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Enhanced Problem Translation and Short-Term Memory: Components of Mathematical Talent

Veronica J. Dark and Camilla Persson Benbow
Iowa State University

The performance of mathematically talented 12- and 13-year-olds on various cognitive tasks was compared with that of average-ability youth, verbally talented youth, and college students. In Experiment 1, the hypothesis that mathematical talent includes enhanced problem-translation skills was supported: The mathematically talented students were better than other groups at writing equations expressing complex relationships. Although the mathematically talented group outperformed their average-ability peers, they were no better than the verbally talented group or the college students in rewriting and recalling the propositions in an algebra story problem. In Experiment 2, the hypothesis that mathematical talent includes enhanced ability to represent and manipulate information in short-term memory was strongly supported: the mathematically talented youth outperformed the other youth and, in most cases, performed as well as or better than the college students.

The Study of Mathematically Precocious Youth (SMPY) implemented talent searches for early identification of mathematically talented children. In these searches, seventh-grade students who have scored in the top 3% on standard, grade-appropriate achievement tests take the verbal and mathematics portions of the Scholastic Aptitude Test (SAT-V and SAT-M, respectively). Although designed as a test of the *developed* mathematical reasoning skills of above-average high school students (Donlon & Angoff, 1971), the SAT-M is an especially good measure of mathematical reasoning ability of gifted seventh graders (Benbow & Stanley, 1981, 1983; Stanley & Benbow, 1986). About 15% of talent search participants score 500 or better on the SAT-M (the level of the average college-bound male; Stanley & Benbow, 1986). We operationally define as mathematically talented those seventh graders with SAT-M scores of 500 or higher.

Little is known about the ability called mathematical talent (Benbow, 1988; Rabinowitz & Glaser, 1985). Our research is, therefore, exploratory. We believe, however, that identification of how mathematically talented students are similar to and different from others provides a basis for understanding the concept. In our research, therefore, mathematically talented youth were compared with average-ability peers, verbally talented peers, and undergraduate college students.

We chose to compare mathematically talented youth with others on tasks derived from two very different frameworks: the "cognitive components" and the "cognitive correlates" approaches. The cognitive components approach is basically a top-down analysis of individual problem statements; researchers examine the relation between test performance and domain-specific knowledge and strategies. In Experiment 1, we took this approach. The cognitive correlates approach, in contrast, is basically bottom-up; researchers examine the relation between performance on complex tasks and basic information-processing skills assumed to underlie the complex performance. Experiment 2 exemplified this approach.

Experiment 1

We investigated the comprehension of linguistically presented mathematical information. By definition, mathematically talented youth are better able than other youth to *solve* mathematics problems. But what is the basis of the ability? Approaching the question from a cognitive components view, we considered the four steps in algebra problem solving delineated by Mayer (1985).

Problem translation, the first step in Mayer's (1985) model, is translating each proposition in the problem into an internal representation. The next step, problem integration, is to put the propositions together into a coherent whole. Problem integration is enhanced by specific knowledge of problem types (problem schemata) that students acquire as they learn to solve algebra problems. Solution planning, the third step in problem solving, is also enhanced by experience; it depends on more general strategic knowledge about what types of operations are likely to work in what situations (see Riley, Greeno, & Heller, 1983). Problem execution, the final step, is carrying out the computations.

The mathematically talented seventh graders had not had formal training in algebra or geometry and therefore had not had the opportunity to develop the schemata or strategic knowledge necessary for Mayer's (1985) problem-integration and solution-planning steps. Neither did the mathematically talented youth always have especially good computational skills (Benbow, 1988). Yet they were successful on the problems of the SAT-M. We reasoned, therefore, that they may be especially good at problem translation, the first step in problem solving. In the first experiment, we tested this hy-

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Correspondence concerning this article should be addressed to either Veronica J. Dark or Camilla Persson Benbow, Department of Psychology, Iowa State University, Ames, Iowa 50011-3180.

pothesis with two comprehension tasks: equation production and rewriting/recall.

A common difficulty in problem translation is moving from a linguistic presentation of a problem to the numeric. For example, students often have difficulty with mathematics problems presented in story form despite being able to solve the same problems presented numerically (e.g., Mayer, 1982a). Furthermore, even undergraduates with advanced training have difficulty with relation statements, such as "There are six times as many students as professors at this university" (e.g., Clement, Lochhead, & Monk, 1980; Soloway, Lochhead, & Clement, 1982). Clement et al. (1980) found that the inability to construct equations from verbal input is a major source of error among engineering students in calculus. Thus a major factor in mathematical talent may be enhanced problem-translation skills, which may be manifested as the ability to go from linguistic to mathematical representations. In our first task, we tested this assumption by examining the extent to which subjects were able to generate the appropriate equations from single sentences.

Some classes of problems are especially difficult (e.g., Greeno, 1980; Lewis & Mayer, 1987; Loftus & Suppes, 1972; Mayer, 1981, 1982b; Riley et al., 1983). Mayer (1982b) suggested that part of the difficulty stems from the types of propositions constituting the problem, especially assignment versus relation propositions. Assignment propositions assign a numeric value to a variable (e.g., "The car traveled 120 miles"); relation propositions express a numeric relation between two variables (e.g., "Car A was twice as fast as Car B"). Studies of college students have shown that assignment propositions are easier to recall than relation propositions, which suggests that assignment propositions are in some sense psychologically more basic (Mayer, 1982b).

If one assumes, then, that recall depends on understanding the material and that understanding of mathematical information is a component of mathematical talent, then mathematically talented youth should show better recall than others of both types of propositions. In addition, relation propositions may be as psychologically basic as assignment propositions for such youth. Our second task, designed to investigate these hypotheses, was modeled after Mayer's (1982b) Experiments 1 and 2. Subjects' understanding of assignment and relation information in complex story problems was measured through rewriting and recalling the problems.

We emphasize that subjects were not asked to perform computations or to actually solve problems in either the equation or rewrite/recall task. The tasks were designed to reflect the subjects' *understanding* of the stimuli presented. We assumed that the tasks tapped into the problem-translation stage of Mayer's (1985) analysis.

Method

Subjects

Gifted youth. Talent search students participating in a summer program at Iowa State University served as the gifted subjects. They were expected to participate in the project as part of their educational experience and in return for the financial subsidy given to the program. Most students had just completed the seventh grade.

The *mathematically talented* group was composed of 20 students (2 girls and 18 boys) with the highest SAT-M scores. They had mean scores as seventh graders (some scores had been grade-adjusted) of 651 on SAT-M and 452 on SAT-V. Their mean age was 12.8 years. The *verbally talented* group was composed of the 20 students scoring highest on SAT-V (11 girls and 9 boys). The group had mean scores of 499 on SAT-M and 553 on SAT-V. Their mean age was 13.5 years.

Average-ability peers. Twenty age-level peers of the gifted students were recruited through a local newspaper ad for 12- to 13-year-olds to participate in memory and attention research. The average-ability youth (7 girls and 13 boys) were paid $10. Their mean age was 12.8 years.

College students. Twenty students (11 women and 9 men) were recruited from an introductory psychology class. They received extra credit toward their final course grade in exchange for participation. Ninety percent either had completed or were currently enrolled in college-level mathematics classes.

Design

Our research was intended to explore the ways in which the mathematically talented group differed from the others. Therefore, for each hypothesis tested, a set of three nonorthogonal, planned comparisons was performed (see Hays, 1973, chap. 14). In the first comparison, we determined whether the mathematically talented youth performed better than their average-ability age-level peers. We planned no further comparisons when we found that the more talented youth were not superior. In the second comparison, we determined whether the mathematically talented youth performed better than the verbally talented youth. We assumed that this comparison would indicate whether the difference in the first comparison was due to a general ability or to a specific mathematical ability. In the final comparison, we determined whether there was a difference between the mathematically talented youth and the college students; this comparison was nondirectional. We assumed that it would indicate the extent to which the performance reflected skills that could be acquired through maturation and/or a general educational experience.

Stimuli and Procedures

Two sets of written verbal stimuli were presented to subjects. The gifted subjects were available only one afternoon and were therefore tested in one large group. All other subjects were tested in smaller groups of 4–12.

Equation stimuli. The first task for all subjects consisted of writing equations to represent the numeric relation expressed in sentences. Each sentence was followed by a statement defining the two variables necessary to write the equation. For example:

> Randy has three times as many transformers as gobots.
> Let T represent transformers and G represent gobots.

Instructions were printed on the front of the subjects' booklets and were read aloud. The instructions contained two completed examples, one of which the subject attempted to write before reading the solution.

Nine problems, each written on a separate sheet, were a mixture of statements from previous studies (Clement et al., 1980; Soloway et al., 1982) and statements that we generated (see Table 1). Subjects were given 20 s to write each equation. The equations were presented in the same order to all subjects.

Table 1
Equation Stimuli

1. Danny has seventeen less baseball cards than Garbage Pail Kids cards.
2. Eighty percent of the kids at the amusement park preferred the Wild Goose ride to the Ferris Wheel.
3. There are six times as many students as professors at this university.
4. In the dormitory cafeteria, for every four people who take cake, five people take ice cream.
5. When Batman and his crew joined the parade, the number of vehicles increased by one hundred and thirty percent.
6. Randy has three times as many transformers as gobots.
7. At the last dormitory party, for every six people who preferred Coke, there were 13 people who preferred Pepsi.
8. Kathy has half as much money today as she had yesterday before her trip to the races.
9. At the horse show the ratio of pintos to bays was seven to eleven.

Algebra story problems. The second set of stimuli (see Table 2) consisted of seven complete algebra story problems selected from Mayer's (1982b) study. The problems were fairly complex, and each contained at least one assignment and one relation statement.

Subjects were told that the study was designed to measure comprehension of story problems and that they would not be asked to solve the problems. They were instructed to read each problem and then to rewrite the information in simple propositions expressing just one idea each. The written instructions, which contained two examples, were read aloud. Subjects were asked to rewrite the second example before an explanation was provided. They were given 2 min to complete rewriting of each story problem. Seven orderings of the problems were used.

After the rewriting booklets were collected, a second set of booklets was passed out. Each page of this booklet was blank except for a title identifying one of the seven problems from the previous booklet. Subjects were instructed to recall the information presented in the original story problems. They were given 2 min to recall as much of each problem as possible. Seven orderings of the problems were used.

Results and Discussion

For all analyses reported in both experiments, the alpha level for tests of significance was .05. In each case, planned comparisons were evaluated with a pooled error term derived from the overall analysis of variance (ANOVA). We computed a conservative test of each comparison by restricting the degrees of freedom to those associated with the group factor (Kirk, 1968, pp. 265–268).

Equations

The nine equations were split into two sets for analysis: (a) the simple equations (1, 3, 6, and 8), which expressed one variable in terms of the second variable and a single whole number, and (b) the complex equations (2, 4, 5, 7, and 9), which required the use of fractions or percentages. We calculated the proportions correct for each type of equation (see Table 3). An ANOVA revealed a significant effect of group, $F(3, 76) = 18.03$, $MS_e = 0.071$; a significant effect of complexity, $F(1, 76) = 148.11$, $MS_e = 0.065$; and an interaction between the two factors, $F(3, 76) = 2.79$, $MS_e = 0.065$.

The planned comparisons revealed that (a) the mathematically talented group performed better than average-ability youth on both the simple equations, $t(76) = 5.75$, and the complex equations, $t(76) = 4.24$; (b) the mathematically talented youth performed better than the verbally talented youth on only the complex equations, $t(76) = 2.29$; and (c) the mathematically talented youth performed better than the college students on both the simple equations, $t(76) = 2.42$, and the complex equations, $t(76) = 2.90$.

Although the mathematically talented youth showed a better understanding than the average-ability youth of even simple linguistically expressed relations, they were not better than the verbally talented youth. Somewhat unexpected was that the college group performed at a lower level. The pattern with the simple equations therefore suggests that general ability is the more important factor. With more complex relations, each group exhibited diminished performance, but the mathematically talented group performed at the highest level. With complex stimuli, then, performance depends more on a specific talent in mathematics. Thus the equation data provide support for the hypothesis that mathematically talented youth show enhanced problem-translation skills.

Table 2
Algebra Story Problems

The River Problem

A river steamer travels 36 miles downstream in the same time that it travels 24 miles upstream. The steamer's engine drives in still water at a rate of 12 miles per hour more than the rate of the current. Find the rate of the current.

The Freeway Problem

A truck leaves LA en route to SF at 1 pm. A second truck leaves SF at 2 pm en route to LA going along the same route. Assume the two cities are 465 miles apart and that the trucks meet at 6 pm. If the second truck travels at 15 mph faster than the first truck, how fast does each truck go?

The Frame Problem

The area occupied by an unframed rectangular picture is 64 square inches less than the area occupied by the picture mounted in a frame 2 inches wide. What are the dimensions of the picture if it is 4 inches longer than it is wide?

The Work Problem

Mr. Russo takes 3 minutes less than Mr. Lloyd to pack a case when each works alone. One day, after Mr. Russo spent 6 minutes in packing a case, the boss called him away and Mr. Lloyd finished packing in 4 more minutes. How many minutes would it take Mr. Russo alone to pack a case?

The TV Problem

The entertainment portion of a 30 minute TV program lasted 4 minutes longer than 4 times the portion devoted to advertising. How many minutes were devoted to advertising?

The Race Problem

In a sports car race, a Panther starts the course at 9 am and averages 75 mph. A Mallotti starts 4 min later and averages 85 mph. How many miles will the first car have driven when it is passed?

The Fence Problem

Mr. Zecha has just fenced his rectangular lot using 350 feet of chain fencing. If the length is 2½ times the width, find the area of the lot.

Table 3
Proportions of Correct Equation Production for Each Group on Simple and Complex Equations

Type of equation	Group: Mathematically talented	Verbally talented	Average ability	College students
Simple	.81	.83	.34	.61
Complex	.35	.16	.00	.11

Algebra Story Problems

Scoring of the rewrite and recall data was modeled on Mayer's (1982b) procedures. For each of the seven problems, we developed a template listing each simple proposition and the category of information that it specified. Each assignment and relation proposition was then scored according to the subjects' rewrite and recall protocols. A proposition was scored as correct if it satisfied a lax criterion that allowed the numeric information to be incorrect as long as the gist of the proposition remained the same (e.g., subjects were not required to remember that the river steamer traveled *36* miles downstream). Although the lax criterion was more important in scoring the recall protocols, it was used also with the rewrite protocols for consistency.

Rewriting. We calculated the proportion correct for the assignment and relation propositions in the rewrite protocols (see Table 4) for each group. An ANOVA revealed significant effects for group, $F(3, 76) = 7.16$, $MS_e = 0.039$, and for type of proposition, $F(1, 76) = 92.26$, $MS_e = 0.010$, but no interaction. The data indicate the importance of the problem-translation stage in arriving at a correct solution. Even with the information readily available to them, subjects had difficulty in identifying the simple propositions in algebra story problems. Especially difficult, for each group, were propositions expressing relation information.

The planned comparisons indicated that the mathematically talented groups performed at a higher level than their average-ability peers for both the assignment propositions, $t(76) = 2.23$, and the relation propositions, $t(76) = 4.14$, but that they did not differ from the other groups. The comparisons suggest that whatever the skills that allow the mathematically talented group to perform better than their average-ability peers, they are not specific and can be obtained through formal education.

Recall. We calculated the proportion correct in the recall protocols (see Table 4). Analysis of the recall data revealed significant effects of group, $F(3, 76) = 12.41$, $MS_e = 0.092$, and type of proposition, $F(1, 76) = 62.89$, $MS_e = 0.010$, but no significant interaction. The planned comparisons showed a pattern identical to that obtained with the rewrite data: The mathematically talented group recalled more than the average-ability group for both the assignment propositions, $t(76) = 5.01$, and the relation propositions, $t(76) = 4.87$, but their recall was no better than that of the other groups.

Better memory among gifted children is a well-documented phenomenon (e.g., Borkowski & Peck, 1986; Keating & Bobbitt, 1978; McCauley, Kellas, Dugas, & DeVillis, 1976). Better memory for schema-relevant information is also well documented (e.g., Bower, Black, & Turner, 1979; Thorndyke & Hayes-Roth, 1979). To the extent that the college students were able to use general schematic knowledge of algebra story problems to enhance their recall, the performance of the gifted children is especially impressive.

The data provide no support for the hypothesis that mathematically talented youth possess a special ability to understand linguistically presented mathematical information when understanding is tapped by rewriting and recall. The mathematically talented group was not distinguishable from the verbally talented and the college groups on either level of performance or the ease with which relation propositions are handled.

Forgetting. Mayer's (1982b) conclusion that assignment information is psychologically more basic than relation information did not distinguish between processes operating during encoding and recall. His subjects showed poorer recall of relation information, but because there was no measure of original encoding, the poorer recall could have resulted from more forgetting, poorer encoding, or both. We were able to obtain a measure of forgetting by subtracting the proportion correct at recall from the proportion correct in the rewrite (see Table 4).

An ANOVA revealed a significant effect of group, $F(3, 76) = 2.79$, $MS_e = 0.114$, but no effect of type of proposition and no interaction. The forgetting data, in conjunction with the rewrite data, clarify the meaning of "psychologically basic." Assignment information is psychologically more basic because of encoding processes rather than memory processes; likewise, relation information is not recalled as well because it is not encoded as well, not because it is more easily forgotten.

As a whole, the results of Experiment 1 provide only partial support for the hypothesis that enhanced problem-translation skills are a component of mathematical talent. When the task is simply to pick out and remember the underlying propositions in an algebra story, mathematically talented youth are no better than their verbally talented peers or college students. When the task requires deriving a more complex equation from a linguistic statement, however, mathematically talented youth are superior to the other groups. Thus mathematically talented students may not have an enhanced understanding

Table 4
Proportions of Correct Rewriting, Recall, and Forgetting for Each Group on Assignment and Relation Propositions

Type of proposition	Group: Mathematically talented	Verbally talented	Average ability	College students
Rewriting				
Assignment	.85	.89	.74	.90
Relation	.75	.75	.55	.73
Recall				
Assignment	.66	.69	.30	.62
Relation	.57	.53	.21	.48
Forgetting[a]				
Assignment	.19	.19	.44	.29
Relation	.20	.22	.39	.25

[a] Forgetting is the rewriting minus recall difference score.

of the mathematical information as much as they have an enhanced ability to represent the information in a *useful* form. When problems must be solved, as on the SAT-M, the ability to translate the information into a useful form gives the mathematically talented students an edge.

Experiment 2

As described earlier, our second experiment fell under the cognitive correlates approach. The assumption was that differences on complex tasks, like those used in Experiment 1, can be understood in terms of basic perceptual and memory processes tapped by simple tasks. Cognitive correlates studies, for example, have shown that individual differences in verbal ability (e.g., Hunt, 1978; Hunt, Lunneborg, & Lewis, 1975) and general intelligence (e.g., Cohn, Carlson, & Jensen, 1985) are correlated with differences in speeded-choice tasks. Because a number of researchers examining correlates of mathematical ability have reported a relation with measures of spatial ability and/or imagery ability derived from paper-and-pencil tests (e.g., Benbow, 1988; Burnett, Lane, & Dratt, 1979; McGee, 1979), we were especially interested in whether the mathematically talented group would show an increased ability to handle spatial information in very simple tasks.

Experiment 2 is conceptualized within Baddeley and Hitch's (1974) framework, in which short-term memory is described as a central working memory that controls two relatively independent buffers: an articulatory loop and a visuospatial scratch pad. The articulatory loop holds verbal information (regardless of its modality of presentation), whereas the visuospatial scratch pad holds primarily spatial information (Baddeley & Lieberman, 1980). Mathematically talented youth were compared with the other groups on a span task, which was assumed to tap the amount of information represented in the buffers, and on a continuous paired-associate task, which was assumed to tap the manipulation of information in central working memory.

Span tasks require immediate reproduction of a set of ordered stimulus items and appear to be measures of basic perceptual processes rather than more complex, strategic processes (e.g., Bachelder & Denny, 1977a, 1977b; Dempster, 1981; Jensen, 1970; Lyon, 1977). Because digits in a digit-span task function as verbal stimuli (i.e., they are treated as stimuli to be named), we assumed that performance on a standard digit-span task reflects the efficiency with which information is represented in the articulatory loop. We also assumed that a spatial analogue of the digit-span task would reflect the efficiency with which information is represented in the visuospatial scratch pad. Therefore, subjects performed a span task with two sets of stimuli. In the digit version, the subject saw a string of four to ten digits. In the spatial version, subjects saw three to nine asterisks in a 10-cell matrix.

Mathematically talented students might not have an advantage in the capacity to represent information, but they might be better able to manipulate it. To assess this possibility, we used a continuous paired-associate task (see Lansman & Hunt, 1982). Subjects attempted to remember five continuously changing letter–response pairs. The responses were either digits or locations in a 10-cell matrix. Although the continuous paired-associate task makes use of information in the buffers, it is primarily an associative task requiring monitoring and updating by central working memory. As a result, it could yield a different pattern of results than the span task.

Method

Subjects and Procedure

The four groups of subjects described in Experiment 1 also participated in this experiment. The same set of planned comparisons across groups was also used.

Each subject completed two tasks with two types of stimuli: digit span, spatial span, letter–digit continuous paired associates, and letter–location continuous paired associates. The combined tasks took 1–1½ hr. Each task was presented on an Apple IIe computer with a standard green phosphor screen. Digit stimuli were presented in the top half of the screen. Taped across the bottom half of the screen was a sheet of transparent plastic with an outline of a symmetric 10-cell matrix drawn in thick black lines. The matrix was 5.9 cm high and 8.3 cm wide in the middle row. There were three cells in the first row, four in the second row, and three in the third row. Each cell of the matrix defined a possible spatial–location stimulus.

The gifted students were tested in two groups of 20 subjects each. The computer facility used with the gifted subjects was not available for use with the other subjects. As a result, they were tested in groups of 1–4. Half of all subjects performed each task with the digit stimuli first, whereas half performed the tasks with the spatial stimuli first. In each case, the span task preceded the continuous paired-associate task.

Span Task

Digit stimuli. Each trial began with a plus sign centered in the top half of the screen for 1 s. This ready signal was followed by the digits displayed one at a time for 1 s each in the same location as the ready signal. After the last digit, the subject wrote the digits in order on an answer sheet.

Spatial stimuli. The ready signal for spatial trials appeared centered on the line between cells in the middle of the matrix. The spatial locations constituting the trial were indicated by a sequence of asterisks. Each asterisk appeared for 1 s centered in one of the 10 cells. After the last asterisk, the subject recorded a response on an answer sheet containing blank matrices. Subjects indicated the order of asterisks by writing the digits 1 through 9 as needed in the appropriate cells of the matrix.

Each digit/location was selected randomly with the restriction that no digit/location occur twice in a row. Lists of digits varied in length from 4 to 10. The set of spatial locations varied in length from 3 to 9. Gifted subjects were presented seven trials at each of the seven lengths for a total of 49 digit trials and 49 spatial trials. Because of time constraints, other subjects were presented six trials at each of the lengths for a total of 42 digit trials and 42 spatial trials. The trials were randomly ordered, and the same order was used for each subject. Each trial was initiated by a keypress, and so there were no time constraints on the responses. There were five practice trials with each stimulus type.

Continuous Paired-Associate Task

Digit stimuli. A ready signal consisting of five plus signs preceded the initial pairing of stimuli. The letters A, B, C, D, and E were presented one at a time along with an equal sign and a digit response term (e.g., A = 4). Each pair was presented for 3 s, and there was a 1-s blank interval between pairs. All stimuli occurred in the same location, centered in the top half of the screen.

The test trials began immediately after the initial presentation of pairs. One of the five letters was randomly selected and presented with a question mark (e.g., A = ?). The display remained on until the subject pressed one of the digit keys at the top of the keyboard. After the subject's response, the new pair was presented for 3 s and testing continued.

Spatial stimuli. In the spatial version of the task, each letter was paired with one of the 10 cells of the matrix. The pair display consisted of the letter's appearing in the center of the cell with which it was paired. The test prompt consisted of a letter followed by an equal sign and question mark presented to the right of the matrix and aligned with the center row. During a test, each cell of the matrix contained a digit. Subjects indicated their choice of cell by typing the digit located in that cell. There were five different random patterns of the 10 digits over the 10 cells. The pattern used was randomly chosen for each test.

The pairing of letters and digits/locations and the selection of letters for testing was completely random and individually generated for each subject. Gifted subjects responded to 60 test stimuli of each type (digit and spatial location). Because of time constraints, other subjects responded to 50 test stimuli of each type. There were eight practice test trials with each stimulus type.

Results and Discussion

Span Task

We calculated proportion correct recall as a function of list length for each type of stimulus (see Figure 1). A response was scored correct only when each stimulus was recalled in the appropriate order. The data from adjacent list lengths (except for the shortest) were combined for analysis. The data of one of the mathematically talented subjects were discarded because he did not follow instructions.

Digit stimuli. The expected decrease in performance with longer lists was confirmed by an ANOVA, $F(3, 225) = 644.63$, $MS_e = 0.019$. There were also significant group differences, $F(3, 75) = 4.14$, $MS_e = 0.063$, and a significant Group × List Length interaction, $F(9, 225) = 2.90$, $MS_e = 0.019$. The planned comparisons showed that (a) the mathematically talented group performed no better than the average-ability group with the longest and shortest list lengths; (b) they did perform at a higher level than the average-ability group with lists of lengths 5 and 6, $t(75) = 3.00$, but were no better than the verbally talented group or the college students; and (c) the mathematically talented group performed better than both the average-ability group, $t(76) = 4.10$, and the verbally talented group, $t(76) = 3.38$, with lists of lengths 7 and 8. The comparisons show floor and ceiling effects but also suggest that the mathematically talented group can handle moderately long lists better than their verbally talented peers can and at a level comparable with that of college students.

We expected that the gifted youth would show better performance than the average-ability youth (e.g., Wechsler, 1974), but the difference between the two gifted groups was not anticipated. Because the digits function simply as stimuli to be named, there was no reason to expect that mathematically talented students would be better than verbally talented students at representing the names in the articulatory loop. We offer a post hoc explanation. It has been shown that span task performance increases with familiarity of stimuli (Case, Kurland, & Goldberg, 1982; Cavanaugh, 1972). Perhaps mathematically talented youth are in some sense more familiar with digit names than are their verbally talented peers. Thus the difference might be attenuated with other alphanumeric stimuli.

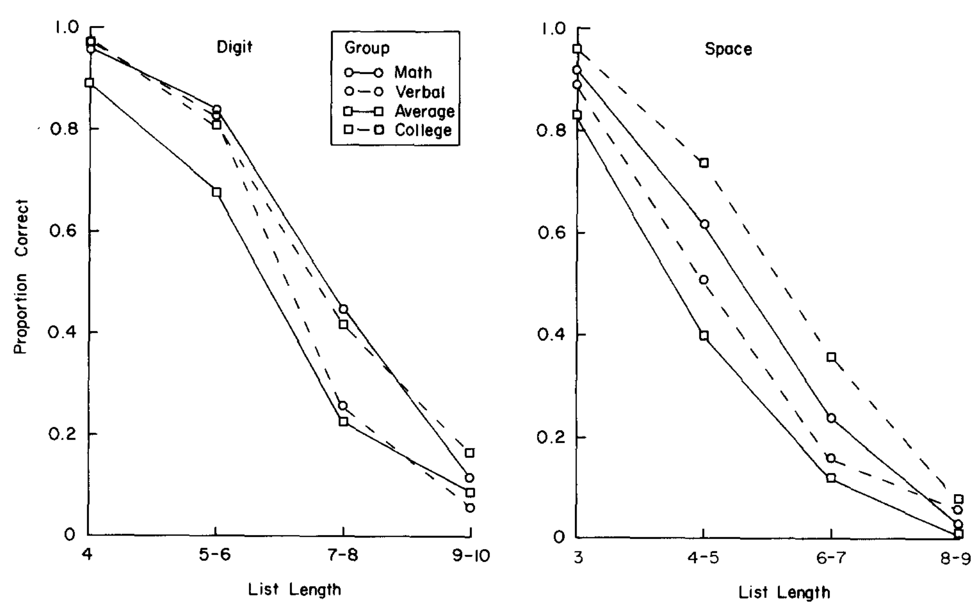

Figure 1. Proportion correct in a span task with digit and spatial location stimuli as a function of list length for each group. (Except for the shortest lists, each point is the average of two adjacent list lengths.)

Although our procedure did not allow a precise determination of individual subjects' digit spans, we estimated them by using Dempster's (1981) definition of span as the length at which a series can be correctly produced 50% of the time. We estimated this length for each subject by identifying the list length at which performance first dropped below .50 and then computing the 50% point by interpolation. The estimated digit spans were 7.5 for the mathematically talented group, 6.7 for the verbally talented group, 6.3 for the average-ability group, and 7.4 for the college group. On the basis of a review of 15 studies, Dempster (1981) estimated that the digit span of 12-year-olds is 6.7 and that of adults is 7.4. Our estimates were identical to Dempster's for the adults and very close for the average-ability youth. The close correspondence suggests that our digit span task is tapping processes similar to those in other studies.

Spatial stimuli. The spatial span data also showed the expected decrease in performance as a function of increasing list length, $F(3, 225) = 561.48$, $MS_e = 0.020$. There were also significant differences between the groups, $F(3, 75) = 7.25$, $MS_e = 0.074$, and a Group × List Length interaction, $F(9, 225) = 2.89$, $MS_e = 0.020$. The planned comparisons revealed that (a) the mathematically talented youth performed no better than the average-ability youth with either the longest or shortest list lengths; (b) they performed better than both their average-ability peers, $t(75) = 3.68$, and their verbally talented peers, $t(75) = 1.81$, with lists of lengths 4 and 5, but not as well as the college students, $t(75) = 2.02$; and (c) they performed better than the average-ability group with lists of lengths 6 and 7, $t(75) = 2.06$, but not better than the verbally talented group. The college students again performed better than the mathematically talented students with lists of lengths 6 and 7, $t(75) = 2.14$.

The spatial span task was more difficult than the digit span task; floor effects become apparent with shorter list lengths. Although the results are not as clear as with the digit stimuli, the mathematically talented youth were better able to remember spatial information than were others their age; this supports the hypothesis that a component of mathematical talent is an enhanced visuospatial buffer. The mathematically talented youth did not perform as well as the college students, however, which suggests that the spatial span task is sensitive to skills acquired with maturation or as the result of a formal educational experience.

We estimated spatial span for each group by using the procedure described earlier. The estimated spatial spans were 5.1 for the mathematically talented group, 4.8 for the verbally talented group, 4.2 for the average-ability group, and 5.8 for the college group.[1] The estimated spatial spans were smaller than the estimated digit spans for all groups. Moreover, the pattern of results differed somewhat for the spatial and digit span tasks. These findings suggest that the tasks tap at least partially different underlying functions and confirm our assumptions that the tasks reflect the efficiency of two separate and independent perceptual buffers (i.e., verbal and spatial).

Continuous Paired-Associate Task

We calculated the proportion of correct responses as a function of lag for the continuous paired-associate data[2] (see Figure 2). *Lag* refers to the number of pairs intervening between the original presentation of the pair and when its association is tested. Data were grouped into Lag 0, Lag 1, Lag 2–3, and Lag 4 or greater to provide approximately equal numbers of observations for each data point. The data were analyzed separately for each type of stimulus.

Digit stimuli. The expected drop in performance as a function of increasing lag was confirmed by an ANOVA, $F(3, 210) = 69.16$, $MS_e = 0.027$. In addition, there was a significant group effect, $F(3, 70) = 8.19$, $MS_e = 0.173$, and a Group × Lag interaction, $F(9, 210) = 2.28$, $MS_e = 0.027$. The planned comparisons showed that (a) the mathematically talented subjects performed better than their average-ability peers at Lag 0, $t(70) = 2.37$; (b) they performed better than both their average-ability peers, $t(70) = 4.37$, and their verbally talented peers, $t(70) = 3.51$, at Lag 1; (c) they performed better at Lag 2–3 than the average-ability group, $t(70) = 4.06$, the verbally talented group, $t(70) = 3.05$, and the college students, $t(70) = 3.16$; and (d) the same was true at Lag 4+, $ts(70) = 5.04$, 3.68, and 3.43, respectively.

Spatial stimuli. An ANOVA confirmed the expected drop in performance with increasing lag, $F(3, 210) = 98.65$, $MS_e = 0.023$. There was also a significant effect of group, $F(3, 70) = 5.78$, $MS_e = 0.144$, but there was no significant Group × Lag interaction. The planned comparisons showed that (a) the mathematically talented group performed better than their average-ability peers at Lag 0, $t(70) = 2.52$, and at Lag 1, $t(70) = 2.56$, but were not superior to their verbally talented peers; (b) the mathematically talented group performed better than both their average-ability and verbally talented peers at Lag 2–3, $ts(70) = 2.66$ and 2.09, and at Lag 4+, $ts(70) = 2.23$ and 1.71; and (c) they did not differ from the college students at any lag.

Performance in the continuous paired-associate task can reflect how well stimuli are represented in the buffers, the use in central working memory of information derived from the buffers, or both. The span tasks, however, are assumed to reflect primarily the buffer representations. Considered in the context of the span data, the paired-associate data suggest that the mathematically talented group has a superior ability to manipulate information in central working memory. Al-

[1] Because only 2 girls were included in the mathematically talented group, gender differences could not be reliably determined across groups. When data were collapsed over groups, however, the only gender difference that emerged was for the spatial span task. Although girls overall had an estimated spatial span of 4.5 and boys had an estimated span of 5.3, the 2 girls in the mathematically talented group did not significantly differ from their 18 male peers (estimated spans of 4.8 and 5.2, respectively).

[2] The failure to include a data-protection code in our computer programs allowed the data from several subjects to be lost when they began experimenting with the keyboard in the middle of the continuous paired-associate task. The digit data from 3 mathematically talented subjects and 3 verbally talented subjects were lost. The spatial data from 4 mathematically talented subjects and 1 verbally talented subject were also lost in this manner. Unfortunately, the gifted children were available on a one-time basis only, and we were not able to obtain more data. In addition, the spatial data from 1 of the average-ability peers were lost as a result of experimenter error.

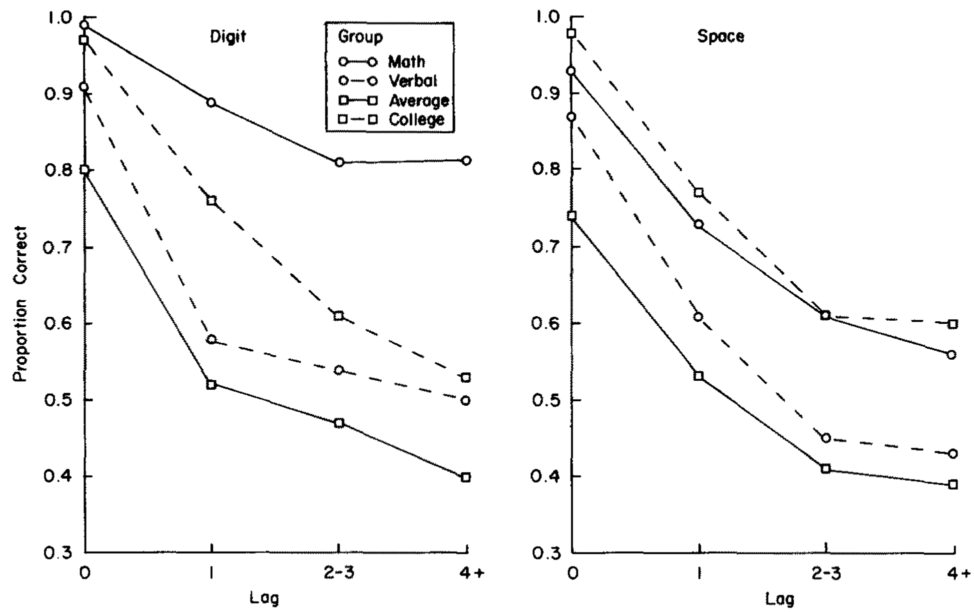

Figure 2. Proportion correct in a continuous paired-associate task with digit and spatial location stimuli as a function of lag between presentation and test for each group. (Data from Lags 2 and 3 are combined, as are data from all lags of 4 and above.)

though the college students could represent more spatial information than the mathematically talented students, the latter were equally good at tasks requiring manipulation of such information. Likewise, although the mathematically talented and college groups could represent equal amounts of digit information, the mathematically talented were clearly superior in manipulating the information.

The finding that mathematically talented youth are better at manipulating information in working memory than are verbally talented youth or college adults is somewhat surprising. Hunt et al. (1975) identified manipulation of information in short-term memory as a characteristic that distinguished college students with high and low verbal ability. Hunt et al. (1975) did not examine groups with different levels of mathematical ability, however. Also, the mathematically talented students in our study had reasonably high verbal ability. In order to better understand both the tasks and the differences between the two gifted groups on them, our future studies will involve various combinations of letters, digits, and other symbols in the span and paired-associate tasks.

General Discussion

In two experiments, we examined the construct of mathematical talent, operationally defined as a high score on SAT-M in seventh grade. In Experiment 1, we considered how mathematically talented youth handled mathematical information in linguistic form. We hypothesized that mathematically talented youth are better at problem translation, the first step in problem solving as described by Mayer (1985). The data provided partial support for the hypothesis. When the problem-translation task was rewriting or recalling material in propositional form, the mathematically talented youth performed no better than the verbally talented youth or the college students. They had the same difficulty as did the other groups in encoding relation propositions. When the problem-translation task was the generation of a simple equation, the mathematically talented subjects exhibited better performance than did either their average-ability peers or the college students, but they performed no better than their verbally talented peers. When the task required generation of more complex equations, however, the mathematically talented youth were clearly superior. It appears, then, that a component of mathematical talent is not so much a better understanding as it is the ability to transform linguistically presented information into a mathematically useful format.

In Experiment 2, we examined information processing at a more basic level. Short-term memory tasks with simple digit and spatial stimuli were designed to ascertain whether mathematically talented youth have an increased ability to represent information in two perceptual buffers, to manipulate information in central working memory system, or to do both. We found support for both. The mathematically talented students performed at a higher level than did the other youth in tasks involving both representation and manipulation of spatial information. The college students performed at least as well as the mathematically talented students with the spatial stimuli, however, which suggests that experience is important in representing matrix-defined spatial information.

Data derived with digit stimuli also clearly showed the enhanced abilities of the mathematically talented youth. Not only did the mathematically talented students outperform both of their peer groups, but their performance was at the level of college students for the digit span task and higher than that of college students on the continuous paired-associate task. Thus the pattern over both digit tasks indicates that in comparison with other talented or older students, the mathematically talented students have enhanced short-term memory capabilities.

In a review of the construct of mathematical talent, Benbow (1988) listed characteristics that have been suggested as correlates of high mathematical ability. The list included spatial ability, field independence, use of images, logic, intuition, flexibility, the ability to recognize unproductive strategies, excellent memory, and high verbal and reasoning skills. Although our investigations were exploratory in nature, the data suggest in addition the following characteristics: (a) a superior ability to represent and manipulate information in short-term memory and (b) a superior ability to translate linguistically presented mathematical information into an equation form necessary for successful problem solution. Both of these abilities appear to set mathematically talented youth apart from other talented youth and, for the most part, from older college students.

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