Descriptor Predictive Control: Tracking Controllers for a Riderless Bicycle

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Abstract

Descriptor Predictive Control (DPC) is a hybrid open-loop closed-loop control strategy for trajectory tracking of nonlinear systems. The intrinsic idea of DPC is the combination of feedback control and the implicit formulation of a control law, the control being obtained by iteratively solving a descriptor system. We consider three models for the riderless bicycle. A cart, a cart plus an inverted pendulum, and a complex model taking into account the complete geometry of a bicycle. We show first on the simple, then on the complex model, how to obtain an output tracking controller using the concept of DPC.

1. Introduction

In this paper, we apply a trajectory tracking control methodology, called Descriptor Predictive Control (DPC), which has been introduced by the authors in [13]. The basic concept of the methodology has initially been introduced in [7]. The underlying idea, is that the controller should compute the control numerically, in an implicit manner using backward differentiation formula, rather than symbolically.

The objective of this paper is to show, on the example of the controller synthesis for a riderless bicycle, described by three models of different degrees of complexity, that nonlinear tracking control is most naturally designed using descriptor systems. Due to the implicit formulation and the use of DPC the resulting control laws remain simple in structure, even in the case of complex dynamical systems.

Consider the nonlinear systems

$$0 = F(\dot{x}, x, t, u) \quad (1)$$

$$y = h(x, u, t) \quad (2)$$

with control u, state x, and output y. Many physical systems, such as constrained mechanical systems, are most naturally initially modeled in the form (1), where $F_{\dot{x}}$ is identically singular, which means that (1) is a system of *Differential Algebraic Equations* (DAE's) or a *Descriptor System*. Classically, a trajectory tracking controller for (1) is obtained by, first, transforming (1)-(2) into the explicit form $\dot{x} = f(x,t) + g(x,t)u$, $y = h(x,u,t)$, and then designing an explicit expression for the controller $u = k(x, y_d, \dot{y}_d, \dots, y_d^{(w)})$, where y_d is the desired output trajectory (see for ex. [6]). Complex symbolic computations are necessary to obtain these explicit expressions, whereas, in many cases, the same control objectives can be obtained by a simpler method. This can be done by considering (1)-(2) to be a DAE in x, y, and u, and computing u numerically. This DAE will in the sequel be referred to as *control DAE.* Clearly, we have to assure that the control DAE has a solution. A necessary condition for that is that (1)-(2) has as many inputs as outputs and be invertible (see for ex. [5, 10] for conditions of invertibility). Furthermore, we need that the control DAE be numerically solvable. To this end we assume that the control DAE is (or has been put into) a DAE of, at least, index three (see [1] for a more detailed discussion of the index of DAS's).

2. Descriptor predictive Control

In order to compute the control (and if necessary approximations for higher order derivatives), the controller uses the solution of the control DAE, which is composed out of the nonlinear plant and stabilized path constraints expressing the tracking requirements.

$$0 = \begin{bmatrix} F(\dot{x}, x, t, u) \\ a(\sigma)[y_d(t) - h(x, u, t)] \end{bmatrix} \quad (3)$$

where $a(\sigma)$ is a stable polynomial in the differential operator $\sigma = d/dt$. The stabilization is necessary as the index has to be reduced to, at least, three [1] to insure safe numerical integrability of the DAE. The tracking control is computed as follows: the measured state is used as an initial condition for the controller DAE which is solved numerically for a short time period (prediction horizon) yielding a prediction of the state and the control. The control is applied for a short period of time as an open loop control, the state is measured again, and the process repeated. The resulting control is a feedback control since the integration is initialized by the measured state of the plant but open-loop over short periods of time. It is also in a sense predictive because the control is obtained by "looking ahead". However, it is different from the "classical" concept of predictive control as used in [8] in that it does not involve optimization.

A linear analysis of the controller has been carried out in [11, 13]. Here, we note only that the application of this controller as proposed in [7] can yield non stabilizing controllers, a problem which can be avoided by using a preliminary feedback, provided the system is minimum phase. The controllers used throughout this paper will be designed such that preliminary feedback will not be necessary.

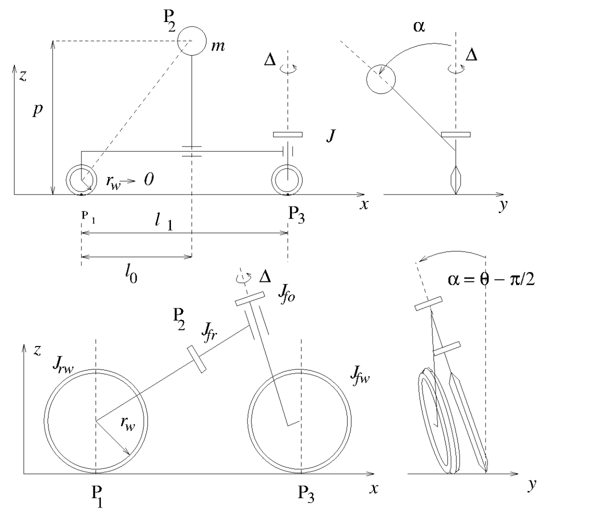

Figure 1 *The simple bicycle model of [4] and the "full" bicycle model of [12].*

3. The bicycle model

We consider three bicycle models. A cart, a cart plus an inverted pendulum, and a full bicycle model taking into account the complete geometry. The two latter models are depicted in Figure 6. Due to the physical constraints expressing the tire road contacts, which are supposed to be ideal, the bicycle is a nonholonomic system.

3.1. The cart model

A standard example for a nonholonomic system is the cart model, which has extensively been studied in the literature (see for example [2]). We denote the xy-position of the cart by the pair (x, y), its orientation by ϕ and its velocity by v. The steering angle is denoted by Δ. Setting $\kappa = \tan(\Delta)/l_1$ and using the assumption that we directly control the steering velocity (that is, $\dot{\Delta} = u_1$) the cart model becomes

$$\begin{array}{ll} \text{Dynamic part} & \left\{ \begin{array}{rcl} l_1\, J\, \dot{\kappa} &=& (1 + l_1^2 \kappa^2)\, u_1 \\ m\, \dot{v} &=& u_2 \end{array} \right. \\ \text{Kinematic part} & \left\{ \begin{array}{rcl} \dot{\phi} &=& v\, \kappa \\ \dot{x} &=& v \cos(\phi) \\ \dot{y} &=& v \sin(\phi) \end{array} \right. \end{array} \qquad (4)$$

where $x = (\kappa, v, \phi, x, y)^T$ is the state and $u = (u_1, u_2)^T$ the control input.

3.2. The simple bicycle model

The simplest bicycle model describing leaning and falling over is a cart extended by an inverted pendulum. This model is studied in [3, 4]. The resulting equations of motion are sufficiently simple to be manipulated by hand. This will help us studying the problem of stabilizing the bicycle and tracking a path in the xy-plane. The simple bicycle model is the cart model plus the equation of motion of an inverted pendulum. As in the cart model, we assume here that we control the angular velocity of the steering angle[1]. The model is equivalent to that in [3] except for an appropriate feedback transformation.

$$\begin{array}{ll} \text{Dynamic part} & \left\{ \begin{array}{rcl} l_1 J \dot{\kappa} &=& (1 + l_1^2 \kappa^2) u_1 \\ m\, \dot{v} &=& u_2 \\ p^2 \ddot{\alpha} - g\, p\, \mathrm{s}_\alpha &=& \mathrm{c}_\alpha \xi \end{array} \right. \\ \text{Kinematic part} & \left\{ \quad \text{see cart model} \right. \end{array} \qquad (5)$$

$$\xi = \kappa\, \dot{v}\, p\, l_0 + (\kappa^2 p^2 \mathrm{s}_\alpha + \kappa\, p) v^2 + v\, \dot{\kappa}\, p\, l_0$$

where α is the bank angle, $\mathrm{s}_\alpha = \sin(\alpha)$ and $\mathrm{c}_\alpha = \cos(\alpha)$. The constant p denotes the length of the inverted pendulum on the cart, l_0 is the distance between the rear wheel and the turning point of the pendulum on the cart, and g is the gravity constant.

3.3. The full bicycle model

Experiments on a real bicycle show that a bicycle has beyond a certain critical velocity and below a second critical velocity a stable movement, which is the straight line movement. Since the inverted pendulum on the cart is unstable for any movement, the simple bicycle model is too incomplex to be (on this point) physically correct and we need to extend the model to a more complex one. The equations of motion of the extended bicycle model (in the sequel referred to as *(full) bicycle model*) are computed in a standard parameterization (q denotes the generalized coordinates) using the Euler Lagrange formalism with constraint equations (λ and γ denote the Lagrange multipliers). The resulting model is a constrained

[1]For some comparisons with the full bicycle model we will need to replace this assumption by the new assumption that we control the angular acceleration of the steering angle.

mechanical system of the form

$$0 = \begin{bmatrix} M(q)\ddot{q} - F(q,\dot{q}) - K(q)u - f_q^T(q)\lambda - G^T(q)\gamma \\ f(q) \\ G(q)\dot{q} \end{bmatrix}$$

where $f_q = \partial f(q)/\partial q$. In the following we suppress the arguments q and $\dot{q}$ in the matrices M, F, K, G, and f, and reduce the index of the constrained system to obtain the index one DAE

$$0 = \begin{bmatrix} \ddot{q} - M^{-1}\left[F - Ku - f_q^T\dot{\hat{\lambda}} - G^T\dot{\hat{\gamma}}\right] \\ cf_q\dot{q} + f \\ G\dot{q} \end{bmatrix}$$

where c is a positive constant, $\dot{\hat{\lambda}} = \lambda$ and $\dot{\hat{\gamma}} = \gamma$. The control inputs are a pedalling torque (generated by an actuator placed between the frame and the rear wheel) and a steering torque (generated by an actuator between the handlebars and the frame). Let $\omega = (\hat{\lambda}^T, \hat{\gamma}^T)^T$, in the following we will denote the full bicycle model as

$$0 = \begin{bmatrix} \ddot{q} - f_{\text{bike}}(q,\dot{q},\dot{\omega},u) \\ g_{\text{bike}}(q,\dot{q}) \end{bmatrix} \tag{6}$$

4. Asymptotic tracking controller

We say a control asymptotically tracks the output y to the desired output trajectory y_d if $y \to y_d$ for $t \to \infty$. In this section we will show that asymptotic tracking of the xy-position is impossible if we allow the bicycle to lean to the side (and fall over).

4.1. Cart model

It is shown in [9] that the position of the center of mass of the rear wheel of the cart, given by $P_1 = (x,y)$, allows to reconstruct the entire state and the control input $u = (u_1,u_2)^T$ by pure differentiation. This implies that we can asymptotically track the xy-position of the cart provided the cart does not stop. (An isolated point in the xy-plain cannot be stabilized by smooth state feedback.) The asymptotic tracking control can be implemented as DPC, which involves the solution[2] of the following controller DAE.

$$0 = \begin{bmatrix} \dot{x} - f_{\text{cart}}(x,u_1,u_2) \\ a(\sigma)(x - x_d) \\ b(\sigma)(y - y_d) \end{bmatrix} \tag{7}$$

where $\dot{x} = f_{\text{cart}}(x,u_1,u_2)$ denotes the nonlinear plant (4). The index of (7) is assured by [9]. $a(\sigma) = a_0 + a_1\sigma + a_2\sigma^2 + a_3\sigma^3$ and $b(\sigma) = b_0 + b_1\sigma + b_2\sigma^2 + b_3\sigma^3$. While the computation of $(x^{(3)}, y^{(3)})$ we involve $\dot{u}_1$ and $\dot{u}_2$.

[2] To solve (7) we may use DASSL[1]. (7) is an index one DAE that can safely be integrated by DASSL. The integrator fails if we set $b_3 = a_3 = 0$. In this case (7) is and index two DAE; DASSL does not support the integration of index two DAE's.

4.2. Simple bicycle model

As we have seen on the cart model, a desired path for the output (x,y) can be expressed in terms of v and κ. Consequently, to study the effect of instability while tacking (x,y), we may disregard the kinematic part in (5) and consider only the dynamic part having has output $y = (\kappa, v)^T$. The zero dynamics of (5) for this output is

$$\begin{aligned} &p^2\ddot{\alpha} - g\,p\,\sin(\alpha) = \cos(\alpha)\,\xi \\ &\xi = \kappa\,\dot{v}\,p\,l_0 + (\kappa^2 p^2 \sin(\alpha) + \kappa\,p)v^2 + v\,\dot{\kappa}\,p\,l_0 \end{aligned} \tag{8}$$

For $\xi =$ const. (8) has in α, in the neighborhood of $\alpha = 0$ and $\xi = 0$, unstable equilibria at $\alpha =$ const.. Consequently, we cannot asymptotically track the output $y = (\kappa, v)^T$ to a desired output trajectory. That is, tracking of the xy-position is not possible with our controller. For the new output $\overline{y} = (\alpha, v)^T$ we obtain again (8) as zero dynamics. However, in κ, (8) has stable equilibria for $\kappa =$ const., in the neighborhood of $\kappa = 0$, and $v =$ const.. That is, asymptotic tracking of $\overline{y} = (\alpha, v)^T$ is possible. Let the nonlinear plant (5) be denoted by $\dot{x} = f_{\text{simp}}(x,u_1,u_2)$, where $x = (\kappa, \alpha, v, \dot{\alpha}, \dot{\kappa})^T$. DPC for the output $\overline{y}$ involves the solution the following controller DAE[3].

$$0 = \begin{bmatrix} \dot{x} - f_{\text{simp}}(x,u_1,u_2) \\ a_0(v - v_d) + a_1(\dot{v} - \dot{v}_d) \\ b_0(\alpha - \alpha_d) + b_1(\dot{\alpha} - \dot{\alpha}_d) + b_2(\ddot{\alpha} - \ddot{\alpha}_d) \end{bmatrix} \tag{9}$$

4.3. Full bicycle model

The same holds for the full bicycle model: we cannot asymptotically track $y = (\kappa, v)^T$, but $\overline{y} = (\alpha, v)^T$. However, new coupling terms between κ, v and α may change the index of the control DAE. In our case, the index of the control DAE can be studied by looking at the linearization, here, the index being equal to the number of differentiations necessary to compute the inverse dynamics, plus one. Note that we need to know the index to chose the degree of the polynomial matrix for the stabilization of the path constraints.

4.3.1 Linear considerations: Consider the minimal representation of the linear model of (6) for the straight line movement in the operation point $\alpha_{\text{equi}} = 0$ rad, $v_{\text{equi}} = 2$ m/s, and $(u_{1_{\text{equi}}}, u_{2_{\text{equi}}}) = (0,0)$. We chose as output the bank angle and the velocity: $\delta\overline{y} = (\alpha - \alpha_{\text{equi}}, v - v_{\text{equi}})^T$. The poles are $\{-15.2,\ 2.15 \pm j1.68,\ -3.8,\ 0\}$ and the zeros for the given output are $\{-55.69, -2.72\}$. The transfer matrix in this operation point is

$$\delta\overline{y} = \text{diag}\left(\frac{c_1(z_1+s)(z_2+s)}{(p_1+s)(p_2+s)(p_3+s)(p_4+s)}, \frac{c_2}{s}\right)\delta u \tag{10}$$

where diag stand for diagonal matrix, p_i, z_i and c_1, c_2 are constants, and $\delta u = u - u_{\text{equi}}$. The two output

[3] The DAE is index one and can safely be integrated by DASSL.

Figure 2 *Example of typical tracking error encountered using DPC. Tracking errors for the bank angle (denoted by the solid line e_1) and speed (denoted by the dotted line e_2).*

channels become coupled as soon as $\alpha_{\text{equi}} \neq 0$, but the number of zeros and poles of the linear system remain the same. As we have four poles and two zeros for the input-output channel $u_1 \to \alpha$, and no zero and one pole for the input output channel $u_2 \to v$, the bank angle has to be differentiated twice and the speed once to inverse the system. This is not consistent with the simple bicycle model. Assume we control the angular acceleration of the the steering angle (and not its velocity). To compute a tracking control for $\overline{y}$ for the simple bicycle model we have to invoke the third derivative of α and the second derivative of v to get an explicit expression for u_1 and u_2. The difference between the two models is due to coupling terms between κ and α, which occur in the full bicycle model and which are absent in the simple bicycle model.

4.3.2 Implementation as DPC: Based on the linear considerations we use the control DAE

$$0 = \begin{bmatrix} \ddot{q} - f_{\text{bike}}(q, \dot{q}, \dot{\omega}, u) \\ g_{\text{bike}}(q, \dot{q}) \\ a_0(\alpha - \alpha_d) + a_1(\dot{\alpha} - \dot{\alpha}_d) + (\ddot{\alpha} - \ddot{\alpha}_d) \\ b_0(v - v_d) + (\dot{v} - \dot{v}_d) \end{bmatrix} \qquad (11)$$

which is (at least in a neighborhood of $\alpha_{\text{equi}} = 0$ and $\kappa_{\text{equi}} = 0$) of index one. Note that, due to the use of the implicit description of the controller, $\dot{v}$ and $\ddot{\alpha}$ are available while the solution of (11).

Computer simulations show that DPC stabilizes the unstable bicycle dynamics and assures asymptotic tracking of the desired output trajectories.

Figure 2 shows an example of a tracking error which might be encountered using DPC. The nonzero tracking errors are due to the piecewise constant implementation of the controller. DPC tracks the output $\overline{y} = (\alpha, v)^T$ to a step function starting at $\alpha = 0$deg. and $v = 2$ m/s. The initial conditions are chosen to be consistent with the desired trajectory. During the computer simulation the bicycle has a speed varying between 1.75 m/s and 2 m/s and a bank angle varying between 0 deg. and 8 deg.. (for more figures and implementation issues see [11]). The bicycle is unstable for speeds below 3.95 m/s and becomes nonminimum phase for speeds below 0.95 m/s, that is, DPC stabilizes the unstable bicycle dynamics and assures asymptotic tracking of the desired output trajectories. The computer simulations show that a sampling period of $h = 0.02$ sec. suffices to stabilize the bicycle and to assure good tracking performance. Good tracking performance of the asymptotic tracking controller is a necessary condition for the next controller, which approximately tracks the “nonminimum phase output” $y = (\kappa, v)^T$.

5. Approximate tracking controller

We have seen that asymptotic tracking of the output $y = (\kappa, v)^T$ is impossible due to unstable zero dynamics. However, if asymptotic tracking is not possible, we may design an approximate tracking controller using a kind of prefilter for the desired output trajectory.

5.1. The idea of approximate tracking

The approximate tracking approach of nonmiminum phase systems considered here has initially been introduced in [4] and is extended in [11]. Its extended version of [11] is based on the following idea. We call an output *(non-) minimum phase output* if the plant is (non-) minimum phase for the output. Consider the plant

$$\dot{x} = f_{\text{plant}}(x, u), \quad y = h(x), \quad \overline{y} = \overline{h}(x)$$

where y is a nonminimum phase output and $\overline{y}$ a minimum phase output. To compose the control DAE for approximate tracking, we add a copy of the plant to the control DAE (involved in asymptotic tracking) and modify the copy such that, for the copy, y becomes a minimumphase output and we do not alter the equilibrium point. That is, if $\dot{x} = f^{\text{copy}}_{\text{plant}}(x, u)$ denotes the copy of the plant, we assure for all x and u that $f_{\text{plant}}(x, u) = f^{\text{copy}}_{\text{plant}}(x, u)$, which implies that we need to determine a matrix E, such that

$$E\dot{x} = f^{\text{copy}}_{\text{plant}}(x, u), \quad y = h(x)$$

is of minimum phase. Then we generate through the modified copy a reference trajectory $\overline{y}_d$ for the minimum phase output $\overline{y}$ which we can asymptotically track to $\overline{y}_d$ using DPC. The extended control DAE for approximate tracking is

$$0 = \begin{bmatrix} E\dot{x} - f^{\text{copy}}_{\text{plant}}(x, u) \\ a(\sigma)[y_d - h(x)] \\ \dot{x} - f_{\text{plant}}(\overline{x}, \overline{u}) \\ \overline{a}(\sigma)[\overline{h}(\overline{x}) - h(x)] \end{bmatrix} \qquad (12)$$

where $a(\sigma)$ and $a(\sigma)$ are two stable polynomials in $\sigma = d/dt$. The resulting control is $\overline{u}$, which applied as DPC tracks y approximately to y_d provide $y_d(t)$ is of sufficiently "low frequency". The control strategy admits a complete analysis for the linear case, which is given in [11]. The approach of [4] is equivalent with [11] for a particular choice of $\overline{h}(x)$, $h(x)$ and E.

5.2. Simple bicycle model

Consider the dynamic part of the simple bicycle model. The following modified bicycle dynamics are clearly locally minimum phase for the output $y = (\kappa, v)^T$ in the neighborhood of $\alpha = 0$ and sufficiently small ξ.

$$\begin{cases} l_1 J\ddot{\kappa} &= u_1 \\ m\dot{v} &= u_2 \\ p^2 e_2\ddot{\alpha} - e_1\dot{\alpha} - g\,p\,\mathrm{s}_\alpha &= \mathrm{c}_\alpha \xi \end{cases}$$

$$\xi = \kappa\,\dot{v}\,p\,l_0 + (\kappa^2 p^2 \mathrm{s}_\alpha + \kappa\,p)v^2 + v\,\dot{\kappa}\,p\,l_0$$

where e_1 and e_2 are chosen such that the polynomial $p^2 e_2 \sigma^2 + e_1\sigma + g\,p = 0$ in σ has stable roots. The modified simple bicycle dynamics will be referred to as $E\dot{x} = f_{\mathrm{simp}}^{\mathrm{copy}}(x, u)$. Now the solution of

$$0 = \begin{bmatrix} E\dot{x} - f_{\mathrm{simp}}^{\mathrm{copy}}(x,u) \\ a_0(v - v_d) + a_1(\dot{v} - \dot{v}_d) \\ b_0(\kappa - \kappa_d) + b_1(\dot{\kappa} - \dot{\kappa}_d) + (\ddot{\kappa} - \ddot{\kappa}_d) \end{bmatrix} \quad (13)$$

yields a desired output trajectory for the minimum phase output $\overline{y}_d = (\alpha, v)^T$, which we may track by the controller (9), which together with (13) forms the extended control DAE (12). In the case of the simple bicycle model we may use a simplified version of the extended control DAE (12). In fact, if we assume that in the modified dynamics (13) $\kappa = \kappa_d$ and $v = v_d$, we obtain a simplified version of (12) by replacing in (9) the path constrain on α by the stabilized zero dynamics (8) in which we replace κ by the desired output κ_d. The new path constraint on α is a second order differential equation and the resulting DAE is still index one, which can safely be solved by DASSL.

$$0 = \begin{bmatrix} \dot{x} - f_{\mathrm{simp}}(x, u_1, u_2) \\ a_0(v - v_d) + a_1(\dot{v} - \dot{v}_d) \\ p^2 e_2\ddot{\alpha} + e_1\dot{\alpha} + g\,p\,\mathrm{s}_\alpha + \mathrm{c}_\alpha\xi \end{bmatrix} \quad (14)$$

$$\xi = \kappa_d\,\dot{v}\,p\,l_0 + ({\kappa_d}^2 p^2 \mathrm{s}_\alpha + \kappa_d\,p)v^2 + v\,\dot{\kappa}_d\,p\,l_0$$

In the computer simulation in Figure 3 we use the controller (14) to track κ to the slew function.

$$\kappa_d(t) = \Delta\sigma\left(\tfrac{70}{t_f^9}t^9 - \tfrac{315}{t_f^8}t^8 + \tfrac{540}{t_f^7}t^7 - \tfrac{420}{t_f^6}t^6 - \tfrac{126}{t_f^5}t^5\right)$$

where $\Delta\kappa = 0.1\,\mathrm{rad}$ and $t_f = 2\,\mathrm{s}$ and the speed is held constant at the value $v_d = 4$ m/s. As κ_d varies too fast we have poor tracking performance. For a "low frequency" sin function we obtain good tracking performance (see Figure 4). We use the controller parameters $b_0 = 5$, $b_1 = 4$, $b_2 = 1$, $a_0 = 2$, $a_1 = 1$ and the model parameters $m = 1$, $J = 1$, $p = 1$, $l_1 = 1$, $l_0 = 0.5$, $g = 9.8$.

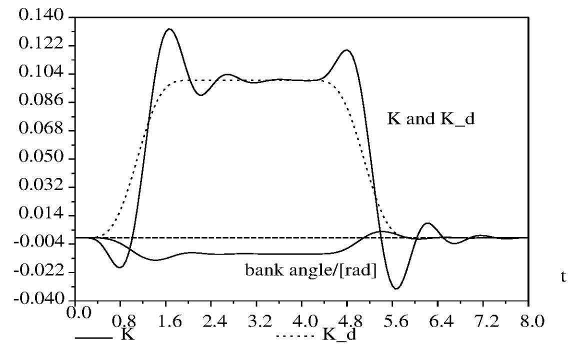

Figure 3 *Simple bicycle model: approximate tracking for the output κ by a desired trajectory composed from the step function.*

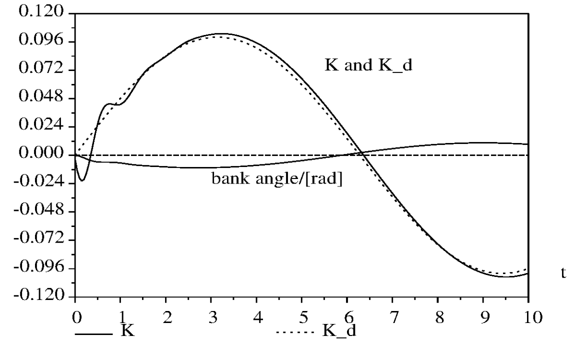

Figure 4 *Simple bicycle model: approximate tracking for the output κ by the sin function $\kappa_d = 0.1\sin(0.5\,t)$.*

5.3. Full bicycle model

We have seen that we can asymptotically track the output $\overline{y} = (\alpha, v)^T$, which is a minimum phase output, as for the simple bicycle model. In analogy to the simple bicycle, $y = (\kappa, v)^T$ is a non minimumphase output. For the full bicycle model κ (or equivalently, the steering angle Δ) has not been chosen as generalized coordinate but can be computed by the kinematic equation $\dot{\phi} = \kappa v$. Consequently, tracking $y = (\kappa, v)^T$ is equivalent with tracking $y^* = (\dot{\phi}_1, v)^T$, where ϕ_1 is the yaw angle of the rear wheel. Clearly, $y^* = (\dot{\phi}, v)^T$ is also a nonminimum phase output. To apply the idea of approximate tracking we need to determine two matrices E_1 and E_2 such that

$$\begin{cases} E_2\ddot{q} + E_1\dot{q} &= f_{\mathrm{bike}}(q, \dot{;}\omega, u) \\ 0 &= g_{\mathrm{bike}}(q, \dot{;}q) \\ y &= h(x) \end{cases}$$

has locally stable zero dynamics. In analogy to the simple bicycle, we may assume that the zero dynamics can approximately be written in terms of the bank angle of the rear wheel and we chose E_1 and E_2 such that we obtain the same stabilization of the bank angle as in the case of the simple bicycle. Computer simulations of the approximate tracking controller for the full bicycle model for different desired output trajectories are shown in Figures 5 - 7.

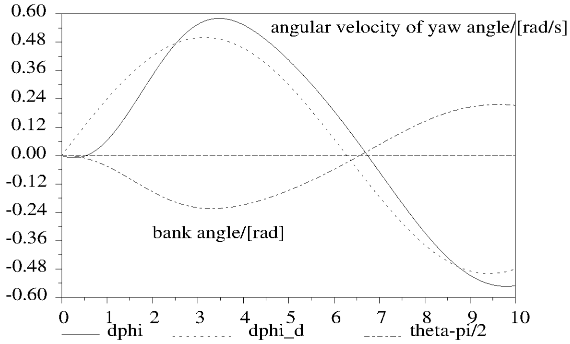

Figure 5 *Full bicycle model: approximate tracking of* $\dot{\phi}_1$ *by* $\dot{\phi}_d = 0.5\sin(0.5\,t)$.

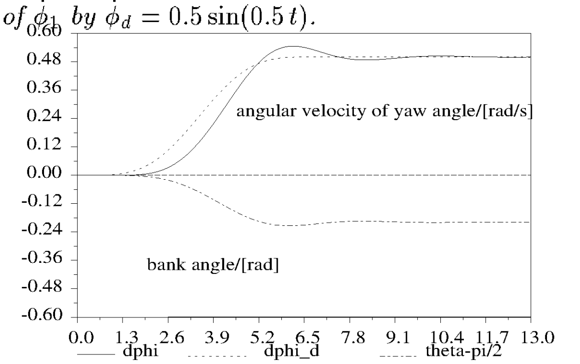

Figure 6 *Full bicycle model: approximate tracking of* $\dot{\phi}_1$ *by a desired output function composed from a step function. The second output is tracked by* $\psi_{1_d} = -v_d/r_w$*, where* $v_d = 4$ *m/s for all* t.

6. Conclusion

We have shown that the control of a riderless bicycle can be approached by systematically extending the cart model (which is a bicycle that does not lean) to more complex descriptions. We have shown on three different models of a riderless bicycle of different complexity how to apply DPC for output trajectory tracking . The tracking controllers for the full bicycle model, remain well structured and require no symbolical differentiation of the output equation. We have shown the design of an asymptotic tracking controller for the bank angle and the velocity of the bicycle model. Finally, we have shown that asymptotic tracking of the xy-position of the bicycle using DPC is not possible, but that approximate tracking can be achieved.

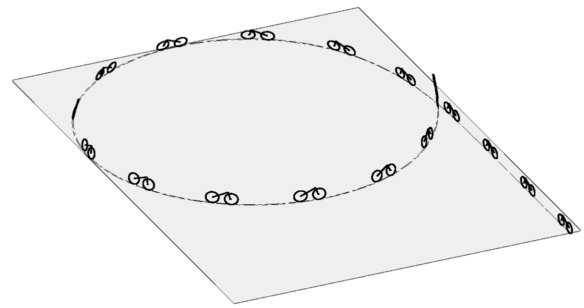

Figure 7 *The resulting maneuver in the xy-plane is a circle.*

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