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IDENTIFICATION AND FOSTERING OF MATHEMATICALLY GIFTED STUDENTS

Rationale of a Pilot Study

ABSTRACT. In a three year research project, annual mathematics talent searches for highly able and motivated twelve year old students were conducted. Of these, 150 took part in a long term Saturday enrichment program to train their mathematical abilities in problem finding and problem solving. The article first discusses the educational and organizational constraints of programs for gifted children. Mathematical giftedness is defined by high achievement in two tests: The Scholastic Aptitude Test (SAT-M) and the HTMB, a set of seven problems specially devised for the talent search. The philosophy of the teaching program is explained and illustrated by examples. Preliminary results indicate the considerable success of the program. Possible consequences for normal classroom teaching are indicated.

1. INTRODUCTION

This article describes a project which has been conducted at the University of Hamburg since 1982. Inspired by the work at the Johns Hopkins University in Baltimore of Julian Stanley and his colleagues (cf. Benbow and Stanley, 1983), who initiated the "Study of Mathematically Precocious Youth (SMPY)" in 1971, a systematic search for mathematically talented twelve year old girls and boys from the Hamburg area was carried out, and a long term Saturday program offered to the top scoring students.

In contrast to the SMPY approach, which favors extreme acceleration through fast paced mathematics classes, a challenging enrichment program was set up.

2. DETERMINANTS AND PHILOSOPHY OF THE STUDY

Any concept for identifying and fostering mathematically gifted students necessarily depends on specific administrative determinants (e.g., the school system of a country) and a "system of values" (e.g., educational objectives). When discussing projects on gifted students it is not only important to know something about the goals, procedure and results, but it is also important to learn something about the specific "boundary conditions" of the study. In this way misunderstandings can be avoided and benefits optimized. To take an example, different results of such studies in different countries can often be explained by different social environments, by different philosophies of mathematics underlying the testing and/or teaching material, by different educational objectives or a mixture of all of these or

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additional factors. Consequently, the following general remarks seem to be very important for the drawing of specific practical conclusions.

(1) The German and the American school systems differ from each other in many respects. In Germany for instance, mathematics is taught to all children from grade 1 to 9 for at least three hours and at most five hours a week. In the "Gymnasium" (most similar to the college bound stream in American High Schools) mathematics is taught to all students up to at least grade 12. Only in grades 12 and 13 do the students have the possibility of choosing additional or more demanding courses. There are no algebra or geometry classes formed of students with a broader age range. No credit is given for accelerated learning in a special subject. If a student scores high in nearly every subject, he/she can skip one grade.

The minimum time for studying mathematics or a related subject at a German University is four years. Grants can be given if a student was a winner of a national or international competition (e.g., *mathematical olympiads*) or if he/she is suggested by the school and accepted by an expert team from a foundation (e.g., Studienstiftung des deutschen Volkes).

As cooperation with the local School Board and the teachers of the students is absolutely necessary, in our situation an enrichment program with as little overlap as possible with the normal mathematics curriculum was the logical consequence.

(2) The age of the students in the program is another important determinant. Tests, teaching material and teaching methods have to take into account that students from grade 6 – not only according to Piaget – are in the transitional stage from the concrete-operational to the formal-operational stage.

(3) At least in Germany, various deficits exist in the school system, normal mathematics teaching and curriculum and in existing concepts of mathematical giftedness. Some examples:

(a) There is too much emphasis on "linear" thinking and too little on the construction of connecting and connected thinking processes which are involved in complex problem areas and are more typical of real life situations as well as mathematical situations. Consequences of this almost worldwide deficit can be seen, for instance, in the fields of ecology and administration. Dörner *et al.* (1983) demonstrated by very comprehensive experiments, that there was only minimal correlation between I.Q. and optimal behaviour in complex problem areas.

(b) Schoolbooks, teaching materials and methods, and in particular multiple choice tests (including the SAT) are oriented too much to knowledge and thinking *products*. To demonstrate the problems with standardized tests, two quotations are given here:

Standardized objective multiple choice tests are primitive, much abused measuring devices. They neither set nor uphold standards. (Lax and Groat in Steen, 1981, p. 85)

Tests tyrannize us... they encourage the false view that mathematics can be separated out into tiny watertight compartments; they teach the perverted doctrine that mathematical problems have a single right answer and that all other answers are equally wrong. Particularly perverse and absurd is the multiple-choice format. I have been doing mathematics now as a professional for nearly 40 years and have never met a situation (outside finite group theory!) in which I was faced with a mathematical problem and knew that the answer was one of five possibilities. (Hilton in Steen, 1981, p. 79).

Nevertheless, knowledge, isolated techniques, and heuristics are necessary and important, and so are corresponding tests like the SAT (we used it, too) and the tests of Krutetskii (1976). They are, however, not sufficient. The whole is not only more than the sum of its parts but is also different. On the other hand, identifying mathematically gifted students does not necessarily mean that one looks for future pure mathematicians only.

(4) In addition to the determinants already mentioned, the specific *philosophy of mathematics* and the objectives of an educational approach are of great importance for every study of mathematical giftedness (as is the explicit or implicit definition of mathematical giftedness). Views on mathematics and reasons for dealing with it can vary considerably. Consequently, the underlying philosophy of mathematics should be made explicit. For instance, mathematics can be seen as

- the science of formal structures and systems (formalism);
- the science of exact proofs and eternal truths ("Euclidian science", cf. Lakatos, 1976);
- a tool for other areas like natural sciences and technology;
- a game or art;
- a bag of tricks and formulae (very well known!);
- the science which can be treated best on a computer (emphasis on discrete mathematics and algorithmic thinking), cf. e.g., Ralston (in Steen, 1981, p. 213–220) and CMJ (1984; 1985).

Our philosophy of mathematics can be characterized in the following way: We place emphasis on informal mathematics and mathematical ideas rather than on abstract structures; we look on mathematics as an open process of thought rather than a universe of fixed products; we stress that mathematics can be taken as a game and as an art, too; "good mathematics" is not defined mainly by a lack of mistakes (the main criterion in standardized tests) but by the quality of ideas; rigour and precision should be developed in students according to their maturity; we have a holistic view of mathematics.

(5) Another important factor in a project on mathematically gifted students is constituted by the *educational objectives* of the corresponding teaching program. Our objectives are the following (there are relations, of course, to our philosophy of mathematics): Exploring mathematics by "micro-research processes" with as little guidance as possible – the focus should be more on problem finding and creating than on problem solving; having fun by playing with mathematics; having as many successes as possible in a given time – the teaching material should have a high "success density"; learning something about applications of mathematics – e.g., by using a computer; learning something about precise written and oral communication in mathematics – e.g., the students should develop their own journal; learning something about social behaviour and responsibility.

3. A DEFINITION OF MATHEMATICAL GIFTEDNESS

After a careful review of the literature, especially Krutetskii (1976) and Polya (1945), and taking into account Section 2, we arrived at the following *definition of mathematical giftedness* (cf. Kiesswetter, 1985):

Mathematical giftedness is a set of testable abilities of an individual. If he/she scores high in nearly all of these abilities, there is a high probability of successful creative work later on in the mathematical field and related areas. These abilities are defined by the mathematical parts of the SAT and by a new test for mathematical giftedness (Hamburger Test für Mathematische Begabung – HTMB), stressing the following complex mathematical activities:

(1) organizing material;
(2) recognizing patterns or rules;
(3) changing the representation of the problem and recognizing patterns and rules in this new area;
(4) comprehending very complex structures and working within these structures;
(5) reversing processes;
(6) finding (constructing) related problems.

These six categories of the HTMB will now be explained with reference to a special version of the game "Nim" (it was part of the teaching program but not an item of the HTMB).

Two players, *A* and *B*, take alternately stones from two piles of stones:

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* * * * * * * * * *

The rules of the game are:

(1) take at least one stone and at most two stones from either pile or from both piles;
(2) you win, if you can take the last stone.

Problem: Try to find a winning strategy for the beginner *A*.

Some ideas are:

(1) The problem can be represented in a different way (cf. Category 3):

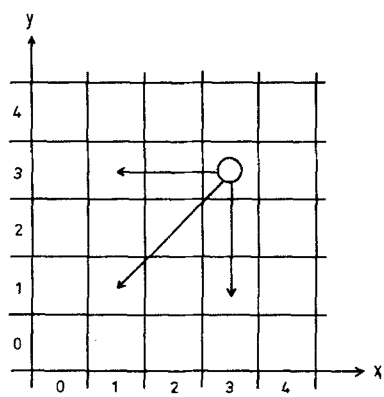
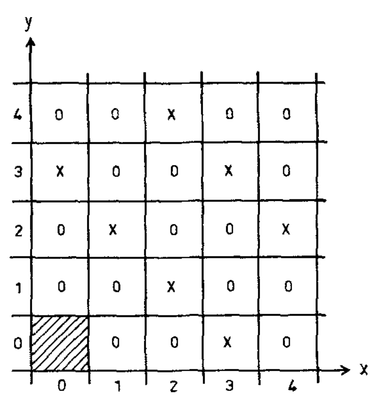


Fig. 1.

(2) "Working backwards", one can organize the material (cf. Categories 1 and 5):



0: winning position
x: loosing position

Fig. 2.

(3) This newly organized material leads to the recognition of a pattern (Category 2): A will win, iff A has the starting position $(x; y)$ with the property that $x+y$ cannot be divided by 3.
(4) One can find new problems (Category 6) by changing the rules of the game or taking more piles/dimensions.
(5) To develop a computer program for this game with highly sophisticated graphic design would demand the creation of a complex structure and working within this structure (Category 4).

For the HTMB, Kießwetter (1985) constructed seven problems, each of which includes at least one of these categories as dominating idea. Two sample problems will be given here which are very similar to corresponding test items.

Problem 1, constructed with focus on Category 2:
(a) Find the number of squares in the following diagram (Figure 3).

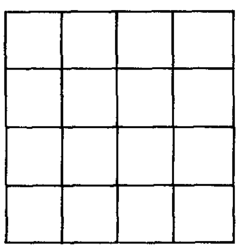


Fig. 3.

(b) How many squares would there be on a 8×8 chessboard?

Problem 2, constructed with focus on Category 4:
For each natural number n, $A(n)$ denotes the alternating total of the digits of that number.
Examples:

$$n=571 \rightarrow A(n)=1-7+5=-1$$
$$n=1027 \rightarrow A(n)=7-2+0-1=4$$

For each natural number n you can find a number $f(n)$ by the following means:

1. Derive $A(n)$
2. Calculate $f(n)$ according to $F(n)=(n-A(n))/11$

Example:

$$n=1027$$
$$f(n)=f(1027)=(1027-A(1027))/11=(1027-4)/11=93$$

The change from n to $f(n)$ is defined as one "step". One can conclude, that after a finite number of steps the number zero will be reached.
Examples:

$$\left.\begin{aligned} n=89\rightarrow f(89)&=(89-1)/11=8 \\ f(8)&=(8-8)/11=0 \end{aligned}\right\}\quad \text{2 steps}$$

$$\left.\begin{aligned} n=571\rightarrow f(571)&=(571+1)/11=52 \\ f(52)&=(52+3)/11=5 \\ f(5)&=(5-5)/11=0 \end{aligned}\right\}\quad \text{3 steps}$$

You can see that in these examples the number of steps required to arrive at zero equals the number of digits in the original number. We will call these numbers "obedient numbers". Unfortunately, not all numbers are "obedient".

$$\left.\begin{aligned} n=1029\rightarrow f(1029)&=(1029-6)/11=93 \\ f(93)&=(93+6)/11=9 \\ f(9)&=(9-9)/11=0 \end{aligned}\right\}\quad \begin{array}{l}\text{3 steps for a} \\ \text{4 digit number}\end{array}$$

It is your task to determine the largest four digit number that is not an "obedient number".

Of course, more than one of the six categories is involved in each of the seven problems, e.g., Problem 1 includes additionally Category 1 and Problem 2 Category 6. The other problems demand even more divergent thinking. In two cases the students had to create their own problems in relation to the given ones (Category 3).

Consequently, the evaluation has to be more holistic and must be less standardized. But in our opinion the test has to meet the requirements of the content area and the overall goals (cf. Section 2), and not vice versa. Scores were given not only for a correct solution but also (mainly) for reasonable use of the six categories. Additional scores were given for appropriate memorized and applied knowledge and routines.

4. THE SELECTION PROCEDURE

Criteria for selecting gifted students were developed with respect to our specific "boundary conditions" and in cooperation with the Johns Hopkins group by combining a German translation of the mathematical parts of the SAT (GSAT) with the new test for mathematical talent just described.

Since 1983, annual talent searches have been conducted among 6th grade students of the Hamburg Gymnasium and Comprehensive Schools. The mathematics teachers were asked to nominate students with exceptional mathematical reasoning ability. Self nomination by students was possible as well. After returning a postcard indicating their interest in the talent search, the students received a preparation booklet that was adapted from the American version of "Taking the SAT". The booklet contains detailed information on the mathematical contents of the test items and a complete version of the mathematical parts of the SAT. Subsequently, formal registration for the test session was requested. About 75 percent of the students who ordered the preparation material decided to take part in the examination. Participation in the talent search was completely voluntary and free of charge.

Table I gives an overview of the results of the three talent searches. The total number of participants represents 1–1.5 percent of the student population of the respective age cohort.

It will be noted that far more boys than girls participated: the percentage of boys varies between 70 and 73. As far as the mean GSAT scores are concerned, the results of the three talent search groups are nearly identical; only the girls show some variation. The boys score slightly higher than the girls. On the HTMB sex differences seem to be less important. In the 1985 group, the girls do even somewhat better than the boys. Due to different scoring procedures, the 1983 results of the HTMB are not directly comparable with those of the following years. For the sake of comparison the GSAT was given to 357 unselected 10th graders from twelve Hamburg

TABLE I

Results of the first three talent searches (Number of participants and mean scores of the two tests)

Talent search	Total			Boys			Girls		
	n	HTMB	GSAT	n	HTMB	GSAT	n	HTMB	GSAT
1983	192	10.9	461	136	10.9	466	56	11.0	450
1984	294	13.3	456	205	13.5	467	89	12.8	432
1985	289	11.2	456	211	10.9	461	78	12.0	442
10th grade students	357		455	168		466	189		443

The mean scores for the HTMB are given as raw scores, while he GSAT results have been converted into the American Standard Scale ($M = 500$, $s = 100$).

TABLE II
Test results of the selected groups

Talent search	Total			Boys			Girls		
	n	HTMB	GSAT	*n*	HTMB	GSAT	*n*	HTMB	GSAT
1983	42	18.1	557	34	18.1	553	8	18.1	578
1984	65	21.6	557	52	21.0	563	13	24.0	531
1985	52	19.6	563	37	19.0	575	15	21.0	530

Gymnasium Schools. The talent search participants produced exactly the same mean score as the 10th graders thus indicating an achievement four years ahead of their class mates.

The test scores were transformed into a common T-scale ($M = 50$, $s = 10$), added, and ranked in order to find the top scoring students to be invited to take part in the course. Table II gives details of the selected groups.

The percentage of boys in the top scoring group was even higher than among the talent search participants. The boys outnumber the girls by 4 to 1. Only in the 1985 talent search are the selected girls represented according to their proportion in the talent search group. The lower mean scores of girls on mathematical aptitude tests and their considerable underrepresentation in the top scoring group are well established observations reported from several countries. Possible reasons for these differences are still under discussion. Differential social and early educational influences as well as role models, vocational expectations, interests etc. are certainly important (cf. Schildkamp-Kündiger, 1974) but may not be the only relevant factors. A recent study by Benbow and Benbow (1984) claims hormonal factors to be responsible for differences in the development of high mathematical reasoning abilities.

All talent search participants received a detailed written report on the results of the talent search and on their individual scores. It was stressed that simply taking the two very difficult tests was itself highly commendable. This was honoured by giving all participants a certificate of appreciation.

5. THE TEACHING PROGRAM

The course work with the selected group consists of some 25 units during the school year. The students gather in lecture rooms of the Faculty of Education on Saturday mornings. A typical session might run as follows.

At the beginning (9.15 a.m.), additional information about the problems of the previous session may be given to the students, and the new problem area introduced by a short lecture. After this three to five groups of about ten students start working on the problem, guided by a teacher. The students may choose their working methods and the kind of problems they want to tackle after solving the initial problem. At 12 o'clock the groups gather for the final plenary session to report and compare their results. The session is finished at 12.30.

Although homework is not required, the students are encouraged to continue their work on the problem area during the week. Outstanding ideas of the students (e.g., proofs, computer programs, games) are published in their own journal. Some of the Saturday sessions are used to invite experts for lectures on topics such as famous mathematicians, number systems in different cultures, finding large prime numbers with the help of computers, computer graphics for fractals etc.

6. THE TEACHING MATERIAL

The teaching material contains problems or sources of problems from elementary mathematics (graph theory, geometry, combinatorics, number theory, game theory). To give some impression of the mathematical content which is preferred in this project and which should correspond to our philosophy of mathematics and educational objectives, a typical example will be presented. It was given to the students at the end of their first year of course work.

Problem: Given an equilateral triangle and a point P in its interior or on its boundary (cf. Figure 4):

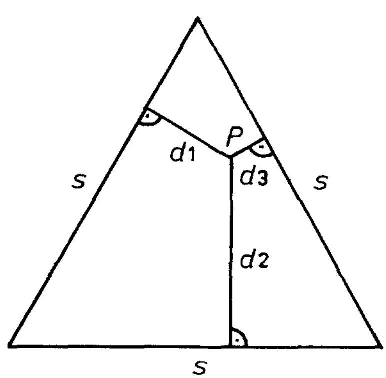


Fig. 4.

You can show with the help of a few examples that the sum of the distances from P to all three sides seems to be always the same:

$$d1+d2+d3=\text{const.}$$

(a) Can you find a proof for this conjecture?

(b) Can you think of any related problems?

Solution for (a). A proof can be carried out in at least two different ways.

(1) Most students applied the strategies "examine special cases" and "reduce the general to a special case".

Case 1: P coincides with one of the vertices of the triangle. The proof is obvious: The "sum of the distances" equals the length of the altitude of the triangle (cf. Figure 5):

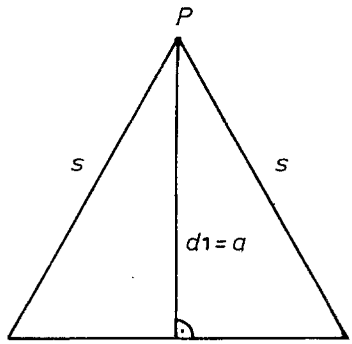


Fig. 5.

Case 2: If P is placed on one of the sides of the triangle, this case can be reduced to Case 1 by a line through P which is parallel to one of the other sides of the triangle. By this, a smaller equilateral triangle is created within the original one. The equation $a=d1+d2$ holds for any placement of P on any side of the triangle (cf. Figure 6):

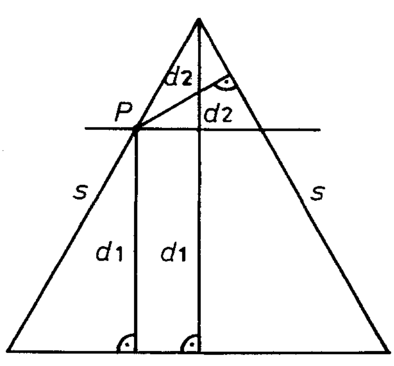


Fig. 6.

Case 3: The general case can be reduced to Case 2 in the same way as Case 2 could be reduced to Case 1 (cf. Figure 7):

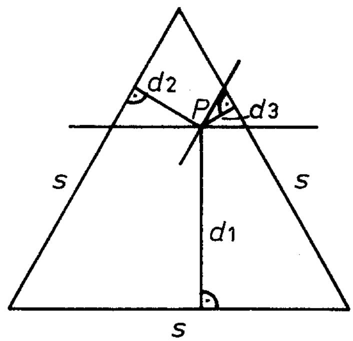


Fig. 7.

Thus, the theorem is proved.

(2) Another possible proof which has not yet been found by any of our students, would be to compute the areas of the three small triangles formed by P and the three sides of the given triangle. The total area must be equal to the sum of the small areas. This proves the proposition $d1 + d2 + d3 = (2*\text{total area})/s = \text{const}$ (cf. Figure 8):

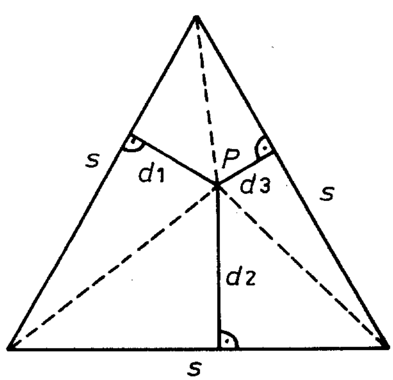


Fig. 8.

Solution for (b). The problem can be generalized and modified in many ways, all of which were discovered by our students.

(1) The point P can be placed outside the triangle (cf. Figure 9):

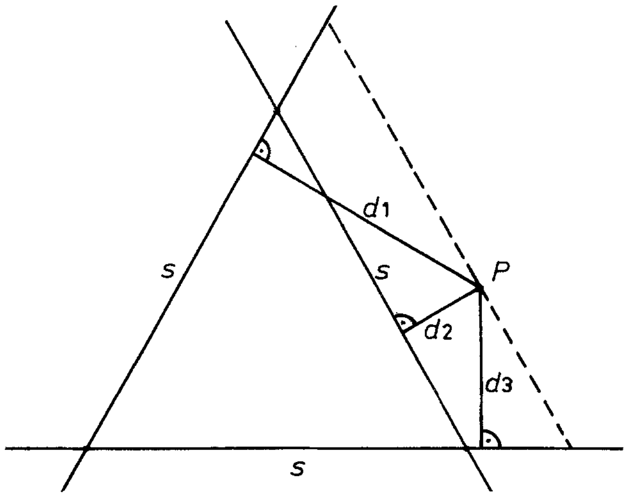


Fig. 9.

In this case the length of the altitude can be computed by a combination of addition and subtraction of the three distances. Each line containing one of the three sides of the triangle defines two different half-planes of the whole plane. One of these half-planes, p_i, contains the interior of the given triangle, the other one (p_0) does not. The sign "+" must be placed in front of the distance d, if P belongs to p_i, otherwise the sign must be "−". In the situation presented in Fig. 9 the equation $a = d1 - d2 + d3$ holds. For each case the proof can be found with the methods presented in (a).

(2) The problem can be generalized to all regular polygons. As every regular polygon can be divided into triangles which all have P as a vertex and one side of the polygon as their base, one can prove by the same area argument as in (a) that the sum of all distances from P to all sides of the regular n-gon equals $n*r(=\text{const})$, where r is the radius of the incircle.

(3) The problem can be generalized into space, too, for regular polyhedra. For instance, in the case of a tetrahedron, the "cutting-off" method as well as the "volume" method (derived from the "area" method) proves $d1 + d2 + d3 = \text{const} = \text{altitude of the tetrahedron}$.

7. ASPECTS OF EVALUATION

Since it was not possible to establish a control group of equally able and motivated students from among the talent search participants without lowering the level of the selection criterion considerably, alternative strategies of program evaluation had to be developed. Furthermore, it seemed unwise to deal with a number of highly talented students only as a control group.

Participation in the program is voluntary. Attendance at the sessions and the drop out rate might therefore be used as indicators of the student's involvement and interest in the program. The mean rate of attendance was about 90 percent, while the drop out rate was seven out of 107 during the first two years of course work. Additionally, the students invested a considerable amount of their free time in very challenging course work over a long period. We consider these facts to be strong clues to an apparently successful program that meets needs of highly motivated and talented young people which otherwise would not have been served.

The selected students are by no means onesided in their talents and interests. This is demonstrated by the fact that an intelligence test yielded a mean I.Q. of 145 for the first two groups including very high scores on verbal subtests. Furthermore, the students have a wide range of hobbies, including sports, music, arts, dance, and science.

The assessment of cognitive learning increments is difficult as well. Some clues were received from a "retention" or "transfer" test, which was developed by K. Kiesswetter. This test was presented to the students after the first year of coursework. It was not and will not be used for identification. The test includes six problems which have a similar internal but not necessarily external structure as some of the topics which had been covered in the previous teaching sessions. One has to take into account that neither assignments were given to the students nor was repetition fostered in the teaching sessions. Even in the case of strong external differences between test problem and related classroom topic more than half of all students in our first group demonstrated explicit memory or transfer ability. For example, the game "Nim" (represented by "piles", cf. Section 3) had been a topic in our teaching program six months before the test, in which they had to cope with a similar problem represented in a coordinate plane.

The results of this test, which was administered in June 1984 to 38 participants, are presented in Table III.

Besides the restrictions which have already been mentioned above, one has to take into account the following issues when interpreting the results of this test:

(1) Sometimes up to 50 percent of the students solved a test problem without apparent memory effects (although use of memory while solving the problem can, of course, not be excluded).

TABLE III
Results of the first Retention/Transfer Test

Problem area	*d*	*a*	*f*	*m*	*fm*	*s*	*sm*	*n*
graph theory	9/83, 10/83	2	2	4	4	0	28	36
game "Nim"	11/83, 1/84	4	11	6	1	8	12	19
perfect numbers	2/84	6	11	1	0	9	17	18
Pick's theorem	4/84	7	9	4	1	17	7	12
triang. numbers	4/84	6	13	3	0	14	8	11
Pascal's triangle	5/84	7	7	1	2	0	28	31

d = date of the corresponding lesson
a = number of students absent from this lesson
f = no solution, no apparent use of memory
m = apparent memorizing, but no solution
fm = wrong solution, apparent memorizing
s = solved without apparent memorizing
sm = solved and apparent memorizing
n = total number of students who memorized

(2) The ability to memorize ("mathematical memory", cf. Krutetskii, 1976) cannot be separated from transfer ability.

The observation and analysis of learning and teaching processes not only contributes to a better understanding of these processes, but should also give us more information about the effects of our teaching program. These observations (with respect to Sections 2 and 3) have not yet been completed and evaluated. Methods applied are similar to those developed by Zimmermann (1977; 1980).

8. POSSIBLE CONSEQUENCES FOR "NORMAL" CLASSROOM TEACHING

Several strategies are being pursued to communicate the experiences of this study to a broader audience.

(1) In autumn of 1985 a series of workbooks for students named "Heureka" was launched.

(2) The material we used in our teaching sessions will be published by K. Kießwetter and will thus soon be available for all interested persons (cf. Kießwetter, 1985).

(3) Systematic studies will show whether carefully selected parts of our teaching material can stimulate and improve "normal" classroom teaching (at least in German Gymnasium Schools). This should, by the way, include some revision and augmentation of curricula and teacher education!

(4) One of the authors – teaching our gifted as well as normal students – has had very encouraging experiences with many topics from our material with normal 7th graders. Two examples will be presented here:

Problem 1: Find all possible tesselations of the plane with regular polygons.

In the previous lessons the students discovered (and proved) the angle theorem for triangles and its generalization for polygons (sum of all interior angles $= (n-2)*180°$). All students discovered without any further stimulus: (a) that equilateral triangles, squares and hexagons will do; (b) new methods to construct regular polygons (pentagons, heptagons, octagons); (c) that three pentagons would leave empty spaces at those points, where they meet, four pentagons are too much, they overlap at the corner points like three heptagons; (d) that the "crucial" points are those, where the polygons meet. Several students discovered: The angle around these points (360°) must be divisible by the interior angle of the corresponding polygon $((n-2)*180°/n)$.

Problem 2: Figure 10 represents a road map (towns *A*–*L*, distances in miles). The roads are covered with snow. The road net is to be cleared so that all towns are connected and the total length of all roads to be cleared is a minimum (cf. Kießwetter and Rosenkranz, 1982).

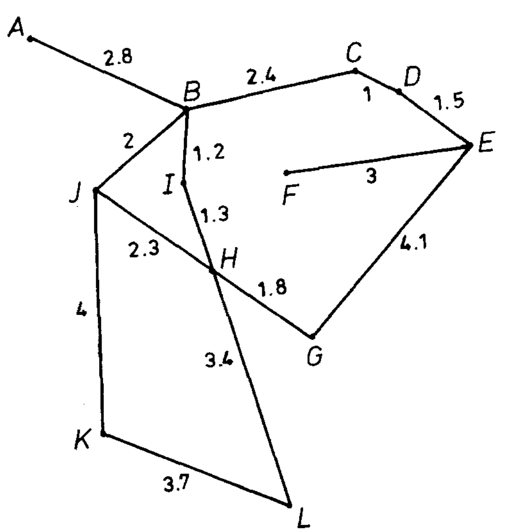


Fig. 10.

Several students discovered the two main algorithms:
(a) the "building up" algorithm: one can start in any town and must look always for the shortest road to a new town;
(b) the "reduction" algorithm: one looks for "loops" (closed paths) and one omits the longest road in this circle. One continues in this way until there are no more "loops".

Additionally, computer programs can be developed for both algorithms.

One student in our gifted program told us: "If more material of this type were presented to the students in the normal classroom they would have much more fun, more success and fewer frustrations!"

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