 * *$ $67 \%$ | $\begin{aligned} & .67^{* *} \\ & 67 \% \end{aligned}$ |
| $\stackrel{2}{N}=150$ | ${ }_{.} 57{ }^{* *}$ | $.75 * *$ $78 \%$ | . $80 \% * *$ | $\begin{aligned} & 80^{* *} \\ & 86 \% \end{aligned}$ | $\begin{aligned} & .40 * * \\ & 72 \% \end{aligned}$ | $\begin{aligned} & .62^{* *} \\ & 64 \% \end{aligned}$ |
| $\begin{gathered} 3 \\ N=100 \end{gathered}$ | $\begin{aligned} & 1.00^{* *} \\ & 67 \% \end{aligned}$ | $.67 * *$ $83 \%$ | . $33^{* *}$ | $\begin{aligned} & .25 \\ & 78 \% \end{aligned}$ | $\begin{aligned} & .53(* *) \\ & 58 \% \end{aligned}$ | $\begin{aligned} & .67(* *) \\ & 50 \% \end{aligned}$ |
| 4 $N=150$ | . $43^{*}$ | . 4 75* | .00 <br> $89 \%$ | $.80 * *$ $86 \%$ | . $33 *$ | $\begin{gathered} .43 * \\ 61 \% \end{gathered}$ |
| $\stackrel{5}{\mathrm{~N}=150}$ | $.62 * *$ $64 \%$ | . $57{ }^{\text {\% }}$ * | .00 $89 \%$ | $.67 * *$ $83 \%$ | $\begin{aligned} & .86 * * \\ & 61 \% \end{aligned}$ | $.77^{* *}$ |

Note: Parentheses around asterisks indicate that the s-value significance had to be estimated because of limited coverage in the significance table.
*p<. 05
$* p<.01$

The six factors compared between samples in five groups produced 30 comparisons for PFA. The same 30 comparisons for PCA led to a total of 60 comparisons in Tables 4 and 5. When such a large number of comparisons are done some significant results will be found by chance. When 60 comparisons are done at the . 05 level one would expect three comparisons ( $60 \times .05=3$ ) to be significant by chance. Therefore, of the 55 comparisons in Tables 4 and 5 which were significant at the .05 level, three of them could have been due to chance. Using the same reasoning when comparisons are done at the . 01 level one would expect less than one comparison to be significant due to chance ( $60 \times .01-.6$ ). Therefore, one of the 48 comparisons in Tables 4 and 5 significant at the . 01 level might have been due to chance.

As was mentioned earlier, 12 of the 60 comparisons in Tables 4 and 5 produced values of the s-index which were not included in the tables of significance. The alternate procedure, in which approximately the same number of items was included as salient for all comparisons, was used. All PCA and PFA comparisons for Groups 1 to 5 were performed using this procedure and the values of the s-index are summarized in Table 6 as well as in Tables 10 and 11.

Tables 7 and 8 give the coefficients of congruence and estimated per cent of shared variance for Groups 1 to 5.

## Table 6

Summary of the $s$-indexes for the components and factors from tables 10 through 13 for the different groups studied. The figures represent the degree of agreement between the two random samples within each group. The higher the figure, the more likely it is that the same Stanford-Binet test items comprised the factors or components in each sample.

| Group | Variable | No. |  | Components or Factors |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 1 | 2 | 3 | 4 | 5 | 6 |
| 1.5 | Sample size | 300 | PCA | . $60 * *$ | . 31 | .67** | . 50 ** | . $86 * *$ | . $62 * *$ |
|  |  |  | PFA | .83** | . $77^{* *}$ | . 50 ** | . $40 *$ | . $67^{* *}$ | . $67 * *$ |
| 2 | Sample size | 150 | PCA | . $33 *$ | .83** | .55** | . $86 * *$ | . $36 *$ | .71** |
|  |  |  | PFA | . $57^{* *}$ | .92** | . $44 *$ | . $80 * *$ | . $57^{* *}$ | .61** |
| 3 | Sample size | 100 | PCA | . $80 * *$ | .60** | . 20 | . 22 | . 25 | . 55 ** |
|  |  |  | PFA | 1.00** | . $57 \% *$ | .73** | .43* | . $32^{*}$ | . $33^{*}$ |
| 4 | Low IQ | 150 | PCA | . $40 *$ | . 50 ** | . $67 * *$ | . 54 ** | . $50 * *$ | . $33 *$ |
|  |  |  | PFA | . $43 *$ | . $77^{* *}$ | . $36 *$ | .67** | . $33^{*}$ | .43* |
| 5 | Hieh IQ | 150 | PCA | . $62 * *$ | .33* | . 18 | . $46 * *$ | . $89 * *$ | . $67 * *$ |
|  |  |  | PFA | . $62^{* *}$ | . $57 * *$ | .43* | . $57 * *$ | . $86 * *$ | . $77 * *$ |
| 6 V | Very low IQ | 100 | PCA ${ }^{\text {a }}$ | . $50 * *$ | . $73 * *$ | . $67 * *$ | .73** | . 50 ** | . $67 * *$ |
| 7 V | Very high IQ | 100 | PCA ${ }^{\text {a }}$ | . $73 * *$ | . $33 *$ | . 17 | . 29 | . 44 | . $57^{* *}$ |

a Only Principal Components Analysis done on these groups
*p<05
**p<. 01

Table 7
Coefficients of Congruence for the Principal Components Analysis: The estimated percent of shared variance was obtained by squaring the coefficient of congruence.

| Group | Components |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | Average |
| 1 | . 838 | . 678 | . 878 | . 821 | . 947 | . 889 |  |
| $N-300$ | 70\% | 46\% | 77\% | 67\% | 90\% | 79\% | 71.5\% |
| $\stackrel{2}{2}$ | . 809 | . 819 | . 782 | . 757 | . 751 |  |  |
| $N=150$ | 65\% | 67\% | 61\% | 57\% | 56\% | 83\% | 64.8\% |
| $N=100$ | . 923 | .742 $55 \%$ | .630 $40 \%$ | .543 $29 \%$ | . 818 | $\begin{aligned} & .884 \\ & 78 \% \end{aligned}$ | 59.0\% |
| $\stackrel{4}{4}$ | . $747 \%$ | . 558 | . 517 | . 644 | . 719 | . $828 \%$ |  |
| $N=150$ | 56\% | 31\% | 27\% | 41\% | 52\% | 68\% | 45.8\% |
| $\stackrel{5}{N}=150$ | . 8318 | . 773 | .603 $36 \%$ | .733 $54 \%$ | .950 $90 \%$ | .945 $.89 \%$ | 66.3\% |
| Average | 69\% | 52\% | 48\% | 49\% | 71\% | 79\% | 61.5\% |

Table 8
Coefficients of Congruence for Principal Factor Analysis: The estimated percent of shared variance was obtained by squaring the coefficient of congruence.

| Group | Factors |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | Average |  |
| 1 | .912 | .791 | .788 | .844 | .945 | .978 |  |  |
| $\mathrm{~N}=300$ | $83 \%$ | $63 \%$ | $62 \%$ | $71 \%$ | $89 \%$ | $84 \%$ | $75.3 \%$ |  |
| 2 | .924 | .887 | .789 | .812 | .840 | .914 |  |  |
| $\mathrm{~N}=150$ | $85 \%$ | $79 \%$ | $62 \%$ | $66 \%$ | $71 \%$ | $84 \%$ | $74.5 \%$ |  |
| 3 | .937 | .726 | .884 | .431 | .848 | .876 |  |  |
| $\mathrm{~N}=100$ | $88 \%$ | $53 \%$ | $78 \%$ | $19 \%$ | $72 \%$ | $77 \%$ | $64.5 \%$ |  |
| 4 | .830 | .648 | .551 | .732 | .832 | .764 |  |  |
| $\mathrm{~N}=150$ | $69 \%$ | $42 \%$ | $30 \%$ | $54 \%$ | $69 \%$ | $58 \%$ | $53.7 \%$ |  |
| 5 | .907 | .850 | .515 | .755 | .939 | .889 |  |  |
| $\mathrm{~N}=150$ | $82 \%$ | $72 \%$ | $27 \%$ | $57 \%$ | $88 \%$ | $79 \%$ | $67.5 \%$ |  |
| Average | $81 \%$ | $62 \%$ | $52 \%$ | $53 \%$ | $77 \%$ | $76 \%$ | $67.1 \%$ |  |

## Comparison of the PCA and PFA

One of the major aspects of this study was to determine which method of analysis was more reliable, and then to use this method for the remaining 12 analyses. In order to determine whether there were significant differences between the two types of analyses, Groups 1 through 5 (Table 4) were compared using McNemar's exact test of correlated proportions. First, all the components and factors reliable at the . Ol level were considered and then all components and factors rellable at the .05 level were considered. When the reliable components in Group 1, Table 4, were compared to the reliable factors in Group 1 , Table 5, by MeNemar's exact test, a .05 level of significance was used.

As can be seen in Table 4, PCA Group 1 has six components significant at the . Ol level. Table 5 shows the same result for PFA with six factors significant at the . Ol level. In this circumstance the use of McNemar's test shows no significant difference between them since, in fact, there is no difference at all.

When the second procedure for the $s$-Index was employed both PCA and PFA had five components and factors significant at the . 01 level. Components $1,3,4,5$, and 6 were significantly reliable while factors $1,2,3,5$, and 6 were significantly reliable. Four of the reliable factors were also reliable components (1, 3, 5, and 6). No difference was found between the two types of analyses with regard to the number of significantly reliable factors.

## P.C.A. Group 2 has five components significant at

 the . Ol level and PFA has six factors significant at the . O1 level. For this comparison the McNemar's $2 \times 2$ table is as shown below:
## PFA

PCA

| PFA |  |  |  |
| :--- | :--- | :--- | :--- |
|  | NS | Sign. |  |
| NS | 0 | 1 | 1 |
| SIgn. | 0 | 5 | 5 |
|  | 0 | 6 | 6 |

The significance of this four-fold table is evaluated by the procedure suggested by Hays (1963, p.602), using a two-tailed test. The results show that the difference between the two methods is not significant at the .05 level. Since PCA Group 2 has six components significant at the .05 level and the same is true of PFA Group 2, there are no significant differences between the methods when using this criterion of significance.

Reapplication of the s-index for Group 2 in Table 6 produced significant results at the . Ol level for four components and five factors. The McNemar's table for this comparison is:


Hays' procedure shows this result to be non-significant.

PCA Group 3 has four factors significant at the . Ol level and PFA Group 3 has five factors significant at the . O1 level. For this comparison the Mc Nemar's $2 \times 2$ table is as shown below:

PCA

| PFA |  |  |  |
| :--- | :---: | :---: | :---: |
|  | NS | Sign. |  |
| NS | 1 | 1 | 2 |
| Sign. | 0 | 4 | 4 |
|  | 1 | 5 | 6 |

McNemar's test shows that the difference between the two methods is not significant at the .05 level.

Readministration of the s-index for Group 3 produced three components (1, 2, and 6) and three factors (1, 2, and 3) significant at the .01 level for Group 3 as seen in Table 6. These were not significantly different.

In view of the fact that six components or factors were extracted each time, only a limited number of combinations could be significant using McNemar's test as a twotalled test. In fact the only combination is 6 components or factors in one group and zero components or factors in another group. All other combinations would be nonsignificant.

As was noted in the original application of the s-index, the only difference between the use of the .01 level and the .05 level for Groups 1 to 3 was that the six components were significantly reliable for Group 2 at the .05 level while only five were significant at the . Ol level. Since there were also six factors in Group 2 significant at the .05 level, the number of significantly reliable components and factors still
do not differ significantly.
Groups 4 and 5 contain low IQ and high IQ samples respectively. Comparison of the samples within these groups provides reliability estimates for PCA and PFA for these restricted IQ groups. When PCA Group 4 is compared with PFA Group 4 it is noted that the former has six components significant, but the latter has only one factor significant at the . O1 level. Using McNemar's test a nonsignificant difference was found. Table 6 shows that the readministration of the s-index produced only four components but two factors significant at the .01 level. This was not a significant difference. Using the .05 level reveals 6 significant components and 5 significant factors by the original use of the s-index as shown in Table 4. Readministration of the s-index led to four components and two factors significant at the .05 level, as shown in Table 6. With both uses of the s-index the number of rellable factors and components was not found to be significantly different.

PCA Group 5 has four components significant at the .O1 level and PFA Group 5 has five components significant at the . Ol level. The difference between the two methods is nonsignificant at the . 05 level. PCA Group 5 has six components significant at the .05 level and PFA Group 5 has five factors significant at the .05 level. The difference between the two methods at this level is nonsignificant.

Readministration of the s-index led to four components and five factors significantly reliable at the .01 level. This difference was nonsignificant. The same comparison at
the .05 level led to five reliable components and six reliable factors which was also a nonsignificant difference.

As can be seen, PCA and PFA were compared in five groups for both the . 05 and . 01 levels, therefore, ten comparisons were made. Since two methods of using the s-index were also employed, the total number of comparisons of PCA and PFA was 20 . Using the .05 level, one would expect one significant result ( $20 \mathrm{x} .05=1$ ) by chance. No significant results were found.

Tables 7 and 8 give the coefficients of congruence between factors from each sample for Groups 1 to 5. Table 7 gives the results for PCA while Table 8 gives them for PFA. The squared coefficients are also included in the tables as estimates of the per cent of shared variance. Average per cents are also given for each group and factor.

From Table 7 it can be seen that the average per cent of shared varlance for the components ranges from $48 \%$ to $79 \%$. Table 8 shows that the shared variance of the factors ranges on the average from $52 \%$ to $81 \%$. From this it can be concluded that both factors and components from one half of a split sample share at least about half of their variance with corresponding factors or components from the other half of the split sample.

## Effects of Sample Size

Since one would hypothesize greater reliability for larger samples, a one-tail test was justified when using the McNemar's exact test to compare the number of rellable factors or components between groups composed of samples of different
sizes. Use of the one-tall test resulted in a greater number of combinations found to be significant. These combinations were: 6 components or factors rellable in one group and zero in another; 5 in one group and zero in another.

PCA Group 1, sample size 300 has 6 components significant at the . Ol level and PCA Group 2, sample size 150 has 5 components significant at the . Ol level; thus the difference between them was found to be nonsignificant. This was also true when the components found to be reliable at the .05 level were compared.

Readministration of the s-index led to five components significantly reliable at the . Ol level for Group 1 and four for Group 2 as shown in Table 6. This was not a significant difference. When the .05 level was used Group 1 had five significant components while Group 2 had six. This was a nonsignificant difference.

When PCA Group 1, sample size 300 was compared with PCA Group 3 sample size 100 , the results showed that PCA Group 1 with 6 components significant was not significantly different from PCA Group 3 with 4 components significant at the .01 level. No new components were significant when the .05 level was used.

Readministration of the s-index produced five components significant at the . Ol level for PCA Group 1 , sample size 300 , and three components significant for PCA Group 3, sample size 100. According to the McNemar's exact test, this was not a significant difference. No new components were significant when the .05 level was used. When the same procedures were used with PFA there were no significant differences in the number
of reliable factors for different sample sizes. In all, 16 comparisons were made and although one might have expected .8 comparisons to be significant by chance, none were significant. Effects of IQ Eevel

Rellability indices for low IQ levels, Group 4, and high IQ levels, Group 5, are shown in Tables 4 and 5. Comparisons were made between the reliable factors for low IQ and full range $I Q$, for high $I Q$ and full range $I Q$ and for low $I Q$ and high IQ, both at the . 01 and the . 05 levels. These comparisons were made for PCA and PFA and for both calculations of the s-index. There was a total of 24 comparisons in all. When 24 comparisons are done at the .05 level one would expect 1.2 comparisons ( $24 \mathrm{x} .05=1.2$ ) to be significant by chance. In actual fact there was no significant difference found.

## Comparison of Different Subject Groups

Only the PCA was used in comparisons of very high IQ vs very low IQ samples; male samples vs female samples; white samples vs black samples; and large $S D$ samples vs small $S D$ samples. It has already been noted that there were no significant differences between PCA and PFA.

## Sex: Male-Female Groups

Group 8 involved a comparison between low IQ males and females. Table 9 shows that four components are significantly related (three at the . 01 level and one at the .05 level). The low IQ sample (Group 4, Table 6) shows that all 6 of the IQ components and factors are significantly related at the . 05 level and therefore rellable. Therefore, four components agree between low IQ males and females, whereas six components and

Table 9
Summary of s-index for the components and factors from Tables 12 through 23, when the two samples to be compared have been selected on the basis of IQ level, sex, race and size of standard deviation.

$* p .<05$
$* *$
factors agree in the two low $I Q$ samples at the .05 level. According to McNemar's test the difference between the low IQ males vs females and the low $I Q$ samples was nonsignificant. Comparisons Across Race

Group 10, white and black samples (Table 9), showed fairly good agreement across races since three components were significantly related at the .01 level and the other three at the . 05 level. This occurred despite the fact that in Group 3 (Table 6), the group set up to test reliability in samples with an $N$ of 100, only three components were significant at the .01 level and three were nonsignificant. The McNemar's exact test indicates that the difference between the white-black sample and the sample size of 200 is nonsignificant. It should be noted from Table 3, that the black and white samples are relatively low $I Q$ samples. Comparisons with low IQ samples such as those found in Groups 4 and 6 in Table 6 show no significant diffference between these groups. The results suggest that when IQ 1s held constant almost no differences are found between races with respect to factorial structure. Comparisons Across Different Standard Deviations

The results in Table 9, Group 11, indicate that the samples with different standard deviations (sample size 300) have five components significant at the . $O 1$ level and one at the .05 level. This is also true of Group 1 PCA, sample size 300, so there are no significant differences. There would appear to be little difference in the composition of components die to a change in the standard deviation of the sample.

## Cross-IQ Comparisons

In comparisons reported previously on page 56, the number of reliable components and factors in the low IQ groups was compared with the number of reliable components and factors in the high $I Q$ groups. The present comparisons were done to determine whether the same subtest items loaded in the same fashion on the components and factors of the low IQ groups and high IQ groups. The results of these comparisons are indicated by Groups 12 through 21 in Table 9. For example, in Group 12, where PCA sample $A$ low $I Q$ was compared to $P C A$ sample $A$ high $I Q$, the five components were significant at the . Ol level, which means that five of the low IQ components can be considered to be the same in structure as the corresponding ilve of the high IQ components. The question to be answered is whether the low IQ samples (Group 12, A low) resemble the high IQ samples (Group 12, A high) to the same extent that the two low IQ samples in Group 4 resembled each other. As can be seen, five components were significantly related in Group 12 PCA , and in Group 4, Table 6, four components were significantly related at the . Ol level. According to the McNemar's test this was not a significant difierence.

If this type of comparison is made for Groups 12 through 21 there will be 64 comparisons in all, derived as follows: (1) there are 32 comparisons for Groups 12 through 15 because each of these is compared to Groups 4 and 5 at the . O1 level and . 05 level and each includes a PCA and PFA result; (2) Groups 16 through 19 include comparisons with Groups 6 and 7 at the .01 and the .05 levels, a total of 16 comparisons,
and (3) there are no exact reference groups for Groups 20 and 21 because low IQ Group 4 and high IQ Group 5 contain a larger sample size and very low IQ Group 6 and very high Group 7 differ in mean IQ. Comparisons were made with both groups which brings the number made to 16 .

Since all 64 comparisons were done at the .05 level one would expect about three ( $64 \mathrm{x} .05=3.2$ ) comparisons to be significant by chance. In fact no comparison was found to be significant.

In the present study Factor 1 was called Visual Judgment. Dean (1951) and Ramsey (1968) found visual or perceptual factors at this age level. No direct comparison can be made with Dean's work, however, because he used a different version of the Binet Scale. In the previous study by Ramsey there was a visual judgment factor which contained four items which loaded at the .30 level or higher. Three of these items, namely Copying a Square, Paper Folding, and Maze Tracing, also comprise three of the six items which loaded at the .30 level or higher on the visual judgment factor in this study.

Although some of the items which load on this factor in the present study have a motor component, namely Copying a Square, Paper Folding, Maze Tracing, and Picture Completion, two of the other items do not depend upon motor ability, namely Number Concepts and Pictorial Similarities and Differences. In Number Concepts the child must not only know how to count, but must be able to visualize when he has selected a sufficient number of blocks to complete the number requested. In other words, if asked for three blocks, he must not place more than three on the paper. Many children tend to add additional blocks after they have counted the correct number. Failure on this item may also be due to poor impulse control, namely the inability to stop counting at the required number, or forgetting the number requested and just continuing to count.

In view of the complex nature of this item, it is not surprisIng that it loads on several other factors with PCA and PFA loadings respectively of $.37, .35$ on Factor IV, .38, . 33 on Factor V, and .43, . 31 on Factor VI for sample size 600.

Pictorial Similarities and Differences II is an item which loads above .30 only on Factor I. This item involves a number of cards which show similar or different items and the child is asked, "Now look at these two. Are they alike? Are they the same?" The child must make a visual judgment, and reply "yes" or "no." No motor activity is involved. The fact that this item loads . 62 and .43 on PCA and PFA for sample size 600 and loads quite consistently on this factor in all other groups (as can be seen in Table 30 ), suggests that visual judgment, rather than visual motor ability is being tapped by Factor I.

This suggestion is further supported by the fact that In the former study (Ramsey, 1968) Paper Folding, Copying a Square, and Maze Tracing, which have both a visual and a motor component, all loaded on a similar factor, but so did jesthetic Comparisons, with a loading of .84 . To succeed with Aesthetic Comparisons, a child must look at three cards containing two faces each, one attractive and one unattractive, and indicate "which is prettier." Again visual judgment appears to be the major determinant for success, assuming that the child knows the meaning of the word, "prettier." On the whole the items loading on Factor $I$ in the present study and the items loading on the visual fudgment factor in the former study suggest that visual judgment is an ability which is tested at age levels 5,6 , and 7 on the Stanford-Binet Scale, 1960 Revision.

Factor 2 was called Verbal Abstract Ability. A reasoning or abstract ability factor was also reported in previous analyses of the Binet by Jones (1961), Dean (1951), and Ramsey (1968). The only two items which loaded on this factor in the total sample of 600 were Similarities: Two Things, . 85, . 53 and Opposite Analogies III, .57, . 45 (PCA and PFA respectively). In the similarities item, the child is asked "In what way are wood and coal alike?" Other similarities are: an apple and a peach, a ship and an automobile, and iron and silver. In the Opposite Analogies III item the child is asked to complete a sentence with the correct word, such as; "The rabbit's ears are long, the rat's ears are $\qquad$ ." Since traditionally the similarities type of item has been considered to be an example of abstract reasoning, the factor has been so labeled, but it could be argued that the item was a general knowledge factor, since unless the child has the information within his repertoire he cannot succeed on this item. Dean (1951) found that this item, Similarities: Two Things, loaded . 63 on a factor he called a reasoning factor, and Ramsey (1968) found Opposite Analogies II loaded . 88 on a factor called Verbal Fluency. These two items appear in Factor 2, (see Table 31) in almost all subsample analyses in rather striking contrast to other items which were quite erratic, appearing in some analyses and not in others.

Factor 3 consisted of only one item, Definitions, which loaded . 93 in PCA and . 46 in PFA. The f'actor was accordingly named after the item. For the present study this item is
considered factorially pure since it does not load above .30 on any other factor. In one or two subsamples Definitions did load above .30 but never above .50 on any factor other than Factor 3. Table 32 shows that Definitions is the only item to load consistently on Factor 3 throughout all subsample analyses. Only one previous analysis was done at an age level which included this item (Ramsey, 1968). In that study Definitions was found to load on three factors but these factors were composed primarily of items at age level IV-6 and V. It would appear from these results that Definitions, which appears at age level V-3, may involve several abilities but only one is shared with other items in the age levels V, VI, and VII of the present study. Factor 4, Numeric Memory, contains Repeating 5 Digits, which loaded . 88 on PCA and . 53 on PFA, and Number Concepts, which loads .37 in PCA and . 35 in PFA. Two previous analyses have had a memory factor (Jones, 1949 and Dean, 1951), for this age level.

Factor 5, Difficulty Level, contained seven items loading above . 30 in PCA and six items in PFA. These were: Picture Absurdities I, .68, .51; Comprehension IV, .61, .45; Copying a Diamond, .60, .43; Patience Rectangles, .39, .33; Number Concepts . 38, .33; Mutilated Pictures .32, .31; and Paper Folding .31, .23. The fact that Picture Absurdities, Comprehension IV and Copying a Diamond all occurred at age level VII and all the items loading on this factor did not seem to measure any specific ability, led to the naming of the factor, Difficulty Level.

Table 34 shows that these three items are the only ones with consistently heavy loadings on Factor 5. This type of factor has not of ten been found in previous analyses but did occur in at least one study. Burt and John (1942), working with ages X and XII of an earlier revision of the Binet, found one factor they labeled "Age" because it loaded only on items at age level XII and not $X$.

Factor 6 had nine loadings above .30 in PGA and eight in PFA. They were: Opposite Analogies II .67, .49; Differences .63, .45; Vocabulary .61, .43; Picture Completion .60, .41; Mutilated Pictures .51, .36; Opposite Analogies III .47, .32; Number Concepts .43, . 31 ; Maze Tracing .41, . 30; and Patience Rectangles . 37, .29. The first three items are strongly dependent upon verbal ability and most of the others require some verbal comprehension. This factor, however, is somewhat complex and may be a combination of verbal ability and attention. It is possible, if more than six factors had been extracted, that some of the items in this factor might not have appeared in this factor but might have appeared in another factor.

All six of the factors in the present study were found to be rellable. Although some of them seemed to drop below significance when compared between split samples with a sample size of only 100 , the number of reliable factors for small sample sizes was not significantly less than that for large sample sizes. The apparent loss of rellability could be due to chance variations in the factors.

One of the purposes of the present study was to determine which method of analysis, PCA or PFA, was more reliable.

On the basis of the present results no significant differences were found between the two methods. When the varimax solutions of the total sample are compared for PCA and PFA, the similarities are striking. There was a tendency for PFA to have lower loadings on any given item but the same items appeared on almost every factor for both PCA and PFA. The higher loadings of PCA were considered to be reasonable since the only difference between the two methods is that PCA employs ones in the diagonal while PFA employs some number less than one such as a reliability estimate. The use of ones in PCA implies that all the variance of each item is analyzed so one might reasonably expect to have higher loadings on the factors.

Another purpose of the present study was to investigate the rellability of factors and components for different sample sizes. Perhaps one of the most strongly established findings in psychology is the relationship betwen sample size and reliability. The fact that significant differences in the number of reliable factors were not found may warrant some explanation. The largest sample size used to check reliability in the present study was 300 . Whether the difference between a sample size of 300 and a sample size of 100 is large enough to demonstrate differences in reliability is questionable. A sample size of 100 is usually considered adequate to insure rellability for most psychologicai varlables. Another point that should be made is that the phi coefficient was used instead of the unstable tetrachoric. Investigators using the tetrachoric correlation may not find samples of size 100 large enough for the occurrence of stable factors.

When comparisons were made between the number of rellable components or factors at different $I Q$ levels, there was no significant difference. In view of this it can be said that no differences were found in the number of reliable components or factors when different IQ levels were compared.

Comparison of Different Subject Groups
In order to determine the effects of race, sex, and IQ upon the components (only PCA was used in these analyses), comparisons were made between components found in different subject groups. The number of components which were found to agree between samples differing in race, sex, or IQ were then compared to the number of reliable components found for that sample size. Mc Nemar's exact test was used to determine whether the number of components which were related between samples of different race, sex, or IQ was significantly different from the number of reliable components.

## Comparisons Across Sex

The components found for males were compared to the components found for females for both low and high IQ samples. Three components were found to agree between low IQ males and females, using the . 01 level and four using the .05 level (see Group 8, Table 9). These were not found to differ significantly from the number of reliable components, six at the .05 and six at the . O1 (see Group 6, Table 6). A similar result was found for male-female comparisons at the high IQ level. The number of components which were found to agree between high IQ males and females, five using the . Ol level and six using the . 05 level (see Group 9, Table 9), was not significantly different from
the number of reliable components, two using the . 01 level and four using the . 05 level (see Group 7, Table 6). From these comparisons it was concluded that no differences could be demonstrated between males and females at these age levels in the components found. These results are in agreement with Lindsey (1966), who also found no difference between sexes in the factors obtained.

## Comparisons Across Race

The number of components found to agree when the black and white samples were compared, three using the . 01 level and six using the .05 level (see Group 10, Table 9), was not significantly different from the number of rellable factors, six at the .01 and six at the .05 (see Group 6, Table 6).

The previous study by Lindsey (1966) used the WISC with white and black pre-school through third grade children. The same ch1ldren were followed through several grades. These groups were compared as to factor structure and no differences were found. Lindsey reported no difference in factors since his coefficient of congruence ranged from . 66 to .82 . As has been pointed out earlier, this coefficient has no test of significance. In the present study the s-index was used to test the significance of factors between samples. The results of the present study support Lindsey's results in that no differences were found which could be attributed to race.

Comparisons Across Different Standard Deviations
Although no previous work has been found concerning the differences between factors when samples differ in standard deviation, the standard deviation is an important characteristic
of a group and, as such, was investigated. The number of components which agreed between groups differing in standard deviation, five at the . 01 level and six at the . 05 level (see Group 11, Table 9), was not significantly different from the number of reliable components, five at the .01 level and five at the .05 level (see Group 1, Table 6). On the basis of these results no differences seem to have occurred because of different standard deviations.

## Comparisons Across IQ Levels

A number of comparisons were made between groups which differed in $I Q$ level (see Groups 12 to 21, Table 9). Usine the .05 and .01 levels, several reference groups, and in some cases both PCA and PFA, a total of 64 comparisons were performed. Determining the significance of these comparisons at the .05 level with McNemar's exact test should have led to about three significant results by chance. In fact none of the 64 comparisons were significant. On this basis it would appear that the differences between factors found at high $I Q$ levels and factors found at low IQ levels were due to chance.

In a previous analysis of the Binet Stormer (1966)
suggested that some differences might exist between the factors found for low IQ as opposed to high IQ groups. Since his results neither quantified nor tested for statistical significance, they may have also been due to chance. The present results point out the necessity for statistical tests when interpreting results.

Suggestions for Future Research
Difficulty was encountered in accurately identifying the abilities associated with the factors found. This difficulty
might be alleviated by the use of reference tests whose factor structure is known.

Another reason for the difficulty of factor identification might have been that too few factors were extracted. That possibility could be investigated by extracting various numbers of factors and using split sample reliability to determine which number of factors gives the most acceptable results.

In the present study the phi coefficient was chosen over the tetrachoric correlation. The reliability of factors when each of these is used might also be a worthwhile subject of investigation. Other measures of association for dichotomous varlables such as lambda are also suggested by Hays (1963).

## CHAPTER V

SUMMARY AND CONCLUSIONS

The present study has attempted to investigate the following questions:
(1)

Will the same factors of intellectual ability be found in two randomly divided samples from the same population and will this reliability estimation hold for both Principal Components Analysis and Principal Factor Analysis?
(2) Will significantly different factors be found across IQ levels?
(3) Will significantly different factors be found between sexes on the same $I Q$ level?
(4) Do racial differences on the same $I Q$ level lead to significantly different factors?
(5) If two groups on the same mean IQ level are systematically divided so that they differ only in the standard deviations of their IQ's, will they differ in the factors found?

To investigate these questions 18 items from the Stanford-Binet Intelligence Scale were used. These items covered age levels V, VI, and VII of the Binet. Test results from previously tested children were collected from schools and day care centers (not including hospitals and clinics). Only children with MA's in the range from 4 years, 6 months to 7 years, 6 months were used. Quota sampling was used to construct a normal distribution of $I Q$ based upon a sample size of 600 . The 600 subjects used had a mean $I Q$ of 99.4 and a standard deviation of 16.2 . The IQ's ranged from 43 to 157. There were 499 whites, 332 boys and 167 girls. Among the 101 black
children 52 were boys and 49 were girls. The mean MA for all children was 5.8 years with a standard deviation of .9 years. Chronological age ranged from 3 years 1 month to 11 years 2 months with a mean of 5.9 years and a standard deviation of 1.2 years.

The total sample was factor analyzed by both Principal Components Analysis (PCA) and Principal Factor Analysis (PFA) and rotated to Kaiser's varimax criterion of simple structure. The unit eigenvalue rule was used to determine the number of factors to extract and this number was used in all later analyses. The factors were identified on the basis of the analyses of the total samples. For the purposes of comparing factors from various samples the results of all later analyses were first rotated to a least squares fit to the total sample for the particular type of analysis used for that sample (elther PCA or PFA).

The total sample was repeatedly divided randomly into reliability samples of sizes 300,150 , and 100. Each of the pairs of samples of the various sizes was analyzed by both PCA and PFA. The two largest samples were again divided into high and low IQ groups forming two low IQ samples of 150 each and two high IQ samples of 150 each. All ten corresponding pairs were analyzed by both PCA and PFA and after rotation were compared by both the salient variable similarity index and the coefficient of congruence. For the salient variable similarity index all loadings above .30 were considered significant. It was noted, however, it might be more accurate to use various saliency levels such as . 20 , .40, and . 50 in
addition to .30 and select the s-index value which produced the most salient items but which s-index was still included in the significance table. Application of McNemar's exact test led to the conclusion that PFA produces results that are not significantly different from PCA and the two methods appear to be more alike than different. PCA was arbitrarlly chosen to use with the remaining groups.

Later samples were selected on the basis of: (1) sex and $I Q$ level to form four groups of low IQ males, low IQ females, high $I Q$ males, and high IQ females; (2) race by selecting a white sample of 101 children to fit the same distribution of IQ's of the blacks in the total sample; and (3) standard deviation by randomly selecting from a larger sample of 793 , From which the 600 sample was constructed, two samples to fit normal alstributions of 100 IQ means but with standard deviations of 12 and 21.

The results were as follows:
(1) Six factors were found for the Stanford-Binet in the present study. They were: Visual Judgment, Abstract Ability, Definitions, Numeric Memory, Difficulty Level, and Verbal. All six were found to be significantly reliable when factors from split samples were compared statistically using Cattell, et. al.'s (1969) s-index. The rellability holds for both PCA and PFA, and significant differences were not found when using either method. The methods produce results that are more alike than different.
(2) Significantly different factors were not found across IQ levels.
(3) Significantly different factors were not found between sexes on the same IQ level.
(4) Racial differences on the same IQ level did not lead to significantly different factors.
(5) Two groups on the same mean IQ level, when systematically divided so that they differ only in standard deviations of their IQ's, did not differ in the factors found.

Table 10
Salient variable similarity index for the Principal Components Analysis: Salience values at . 20, . 30, . 40, and . 50 were used. The value with the smallest hyperplane percent above $60 \%$ is used.

| Group | Components |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| $N=\stackrel{1}{300}$ | $.60^{* *}$ | . $63 \%$ | .$^{.67 \% *}$ | $.50 * *$ | $\begin{aligned} & .86 * * \\ & 61 \% \end{aligned}$ | $.62 * *$ |
| $\stackrel{2}{N-150}$ | . $33 *$ | $\begin{aligned} & .83 * * \\ & 67 \% \end{aligned}$ | ${ }_{.}^{.55 * *}$ | $\begin{aligned} & .86 * * \\ & 81 \% \end{aligned}$ | $\begin{gathered} .36 * \\ 69 \% \end{gathered}$ | $\begin{aligned} & .71^{* *} \\ & 61 \% \end{aligned}$ |
| $\stackrel{3}{\mathrm{~N}=100}$ | $\begin{aligned} & .80 * * \\ & 72 \% \end{aligned}$ | $\begin{aligned} & .60 * * \\ & 72 \% \end{aligned}$ | $.20$ | .22 $75 \%$ | $\begin{aligned} & .25 \\ & 78 \% \end{aligned}$ | $\begin{aligned} & .55 * * \\ & 69 \% \end{aligned}$ |
| $\begin{gathered} 4 \\ N=150 \end{gathered}$ | $\begin{aligned} & .40^{*} \\ & 72 \% \end{aligned}$ | $\begin{aligned} & .50 * * \\ & 78 \% \end{aligned}$ | $.67 * *$ | $\begin{aligned} & .54 * * \\ & 69 \% \end{aligned}$ | $\begin{aligned} & .50 * * \\ & 67 \% \end{aligned}$ | $\cdot \frac{33^{*}}{67 \%}$ |
| $\stackrel{5}{N}=150$ | $\begin{aligned} & .62 * * \\ & 64 \% \end{aligned}$ | $\cdot \frac{33 *}{67 \%}$ | $.18$ | $\begin{aligned} & .46 * * \\ & 64 \% \end{aligned}$ | $\begin{aligned} & .89 * * \\ & 75 \% \end{aligned}$ | $\begin{aligned} & .67^{* *} \\ & 75 \% \end{aligned}$ |

[^0]Table 11
Salient variable similarity index for the Principal Factor Analysis: Salience values at .20, 30,40 , and .50 were used. The value with the smallest hyperplane percent above $60 \%$ is used.

| Group | Factors |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | . 83** | . $77^{* *}$ | . 50 ** | . $40 *$ | . $67 * *$ | $.67 * *$ |
| $N=300$ | 67\% | 63\% | 89\% | 72\% | 67\% | 67\% |
| $\stackrel{2}{2}$ | $.57 * *$ $61 \%$ | . $932^{* *}$ | . $44 *$ | . $80 \%$ \% | $.57 * *$ $61 \%$ | $.61 * *$ |
| $N=150$ | 61\% | 63\% | 75\% |  |  |  |
| $\begin{gathered} 3 \\ \mathrm{~N}=100 \end{gathered}$ | $\begin{aligned} & 1.00 * * \\ & 66 \% \end{aligned}$ | $.57 * *$ $81 \%$ | -79\%** | $.43 *$ $61 \%$ | . $33 *$ | . $33 *$ |
| $\begin{gathered} 4 \\ N=150 \end{gathered}$ | . 43 \% ${ }^{\text {6\% }}$ | ${ }^{.77 * *}$ | . $36 *$ | . $67 * *$ | . $33 *$ | $\begin{aligned} & .43 * \\ & 61 \% \end{aligned}$ |
| $\stackrel{5}{N}=150$ | . $62 * *$ | . $57{ }^{\text {8 }}$ ** | . 4 61\% | $.57 * *$ $61 \%$ | $\begin{aligned} & .86 * * \\ & 61 \% \end{aligned}$ | $.77^{* *}$ |

$$
\begin{array}{r}
* p<.05 \\
* * p<.01
\end{array}
$$

Table 12
Salient variable similarity index for the Principal Components Analysis: Salience values at .20, . 30, . 40, and . 50 were used. The value with the smallest hyperplane percent above $60 \%$ is used.

| Group | Components |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| $\begin{gathered} 6 \\ N=100 \end{gathered}$ | $.50 \% *$ $67 \%$ | . $73{ }^{* *}$ | $.67 * *$ $75 \%$ | . 73 \% ${ }^{*}$ | . $50 \% *$ | $\begin{aligned} & .67^{* *} \\ & 75 \% \end{aligned}$ |
| $\begin{gathered} 7 \\ N=100 \end{gathered}$ | $.73 * *$ $69 \%$ | . $333 * *$ | $\cdot \frac{17}{67 \%}$ | . 29 * | . $474 *$ | $\begin{aligned} & .57^{* *} \\ & 61 \% \end{aligned}$ |

[^1]Table 13
Coefficients of Congruence for the Principal Components Analysis: The estimated percent of shared variance was obtained by squaring the coefficient of congruence.

| Group | Components |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | Average |
| $\begin{gathered} 6 \\ N=100 \end{gathered}$ | $\begin{array}{r} .767 \\ 59 \% \end{array}$ | $\begin{array}{r} .731 \\ 53 \% \end{array}$ | $.651$ | $\begin{aligned} & .850 \\ & .72 \% \end{aligned}$ | $\begin{aligned} & .857 \\ & 73 \% \end{aligned}$ | $\begin{aligned} & .834 \\ & 70 \% \end{aligned}$ | 61. $6 \%$ |
| $\stackrel{7}{N}=100$ | $\begin{aligned} & .827 \\ & 68 \% \end{aligned}$ | $\begin{aligned} & .516 \\ & 27 \% \end{aligned}$ | $\cdot \begin{array}{r} 594 \\ 35 \% \end{array}$ | $.702$ | $\begin{aligned} & .751 \\ & 56 \% \end{aligned}$ | $\cdot \frac{831}{69 \%}$ | 50.8\% |
| Average | 64\% | 40\% | 39\% | 61\% | 65\% | 70\% | 56.2\% |

Table 14
Salient variable similarity index for the Principal Components Analysis: Saliency values at .20, . 30, . 40 and .50 were used. The value with the smallest hyperplane percent above $60 \%$ is used.

| Group | Components |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| 8 | $.46 *$ | .20 | .31 | $.57^{* *}$ | $.86 * *$ | $.71 * *$ |
| $\mathrm{~N}-100$ | $64 \%$ | $72 \%$ | $64 \%$ | $81 \%$ | $61 \%$ | $61 \%$ |
| 9 | $.71^{* *}$ | $.62 * *$ | $.33^{*}$ | $.57^{* *}$ | $.62^{* *}$ | $.62 * *$ |
| $\mathrm{~N}-100$ | $61 \%$ | $64 \%$ | $67 \%$ | $81 \%$ | $64 \%$ | $64 \%$ |

* $p<05$
**p<. 01

Table 15
Coefficients of Congruence for the Principal Components Analysis: The estimated percent of shared variance was obtained by squaring the coefficient of congruence.

| Group | Components |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | Average |
| $\begin{gathered} 8 \\ N=100 \end{gathered}$ | .547 $30 \%$ | . 582 | . 5388 | $\begin{gathered} .789 \\ 62 \% \end{gathered}$ | . 790 | $\begin{aligned} & .883 \\ & 78 \% \end{aligned}$ | 49.2\% |
| $\begin{gathered} 9 \\ \mathrm{~N}=100 \end{gathered}$ | $\begin{array}{r} .773 \\ 60 \% \end{array}$ | . 751 | $.253$ | $\begin{aligned} & .622 \\ & 39 \% \end{aligned}$ | $\begin{aligned} & .862 \\ & 74 \% \end{aligned}$ | .892 $80 \%$ | 52.5\% |
| Average | 45\% | 45\% | 17.5\% | 50.5\% | 68\% | 79\% | 50.9\% |

Table 16
Salient variable similarity index for the Principal Components Analysis: Saliency values at . 20, . 30, . 40 , and .50 were used. The value with the smallest hyperplane percent above $60 \%$ is used.

| Group | Components |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| $\begin{gathered} 10 \\ \mathrm{~N}=101 \end{gathered}$ | . $33 *$ | $.50 * *$ $78 \%$ | $.75 * *$ $78 \%$ | $.40 *$ $72 \%$ | $.50 \% *$ $67 \%$ | $\begin{aligned} & .40 \\ & 72 \% \end{aligned}$ |
| 11 $N=300$ | $.71 * *$ $61 \%$ | $\xrightarrow{.67 * *}$ | $.55 * *$ $.69 \%$ | $.40 *$ $72 \%$ | . $50{ }^{\text {67\% }}$ | $\begin{aligned} & .67 * * \\ & 75 \% \end{aligned}$ |

$\begin{aligned} * p & <.05 \\ * * p & <.01\end{aligned}$

Table 17
Coefficients of Congruence for the Principal Components Analysis: The estimated percent of shared variance was obtained by squaring the coefficient of congruence.

| Group | Components |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | Average |
| 10 | .743 | .719 | .703 | .585 | .757 | .882 |  |
| $\mathrm{~N}-101$ | $55 \%$ | $52 \%$ | $49 \%$ | $34 \%$ | $57 \%$ | $78 \%$ | $54.3 \%$ |
| 11 | .885 | .718 | .701 | .755 | .815 | .940 |  |
| $\mathrm{~N}-300$ | $78 \%$ | $52 \%$ | $49 \%$ | $57 \%$ | $66 \%$ | $88 \%$ | $65.1 \%$ |

Table 18
Salient variable similarity index for the Principal Components Analysis: Saliency values at .20, .30, . 40, and . 50 were used. The value with the smallest hyperplane percent above $60 \%$ is used.

| Group | Components |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| $\begin{gathered} 12 \\ \mathrm{~N}-150 \end{gathered}$ | . 4 6* | $.60 * *$ $72 \%$ | $\cdot \frac{18}{69 \%}$ | $.57 * *$ $61 \%$ | $\begin{aligned} & .67^{* *} \\ & 67 \% \end{aligned}$ | $.62^{* *} 64 \%$ |
| $\begin{gathered} 13 \\ \mathrm{~N}-150 \end{gathered}$ | $.67 * *$ $75 \%$ | . $50 \% *$ | . 40 * | $.71^{* *}$ | $.57 * *$ $61 \%$ | $\begin{aligned} & .57^{* *} \\ & 61 \% \end{aligned}$ |
| $\begin{gathered} 14 \\ \mathrm{~N}-150 \end{gathered}$ | . 57 \% ${ }^{\text {8\% }}$ | $.60 * *$ $72 \%$ | . $141 \%$ | . $52 \% *$ | . 71 \% ${ }^{*}$ | $.43 *$ $61 \%$ |
| $\begin{gathered} 15 \\ \mathrm{~N}-150 \end{gathered}$ | . $57{ }^{\text {6\%* }}$ | . 25 | . $33 *$ | $.44 *$ $75 \%$ | $\begin{aligned} & .50^{* *} \\ & 72 \% \end{aligned}$ | $\begin{aligned} & .62^{* *} \\ & 63 \% \end{aligned}$ |

$\begin{aligned} & * p \text { * }<.05 \\ & * * p<.01\end{aligned}$

Table 19
Salient variable similarity index for the Principal Factor Analysis: Salience values at .20, . 30 , . 40 , and .50 were used. The value with the smallest hyperplane percent above 60 percent is used.

| Group | Factors |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| $\begin{gathered} 12 \\ \mathrm{~N}-150 \end{gathered}$ | $\frac{.62 * *}{64 \%}$ |  | $\begin{aligned} & .60 * * \\ & 72 \% \end{aligned}$ | $.62^{* *}$ | $\begin{aligned} & .71^{* *} \\ & 61 \% \end{aligned}$ | $.29$ |
| $\begin{gathered} 13 \\ \mathrm{~N}-150 \end{gathered}$ | $.86 * *$ $61 \%$ | . 44 * | .00 $88 \%$ | $.62 * *$ $64 \%$ | $.50 * *$ $67 \%$ | $\begin{array}{r} .46 * \\ 64 \% \end{array}$ |
| $\begin{gathered} 14 \\ \mathrm{~N}-150 \end{gathered}$ | $.50 * *$ $66 \%$ | $.62 * *$ $64 \%$ | $.43 *$ $61 \%$ | $\begin{aligned} & .62 * * \\ & 64 \% \end{aligned}$ | $\begin{aligned} & .83 * * \\ & 67 \% \end{aligned}$ | $\begin{gathered} .46 * \\ 64 \% \end{gathered}$ |
| $\begin{gathered} 15 \\ \mathrm{~N}-150 \end{gathered}$ | $\begin{aligned} & .75 * * \\ & 77 \% \end{aligned}$ | . $60 \% *$ | . $36 \%$ | $.62 * *$ $64 \%$ | . $43 *$ | $\begin{aligned} & .57^{* *} \\ & 61 \% \end{aligned}$ |

$$
\begin{array}{r}
* p<05 \\
* * p<.01
\end{array}
$$

Table 20
Coefficients of Congruence for the Principal Components Analysis: The estimated percent of shared variance was obtained by squaring the coefficient of congruence:

| Group | Components |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | Average |
| 12 | .770 | .770 | .613 | .749 | .872 | .813 |  |
| $\mathrm{~N}=150$ | $59 \%$ | $59 \%$ | $38 \%$ | $56 \%$ | $76 \%$ | $66 \%$ | $59.1 \%$ |
| 13 | .808 | .629 | .603 | .716 | .735 | .797 |  |
| Na 150 | $65 \%$ | $40 \%$ | $36 \%$ | $51 \%$ | $54 \%$ | $64 \%$ | $51.7 \%$ |
| 14 | .747 | .809 | .580 | .687 | .890 | .829 |  |
| $\mathrm{~N}=150$ | $56 \%$ | $65 \%$ | $34 \%$ | $47 \%$ | $79 \%$ | $69 \%$ | $58.3 \%$ |
| 15 | .740 | .623 | .580 | .704 | .819 | .853 |  |
| $\mathrm{~N}=150$ | $55 \%$ | $39 \%$ | $34 \%$ | $50 \%$ | $67 \%$ | $73 \%$ | $52.9 \%$ |
| Average | $58.8 \%$ | $50.8 \%$ | $35.5 \%$ | $51.0 \%$ | $56.5 \%$ | $68.0 \%$ | $55.5 \%$ |

Table 21
Coefficients of Congruence for Principal Factor Analysis: The estimated percent of shared variance was obtained by squaring the coefficient of congruence.

\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{Group} \& \multicolumn{7}{|c|}{Factors} <br>
\hline \& 1 \& 2 \& 3 \& 4 \& 5 \& 6 \& Average <br>
\hline ${ }_{N=12}^{12}$ \& . 85 \& . $841 \%$ \& . 553 \& . $781 \%$ \& . 8285 \& . 748 \& 62.8\% <br>
\hline 13
$N=150$ \& . 9387 \& . 823 \& . 520 \& .770

$59 \%$ \& . 840 \& . $797 \%$ \& 62.7\% <br>

\hline $$
\mathrm{N}=150
$$ \& . 866 \& . 791 \& .415

$17 \%$ \& .767

$59 \%$ \& . 924 \& .819 \& 61.0\% <br>

\hline $$
\begin{gathered}
15 \\
\mathrm{~N}=150
\end{gathered}
$$ \& . 8488 \& .709

$50 \%$ \& $\begin{array}{r}.455 \\ 21 \% \\ \hline\end{array}$ \& .831

$69 \%$ \& \[
$$
\begin{array}{r}
.884 \\
78 \%
\end{array}
$$

\] \& \[

$$
\begin{aligned}
& .793 \\
& .63 \%
\end{aligned}
$$
\] \& 59.1\% <br>

\hline Average \& .77.8\% \& 63.0\% \& 24.0\% \& 62.0\% \& 80.0\% \& 62.5\% \& 61.4\% <br>
\hline
\end{tabular}

Table 22
Salient variable similarity index for the Principal Components Analysis: Saliency values at $.20, .30, .40$, and .50 were used. The value with the smallest hyperplane percent above $60 \%$ was used.

| Group | Components |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| $\begin{gathered} 16 \\ N=100 \end{gathered}$ | $.71 * *$ 61\% | . $50 \% *$ | . $33 *$ | . $33 \%$ | . $57{ }^{\text {81\% }}$ | . $57{ }^{* *}$ |
| $\begin{gathered} 17 \\ \mathrm{~N}=100 \end{gathered}$ | . $774 \%$ | . $50 \% *$ | . 22 | $\begin{aligned} & .67 * * \\ & 83 \% \end{aligned}$ | $\cdot \frac{77^{* *}}{64 \%}$ | $\begin{aligned} & .60 * * \\ & 72 \% \end{aligned}$ |
| $\begin{gathered} 18 \\ N=100 \end{gathered}$ | . 55 \%** | . $57{ }^{*}{ }^{*}$ | . 20 | . $44 *$ | $\stackrel{.55 * *}{69 \%}$ | $\begin{aligned} & .53^{* *} \\ & 61 \% \end{aligned}$ |
| $\begin{gathered} 19 \\ \mathrm{~N}=100 \end{gathered}$ | $\begin{aligned} & .73 * * \\ & 69 \% \end{aligned}$ | $.46 *$ $64 \%$ | . 600 | . 4 4** | $.75 * *$ $78 \%$ | $\begin{aligned} & .89 * * \\ & 75 \% \end{aligned}$ |
| $\begin{array}{rl} * p<05 \\ * * p & 01 \end{array}$ |  |  |  |  |  |  |

Table 23
Coefficients of Congruence for the Principal Components Analysis: The estimated percent of shared variance was obtained by squaring the coefficient of congruence.

| Group | Components |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | Average |
| $\begin{gathered} 16 \\ \mathrm{~N}-100 \end{gathered}$ | .771 $59 \%$ | $\begin{aligned} & .540 \\ & \cdot 29 \% \end{aligned}$ | .347 $12 \%$ | $\begin{array}{r} .688 \\ 47 \% \end{array}$ | $\begin{array}{r} .703 \\ 49 \% \end{array}$ | $\begin{aligned} & .817 \\ & 67 \% \end{aligned}$ | 44.0\% |
| $\begin{gathered} 17 \\ \mathrm{~N}-100 \end{gathered}$ | . 823 | .790 $.62 \%$ | .598 $36 \%$ | .690 $48 \%$ | $\begin{aligned} & .909 \\ & 83 \% \end{aligned}$ | $\begin{array}{r} .845 \\ 71 \% \end{array}$ | $61.3 \%$ |
| $\begin{gathered} 18 \\ \mathrm{~N}-100 \end{gathered}$ | . 827 | $\begin{aligned} & .783 \\ & .71 \% \end{aligned}$ | $.$ | $\cdot \begin{array}{r} 599 \\ 36 \% \end{array}$ | $\begin{array}{r} \cdot 782 \\ 61 \% \end{array}$ | $\begin{array}{r} .731 \\ 53 \% \end{array}$ | 47.2\% |
| $\begin{gathered} 19 \\ \mathrm{~N}-100 \end{gathered}$ | $\begin{array}{r} .785 \\ 62 \% \end{array}$ | $\begin{aligned} & .455 \\ & 21 \% \end{aligned}$ | $\begin{gathered} 704 \\ \\ 50 \% \end{gathered}$ | $.818$ | $\begin{array}{r} 829 \\ 69 \% \end{array}$ | $.826$ | 56.0\% |
| Average | 64\% | 43\% | 26\% | 50\% | 66\% | 65\% | 52.1\% |

Table 24
Salient variable similarity index for the Principal Components Analysis: Saliency values at . 20, . 30, . 40 , and . 50 were used. The value with the smallest hyperplane percent above $60 \%$ is used.

| Group | Components |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| $\begin{gathered} 20 \\ N=100 \end{gathered}$ | $\begin{gathered} .46^{*} \\ 64 \% \end{gathered}$ | . $73 \% *$ | $\cdot \frac{18}{69 \%}$ | $\begin{gathered} .43 * \\ 61 \% \end{gathered}$ | $\begin{aligned} & .67 * * \\ & 75 \% \end{aligned}$ | $\begin{gathered} \cdot 36 * \\ 69 \% \end{gathered}$ |
| $\begin{gathered} 21 \\ N=100 \end{gathered}$ | $\begin{gathered} .43^{*} \\ 61 \% \end{gathered}$ | $.50 * *$ $67 \%$ | $\cdot \frac{14}{61 \%}$ | $\begin{gathered} .40 * \\ 86 \% \end{gathered}$ | $\begin{array}{r} .33 * \\ 67 \% \end{array}$ | $\cdot \frac{.46 *}{64 \%}$ |

${ }^{*} \mathrm{p}<.05$
$* * \mathrm{p}<.01$

Table 25
Coefficients of Congruence for the Principal Components dnalysis: The estimated percent of shared variance was obtained by squaring the coefficient of congruence.

| Group | Components |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | Average |
| $\begin{gathered} 20 \\ \mathrm{~N}-100 \end{gathered}$ | . 691 | $\begin{gathered} .838 \\ 70 \% \end{gathered}$ | .531 $28 \%$ | $\begin{array}{r} .628 \\ 39 \% \end{array}$ | $\cdot \begin{aligned} & .829 \\ & 69 \% \end{aligned}$ | $\begin{array}{r} .770 \\ 59 \% \end{array}$ | 47.8\% |
| $\begin{gathered} 21 \\ \mathrm{~N}-100 \end{gathered}$ | .576 $33 \%$ | . 555 | $\begin{array}{r} .085 \\ 1 \% \end{array}$ | $\begin{gathered} .687 \\ 47 \% \end{gathered}$ | $\begin{array}{r} .636 \\ 40 \% \end{array}$ | $\begin{array}{r} .761 \\ 58 \% \end{array}$ | 35.0\% |
| Average | 41\% | 51\% | 15\% | 43\% | 55\% | 59\% | 41.4\% |

Table 26

Biserial Correlations for Stanford-Binet Items used in the present study. Values based on the total sample are reported as well as the values presented by the test authors (Terman and Merrill, 1960, 343-344).

| Item |  | Biserial Correlation |  |
| :---: | :---: | :---: | :---: |
|  |  | Present Study | Terman \& Merrill |
| V-1 | P1cture Completion: Man | . 73 | . 46 |
| 2 | Paper Folding: Triangle | . 58 | . 54 |
| 3 | Definitions | . 53 | . 57 |
| 4 | Copying a Square | . 74 | . 62 |
| 5 | Pictorial Similarities and Differences II | . 63 | . 73 |
| 6 | Patience: Rectangles | . 60 | . 57 |
| VI- 1 | Vocabulary | . 80 | .67 |
| 2 | Differences | . 81 | . 71 |
| 3 | Mutilated Pictures | . 75 | . 65 |
| 4 | Number Concepts | . 86 | . 77 |
| 5 | Opposite Analogies | . 74 | . 67 |
| 6 | Maze Tracing | . 79 | . 69 |
| VII-1 | Picture Absurdities I | . 72 | . 64 |
| 2 | Similarities: Two Things | . 63 | . 65 |
| 3 | Copying a Diamond | .72 | . 62 |
| 4 | Comprehension IV | .76 | . 48 |
| 5 | Opposite Analogies III | . 73 | . 62 |
| 6 | Repeating 5 Digits | . 57 | . 59 |

## Table 27

Significance values for the salient variable similarity index (s-index). Values reported are for analysis involving 18 variables and were obtained by interpolation from Cattell, et. al. (1969, 788-790).

| Percent | Significance Level |  |
| :---: | :---: | :---: |
|  | .05 | .01 |
| 60 | .39 | .50 |
| 70 | .31 | .49 |
| 80 | .27 | .48 |
| 90 | .24 | .47 |

Table 28

Distributions of $I Q$ scores for total sample, samle $G$, sample $H$, Black sample and White sample.

| IQ | Total <br> Sample $N=600$ | $\begin{gathered} \text { Sample } \\ G \\ N=300 \end{gathered}$ | $\begin{gathered} \text { Sample } \\ H \\ N=300 \end{gathered}$ | $\begin{aligned} & \text { Sample } \\ & \text { Black } \\ & \mathrm{N}=101 \end{aligned}$ | Sample <br> White $\mathrm{N}=101$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 160-169 |  |  | 1 |  |  |
| 150-159 | 1 |  | 2 |  |  |
| 140-149 | 3 | 1 | 6 |  |  |
| 130-139 | 15 | 2 | 14 |  |  |
| 120-129 | 45 | 12 | 28 | 2 | 2 |
| 110-119 | 96 | 47 | 44 | 12 | 12 |
| 100-109 | 140 | 88 | 55 | 21 | 21 |
| 90-99 | 140 | 88 | 55 | 22 | 22 |
| 80-89 | 96 | 47 | 44 | 26 | 26 |
| 70-79 | 45 | 12 | 28 | 16 | 16 |
| 60-69 | 15 | 2 | 14 | 2 | 2 |
| 50-59 | 3 | 1 | 6 |  |  |
| 40-49 | 1 |  | 2 |  |  |
| 30-39 |  |  | 1 |  |  |

Table 29
Stanford-Binet Items used in the present study and their computer abbreviations.

|  | Name of Item | Computer <br> Abbreviations |
| :---: | :---: | :---: |
| V- | 1 Picture Completion: Man | P C MN |
|  | 2 Paper Folding: Triangle | PPR FL |
|  | 3 Definitions | DEFINS |
|  | 4 Copying a Square | CPY SQ |
|  | 5 Pictorial Similarities and Differences II | P S\&D2 |
|  | 6 Patience: Rectangles | PA RCT |
| VI- | 1 Vocabulary | VOCABU |
|  | 2 Differences | DIFFRS |
|  | 3 Mutilated Pictures | MUTL P |
|  | 4 Number Concepts | NUM CN |
|  | 5 Opposite Analogies | OPP A2 |
|  | 6 Maze Tracing | MAZ TR |
| VII- | 1 Picture Absurdities I | P ABSI |
|  | 2 Similarities: Two Things | SML 2 T |
|  | 3 Copying a Diamond | CPY DI |
|  | 4 Comprehension IV | COMP 4 |
|  | 5 Opposite Analogies III | OPP A3 |
|  | 6 Repeating 5 Digits | REP 5D |

Table 30

\# Indicates loadings of .50 or higher
: Indicates loadings of .30 to .49
Table 31

[^2]Table 32

| Group | Total <br> Sample | 1 <br> Large <br> Sample | 2 Medium Sample | 3 Small Sample | $\begin{gathered} \hline 4 \\ \text { Low } \\ \text { IQ } \end{gathered}$ | $\begin{array}{\|c} 6 \\ \hline \text { V. Low } \\ \hline \end{array}$ | $\begin{gathered} 8 \\ \text { Low } \\ \text { IQ } \end{gathered}$ | \| 10 | $\begin{gathered} 5 \\ \text { High } \\ I Q \end{gathered}$ | $\begin{array}{\|c\|} \hline 7 \\ \text { V. High } \\ \text { IQ } \\ \hline \end{array}$ | $\begin{gathered} 9 \\ \text { High } \\ \text { IQ } \end{gathered}$ | $\begin{aligned} & 11 . \\ & \text { S.D. } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Slze | 600 | 300 | 150 | 100 | 150 | 100 | 100 | 101 | 150 | 100 | 100 | 300 |
| Sub-Sample |  | A B | C D | C2 D2 | A B | E F | M F | W B | A B | E F | M F | G H |
| Item Name | \# | \# \# | \# \# | \# \# | \# \# | : \# | \# \# | \# \# | \# \# | \# \# | \# \# | \# \# |
| Mutil Pict |  | : |  | : | : : |  |  |  | \# |  | \# | : |
| Copy Squar |  |  | : : |  |  |  |  |  |  |  |  |  |
| Maze Treng |  |  | : | : |  |  | : | : |  |  |  |  |
| Pict S\&D 2 |  |  |  | : |  |  |  |  | : | : | : |  |
| Opp Anal 2 |  |  |  |  | : |  |  |  |  |  |  |  |
| Vocabulary |  |  |  |  |  | \# : | : |  |  |  |  | : |
| Papr Fldng |  |  |  |  |  | \# |  |  |  |  |  |  |
| Numbr Cncp |  |  |  |  |  |  | : |  |  |  |  |  |
| Pict $C$ Man |  |  |  |  |  |  |  | : : |  |  |  |  |
| Differencs |  |  |  |  |  |  |  | : |  | : |  |  |
| Pat Retngl |  |  |  |  |  |  |  |  |  |  | : |  |
| Copy Diamn |  |  |  |  |  |  |  |  |  |  | : |  |

[^3]Table 33

| Group | Total <br> Sample | 1 Large Sample | $\begin{gathered} 2 \\ \text { Medium } \\ \text { Sample } \end{gathered}$ | $\begin{gathered} 3 \\ \text { Small } \\ \text { Sample } \end{gathered}$ | $\begin{gathered} 4 \\ \text { LOW } \\ \text { IQ } \end{gathered}$ | $\begin{gathered} 6 \\ V \cdot \text { Low } \\ \text { IQ } \end{gathered}$ | $\begin{gathered} 8 \\ \text { LOW } \\ \text { IQ } \end{gathered}$ | $\begin{gathered} 10 \\ \text { Race } \end{gathered}$ | $\begin{gathered} 5 \\ \text { HIgh } \\ \text { IQ } \end{gathered}$ | $\begin{gathered} 7 \\ \text { V.High } \\ \text { IQ } \end{gathered}$ | $\begin{gathered} 9 \\ \text { High } \\ \text { IQ } \end{gathered}$ | $\frac{11}{\mathrm{~S} . \mathrm{D} .}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Size | 600 | 300 | 150 | 100 | 150 | 100 | 100 | 101 | 150 | 100 | 100 | 300 |
| Sub-Sample |  | A B | C D | C2 D2 | A B | E F | M F | W B | A B | E F | M F | G H |
| Item Name | \# | $\begin{gathered} \text { \# \# } \\ \vdots: \\ : \\ : \end{gathered}$ | $\begin{aligned} & \text { \# \# } \\ & : ~: ~ \\ & : ~: ~ \\ & : ~: ~ \end{aligned}$ | \# : | \# \# <br> \# : <br> : : <br> : <br> : <br> : | $\begin{aligned} & \text { \# \# } \\ & : ~: ~ \\ & : ~: ~ \\ & : \end{aligned}$ | $\begin{gathered} \# \# \\ : ~: ~ \\ \# \end{gathered}$ | \# \# | \# \# | \# \# | \# \# | \# \# |
| Rep 5 Dgts |  |  |  |  |  |  |  |  |  |  |  |  |
| Numbr Cnep |  |  |  |  |  |  |  |  |  |  |  | : : |
| Copy Diamn |  |  |  |  |  |  |  | : | : | : | : | : |
| Maze Treng |  |  |  |  |  |  |  |  |  |  |  |  |
| Opp Anal 2 |  |  |  |  |  |  |  | : |  |  |  |  |
| Pat Retngl |  |  |  |  |  |  |  | : : | : | : |  | : |
| Comprehn 4 |  |  |  |  |  |  | : |  |  |  |  |  |
| Differencs |  |  |  |  |  |  |  |  |  |  |  |  |
| Pict C Man |  |  |  |  |  |  |  |  |  |  |  |  |
| Pict S\&D 2 |  |  |  |  |  |  | : | : |  |  |  |  |
| Opp Anal 3 |  |  |  |  |  | : : |  |  | : |  |  | : |
| Mutil Pict |  |  |  |  |  |  |  | : |  |  |  |  |
| Vocabulary |  |  |  |  |  |  |  |  |  |  | : |  |

> \# loading of .50 or better
> loading between . 30 and .49
Table 34

| Group | Total Sample |  | $\begin{gathered} 2 \\ \text { Medium } \\ \text { Sample } \end{gathered}$ | $\frac{3}{\text { Small }}$ Sample | $\begin{gathered} \hline 4 \\ \text { Low } \\ \text { IQ } \\ \hline \end{gathered}$ | $\begin{gathered} 6 \\ \text { v. Low } \\ \text { IQ } \end{gathered}$ | $\begin{gathered} 8 \\ \text { Low } \\ \text { IQ } \end{gathered}$ | $\begin{aligned} & 10 \\ & \text { Race } \end{aligned}$ | $\begin{gathered} 5 \\ \mathrm{High} \\ \mathrm{IQ} \\ \hline \end{gathered}$ | $\begin{gathered} 7 \\ \text { v. } \mathrm{HIGh} \\ \hline \end{gathered}$ | $\begin{gathered} 9 \\ \text { High } \\ \text { IQ } \end{gathered}$ | $\begin{aligned} & \text { IT } \\ & \text { S.D. } \\ & \text { IQ } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Size | 600 | 300 | 150 | 100 | 150 | 100 | 100 | 101 | 150 | 100 | 100 | 300 |
| Sub-Sample |  | A B | C D | C2 D2 | A B | E F | M F | W B | A B | E F | M F | G H |
| Item Name |  |  |  |  |  |  |  |  |  |  |  |  |
| Plct Absrl | \# | \# \# | \# \# | : \# | : \# | \# \# |  |  |  |  | \# \# | \#\# |
| Comprehn 4 | \# | \# \# | \# \# | \# | \# \# | \# \# | \# \# | \# \# | \# : | \# \# |  | \# \# |
| Copy Diamn | \# | \# \# | \# | \# \# | : : | \# | \# \# |  | \# | \# \# | :\# | \# \# |
| Pat Retng1 | : | \# : | : |  | \# | : \# | : : | \# | \# \# | \# | :\# |  |
| Numbr Cncp | : | : : | : | : : | : | : \# |  |  | : : | \# |  | : |
| Papr Fldng | : | : |  | : : | : : |  | \# : |  | : | : | : |  |
| Vocabulary |  |  | : | : | : |  | : : | : |  | - | : | : |
| Pict S\&D 2 |  |  |  |  |  |  |  |  |  |  |  |  |
| Copy Squar |  |  |  |  | : |  |  |  | : | : |  |  |
| Simil 2Tng |  |  |  | : | - | - |  |  |  |  |  | - |
| Maze Treng |  |  |  | - |  | : |  |  |  | : |  | : |
| Rep 5 Dgts |  |  |  |  | : |  |  | : |  |  |  | : |
| Differencs |  |  |  |  |  |  |  |  | : | : ${ }^{\text {P }}$ | : |  |

Table 35

\# Indicates loadings of .50 or higher
: Indicates loadings of .30 to .49

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ITEM LOADINGS OM PRINCIPAL COMPONENTS for stanforo-binet age levels v, vi, vil analrsis for the total sample with nowoo


TABLE 38
item loidings on primitpal components for stanford-binet age levels y, vif, yit group 1 - sample a - ne300 after rotation to fit the total sample


TABLE 39




TADLE 42
ITER LOADIMGS ON PRINCIPAL COMPONENTS FOR STANFDRD-BINET AGE LEVELS $V_{0}$ VI, vil group 2 - Sample c - Neliso after rotailan to fit the total sample




## TABLE 44

ITEM LOADINGS ON PRINCIPAL FACTORS FOR STANFORD-BINET AGE LEVELS V. VI. VII GROUP 2 - SAMPLE C - Nelso after rotation to fit the total sample


## TRQLE 45

 factors


TABLE 4B
ITEN LOADIMAS ON PRIMCIPAL COMPONENTS FOR STANFORD-BINET AGE LEVELS V, VI, VII GROUP 3 - SAMPLE C-2 - N=100 AFTER ROTATION TO FIT THE TOTAL SAMPLE .


GROUP 3 - SAMPLE $0=2-N=100$ AEIER RAIATLOA TO FLIT THE TOTAL SAMPLE
COMPDNENTS



 TABLE 53

FACTORS
iten loadimes on principal components for stanford-binet age levels V , vi, vil
group 5 - Sample a high ie - n=150 after rotation to fit the total sample


## TABLE 55



C..
TABLE 58
ITEN LOADINGS ON PRIHCipal Components for stanford-binet age levels vi vi, vil
GROUP 6 - SAMPLE E very lon IG - Ne 100 after rotation to fit the total sample
 Thale sg
 COMPONENTS


TABLE 60
ITEM LDADINGS ON PRINCIPAL COMPONENTS FOR STAMFORD-BINET AGE LEVELS V VI. VII
GROUP 7 - SAMPLE E VERY HIGH 10 - NaI 00 AFTER ROTATION TO FIT THE TOTAL SAMPLE
 TAQLE-61

item loadings om principal components for stanford-binet age levels v. vi, vil
group a - sample males low 10 - ne 100 after rotation to fit the total sample




TABLE 66
ITEM LOADINES ON PRINCIPAL COMPONENTS FOR STANFORD-BINET AGE LEVELS Y, YI, VII
GROUP 10 - UHETE SAMPLE M=100 AFPER ROTATION TO FIT THE TOTAL SAMPLE
GROUP 10 - WHITE SAMPLE $=N=100$ AFTER ROTATION TO FIT THE TOTAL SAMPL


TABLE-67

GROUR 10- BLACK SAMPLE- $=$ H=100-AFIER-ROTAFLON-TOLEIT-THETOTAL SAMPLE
item loadings om primeipal components for stanford-binet age levels vi vi, vii
gROUP 11 - SAMPLE G SMALL SD - N=300 after ROTATION TO FIt the total sample


MABLE G9




[^0]:    * p :05
    **p<01

[^1]:    * $\mathrm{p}<.05$
    ** $p<.01$

[^2]:    \# loading of .50 or better
    : loading between . 30 and .49

[^3]:    \# loading of .50 or better
    : loading between .30 and .49

