Making Mathematics—The Coffee Connection

Agnes Arvai Wieschenberg

We college teachers are endlessly concerned about motivating our students and are constantly looking for better ways to stimulate students to achieve excellence. The focus of many research efforts today is the classroom presentation. Should it be a lecture, discussion, multimedia, or some combination of them? This essay takes a look at some unorthodox methods that could be used outside the classroom to help students become scholars.

Although the focus is on mathematics, most of the ideas can be transferred to other academic subjects. The stories are unconventional, but they are very successful examples from the twentieth century literature. I believe that they may help spark new ideas that can be applied in different college settings.

The Premise

In a talk about child prodigies, Paul Erdös(1913–1996), one of the century's most prolific mathematicians, once remarked that "in Hungary many mathematicians drink strong coffee." Erdös admitted that he had trouble doing mathematics without it, claiming that his mathematical notes became blank pieces of paper that he just stared at, unable to work. According to Erdös, Alfred Renyi (1921–1970), another well-recognized Hungarian mathematician, went even further when he stated "a mathematician is a machine which turns coffee into theorems."

It is well documented that Budapest, capital of Hungary, has for centuries had wonderful coffeehouses where one could spend many hours in the company of friends drinking coffee, smoking, and discussing current events from politics to local gossip. But who would talk mathematics in these establishments? The comments by Erdös and Renyi intrigued me and started me on an unusual research project to find evidence that this mathematics-coffee-and-coffeehouse connection is indeed a valid one and to ponder whether or not a creative atmosphere—with coffee as a main ingredient—has an effect on formulating mathematical thought.

In the winter of 1983, I had a personal interview with George Pϕlya in Palo Alto, California. We discussed mathematicians, and in particular, why so many brilliant mathematicians have emerged from Hungary since the turn of the century. Polya maintained that because Hungary was a poor country and mathematics is the cheapest science—all you need to do mathematics is a pencil, some paper, and a waste paper basket—a lot of talented young people turned to mathematics.

In fact, that was true of Poland as well because Hungary and Poland were part of the Austro-Hungarian Empire until 1918. The only requirement that may be needed is an appropriate mathematical atmosphere, which was definitely present in the turn-of-the century Austro-Hungarian Empire, which included Lwow in mathematician Banach's time, and Budapest during Professor Fejer's years.

Coffeehouses

The exotic drink coffee, followed by a coffeehouse culture, swept Europe with a vengeance. The first coffeehouse opened in the middle of the seventeenth century in Venice, with the name of "La Bottega del Caffe," under the arches of St. Mark's Square. Other great European cities followed suit, Oxford in 1650, London in 1652, Paris in 1660, Marseilles in 1671, and Vienna in 1683. Actually, the very first European coffeehouse opened its doors in Hungary, in Buda (Buda and Pest became Budapest in 1898), during the century and a half of Turkish occupation. But its patrons were restricted to men of Turkish origin. Hungarians—to protest the Turkish occupation—showed no interest in sharing the coffee drinking habit with the Turks at that time. The number of coffeehouses, however, kept growing all over Europe.

Because coffee came from Turkey, it was first referred to by many as "the drink of the devil." Shakespeare (1564–1616) referred to it as the "hot and rebellious liquor" in *As You Like It.* In the beginning it was vehemently rejected by the European society, especially the Hungarians.

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In 1721, Montesquieu (1689–1755) the French jurist and political philosopher, protested against the consumption of coffee. He claimed that those who frequented coffeehouses become revolutionaries because coffee induces a certain kind of drunkenness, which is dangerous for the future of the country.

At the time of his protest, there were already about three hundred coffeehouses in Paris. Legend has it that Honoré de Balzac (1799-1850) carried with him a special blend of coffee and coffee-maker wherever he traveled. According to his own account, he consumed 50,000 cups of coffee while working on his play *La Comédie Humaine.* Rousseau and Voltaire were also avid coffee drinkers. Stendhal even said that he would rather forgo love than give up coffeehouses.

Who went to coffeehouses? Painters, writers, philosophers, musicians, politicians, scientists, students, and men from every social status. Women were not part of the scene until the first decades of the twentieth century. For those who frequented the cafes it was a home away from home, a warm, bright, comfortable place with instant company, conversation, coffee, liqueur, smoke, newspapers, and games. (Sounds like a men's club?) Although in the history of coffeehouses one does not find many references to mathematicians frequenting these establishments, there are a few interesting ones.

Coffee, Warmth, and Genius

The best example is that of Stefan Banach (1892–1945), the founder of functional analysis and one of the great mathematicians of the twentieth century. His half-sister noted in an interview that her brother "drank coffee by the gallon." But even more important, Banach was known to have spent most of his waking hours in cafes surrounded most of the time by fellow mathematicians and some students from nearby universities.

Banach was discovered by Hugo Steinhaus, who was a professor of mathematics at Lwow University from 1920 to 1941. These two mathematicians were to be known later as the founders of what we now call the Lwow School of Mathematics. And yet Banach had no college degree. He was a self-taught mathematician, and he needed a legal exemption from the minister of education to be allowed to take a master's examination, which was necessary to obtain his Ph.D.

Curiously, Banach never wrote his dissertation. He had no interest in writing down any of his theorems and proofs in publishable form; he was bored with details. Although he did plenty of mathematics to earn his degree—he had introduced by this time what later became known as Banach Spaces and the Banach Fixed Point Theorem—he never bothered to write his theories in a presentable form. Finally, he had to be literally tricked into completing his thesis, as his biographer tells us:

> Professor Ruziewicz instructed one of his assistants to accompany Banach on his frequent visits to the coffee houses, query him in a discreet fashion on his work, and afterwards write down Banach's theorems and proofs. When all of this information was typed out, the notes were presented to Banach, who edited the text. This is how his Ph.D. dissertation was finally completed (recorded by Turowitz). The title of Banach's dissertation translated from Polish is "On Operations on Abstract Sets and Their Application to Integral Equations. (Kaluza 1996, 32–3)

To receive the doctoral degree, Banach still had to prepare for and pass the doctoral examination. He dutifully read and mastered the material including a required extra topic in astronomy, but he had to be tactfully deceived again to take the exam. Nevertheless, at the age of twenty-eight, Banach was awarded the Ph.D. of Mathematical Sciences under the advisement of Antoni Lomnicki. He received his habilitation in 1922 and thus became a *professor extraordinarius* at the Jan Kazimierz University in Lwow, Poland.

Cafes that were frequented by mathematicians were of the Viennese type offering large comfortable spaces with newspapers hanging on bamboo racks. Chess sets, dominoes, and cards were available for the enjoyment of the patrons. Comfortable seating with marble tables created a warm and congenial atmosphere. Some patrons or groups visited the same cafe daily, and certain tables were reserved for them. Most cafes served coffee, tea, and pastries; some would offer food and other drinks, too. At the turn of the century, most people in eastern Europe had no central heating, and every room had its own wood burning stove that needed to be nurtured all day long to keep warm. In a cafe the chore of keeping the place warm was taken care of by someone else, so that one could sit in comfort with friends, lovers, or colleagues all morning or afternoon for the price of a cup of coffee.

Banach's Lwow had many such cafes. Cafe Roma used to be the unofficial gathering place of the local chapter of the Polish Mathematical Society. After their official weekly Saturday meetings, they would retire to the Cafe Roma on Akademicka Street close to the university. Banach later moved the meeting across Akademicka Street to the by now legendary Scottish Cafe. For the gathered mathematicians, the little marble top coffee tables served as a sketch pad, which had to be washed by the proprietor after the patrons left. After awhile the owners became annoyed by the scribbles, and finally in 1935, Banach's wife purchased a notebook, which Banach then presented to the group.

The notebook over the years became a diary of the mathematical problems and theorems as they were developed over coffee and heated discussions. It was kept in the cloakroom of the Scottish Cafe and was available upon request to the mathematicians frequenting the cafe. The notebook was later published under the name of *The Scottish Book: A Collection of Problems;* it became well known in mathematical circles as *The Scottish Book.*

Just who frequented the Scottish cafe? Almost all the local and visiting mathematicians, most of whom made a lasting mark in mathematics. They came first at random, "but pretty soon a certain rhythm and custom were established, as daily sessions at the Scottish grew into a permanent ritual among many mathematicians" (Kaluza 1996, 61). We can recognize many of the names of the initial participants, which included Banach, Steinhouse, Ruziewicz, Kaczmarz, Zylinski, and Mazur, to whom Banach was closest.

Later the circle expanded to include Ulam, Auerbach, Nikliborc, Eilenberg, Orlicz, Eidelheit, Kac, Birnbaum, Schreier, Schauder, Kuratowski, and Nikodym. As Kaluza writes,

> Usually they began arriving between 5 and 7 pm—always occupying the same

> tables—and for the next several hours they worked with total concentration, covering the marble tabletops with mathematical formulas. But saying that "they worked with total concentration" is not completely accurate, as there were no meetings without jokes, heated discourse, shouting, and drinking. Banach, for example, drank enormous amounts of coffee and cognac and smoked dozens of cigarettes. (61–2)

In the smoke-filled parlors, some played chess while others drank coffee and watched. Ulam (1975) recalls:

> Needless to say such mathematical discussions were interspersed with a great deal of talk about science in general (especially physics and astronomy), university gossip, politics, the state of affairs in Poland; or to use one of John von Neumann's favorite examples, "the rest of the universe." (35)

The variety of characters in the energetic group generated a mood of high spirits, and the hard work was accomplished in the presence of the wry humor that is usually associated with people of great minds.

In his autobiography, Stanislaw Ulam fondly remembers his youth, spending countless hours in cafes with friends in his home-town of Lwow. Ulam was born in 1909 and had the good fortune to attend the Lwow Polytechnic Institute, which at the time had the most outstanding mathematics faculty, including Professor Kuratowski (Kuratowski studied in Warsaw under Mazurkiewitcz, Janiszewski, and Spierpinski). Ulam was particularly impressed by the lectures of Kuratowski, and he participated in the discussions Kuratowski had with some of his senior students. Ulam attributes his mathematical development to those stimulating discussions.

Later, Ulam became a key member of the well-known Polish School of Mathematics. Mathematician Marc Kac remarked on the unusual circumstance that Lwow offered: a "culture based on leisure and discourse"—a "peculiar kind of Polish existence where you were in cafes all hours of the day or night drawing diagrams on small pieces of paper." Ulam (1975) recalled how some important mathematical theorems were developed as a result of these meetings.

Paul Erdös visited colleges around the world. He spent most of his money on prizes for students who found the best solution to a problem that he proposed. Many students benefited.

> Sometimes we would sit for hours in a coffee house. He (Mazur) would write just a symbol or a line like y = f(x) on a piece of paper, or the marble table top. We would both stare at it as various thoughts were suggested and discussed. These symbols in front of us were like a crystal ball to help us focus our concentration. (31)

Ulam recalled a meeting with Banach and Mazur in the Scottish Cafe that went on for seventeen hours interrupted only by necessities such as meals. Ulam was extremely impressed by the way Banach could discuss mathematics, reason about it, and find proofs—all in the same discussion. Ulam claims that "most of my mathematical work was really started in conversations with Mazur and Banach" (33). Ulam had the good fortune of finding himself in the midst of this cafe society, which he enjoyed immensely and missed greatly later in the United States.

Years later during the Second World War, working on the Manhattan Project in Los Alamos, Ulam once again found the atmosphere that resembled that of Lwow. Working with some of the greatest scientific minds of the twentieth century and living in a small secured area reserved for the scientists and their families reminded him of his student years and the heated discussions in the cafes of Lwow. Ulam's most valuable contributions at Los Alamos came after the war in the development of the hydrogen bomb.

In Budapest, the cafes flourished. At the turn of the century there were six hundred coffeehouses in Budapest; some of them were open twenty-four hours a day and some for 365 days a year. Many of them were the centers of intellectual life. Students spent countless hours in cafes, sometimes with professors who stimulated the conversation. Professor Lipot Fejer (1880–1959) spent many afternoons at the neighborhood cafes with some of his students. Fejer has a remarkable legacy of brilliant students, among them George Pølya, Eugene Wigner, Gabor Szego, Marcel Riesz, and later Paul Erdös and Pal Turan. John von Neumann prepared his doctoral dissertation under Fejer. While still in the Lutheran Gymnasium (secondary school), Neumann frequented the gathering of the top mathematicians of the time. Their meetings were held at a coffeehouse in Budapest (Radnai and Kunfalvi 1988, 48).

Eugene Wigner, a future Nobel laureate, after arriving in the United States, like Ulam, missed the congenial meetings and discussions at the cafes. As a student in Berlin in 1920 Wigner had attended Max Wolmer's colloquium; he remembered that afterwards "many of us went out to a coffeehouse and sat around a large table, talking further. Conversation was not always rigorous, but it touched on all of the things we loved, not only physics but nature, family and culture" (Szanton 1992, 75).

Fejer, who worked with distinction on Fourier series, was a well-known mathematician with a magnanimous nature. Pϕlya recalled in our 1983 interview, "Almost everybody of my age group was attracted to mathematics by Fejer"; he would sit in a cafe in Budapest with his students and solve interesting problems in mathematics or tell stories about mathematicians he had known. Although his lectures were considered the experience of a lifetime, his influence outside the classroom was even more significant because it was around him that the first mathematical school developed in Hungary (Wieschenberg 1984, 8).

Princeton: Tea Will Do

At Princeton, the first wave of research mathematicians began to arrive in 1930, thanks to the generosity of the Rockefeller Foundation. Among the first to arrive from Hungary were John von Neumann, a brilliant mathematician, and Eugene Wigner, the physicist who later, in 1963, won the Nobel Prize in physics. Wigner often expressed longings for the congenial discussions in the cafes of Europe as well as the informal meetings and chats with the professors about the latest research. He complained, "The town had no coffeehouses in the European sense, where scholars and their students went for a lively, extended conversation" (Szanton 1992, 120).

When John von Neumann joined the Princeton crowd, his house became a gathering place for the mathematicians and physicists. His parties were known to be fun, as well as intellectual events.

The Bambergers' philanthropy allowed Princeton to create an independent Institute for Advanced Study at Princeton. The institute would provide the researchers an environment without any pressure or official duties. There were no students to teach, no responsibilities other than to think. But some scholars did not believe that this sterile atmosphere was the ideal condition for research. Richard Feynman (Macrae 1992), a Nobel laureate who had a great sense of humor, made the following pithy remark upon turning down a professorship at Princeton's Institute for Advanced Study:

> When I was in Princeton in the 1940s I could see what happened to those great minds at the Institute for Advanced Study, who had been specially selected for their tremendous brains and were now given the opportunity to sit in this lovely house by the woods there, with no classes to teach, with no obligations whatsoever. These poor bastards could now sit and think clearly all by themselves. Okay? So they don't get an idea for a while. They have every opportunity to do something and they are not getting any ideas. I believe that in a situation like this a kind of guilt or depression worms inside you, and you begin to worry about not getting any ideas. And nothing happens. Still no ideas come. (176)

In the United States, Princeton became the center of mathematics. When professor Lefschetz arrived in the 1920s, his unusual philosophy changed the focus of teaching and learning. Grades and class attendance lost their importance; his most urgent requirement of his students was that they come to tea. The students were required to attend tea every afternoon, where they would meet the best mathematicians in the world. "Tea was the high point of the day." On Wednesdays it was held at Fine Hallin, the professors' room; on other days in the east common room, otherwise known as the students' room. "It was very much a family gathering, small and intimate" (Nasar 1998, 63).

Although friendly, the atmosphere was highly competitive. They were discussing math, and math problems; they shared their reading of current math papers and gossiped, as well.

> The students who gathered at teatime were as remarkable, in a way, as the faculty. Poor jews, new immigrants, wealthy foreigners, sons of working classes, veterans in their twenties, and teenagers, the students were a diverse as well as a brilliant group. . . ." (Nasar 1998, 64)

Unfortunately not surprisingly, there were no women among the students. In 1948, John Nash, Nobel laureate, who was among the fortunate students at Princeton, remarked, "I was awkward socially, shy and isolated. Everything was wonderful. This was a whole new world. Here was a whole community in which I felt very much at home" (64).

Coffee's Lasting Effects

A stimulating atmosphere can do wonders for mathematics, as well as for other academic disciplines. Coffee *may* have an extra ingredient that can help stimulate the mind. It seems that interaction between students and professors and visiting mathematicians not only develops the interest in the subject but also stimulates new thinking. If one is totally immersed in mathematical topics, it can become a way of life. In fact, Paul Erdös visited many colleges all over the world, saw many students, and recognized the talented. He spent most of the money that he earned on prizes for students who came up with the best solution to a mathematics problem that he proposed. Many students benefited. When he visited Brooklyn College, I heard him lecture on one occasion and saw the way the students looked up to him.

Professors Pϕlya and Szego at Stanford organized competitions for California students, modeled on the Hungarian Eotvos Competition, to discover the very talented ones. Unfortunately, the competition had to be discontinued because of lack of money. But students of Pϕlya will never forget him because of his deep commitment to them and to mathematics.

One of them, John Kemeny, was a favorite among Dartmouth students. They admired him as a professor, as well as the college president. Even during his presidential years, he kept his door open so that students could reach him all the time. He also insisted on teaching an undergraduate mathematics course to keep in touch with the students.

A dedicated professor who is also willing to be a mentor to students outside the classroom provides an invaluable experience. The most important roles of a professor in a college setting are the ones he or she does not get paid for. Each of us can probably remember that one professor who profoundly changed our lives and choice of careers. Those of us who are lucky had a good mentor—some, unfortunately, had none.

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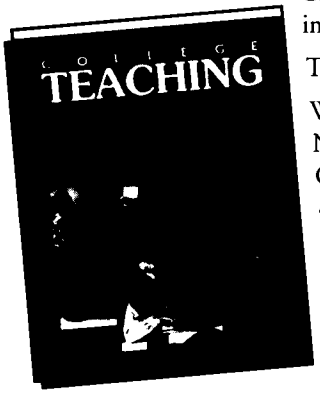

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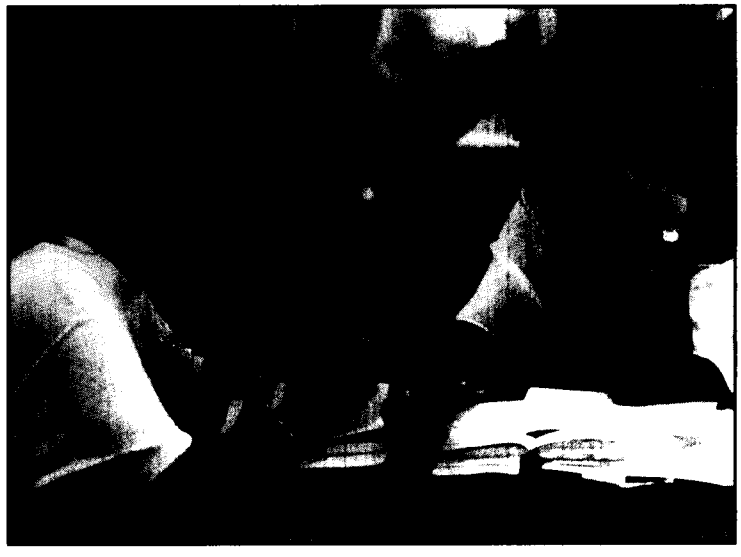

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