Forward Induction in the Battle-of-the-Sexes Games

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This paper provides experimental evidence on forward induction as a refinement criterion. In the basic extensive form, one of the two players chooses to play a battle-of-the-sexes game or to receive a certain payoff. According to forward induction, choosing to play the game is a signal about intended action. Though the presence of the outside option changes play, we find only limited support for the forward-induction hypothesis. The effects of the outside option also reflect the creation of a focal point through the asymmetry created by offering the outside option to one of the two players. (*JEL* C72)

In many games, the concept of Nash equilibrium does not yield a unique prediction regarding agents' play. To resolve the problem of multiple equilibria, game theorists have advanced a number of refinements to the concept of Nash equilibrium. This paper provides experimental evidence on the success of forward induction as a refinement criterion (see Elan Kohlberg and Jean François Mertens, 1986) using a variant of a well-known game with multiple Nash equilibria, the battle-of-the-sexes game.[1]

Figure 1 illustrates a battle-of-the-sexes game, hereafter BOS. Here there are two pure-strategy Nash equilibria: (1,2) and (2,1). The column player prefers the first equilibrium, and the row player prefers the other. There is also a symmetric mixed-strategy equilibrium in which strategy 1 is played with probability $\frac{1}{4}$.

For this game, disequilibrium (i.e., *ex post*, play that does not constitute an equilibrium) may arise from two sources. First, unless one of the equilibria is viewed as a natural outcome (i.e., focal) by both players, coordination of play on a particular pure-strategy equilibrium is unlikely. Second, if the mixed-strategy equilibrium of the battle-of-the-sexes game is viewed as the natural outcome (as suggested by, e.g., Joseph Farrell [1987]) there is a chance that,

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[1]In fact, this is a commonly used example in the discussion of forward induction (see e.g., Eric van Damme, 1989). There are other experimental papers that investigate refinements of Nash equilibrium. For incomplete-information games, these include Jeffrey Banks et al. (1988) and Jordi Brandts and Charles Holt (1992). We comment below on the more closely related papers that focus on forward induction in experimental games with complete information.

		Column Player 1	Column Player 2
Row Player	1	0,0	200,600
	2	600,200	0,0

FIGURE 1. BATTLE-OF-THE-SEXES GAME (BOS)

ex post, neither pure-strategy equilibrium will be observed.

In Cooper et al. (1989), hereafter CDFR1, we provide experimental evidence of *ex post* disequilibria in the battle-of-the-sexes game. For BOS, as reported in CDFR1, play was closest to the mixed-strategy prediction. More importantly, the strategy choices yielded (*ex post*) equilibria less than 42 percent of the time. Thus, there are mutual gains to be realized if the players can avoid disequilibrium outcomes.

It is therefore natural to consider the effects of institutions that might alter behavior and avoid coordination problems. Along these lines, game theorists have suggested two important variations in these games which serve to overcome these problems; both involve adding a stage prior to play of the game.

One approach, which is the focus of this paper, involves the application of forward induction to extensive form games and corresponds to Kohlberg and Mertens' (1986) notion of strategic stability in normal form games.[2] In an extensive form game, the idea is that a player's beliefs about his rival's play from the start of any subgame might depend on play prior to that subgame. For example, suppose that the row player has the choice of an outside option with a certain payoff should he choose not to play a game, but he chooses to play regardless. The fact that he has chosen to play tells the column player something about the row player's intentions, namely, that the row player expects to earn more by playing the game than he would earn by taking the outside option. When the outside option dominates some of the row player's strategies, then the deletion of these strategies may reduce the set of equilibria. The next section provides a more detailed discussion of the logic of forward induction and its relation to strategic stability.

A second approach is to allow players the opportunity to engage in nonbinding, preplay communication.[3] Though cheap talk is, by definition, payoff-irrelevant, it can influence players' beliefs about the likely action of their opponents. CDFR1 provides experimental evidence on the role of pre-play communication in overcoming disequilibrium problems for the battle-of-the-sexes game.

The goal of this paper is to assess the predictive power of forward induction. To do so, we consider a simple variant of BOS in which the row player can choose between an outside option of 300 and playing the game, BOS. This is a leading example in the literature on forward induction and thus serves as a useful starting point for our analysis. Our results in this treatment are in accord with the predictions of forward induction.

In contrasting this extensive form game with BOS, there are two important differences. First, of course, is the presence of the outside option which, through the logic of iterative deletion, leads to play at the row player's preferred outcome. Second, there is an asymmetry in the game created by extending the outside option to only one of

[2]See the extensive discussion in van Damme (1989, 1990) on the relationship between these concepts.

[3]Farrell (1987) argues that, permitted this "cheap talk," players will be able to reduce the frequency of *ex post* disequilibrium outcomes in the battle-of-the-sexes game. Along these same lines, in Cooper et al. (1992a), we extend Farrell's argument to show how cheap talk might overcome coordination failures in simple coordination games.

the players. Thus, there are two influences at work in the extensive form game: iterative deletion and asymmetry.

We consider alternative games to separate these two influences. These additional treatments provide evidence that leads us to question the power of forward induction. First, we consider play of the normal form representation of our extensive form game with an outside option.[4] By iterative deletion of dominated strategies, the same outcome observed in the extensive form treatment is predicted for the normal form. In practice, this was not the case: while the row player did not choose the dominated strategy, the column player did not take the next step in the process of iterative deletion. Under the hypothesis that the normal and extensive form representations are equivalent, this is evidence against forward induction.

This problem with iterative deletion in the normal form led us to consider an alternative hypothesis about the role of the outside option in the first extensive form game. In particular, we consider the possibility that the outside option provides a focal point; that is, if it is made available to the row player, this selects the row player's preferred outcome regardless of the outside option's value. In this case, even if the outside option of the row player is only 100, the fact that the row player had this additional choice could influence play. In contrast, forward induction implies that the presence of this option should be irrelevant to observed play. Our results indicate that even an outside option of 100 influences play of the game (in favor of the row player) though not to the degree of the 300 outside option.

We further investigate this hypothesis by considering an extensive form BOS game in which the row player selects an action prior to the column player but row's choice is not observed by the column player when he moves.[5] The issue is whether the asymmetry in this extensive form provides a strategic advantage to the row player by making his preferred equilibrium focal. Our results indicate that the sequencing of moves matters and does provide an advantage to the row player.[6] Thus, it appears that the effects of the outside option are not solely due to forward-induction arguments but also reflect the creation of a focal point through the asymmetry in the game created by offering the outside option to the row player.

I. Forward Induction and the Outside Option

Through forward induction, a player may be able to infer information about his rival's intended actions from actions taken in earlier stages of a game. In this way, play from the start of a particular subgame could depend upon the choices made that led to that subgame. This section describes this logic and its force in the selection from among the multiple equilibria of the Figure 1 game. For an extensive discussion of the attractive but elusive idea of forward induction and the concept of stability, the reader is directed to Kohlberg and Mertens (1986), In-Koo Cho and David Kreps (1987), van Damme (1989, 1990), and Elhanan Ben-Porath and Eddie Dekel (1992).

Consider the battle-of-the-sexes game with an outside option of 300, BOS-300, illustrated in Figure 2. Suppose that, prior to the play of this game, the row player is offered a choice between receiving 300 (the "outside option") or playing BOS, given in Figure 1. If the row player elects to play the game, the column player might reason as follows: "The row player has passed on an opportunity to collect 300 in order to play

[4]Comparison of the normal and extensive forms was motivated by a similar exercise in Andrew Schotter et al. (1990).

[5]Game theory's prediction here is that this sequencing of moves should be irrelevant. However, as will be discussed, Amin Amershi et al. (1989a, b) develop an equilibrium concept in which the timing of unobserved moves matters.

[6]In private communication, Jack Ochs, Lawrence Ausubel, and Charles Wilson report similar results in a battle-of-the-sexes game.

the game. If he planned to play strategy 1, he would have been better off selecting the outside option. Therefore, he must be planning to play 2." As a consequence of this logic, the column player should play 1, leading to the outcome (2,1). Any outside option paying the row player between 200 and 600 would support the (2,1) outcome as well.

By this logic, the predicted outcome is (2,1), and coordination problems with the battle-of-the-sexes game are completely resolved. The addition of the outside-option stage to this game has created a new game with two subgame-perfect, pure-strategy, Nash equilibria. The first is the one just described; the row player rejects the outside option, and the (2,1) equilibrium of the subgame is played. In the other equilibrium, the row player takes the outside option; this is supported by beliefs that the column player will play strategy 2 at least 50 percent of the time. Though still subgame-perfect, this equilibrium does not survive the refinement of forward induction.

Hypothesis FI: If the outside option for the row player prior to the play of BOS is between 200 and 600, the row player will elect to play BOS, and the outcome will be (2,1).

The process of deleting dominated strategies, used in the evaluation of the extensive form game, can equally be applied to the normal form representation of the game given in Figure 3.[7] According to Kohlberg and Mertens (1986), the normal form representation of a game should yield the same outcome as the extensive form. That is, there is no information lost by focusing exclusively on the normal form representation. For normal form games, they argue that "stable equilibrium" is the appropriate notion of equilibrium and that this captures the logic

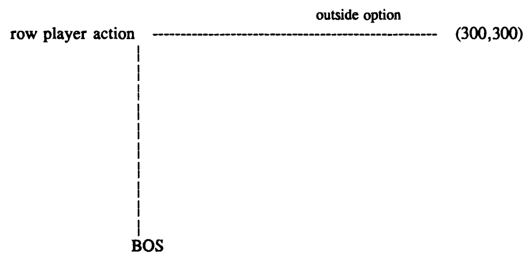

FIGURE 2. BATTLE OF THE SEXES WITH AN OUTSIDE OPTION, BOS-300

of forward induction (proposition 6 in Kohlberg and Mertens [1986]).[8] For the row player, 1 is strictly dominated by 0. After this deletion, 2 is weakly dominated by 1 for the column player. After this deletion, the row player obviously will choose 2 and column 1. This leads to the following hypothesis:

Hypothesis NF: In BOS-300-NF, the outcome will be (2,1).

We will also examine a modified version of this hypothesis. To the extent that subjects have difficulty performing iterative deletion of dominated strategies in the normal form of this game (and we shall see below that they do), it seems natural to ask whether subjects become better at this if they have additional information about the aggregate history of play. With this additional information, players could compute the expected profitability of each strategy. Thus, they should learn that the return to playing a dominated strategy is small.[9] We will refer to this game as BOS-300-NF-HIS, and we modify hypothesis NF in the following way:

Hypothesis HIS: In BOS-300-NF-HIS, the outcome will be (2,1).

[7]Here the strategies for the column player are the same as those in Figure 1. For the row player, strategy 0 is the outside option. Strategy 1 (2) means that the row player elects to play the game and then chooses action 1 (2) in the battle-of-the-sexes game.

[8]For further discussion of this point, see van Damme (1989, 1990).

[9]The set of equilibrium outcomes in the game with history is the same as the set in the game without history.

		Column Player 1	Column Player 2
Row Player	1	300,300	300,300
	2	0,0	200,600
	3	600,200	0,0

FIGURE 3. BOS-300-NF

II. Experimental Design

In the experiment, players participated in games such as those in Figures 1–3. Each player was paired with an anonymous opponent. One was designated the row player, and the other the column player.

Each game was designed to be one of complete information, because each player's payoff matrix was common knowledge, and the numerical payoffs represented a player's utility if the corresponding strategies were chosen. To accomplish this, we induced payoffs in terms of utility using the procedure of Alvin Roth and Michael W. K. Malouf (1979).[10] With this procedure, each player's payoff is given in points; these points determine the probability of the player winning a monetary prize. At the end of each period of each game, we conducted a lottery in which "winning" players received \$1.00 or \$2.00, depending on the session, and "losing" players received \$0.00.[11] The probability of winning was given by dividing the points the player had earned by 1,000. Since expected utility is invariant with respect to linear transformations, this procedure ensures that, when players maximize their expected utility, they maximize the expected number of points in each game, regardless of their attitude toward risk.

The experiment used 21 cohorts of players, each consisting of 11 different players.[12] All players were recruited from upper-division undergraduate and MBA classes at the University of Iowa. Upon their arrival, players in a cohort were seated at separate computer terminals, and each was given a copy of the instructions for the experiment. These instructions are available from the authors upon request. Since these instructions were also read aloud, we assume that the information contained in them is common knowledge.

Each player participated in a sequence of one-shot games against different anonymous opponents within his cohort. All pairing of players was done through the computer using a procedure described below. Since players reported their strategy choices through computer terminals, no player knew the identity of the player with whom he was currently paired. With the exception of BOS-300-NF-HIS no player knew the history of decisions made by any of the other players in the cohort. At the start of each period in BOS-300-NF-HIS, players were informed about the aggregate proportion of play of each strategy in each of the previous periods and the cumulative proportion of play over all periods.

Each cohort participated in two separate sessions.[13] In session I, all players partici-

[10]See Roth and Malouf (1979) and Joyce Berg et al. (1986) for a complete description of the procedure.

[11]All session-I games had \$1.00 prizes, while session-II games had \$2.00 prizes.

[12]In an earlier working paper (Adil Abdalla et al., 1989), we reported our initial results on forward induction in battle-of-the-sexes and coordination games. The results from using the simple battle-of-the-sexes game were reported in CDFR1. With these exceptions, the results testing forward induction are reported here for the first time. We also compare them with data obtained in earlier work on one-way communication reported in CDFR1.

[13]In both sessions, player pairings followed an 11-period sequence in which each player played every other player exactly once, and each alternated between being the row and column player during the periods when he was participating. These pair assignments were randomized at the beginning of each 11 periods

pated in ten symmetric one-shot dominant-strategy games (see Cooper et al. [1990] for a description of session I). During session I, each player played once against every other player. Since there was an odd number of players, one sat out each period.[14] Thus, session I consisted of 11 periods. Also, players alternated being row and column players during the periods in which they were active participants. Session I was conducted for two reasons: first, to provide players with experience with experimental procedures; and second, to see how well the dominant-strategy equilibrium prediction performed.[15]

In session II, all players participated in 20 additional one-shot games which differed from the game played in session I. Each played against every other player twice: once as a row player and once as a column player. As in session I, one player sat out in each period, and players alternated between being row and column players during the periods in which they were participating. Thus, session II consisted of 22 periods.[16]

The four session-II treatments we will focus on initially consist of BOS (see Fig. 1), BOS-300 (see Fig. 2), BOS-300-NF (see Fig. 3), and BOS-300-NF-HIS. Three replications were conducted under each treatment.[17]

III. Results

For the session-I game, as in Cooper et al. (1989, 1990, 1992a), we found that subjects played the dominant strategy over 90 percent of the time. For the session-II games, the data are not independent across all periods. Using Fisher's exact test, we can reject the hypothesis that the data in the first 11 periods are the same as the data from the last 11 periods. We can, however, pool the data across the last 11 periods of all games.[18] That is, we are unable to reject the hypothesis that actions were independent of time in the last half of the games. Thus, when analyzing our results, we restrict attention to the last half of the data in an effort to focus on "equilibrium" play since we have no model of the equilibration process. We also pool results across the replications since, under the null hypothesis

to prevent players from playing against the same opponent in the same order within each 11-period sequence. Players were told that they would play each other player once in session I and twice in session II (once as a row player and once as a column player). They were not told anything more about the sequence of matches in session II.

This matching procedure was used in order to mitigate reputation effects in these games. We have examined another coordination game (game 3 of Cooper et al. [1990]) where we compared the present design with a matching procedure in which contagion and reputation effects were not feasible; 40 players participated in 20 one-shot games. The subjects were divided into two groups of 20 and matched such that each player only played another player once, and the history of their actions could not influence the behavior of agents these players were matched with in the future. The design is analogous to Robert Townsend's (1980) turnpike model and has the property of no contagion and no reputation effects. In comparing the results, we found no differences across the designs.

[14]We wanted the pairings to satisfy two conditions: (i) players were to alternate being row and column players, and (ii) each player was to play each of the other players once (in session I) or twice (in session II). It is impossible to satisfy these two conditions with an even number of players. Having the player who sits out draw the lottery ticket may serve the additional purpose of helping to convince players that the lottery is run fairly.

[15]The ability of players to recognize dominated strategies is particularly important in these games since the logic of forward induction requires the iterated deletion of dominated strategies.

[16]Each cohort completed the two sessions in about 90 minutes. Payments to participants ranged from \$6 to \$33.

[17]Three additional treatments (BOS-100, BOS-SEQ, and BOS-1W) are detailed in the following sections, yielding seven treatments, replicated three times.

[18]We use Fisher's exact test throughout this paper to test for the statistical significance of differences (see Maurice Kendall and Alan Stuart, 1979 pp. 582–6). In rejecting the hypothesis of no differences in play between the first 11 and the second 11 periods for each replication, the largest p value was 0.045. In the test for differences in play in the last 11 periods, the smallest p value was 0.261.

TABLE 1—BATTLE-OF-THE-SEXES GAME: FREQUENCY OF OUTCOMES, LAST 11 PERIODS

	Outcome			
Game	(2,1)	(1,2)	DIS	OO
BOS	31 (19)	37 (22)	97 (59)	—
BOS-300	119 (90)	0 (0)	13 (10)	33
BOS-300-NF	53 (32)	2 (1)	86 (52)	24 (15)
BOS-300-NF-HIS	94 (57)	0 (0)	62 (38)	9 (5)

Notes: Percentages are shown in parentheses; for BOS-300, numbers in parentheses show the percentage of play in the BOS subgame. For BOS-300-NF and BOS-300-NF-HIS, the amounts entered in the OO column correspond to the play of 0 by the row player and 2 by the column player. The (0,1) outcome is aggregated in the DIS column.

of forward induction, there is no reason to suspect the existence of cohort effects.

The outcomes for BOS-300 are given in Table 1.[19] For this treatment, the data support hypothesis FI: 90 percent of the time that the outside option was not taken, equilibrium (2,1) was played. The frequency of play by row and column is given in Table 2. There it can be seen that of the 132 times the row player elected to play the game, strategy 2 was chosen 98 percent of the time, while 92 percent of the time, the column player selected strategy 1.

Inconsistent with hypothesis FI, however, is the fact that the outside option was taken 20 percent of the time. Recall that taking this option is still part of a subgame-perfect equilibrium. The outside option was more attractive to some players than others. In the last 11 periods each player had a chance to take the option five times. Of the 33 players playing this game, 19 never took the option, while two players took it four times, and one took it all five times. These three players (about 9 percent of players) account for almost 40 percent of the selections of the outside option.[20]

Note too that the play of the subgame is quite different from the play of BOS, the baseline game in which there was no outside option. Fisher's exact tests reveal that, in terms of the strategies played, BOS-300 (even if we exclude cases in which the outside option was chosen) is significantly dif-

[19]We require that players pass a "rationality" test in session I and that any cohort must choose the dominant strategy 85 percent of the time before we pool this with the rest of the data. We have required this for all of our published experiments (e.g., Cooper et al., 1989, 1990, 1992a) and work not published. There is only one replication that ever failed to satisfy the 85-percent rule; one replication of the BOS-300 game which we have excluded here. In the first session with this cohort, subjects failed to play their dominant strategy more than 25 percent of the time. In the second session, row players chose a dominated strategy (strategy 1) 29 percent of the time after forgoing the outside option. This, in turn, led column players to choose their most preferred strategy (strategy 2) 47 percent of the time. Since forgoing the outside option failed to provide a clear signal of the row player's intention, the outside option was chosen 55 percent of the time in the last 11 periods of this replication. While this replication is fascinating in its own right, we take the position that the players in this cohort were sufficiently different (as indicated by their session-I behavior) to warrant withholding this replication from the ensuing comparisons.

[20]This pattern of play, in which a few players accounted for a large fraction of the outside option choices, is not present in any of the other games we conducted.

TABLE 2—BATTLE-OF-THE-SEXES GAME: PERCENTAGE OF STRATEGIES PLAYED BY PLAYER, LAST 11 PERIODS

	Row player			Column player	
Game	Strategy 0	Strategy 1	Strategy 2	Strategy 1	Strategy 2
BOS	—	38 (130)	62 (210)	35 (124)	65 (188)
BOS-300	20 (300)	2 (16)	78 (552)	92 (196)	8 (12)
BOS-300-NF	42 (300)	5 (72)	53 (384)	64 (232)	36 (156)
BOS-300-NF-HIS	27 (300)	1 (42)	72 (474)	79 (213)	21 (87)

Note: Numbers in parentheses are the expected profits to the player from playing the associated strategy given that his opponent is playing according to the observed frequencies.

ferent from the simple BOS game.[21] We can also compare these games with respect to the frequency with which we observe subgame-perfect equilibria *ex post*. There are significantly more subgame-perfect equilibria observed in BOS-300 than in BOS, and different equilibria are observed in these two games. In BOS we observe almost equal numbers of (2,1) and (1,2) outcomes, while in BOS-300 the outside option constitutes 22 percent of all equilibria observed and the (2,1) outcome the other 78 percent. Indeed, as a fraction of all equilibria observed, there is a significantly greater proportion of (2,1) outcomes in BOS-300 than in BOS.

Next, we turn to hypothesis NF: given that forward induction has some power in BOS-300, does this carry over to the normal form representation? The results of BOS-300-NF do not support hypothesis NF. The equivalent strategy to the outside option is taken 42 percent of the time; this is significantly greater than the 20 percent in BOS-300.[22] Further, the (2,1) equilibrium is observed only 32 percent of the time versus 72 percent in BOS-300. The failure of the iterative-deletion argument in the normal form stems from the play of the column player. While the row player selects his dominated strategy 1 only 5 percent of the time in BOS-300-NF, the column player chooses strategy 2, which is weakly dominated by 1 if the row player never plays 1, 36 percent of the time. This is in sharp contrast to what was observed in BOS-300 where the column player chose strategy 2 only 8 percent of the time. Thus, iterative deletion stops after one round. The fact that our design causes row and column players to rotate roles is interesting. Since each can experience the game from the other's perspective, this rotation should enhance the process of iterative deletion.[23]

Before proceeding further, it seems natural to ask whether subjects with additional experience might have become more adept

[21] Here, and throughout the remainder of the paper, all differences which are reported as significant have p values of 0.001 or less. When a difference is not significant, its p value is given in the text.

[22] The (0,1) and (0,2) outcomes were observed 69 times in BOS-300-NF.

[23] There is a second influence here that could promote iterative deletion in the normal form. In contrast to the extensive form, even if the row player chooses the outside option (0 in the normal form), the play of the column player is revealed. This allows the row player to "learn" about the column player's choices even when the row player selects the outside option and might have strengthened the iterative-deletion effects. The fact that BOS-300-NF provides less support for iterative deletion than does BOS-300 shows that this effect was not important.

at forward-induction reasoning in the normal form. If so, the differences between the normal and extensive forms might simply reflect the fact that the extensive form facilitates learning about iterative deletion. To promote the learning process in the normal form, we altered our design to provide information to players about the aggregate history of play. The results of this treatment are reported in Tables 1 and 2 as BOS-300-NF-HIS.

The results lie between those reported for BOS-300 and BOS-300-NF: history influences play, though play is still not consistent with the forward-induction prediction. The strategy which is equivalent to the outside option (strategy 0) is played 27 percent of the time by the row player. Further, the column player plays strategy 2 more than 20 percent of the time in the last 11 periods even though the row player almost never plays strategy 1. As in BOS-300-NF, the second round of iterative deletion is not completed. This leads us to reject hypothesis HIS, though the additional information certainly moved play closer to the prediction of forward induction. Most importantly, the differences between the normal and extensive form results remain even in sessions designed to make the iterative-deletion process more transparent to players.

Overall, the results on forward induction are mixed. We find some support for forward induction in the simple BOS-300 game, but iterative deletion fails in normal form. For BOS-300, we find that the row player chooses 2 and the column player selects 1 quite frequently relative to the baseline game (BOS). To this degree, the presence of the outside option alters play and does so in the direction predicted by the theory. Still, the outside option is chosen 20 percent of the time.[24]

Additional Treatments.—One possible explanation for the difference in outcomes between the extensive and normal form games is that the outside option influences the path by providing an asymmetry in the extensive form game which creates a focal point for play in the battle-of-the-sexes subgame. In this way, there are important presentation effects at work. To evaluate the focal-point argument, we considered a treatment in which the row player received 100 from an outside option or could elect to play BOS (denote this as BOS-100). This is the Figure 2 game with a 100 outside option. According to forward induction, the presence of such a low outside option should have no influence on subsequent play. That is, the outside option should not be taken, and the subgame should be indistinguishable from the play of the subgame in the absence of the outside option. However, the focal-point argument would imply that the outcome of this game would differ from BOS. Our results are summarized in Table 3 along with the previously reported results for BOS, BOS-300, and BOS-300-NF.

The results from BOS-100 are quite different from BOS: the outside option of 100 is not irrelevant to play. While the outside option is taken infrequently (as predicted by the theory), the play of the subgame is dramatically different from BOS. The row player plays strategy 2 significantly more often, while the column player plays strategy 1 significantly more often in BOS-100 than in BOS. The result is a large increase (from 19 percent to 63 percent) in the frequency with which the row player's preferred equilibrium is observed. In fact, the play in BOS-100 is quite close to that observed in BOS-300. In both cases, the outside option focused play on the row player's preferred equilibrium, (2, 1). The main difference between BOS-100 and BOS-300 is in the breakdown of play between DIS and OO. This is supported by the evidence in Table 4, where further detail is given on the frequency with which the row and column players choose each strategy. The row player plays strategy 1 significantly more often in BOS-100, and similarly, the column player plays strategy 2 significantly more often in

[24]These results are consistent with those reported by Brandts and Holt (1993). They find that forward induction appears to work in a simple two-stage game (similar to BOS-300) but fails in a more complicated game. They too find that the outside option is taken too frequently.

TABLE 3—BATTLE-OF-THE-SEXES GAME: FREQUENCY OF OUTCOMES, LAST 11 PERIODS

	Outcome			
Game	(2,1)	(1,2)	DIS	OO
BOS	31 (19)	37 (22)	97 (59)	—
BOS-100	102 (63)	5 (3)	55 (34)	3
BOS-300	119 (90)	0 (0)	13 (10)	33
BOS-SEQ	103 (62)	6 (4)	56 (34)	—
BOS-300-NF	53 (32)	2 (1)	86 (52)	24 (15)

Note: Percentages are shown in parentheses; for BOS-100 and BOS-300, numbers in parentheses show percentage of play in the BOS subgame. For BOS-300-NF, the amounts entered in the OO column correspond to the play of 0 by the row player and 2 by the column player. The (0,1) outcome is aggregated in the DIS column.

TABLE 4—BATTLE-OF-THE-SEXES GAME: PERCENTAGE OF STRATEGIES PLAYED BY PLAYER, LAST 11 PERIODS

	Row player			Column player	
Game	Strategy 0	Strategy 1	Strategy 2	Strategy 1	Strategy 2
BOS	—	38 (130)	62 (210)	35 (124)	65 (188)
BOS-100	2 (300)	13 (54)	85 (438)	73 (176)	27 (78)
BOS-300	20 (300)	2 (16)	78 (552)	92 (196)	8 (12)
BOS-SEQ	—	12 (60)	88 (420)	70 (176)	30 (72)
BOS-300-NF	42 (300)	5 (72)	53 (384)	64 (232)	36 (156)

Note: Numbers in parentheses are the expected profits to the player from playing the associated strategy given that his opponent is playing according to the observed frequencies.

that game than in BOS-300. The results from this treatment lend support to the conjecture that the role of the outside option in BOS-300 was, at least in part, a consequence of the asymmetry created by giving the row player an outside option and thereby making his preferred equilibrium a focal point.

To investigate this logic further, we considered an additional treatment using BOS. In this new treatment, the row player selected a strategy prior to the column player, but this choice is not reported to the column player. Game theory predicts that this ordering of moves should be inconsequential since the column player is not informed of the row player's move at the time he chooses a strategy. However, from the perspective of creating focal points, the asymmetry in moves may be sufficient to support the row player's preferred outcome. Amershi et al. (1989a, b) develop a refinement of Nash equilibrium, which they term MAPNASH, that applies to games with a sequencing of unobserved moves as in our design. They argue that common knowledge about the sequence of moves endows the row player with the ability to take actions he believes that the column player will rationally anticipate and *respond* to optimally. For the battle-of-the-sexes games, this implies that the outcome should be the row player's preferred equilibrium.

Our findings for this treatment, labeled BOS-SEQ are also summarized in Tables 3 and 4. Overall, we find that the sequencing of moves matters. The outcome of BOS-SEQ is significantly different from BOS in that there are more than three times as many plays of the row player's preferred outcome, the (2,1) equilibrium. Further, BOS-SEQ and BOS-100 are indistinguishable ($p = 0.910$): the distribution of outcomes across the two treatments are not significantly different. The focal effects of providing the row player with a payoff-irrelevant outside option (as in BOS-100) are similar to providing the row player with a (theoretically irrelevant) first-mover advantage.

IV. Communication

The predictions of a model with one-way, pre-play communication ("cheap talk") coincide with those arising in this application of forward induction.[25] To see this, consider a game with cheap talk in which the row player has an opportunity to announce one of the feasible strategies. Suppose, following Farrell (1987), that such cheap-talk promises will be believed by the column player if it is in the interest of the row player to honor them should they be believed. Essentially, this allows the row player to pick his preferred outcome from among the set of Nash equilibria. If this interpretation of the meaning of announcements is correct, the row player should announce and play strategy 2 in the battle-of-the-sexes game; yielding the (2,1) outcome.

As emphasized by Ben-Porath and Dekel (1992), rejecting an outside option is a means of communicating an intended action, just as is one-way communication.[26] While there is an exogenous opportunity cost associated with rejecting an outside option, a similar opportunity cost arises endogenously in games with cheap talk. In equilibrium, talk is not cheap, since sending alternative messages leads to different outcomes.[27] Thus, by electing to play the game, the row player is effectively sending the same message as a player announcing 2 in the one-way communication game. This leads to the following hypothesis:

Hypothesis 1W: The outcome of the battle-of-the-sexes subgame with an outside option of 300 should be the same as the outcome of this game with one-way communication, when row announces 2.

To test this hypothesis, we compare the results from CDRF1 on the role of communication in battle-of-the-sexes games with the results reported for these games with an outside option (BOS-300), see Table 5. The design for these communication games was the same as used for the outside-option treatments with the following exceptions. Game BOS-1W is the battle-of-the-sexes

[25]Throughout this discussion we will be considering games in which the outside option falls into the relevant ranges of (200, 600) for the battle-of-the-sexes game.

[26]Kohlberg and Mertens (1986 p. 1013) also argue that forward induction is a very specific form of pre-play communication.

[27]For the battle-of-the-sexes game, the opportunity cost of saying 1 rather than 2 is 200 under the assumption that announcing 1 leads to the (1,2) equilibrium (see Cooper et al., 1989). Note that these opportunity costs differ from the value of the outside option. The theory does not require that these opportunity costs be equal: the outside option must lie between 200 and 600 for the battle-of-the-sexes game.

TABLE 5—ONE-WAY COMMUNICATION VERSUS FORWARD INDUCTION: FREQUENCY OF OUTCOMES, LAST 11 PERIODS

	Outcome			
Game	(2,1)	(1,2)	DIS	OO
BOS-1W	158 (96)	1 (1)	6 (4)	—
(Announce 2)	154 (98)	0 (0)	3 (2)	—
BOS-300	119 (90)	0 (0)	13 (10)	33

Notes: Percentages are shown in parentheses; for BOS-300, numbers in parentheses show percentage of play in the BOS subgame.

game with a one-way pre-play communication stage added to the session-II game. The row player is permitted, prior to the play of the game, to send a message (though he may choose to be silent) to the other player indicating his intended action. He is not bound to honor this promise.

One-way communication successfully coordinated players' strategies. Again, the outcome expected and observed was (2,1). Conditional on not taking the outside option, the play in BOS-300 and BOS-1W supports hypothesis 1W: the difference between the frequencies with which strategies were chosen is not statistically different ($p = 0.62$). Also, BOS-300 and BOS-1W generate significantly more equilibrium observations than BOS.

As in the discussion of BOS-300, a question arises regarding the role of pre-play communication. Does BOS-1W differ from BOS because the pre-play messages have some meaning in equilibrium (as suggested by Farrell's logic) or simply because of the focal point created by allowing only one player to send a message? One could conceive of a treatment with one-way pre-play communication in which messages become meaningless, either because of the message space or because messages are sufficiently garbled. This would allow one to evaluate the effects of communication with meaningless messages.

Our BOS-SEQ is close to that design: the row player has an asymmetric role, but the "message" is never received by the column player. With that in mind it is useful to compare BOS-1W with BOS-SEQ. Comparing the results from Tables 3 and 4, note that BOS-SEQ has fewer plays of (2,1) than does BOS-1W. This leads us to conclude that the messages had content in BOW-1W.[28]

V. Conclusions

The goal of this paper was to assess the predictive power of forward induction. Overall, our results cast serious doubt on the forward-induction hypothesis: observed play was inconsistent with hypothesis FI. Further, there appear to be important differences in play associated with extensive versus normal form representations, implying the presence of presentation effects. Finally, we found evidence that a focal-point effect was operative which partially explained the limited success of forward induction in BOS-300.

These results contrast with those reported in Cooper et al. (1992b) and John van Huyck et al. (1993) on the success of forward-induction types of arguments in games with multiple, Pareto-ranked equilibria. In Cooper et al. (1992b), we find that, in a two-player, two-strategy coordination game, a payoff-relevant outside option alters play in the direction suggested by for-

[28]Further evidence of this arises from the observations, reported in Cooper et al. (1989) that coordination was improved with two-way simultaneous communication.

ward induction and argue, in contrast to the BOS game considered here, that these effects do not reflect the creation of a focal point. Van Huyck et al. (1993) consider an experimental setting in which players participate in an auction for the right to play a coordination game and find that the auction stage eliminates coordination failures. In effect, the price in the auction is high enough that players in the coordination game can eliminate strategies and thus achieve the Pareto-dominant Nash equilibrium.

From a methodological perspective, we, as well as other experimentalists investigating forward induction, have used the concept as a selection criterion. Subjects are given a game to play, and the outcomes are evaluated in light of the selection criterion. However, intertwined with this is the process of arriving at a prescribed equilibrium. In contrast, Kohlberg and Mertens (1986 p. 1005 [n. 3]) outline an experimental procedure in which subjects "...are told a recommended mixed strategy vector, and that it is a stable equilibrium, expected to be adhered to by all participants." From this perspective, the issue is whether play will remain at the suggested equilibrium rather than whether play will evolve to that particular equilibrium.[29] Consideration of this alternative approach may yield additional insights into the stability of particular equilibria while providing less insight into the likelihood of observing predicted outcomes in the absence of pre-play negotiation. Whether the forward-induction prediction is supportable, in both the battle-of-the-sexes and coordination games, through this alternative design remains an interesting open question.

[29]Brandts and W. Bentley MacLeod (1991) use this design and find some support for forward induction in a twice-repeated battle-of-the-sexes game.

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