15 NEW WAYS TO CATCH A LION*

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This, O Best Beloved, is another tale of the High and the Far-Off Times. In the blistering midst of the Sand-Swept Sahara lived a Pride of Lions. There was a Real Lion, and a Projective Lion, and a pair of Parallel Lions; and all manner of Lion Segments. And on the edge of the Sand-Swept Sahara there lived a 'nexorable Lion Hunter...

I make no apologies for raising once again the problems of the mathematical theory of big game hunting. As with any branch of mathematics, much progress has been made in the last decade.

The subject started in 1938 with the epic paper of Pétard [1]. The main problem is usually formulated as follows: *In the Sahara desert there exist lions. Devise methods for capturing them.* Pétard found ten mathematical solutions, which we can paraphrase as follows.

1. The Hilbert Method. Place a locked cage in the desert. Set up the following axiomatic system.

(i) The set of lions is non-empty.
(ii) If there is a lion in the desert, then there is a lion in the cage.

Theorem 1. There is a lion in the cage.

2. The Method of Inversive Geometry. Place a locked, spherical cage in the desert, empty of lions, and enter it. Invert with respect to the cage. This maps the lion to the interior of the cage, and you outside it.

**Seven Years of Manifold/1968–1980*, Ian Stewart and John Jaworski, eds., Cheshire, England, Shiva Publishing Limited, 1981, pp. 36–39. Reprinted by permission.

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3. THE PROJECTIVE GEOMETRY METHOD. The desert is a plane. Project this to a line, then project the line to a point inside the cage. The lion goes to the same point.

4. THE BOLZANO-WEIERSTRASS METHOD. Bisect the desert by a line running N-S. The lion is in one half. Bisect this half by a line running E-W. The lion is in one half. Continue the process indefinitely, at each stage building a fence. The lion is enclosed by a fence of arbitrarily small length.

5. THE GENERAL TOPOLOGY METHOD. Observe that the desert is a separable metric space, so has a countable dense subset. Some sequence converges to the lion. Approach stealthily along it, bearing suitable equipment.

6. THE PEANO METHOD. There exists a space-filling curve passing through every point of the desert. It has been remarked [2] that such a curve may be traversed in as short a time as we please. Armed with a spear, traverse the curve faster than the lion can move his own length.

7. A TOPOLOGICAL METHOD. The lion has at least the connectivity of a torus. Transport the desert into 4-space. It can now be deformed in such a way as to knot the lion [3]. He is now helpless.

8. THE CAUCHY METHOD. Let $f(z)$ be an analytic lion-valued function, with ζ the cage. Consider the integral

$$\frac{1}{2\pi i}\int_C \frac{f(z)}{z-\zeta}\,dz$$

where C is the boundary of the desert. Its value is $f(\zeta)$, that is, a lion in a cage.

9. THE WIENER TAUBERIAN METHOD. Procure a tame lion L_0 of class $L(-\infty,\infty)$ whose Fourier transform [Furrier transform?] nowhere vanishes, and set it loose in the desert. Being tame, it will converge to the cage. By Wiener [4] every other lion will converge to the same cage.

10. THE ERATOSTHENIAN METHOD.* Enumerate all objects in the desert; examine them one by one; discard all those that are not lions. A refinement will capture only prime lions.

Pétard also gives one physical method with strong mathematical content:

11. THE SCHRÖDINGER METHOD. At any instant there is a non-zero probability that a lion is in the cage. Wait.

The next work of any significance is that of Morphy [5]. I confess that I do not find all of his methods convincing. The best are:

*This differs from the *Monthly* version since it refers to a slightly expanded version of the Pétard article that appeared in *Eureka*.

12. SURGERY. The lion is an orientable 3-manifold with boundary and so [6] may be rendered contractible by surgery. Contract him to Barnum and Bailey.

13. THE COBORDISM METHOD. For the same reasons the lion is a handlebody. A lion that can be handled is trivial to capture.

14. THE SHEAF-THEORETIC METHOD. The lion is a cross-section [8] of the sheaf of germs of lions in the desert. Re-topologize the desert to make it discrete: the stalks of the sheaf fall apart and release the germs, which kill the lion.

15. THE POSTNIKOV METHOD. The lion, being hairy, may be regarded as a fibre space. Construct a Postnikov decomposition [9]. A decomposed lion must, of course, be long dead.

16. THE UNIVERSAL COVERING. Cover the lion by his simply-connected covering space. Since this has no holes, he is trapped!

17. THE GAME-THEORY METHOD. The lion is big game, hence certainly a game. There exists an optimal strategy. Follow it.

18. THE FEIT-THOMPSON METHOD. If necessary add a lion to make the total odd. This renders the problem soluble [10].

Recent, hitherto unpublished, work has revealed a range of new methods:

19. THE FIELD-THEORY METHOD. Irrigate the desert and plant grass so that it becomes a field. A zero lion is trivial to capture, so we may assume the lion $L \neq 0$. The element 1 may be located just to the right of 0 in the prime subfield. Prize it apart into LL^{-1} and discard L^{-1}. (Remark: the Greeks used the convention that the product of two lions is a rectangle, not a lion; the product of 3 lions is a solid, and so on. It follows that every lion is transcendental. Modern mathematics permits algebraic lions.)

20. THE KITTYGORY METHOD. Form the category whose objects are the lions in the desert, with trivial morphisms. This is a small category (even if lions are big cats) and so can be embedded in a concrete category [11]. There is a forgetful functor from this to the category of sets: this sets the concrete and traps the embedded lions.

21. BACKWARD INDUCTION. We prove by backward induction the statement $L(n)$: "It is possible to capture n lions." This is true for sufficiently large n since the lions will be packed like sardines and have no room to escape. But trivially $L(n+1)$ implies $L(n)$ since, having captured $n+1$ lions, we can release one. Hence $L(1)$ is true.

22. ANOTHER TOPOLOGICAL METHOD. Give the desert the *leonine* topology, in which a subset is closed if it is the whole desert, or contains no lions. The set of lions is now dense. Put an *open* cage in the desert. By density it contains a lion. Shut it quickly!

23. The Moore-Smith Method. Like (5) above, but this applies to non-separable deserts: the lion is caught not by a sequence, but by a net.

24. For those who insist on sequences. The real lion is non-compact and so contains non-convergent subsequences. To overcome this let Ω be the first uncountable ordinal and insert a copy of the given lion between α and $\alpha + 1$ for all ordinals $\alpha < \Omega$. You now have a *long lion* in which all sequences converge [12]. Proceed as in (5).

25. The Group Ring Method. Let Γ be the free group on the set G of lions, and let $Z\Gamma$ be its group ring. The lions now belong to a ring, so are circus lions, hence tame.

26. The Bourbaki Method. The capture of a lion in a desert is a special case of a far more general problem. Formulate this problem and find necessary and sufficient conditions for its solution. The capture of a lion is now a trivial corollary of the general theory, which *on no account should be written down explicitly.*

27. The Hasse-Minkowski Method. Consider the lion-catching problem modulo p for all primes p. There being only finitely many possibilities, this can be solved. Hence the original problem can be solved [13].

28. The PL Method. The lion is a 3-manifold with non-empty boundary. Triangulate it to get a *PL* manifold. This can be collared [14], which is what we wish to achieve.

29. The Singularity Method. Consider a lion in the plane. If it is a regular lion its regular habits render it easy to catch (e.g. dig a pit). WLOG it is a singular lion. Stable singularities are dense, so WLOG the lion is stable. The singularity is not a self-intersection (since a self-intersecting lion is absurd) so it must be a cusp. Complexify and intersect with a sphere to get a trefoil knot. As in (7) the problem becomes trivial.

30. The Measure-Theoretic Method. Assume for a contradiction that no lion can be captured. Since capturable lions are imaginary, all lions are real. On any real lion there exists a non-trivial invariant measure μ, namely Haar or Lebesgue measure. Then $\mu \times \mu$ is a Baire measure on $L \times L$ by [15]. Since a product of lions cannot be a bear, the Baire measure on $L \times L$ is zero. Hence $\mu = 0$, a contradiction. Thus all lions may be captured.

31. The Method of Parallels. Select a point in the desert and introduce a tame lion not passing through that point. There are three cases:

(a) The geometry is Euclidean. There is then a unique parallel lion passing through the selected point. Grab it as it passes.

(b) The geometry is hyperbolic. The same method will now catch infinitely many lions.

(c) The geometry is elliptic. There are no parallel lions, so every lion meets every other lion. Follow a tame lion and catch all the lions it meets: in this way every lion in the desert will be captured.

32. THE THOM-ZEEMAN METHOD. A lion loose in the desert is an obvious catastrophe [16]. It has three dimensions of control (2 for position, 1 for time) and one dimension of behaviour (being parametrized by a lion). Hence by Thom's Classification Theorem it is a swallowtail. A lion that has swallowed its tail is in no state to avoid capture.

33. THE AUSTRALIAN METHOD. Lions are very varied creatures, so there is a variety of lions in the desert. This variety contains free lions [17] which satisfy no non-trivial identities. Select a lion and register it as "Fred Lion" at the local Register Office: it now has a non-trivial identity, hence cannot be free. If it is not free it must be captive. (If "Fred Lion" is thought to be a trivial identity, call it "Albert Einstein.")

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