4 Mathematics Taught at a Fast Pace: A Longitudinal Evaluation of SMPY's First Class

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Abstract

Fast-paced classes have been advocated in SMPY's proposals for curricular flexibility. To evaluate the long-term effects of such a class, the responses to two questionnaires completed nine years later by both the participants and the nonparticipants of SMPY's first two mathematics classes were analyzed. The participants scored significantly higher in high school on the SAT-M, expressed greater interest in mathematics and science, and accelerated their education much more than the nonparticipants. Gaps in knowledge of mathematics by the participants were not found. All groups attended selective colleges, but the students who completed the fast-paced class chose the most academically difficult. It is concluded that when highly able youths are presented the opportunity, many of them will accumulate educational advantage.

In chapter 11 of this volume Feldhusen argues that "eclectic" educational programming is necessary to meet the needs of gifted students. To meet the special needs of the highly gifted, Feldhusen advocates acceleration. To meet the special needs of highly mathematically precocious students, the Study of Mathematically Precocious Youth devised a "fast-paced" mathematics class (Fox 1974; George & Denham 1976; Stanley 1976; Bartkovich & George 1980). As the name indicates, mathematics in it was taught at a rapid pace geared to the ablest members of the class. The content was the regular precalculus curriculum taught in junior high and senior high school (algebra I and II, geometry, college algebra, trigonometry, and analytic geometry). The first class, designated Wolfson I in honor of its splendid teacher, Joseph R. Wolfson, met from June 24, 1972, until August 11, 1973.

The program was designed primarily for students in Baltimore County public schools who had finished the sixth grade. In order to be eligible, the students had to have scored on the Academic Promise Test (APT) (Psychological Corporation 1959) at the ninety-ninth percentile of sixth-grade norms on the number (arithmetic) subtest and at the ninety-ninth percentile of sixth-grade norms on either the abstract reasoning or the verbal subtest. In addition to the twenty-five students so identified there were six highly recommended, able students known by SMPY. Thus thirty-one students (nineteen boys and twelve girls) were invited to attend the class; fourteen boys and seven girls accepted. One boy [1] dropped out of the class during the first week and a second one did so within the first few weeks. One boy and two girls were added to the class in September. As a result, thirty-four students had the opportunity to attend SMPY's first fast-paced mathematics class; twenty-two stayed long enough to reap some benefits from it.

The initial success and progress of this class have been discussed previously (Fox 1974; Stanley 1976, pp. 156–59). Thus only a brief summary is supplied here. As noted, nineteen students stayed in the program and studied algebra I for nine weeks on Saturday mornings, two hours each week during the summer of 1972. Of those nineteen, fourteen scored high enough on the Educational Testing Service's (ETS) Cooperative Mathematics Tests: Algebra I (ETS 1962) to be able to continue in the program and study algebra II. The other five students were advised to take algebra I as seventh-graders the next year in school. At this time one other student chose to drop the class because her girlfriend did, but three others (two ex-seventh-graders and one ex-sixth-grader) were added. Thus in the fall of 1972 sixteen students (nine boys and seven girls) began the study of algebra II for two hours on Saturday mornings. This group was later split into a "fast" class or group and a "slow" class or group. The members of the slow group (two boys and four girls) had had trouble keeping up with the pace of the class or had scored low on the standardized Algebra II test. The goal of the slow group was to finish algebra II by June, 1973, when most of them would be completing the seventh grade. The goal of the fast group was to complete algebra II, college algebra, geometry, trigonometry, and analytic geometry by August, 1973. Of the ten persons in the fast group, two girls decided not to study plane geometry with the class during the summer of 1973.

Although the goals for this pioneering group were impressive, they were met successfully (see Fox 1974). The original Wolfson I class surpassed SMPY's expectations. In twelve to fourteen months, eight students completed 4½ years of mathematics, two completed 3½ years, and six completed 2 years.

More than nine years had passed since the inception of this class when this evaluation of its fairly long-term effects took place.

Longitudinal Follow-Up Procedure

In May 1980 two questionnaires were mailed to each of the thirty-four students who had been given the opportunity to attend the Wolfson I class. One of the questionnaires was an eight-page follow-up survey that had been sent to SMPY students in the fall when the students would have been graduated from high school (see Appendix 2.1). Most of the students had been mailed this questionnaire as part of the general follow-up conducted by SMPY. For information on how the follow-up was carried out and the general results for the whole SMPY group, see Benbow, chapter 2 of this volume. The students who were not in this follow-up or who had not responded were sent another follow-up questionnaire with a $5 inducement for completion, along with the second questionnaire, in May, 1980 (see Appendix 4.1). The additional three-page questionnaire brought each student's educational progress up to date as of the summer of 1980. An autographed copy of one of SMPY's volumes in the Studies in Intellectual Precocity series was offered as a compensation for completing that questionnaire. The response rates for the two questionnaires were 100 percent for the follow-up questionnaire and 94 percent for the additional Wolfson I questionnaire.

ANALYSIS

The resulting data were coded and keypunched onto the computer by use of the Filgen and Qgen systems (The Johns Hopkins University Computing Center). The students in the study were classified into four groups upon which the data analysis was performed using the SPSS program (Nie et al. 1975). The composition of the four groups can be seen in table 4.1. The student who had attended only one session of Wolfson I was excluded from all analyses. The following year he had attended the second fast-paced mathematics class conducted by SMPY (Wolfson II). It was felt that his inclusion for Wolfson I would bias the results.

TABLE 4.1. Classification of the Students Included in the Analysis Who Were Given the Opportunity to Attend Wolfson I (by Their Course of Action)

Group A: Finished Wolfson I in the Fast Class[a]	Number	Group B: Finished Wolfson I in the Slow Class	Number
Boys	7	Boys	2
Girls	3	Girls	4
Total	10	Total	6

Group C: Dropped Out or Were Asked to Leave Wolfson I	Number	Group D: Did Not Attend Wolfson I	Number
Boys	5	Boys	5
Girls	2	Girls	5
Total	7	Total	10

NOTE: To reduce bias, one boy was excluded from the analysis because he attended only one meeting of Wolfson I and later completed Wolfson II.
[a] Two girls in this group did not complete the whole sequence, because they did not attend the class meetings during the summer of 1973.

Results

SAT SCORES

Most of the students who were extended the opportunity to join Wolfson I participated in one of SMPY's talent searches, where they took the College Board's Scholastic Aptitude Test-Mathematics and -Verbal in January or February of 1973. Three were in SMPY's March, 1972, search. Their performance on the SAT in the talent search as either seventh- or eighth-graders and then later in high school is contrasted by group in table 4.2. Every group's mean SAT-M score in the talent search surpassed the mean score obtained in any of SMPY's talent searches (Benbow & Stanley 1980). Furthermore, the mean scores on the SAT of the groups in high school were much superior to the mean scores obtained in high school by the participants in the first three talent searches (Benbow, chapter 2 of this volume) and by college-bound seniors.

Although there were certain biases between the groups with respect to taking the SAT in a talent search (e.g., all ten members of Group A took the SAT, whereas only four of six – 67 percent – of Group B did), it seems that at the time of the talent search the members of Group A (the ones who completed Wolfson I in the fast class) received the highest SAT scores, followed by Group B (the ones who completed Wolfson I in the slow class). Groups C (who dropped out of or were asked to leave Wolfson I) and D (who did not attend Wolfson I) scored similarly on SAT-M (scoring

TABLE 4.2. SAT Scores at the Time of Talent-Search Participation and as Reported in High School (by Group)

Group[a]	*N*	Mean SAT-M	Standard Deviation	Mean SAT-V	Standard Deviation	Mean of Year Taken[b]
			Talent Search			
A	10	633	64	487	78	73
B	4	563	86	525	76	73
C	5	488	53	382	67	73
D	5	492	85	420	46	73.4
			High School			
A	10	751	47	624	80	75.2
B	6	736	30	645	108	76.3
C	7	708	61	582	56	77.3
D	10	708	39	613	71	77.2

[a] For the meaning of A, B, C, and D here and in tables 4.2–4.8, see table 4.1.
[b] 1973 talent-search SAT scores were used if available.

approximately 140 points lower than Group A or 70 points lower than Group B). The differences between the groups were significant by an Analysis of Variance (ANOVA) ($F = 6.9, p < .01$). The talent search SAT-V scores were also significantly different between groups by an ANOVA ($F = 3.8, p < .05$). Group B, followed by A, had the highest scores.

In high school there is a complicating factor when comparing performance on the SAT—the groups did not take the SAT at the same time. Group A took their SATs in high school one year earlier, on the average, than Group B, who took the SAT one year earlier than Groups C and D. Yet on SAT-M Groups A and B scored essentially the same and superior to Groups C and D, who scored similarly. The differences between the groups were significant by an ANCOVA (analysis of covariance) controlling for year taken ($F = 3.7, p < .05$). On the SAT-V, however, Group B scored better than the other three groups, but the mean difference between groups was not significant.

MATHEMATICS COURSE-TAKING IN HIGH SCHOOL

The mathematics course-taking in high school by the students in the various groups is shown and contrasted in table 4.3. Because of the format of Wolfson I, 80 percent of the fast group would have finished the 4½ years of precalculus mathematics, while 100 percent of the students in the slow group should have finished algebra II. Later in high school all but one (83 percent) of the students in the slow group finished the precalculus sequence, and everyone in the fast group did. This percentage of students reporting a completion of precalculus was much higher than for the other groups.

TABLE 4.3. Reported High-School Mathematics Course-Taking (by Group)

Group	Taking Course/Total	Course Grade[a] Mean	Course Grade[a] Standard Deviation	Number of Years of Precalculus Courses Mean	Number of Years of Precalculus Courses Standard Deviation
A	10/10	–	–	4.5	0
B	6/6	3.4	0.6	4.3	0.3
C	7/7	3.6	0.6	4.0	0.5
D	10/10	3.6	0.6	3.9	0.6
		Course Grade[a] Mean	Course Grade[a] Standard Deviation	School Grade[b] Mean	School Grade[b] Standard Deviation
		Calculus I (Differential)			
A	9/10[c]	3.4	0.5	10.1	1.1
B	4/6	3.8	0.5	11.3	1.0
C	5/7	3.2	1.3	11.8	0.4
D	8/10	3.6	0.5	11.3	0.9
		Calculus II (Integral)			
A	9/10[c]	3.6	0.7	10.4	0.9
B	4/6	3.8	0.5	11.3	1.0
C	2/7	4.0	0	12.0	0
D	5/10	3.8	0.5	11.2	0.8

NOTE: None of the group differences in course grades and school grades was significant by an ANOVA.
[a]4 = *A*; 3 = *B*; 2 = *C*; 1 = *D*; 0 = *F*.
[b]8 = eighth grade, etc.
[c]While still in high school the missing person took calculus at a college.

With regard to the next level of mathematics in high school, 100 percent of Group A completed one year of calculus (one person did so at a community college as a high-school student), whereas 67 percent, 29 percent, and 50 percent of Groups B, C, and D, respectively, did so (table 4.3). Not shown in Table 4.3 is that high-school mathematics enrichment courses were taken mostly by Group B.

The grades earned by the students in the mathematics classes were uniformly high. As expected, Group A students took their mathematics at an earlier age than did students in all the other groups. Group B students took precalculus mathematics, but not calculus or enrichment courses, earlier than either Group C or Group D students (table 4.3).

AP MATHEMATICS EXAMINATIONS

Of the mathematics courses taken in high school, the most advanced and difficult are those that have as their goal the taking of the Advanced Placement Program examinations. Students are offered their choice of two AP mathematics examinations, Level AB and the more advanced Level BC. A high score on the Level AB examination can yield credit for a one-

TABLE 4.4. Performance of the Groups on the Advanced Placement Program Mathematics Examinations (by Group)

Group	Taking Exam/ Total	Mean Score[a]	Standard Deviation	Mean of Year Taken
			Calculus AB	
A	3/10	3.7	0.6	75.3
B	2/6	4.0	1.4	76.5
C	2/7	3.5	0.7	78.0
D	1/10	4.0	–	78.0
			Calculus BC	
A	6/10	4.2	0.8	75.6
D	2/10	3.5	0.7	77.0

[a]Grades on the APP exams can range from 1 (the lowest possible) to 5 (the highest possible).

semester college course in calculus, while two semesters of credit in college calculus can be gained from success on the BC examination. Grades on these examinations range from 1 to 5, where 3, 4, or 5 are considered high. Ninety percent of the students in Group A took these exams, which is a higher percentage than for the other groups (see table 4.4). Group A took mostly the BC exam. This was not true for the other three groups. Furthermore, Group A took these exams earlier, on the average, than the other groups.

COLLEGE COURSES AS A HIGH-SCHOOL STUDENT

Some of the students took college courses on a part-time basis for college credit while they were still in high school. With respect to college courses, Group A, with a mean number of 4.8 taken, was much more active than Groups B, C, and D, which took a mean of 0, .14, and .3, respectively. Group A had taken at least sixteen times as many college courses as the other groups. College courses taken by the students in Groups C and D were mainly in the field of mathematics. Especially disappointing, however, is Group B's lack of use of this educational alternative, coupled with the fact that not one member of the group had taken the AP Calculus BC exam. They had been the poorer achievers in SMPY's first fast-paced mathematics class and continued to be so thereafter.

COLLEGE BOARD ACHIEVEMENT TESTS

The performance of the four groups on nine of the fifteen achievement tests of the College Board can be seen in table 4.5. These tests measure the students' achievement in a high-school subject, usually during the eleventh or twelfth grade.

TABLE 4.5. Reported Performance on the College Board's Achievement Tests Taken by at Least One Person (by Group)

Group	N	Mean Score[a]	Standard Deviation	Mean of Year Taken	National Sample of 1978 College-Bound Students Taking the Test: Mean	National Sample of 1978 College-Bound Students Taking the Test: Standard Deviation	Percentile Rank of SMPY Students' Scores
				Math Level I			
A	1	800	–	76.0			99+
B	3	697	95	74.5	541	99	93
C	2	745	49	77.0			97
				Math Level II			
A	7	778	37	75.8			87
B	3	710	28	76.0	665	95	62
C	1	800	–	75.0			91
D	6	760	48	77.5			81
				English Composition			
A	6	683	102	75.8			94
B	4	655	41	76.3			90
C	4	589	169	76.5	512	105	74
D	4	727	17	77.5			98
				Biology			
A	1	710	–	77.0			92
B	2	555	78	75.0	544	111	50
				Chemistry			
A	4	630	99	75.5			67
B	1	540	–	77.0			40
C	2	620	71	76.5	577	102	63
D	3	727	47	77.7			91
				Physics			
A	2	670	85	77.0			72
B	2	545	92	75.5	591	106	35
				French			
C	2	615	35	76.5			72
D	2	640	28	77.5	552	109	75
				Spanish			
C	1	540	–	78.0	544	120	53
				Russian			
A	2	580	99	76.5	587	148	51

[a] The differences between groups were not significant.

Of special interest is the groups' performance on the mathematics achievement tests, Level I and the more difficult Level II. In table 4.5 it can be seen that all the groups scored extremely high. The mean scores are not far from the maximum score of 800, except for Group B's performance on Mathematics Level I, which they took at an early age. The percentile ranks of the mean scores were also high. Furthermore, the groups' mean score on Mathematics Level I was 189 points superior to the mean of a national sample of college-bound seniors, and on Mathematics Level II 96 points, also more than a standard deviation. Thus the performance of all the groups was excellent. Learning mathematics at a rapid pace is seen not to be detrimental to longer-term retention or achievement in mathematics, because if this were the case, we would expect Groups A and B to receive lower scores than the other two groups.

Several interpretations of the data can be made from the performance on the remaining achievement tests (see table 4.5). A high percentage of the students took the English Composition examination in high school. Group D made the best scores, but the members of Group D were also much older than the other groups' members when the test was taken. The mean difference between groups was significant by an ANCOVA controlling for year taken ($F = 3.6$, $p < .05$).

Another trend in the data of table 4.5 is that of the science examinations; Chemistry (with ten takers) was most popular. Performance on all the examinations was excellent and for the most part was above the means for a college-bound sample of high-school students (CEEB 1979).

AWARDS AND HONORS

In the two questionnaires the students were asked to report any awards and honors won, including National Merit Scholarship Corporation and mathematics contest participation. The National Merit competition is judged on the basis of high scores on the Preliminary Scholastic Aptitude Test.[2] All the groups, except perhaps C, did well in this competition (i.e., 100 percent of Group A, 66 percent of Group B, 75 percent of Group D, and 14 percent of Group C received at least a Letter of Commendation). Two members of Groups A and D did not take the PSAT.

With respect to scholastic awards and honors won in high school, 40 percent, 67 percent, 43 percent, and 80 percent of Groups A, B, C, and D, respectively, reported receiving at least one. In college the percentage of the groups' members reporting having received at least one award or honor ranged between 29 and 60 percent.

With regard to participation in mathematics contests (not including SMPY's talent searches), 60 percent of Group A reported having participated in at least one, while no one in Group B did and only 43 percent and 10 percent of Groups C and D, respectively, did.

COLLEGE ENTRANCE AND STATUS

Every student except one in Group C had entered a college or university on a full-time basis (see table 4.6). The students' ages at college entrance varied greatly, however. Group A was on the average two years ahead of Group B and at least three years ahead of Groups C and D in date of college entrance. Furthermore, the difference between groups in percentage of students entering college early is large (see column 10 of table 4.6). Of Group A, 90 percent entered early, while only 33 percent and 10 percent of Groups B and D, respectively, did so, and no one in Group C did. To test for significant differences between groups on date of college entrance, an ANOVA was performed. The difference was significant ($F = 9.7$, $p < .001$).[3] In addition, Group A students had on the average at least six times as many advanced-standing credits as students in the other groups when they began college. Appendix 4.2 updates where in college or graduate school each student in the four groups was as of the summer of 1980; clearly the students in the fast group are much ahead of members of the other groups.

COLLEGE INTELLECTUALISM AND STATUS SCORES

The colleges attended by the students were given, where available, an intellectualism and a status score obtained from the Astin (1965) scale. Astin (1965, p. 54) defines a four-year college with a high intellectualism score as having a student body that "would be expected to be high in academic aptitude (especially mathematical aptitude) and to have a high percentage of students pursuing careers in science and planning to go on for Ph.D. degrees." A four-year college with a high status score is defined as having a student body that "would be expected to have a high percentage of students who come from high socioeconomic backgrounds and who themselves aspire to careers in enterprising fields (lawyers, business executives, politicians)" (ibid.). The scores are T-scores having a mean of 50 and a standard deviation of 10.

The means of the college ratings by group were all above 50, but they were not significantly different from each other by group. The mean intellectualism scores ranged between 57 for Group C and 67 for Group A. Group A attended the most academically difficult colleges or universities,[4] followed by B, D, and C, in that order. In terms of the status scores, Group D came out highest, with a mean of 59, followed by A, B, and C, all with mean scores of 55. Clearly, the four groups attended intellectually and socially elite schools. Appendix 4.2 lists the colleges attended by the students.

TABLE 4.6. Year of Reported College Entrance and Comparison of Reported College Status at Time of Entrance and Degree of Educational Acceleration (by Group)

	Year of College Entrance										
Group	1974	1975	1976	1977	1978[a]	1979	Mean of Entrance Year[b]	Standard Deviation	Percentage Entering Early	Mean Number of Advanced Standing Credits[c]	Standard Deviation
A	2	1	4	1	2	0	75	1.4	90	24.7	11.9
B	0	0	2	0	4	0	77.3	1.0	33	2.0	3.1
C	0	0	0	0	6	0	78	0	0	3.4	3.8
D	0	0	0	1	6	3	78.2	0.5	10	4.1	7.7

		Degree of Acceleration (Percentage of Group)					
Group	*N*	None	Some, But Less than One Year	At Least One Year, But Fewer than Three Years	Three Years or More	Mean[d, e]	Standard Deviation
A	10	0	10	20	70	2.6	0.7
B	6	50	17	33	0	0.8	1.0
C	7	29	43	29	0	1.0	0.8
D	10	30	50	20	0	0.9	0.7

[a] Expected year for most of these students.
[b] The difference between group means was significant ($F = 9.7, p < .01$).
[c] The difference between group means was significant ($F = 5.0, p < .01$).
[d] Acceleration was coded as follows:
0 = None
1 = Less than 1 year
2 = One year or more but fewer than 3
3 = Three years or more
[e] The difference between group means was significant ($F = 13.7, p < .001$).

TABLE 4.7. Reported Use of Accelerative Options

Group	*N*	Mean Number of Grades Skipped	Standard Deviation	Mean Number of College Credits Received from APP Exams
A	10	2.0	1.2	8.0
B	6	0.7	1.0	2.0
C	7	0.4	0.8	3.4
D	10	0.1	0.3	3.3

ACCELERATION

Each student was rated on the degree to which his or her educational progress had become accelerated eight years after the beginning of the Wolfson I class (see table 4.6). Members of Group A were much more accelerated than members of the other groups. In Group A 70 percent were accelerated by three or more years, while not one person in the other groups was. Members of all the other groups were, however, somewhat accelerated on the average, but no big differences can be seen between Groups B, C, and D. The differences between groups in acceleration were statistically significant ($F = 13.7$, $p < .001$).

The way the students' acceleration was achieved is shown and contrasted by group in table 4.7. Group A made use of all the options and to a much greater extent that did the other groups, which used the accelerative options to about the same degree. Not included in table 4.7 is the fact that Groups A and B had been initially accelerated in mathematics as part of the Wolfson I class. Appendix 4.2, where the status of each student's educational progress as of June, 1980, is shown, highlights the results of tables 4.6 and 4.7.

ACCELERANTS' VIEW OF ACCELERATION

The accelerated students in all groups were asked to rate how they felt their educational acceleration had affected their social and/or emotional development. They were also asked to reconsider their decision to accelerate. Overall, the students felt positive about their acceleration.

With regard to social and/or emotional development, only one (in Group C) of the twenty-two accelerants felt that acceleration had affected him much to the worse. This same person, however, would accelerate his educational progress again if he had a chance to reconsider the decision. In contrast, 18 percent of the accelerants felt acceleration had affected him/her much for the better. On the average, all the groups felt that acceleration had slightly benefited their social and/or emotional development and certainly had not hindered it. There were group differences, but they

Standard Deviation	Mean Number of Credits Received for College Work Completed in High School	Standard Deviation	Percentage Completing College in Fewer Than Four Years	Percentage Receiving Master's Degree Concurrently with Bachelor's
6.0	11.0	8.9	50	10
3.1	0	0	0	0
3.8	3.8	3.8	0	0
7.8	3.2	0	10	0

were not found to be significant by an ANOVA. It must be noted that Group A was much more accelerated than the other groups and still held overall positive feelings.

How did the accelerants reconsider their decision to accelerate? Most students would accelerate at least as much as they had already done. Only one student, who had skipped three grades and received a high number of college credits for AP work, would in retrospect accelerate somewhat less. Thus in conclusion it can be said that accelerated students tend to view acceleration as being beneficial.

COLLEGE MAJORS: SCIENCE VERSUS MATHEMATICAL SCIENCES

At least 50 percent of students are majoring in either science (including engineering) or mathematical sciences in college (ranging from 50 percent for Group B to 80 percent for Group A). Mathematical sciences are most popular for Group A, with 50 percent majoring in them, while science is at least equally as popular for the other groups (ranging between 30 and 40 percent majoring in science for all four groups). For Group A, computer science is by far the most popular field. Each student's major is shown in Appendix 4.2.

COLLEGE COURSE-TAKING

The number of undergraduate courses taken in mathematics, science, and engineering by the summer of 1980 and the mean grades received by group can be seen in table 4.8. Even though they had the most advanced standing in mathematics, members of Group A had taken the greatest number (6.9) of mathematics courses by summer, 1980. This was also true for science and engineering. But of course Group A had been in college much longer. The differences between the groups in grades received were not significant, but the differences in number of courses taken in mathematics and engineering were ($F = 5.1, p < .01$ for mathematics and $F = 4.6, p < .01$ for engineering). Their mean grades were for the most part

TABLE 4.8. Reported Mathematics, Science, and Engineering Course-Taking in College (Full-Time Students, by Group)

Group	*N*	Mean Number of Mathematics Courses[a]	Standard Deviation	Mean Grade in Mathematics	Standard Deviation	Mean Number of Science Courses	Standard Deviation	Mean Grade in Science	Standard Deviation
A	9	6.9	4.1	3.0 ($N = 8$)	0.7	6.6	3.7	3.3 ($N = 8$)	0.4
B	6	4.0	2.9	3.0 ($N = 5$)	0.6	4.3	1.6	3.1 ($N = 6$)	0.7
C	6	3.2	1.2	3.3 ($N = 6$)	0.5	3.8	3.9	3.4 ($N = 5$)	0.5
D	9	1.9	1.5	3.1 ($N = 7$)	1.3	4.1	3.2	3.2 ($N = 9$)	0.6

Group	*N*	Mean Number of Engineering Courses[b]	Standard Deviation	Mean Grade in Engineering	Standard Deviation
A	10	5.7	5.5	3.2 ($N = 7$)	0.7
B	6	1.3	2.4	3.8 ($N = 2$)	0.2
C	6	1.0	2.5	2.5 ($N = 1$)	0
D	9	0.1	0.3	2.0 ($N = 1$)	0

[a] The differences between the groups were significant by an ANOVA ($F = 5.1$, $p < .01$).
[b] The differences between the groups were significant by an ANOVA ($F = 4.6$, $p < .01$).

above a *B,* and many courses were taken, except for in engineering for Groups B through D.

USE OF EDUCATIONAL OPPORTUNITIES

The students were asked to rate how well they had made use of all their available educational opportunities. For the students in all groups, the mean response was "above average." No significant differences emerged between the groups, although it would objectively seem that Group A had made the best use of all available educational opportunities.

EDUCATIONAL AND OCCUPATIONAL ASPIRATIONS

The educational aspirations for all students were for the most part high. The means for all groups were to obtain more than a master's degree. Not one student aspired to obtain less than a bachelor's degree. Only 51 percent of students in the general population aspire to obtain a bachelor's degree or more (Charles Kettering Foundation 1980).

The occupational status of each student's career goal was rated by the Reiss (1961) scale. The average status occupation for the norm group on this scale was 70, which is a score assigned to a nurse. On this scale the highest score given to an occupation was a dentist, with 93 points, and the lowest was to a tobacco laborer, with 20 points. For the students in the four groups the means of the occupational status of their career goal ranged from 81.5 (Group B) to 84.2 (Group C). Occupations falling into that range on the scale include engineers and college professors. Thus the students in all groups have high educational and occupational aspirations, with no significant group differences.

SMPY'S INFLUENCE

The final item of interest is how the students felt their association with SMPY had helped them educationally. Not unexpectedly, the students who had remained in Wolfson I felt that SMPY had helped them educationally more than did the students in the other groups ($t = 2.9, p < .01$). The students in Groups A and B felt that SMPY had helped them considerably, while Groups C and D felt that SMPY had helped them slightly more than not at all.

Summary

Participation in a fast-paced mathematics class for highly mathematically precocious junior-high-school students appears to have many long-term benefits, not only in time needed to complete the study of mathematics but also in time needed to complete a student's education. This has been demonstrated by this study and by the evaluation of the second fast-paced mathematics class, called Wolfson II (see Appendix 4.3). Because of the small number of students, however, most differences between the students participating at various levels were not found to be significant.

Most of the students who were extended the invitation to join the Wolfson I class participated in at least one of SMPY's talent searches. At the time of the talent search the students completing Wolfson I in the fast group received the highest SAT-M scores, followed by the students finishing in the slow group. The difference between groups was significant ($p < .01$). On SAT-V at talent-search participation the slow group, followed by the fast group, had the highest scores. Again, the difference between groups was significant ($p < .05$). In high school, when the SAT was taken again, no significant difference was found between the groups on SAT-V, but a significant difference was found on SAT-M ($p < .05$). Groups A and B received the higher scores. We do not know whether the students in the Wolfson I class earned significantly higher SAT-M scores in the talent search and in high school than the students who dropped out or did not participate because of their participation in the class or because they were initially abler. Most likely it is a combination of the two, since the students in Wolfson I also earned higher SAT-V scores,[5] but their superiority on SAT-V was not as great as that on SAT-M. Furthermore, all the students had met the same ability criteria before the class was begun. Therefore, the fast-math class itself may serve to boost the students' aptitudes for mathematics.

The mathematics course-taking in high school was obviously affected by Wolfson I. Many more of the students who stayed in the program finished calculus in high school. Furthermore, many more of the students who finished in the fast group took the AP mathematics examinations, especially Level BC, and took many more college mathematics courses (and other college courses) while still in high school. When the College Board's mathematics achievement tests were taken in high school, the students in all groups who took them tended to make quite high scores. Because Groups A and B did not score less well than Groups C and D (whose members did not participate in the fast-paced mathematics program), we conclude that having covered the content of the high-school mathematics curriculum in an accelerated manner did not hinder long-term retention or achievement or leave holes and gaps in students' knowledge. The scores on the other, nonmathematics, achievement tests were mostly above the mean score for college-bound high-school students.

The students who completed Wolfson I in the fast group became much more accelerated in their educational progress than the other students. The amount was more than could be accounted for by just the fact that the students were in Wolfson I. The difference between groups was significant

($p < .001$). On the average the accelerated students had positive feelings toward their acceleration. Most would do it over again, perhaps to an even greater extent.

All the students except one are attending or have attended college full time at, on the average, academically and socially elite schools. Most of the students majored in science or mathematically related fields. More of the students in the fast group majored in mathematically related fields than did those in the other groups. Furthermore, so far in college they had also taken more courses in science, mathematics, and engineering than students in the other groups. No significant differences were found between the groups in terms of educational aspirations and status of their career goals.

The most successful students in Wolfson I (i.e., the ten finishing in the fast group) had achieved much more by summer, 1980, than the six students who finished in the slow group. Since the slow group was comprised mostly of girls (4:2) and the fast group mostly of boys (7:3), this difference in achievement could perhaps be due partly to the unwillingness of girls to accelerate their educational progress, especially in mathematics (Fox 1976; Daggett, chapter 9 of this volume).

Thus a fast-paced mathematics class offered to mathematically precocious students does have educational benefits eight years after the class was conducted. The especially successful students in that class have achieved much more in high school and college than the students who did not participate but who had been essentially equally able (see Appendix 4.2).

Conclusions

How much did the actual procedures and content of the Wolfson I class help the members of Group A, who obviously have done extremely well academically thus far? As is inevitable in a study involving highly meaningful, demanding activities with human beings, one cannot fully disentangle the influences of general and special abilities, motivation, and facilitation by the student's family and teachers. Comparisons of the four Wolfson I groups help, especially because (as shown in table 4.2) ability differences in high school were not large.

The five boys and five girls who chose not to attend Wolfson I (Group D) seem to have done so mainly because they had competing activities in the summer of 1972. Not much time elapsed between the invitation to enroll and the June 24 starting date, so some youths – likely, those from the most affluent families – had made other plans. This inference receives some support from inspection in Appendix 4.2 of the colleges they later attended. As already noted, Group D students attended higher-status colleges than did students in Groups A, B, or C. As of the 1980–81 academic year not a single member of Group D was accelerated in college placement by age, and three of the ten were less than age-in-grade. In contrast, only one of the Group A students was not accelerated. All but one of the seven males were more than one year ahead of their age-mates. One earned his master's degree at age 19, another at age 20, and a third at age 21. One is the fourth youngest person to receive a bachelor's degree from Johns Hopkins in its 105-year history; until 1981 he was the youngest graduate since 1887 (Stanley & Benbow 1982).

Therefore, even though it is possible that superior motivation alone accounts for the splendid showing of Group A, we consider such an interpretation most implausible. What has been demonstrated clearly is that when highly able youths are offered the opportunity to forge ahead far faster and more rigorously in precalculus, many will accept the offer, and a considerable percentage of those will make mighty educational and professional strides, probably for the rest of their lives. This observation, based on the Wolfson I class, is amply supported by SMPY's many replications of and extensions of the fast-paced mathematics model in a wide variety of curricular situations (e.g., Keating, Wiegand, & Fox 1974; George & Denham 1976; Stanley 1976; Mezynski & Stanley 1980; Bartkovich & George 1980; Bartkovich & Mezynski 1981; Mezynski, McCoart, & Stanley, chapter 6 of this volume). It demonstrates the multiplicative effect of the accumulative advantage (here, participation in the special mathematics class) that Zuckerman (1977) describes as characterizing Nobel Laureates.

Of the three girls in Group A, two are accelerated one year each, both by finishing high school one year early. One girl in Group B finished college two years younger than average, as did one boy. One boy in Group C is accelerated one year in college—actually, by high-school entering rules in his state, by just two days, because he was born on January 2. As noted earlier, no one in Group D was accelerated. In agreement with these results, SMPY's researchers have usually found only a few girls accelerated even a single year by the time the baccalaureate is awarded; an appreciable percentage of the boys proceed far faster (see Daggett, chapter 9 of this volume).

There are some signs that this gender differential is changing, however. Twenty-five of the Johns Hopkins 632 fall of 1980 entrants were at least two years accelerated in grade placement relative to chronological age; 9 of them (36 percent) were female. This is slightly greater than the percentage of Johns Hopkins students who are female. One of these girls entered at age 13 with sophomore standing, having already completed one year each of college inorganic and organic chemistry, calculus, and biology, and one semester of physics.

Paradoxically, however, whereas seven of the twenty males in Groups

A through D attended Johns Hopkins, not a single one of the thirteen females did. The chi-square for this difference was 3.87, $p < .05$. That unintentional, unexpected side (or main?) effect of the Wolfson I recruitment and instruction suggests that ingratiating effects for an institution of having youngsters study at it may be related to gender. It will be interesting to compare the mathematics course results with those found for fast-paced verbal courses conducted by Johns Hopkins to see whether a similar pattern holds for them.

Notes

1. He later joined SMPY's second fast-paced mathematics class, Wolfson II (see Appendix 4.3).
2. The formula for qualifying as a semi-finalist is 2(PSAT-V score) + 1(PSAT-M score). The minimum composite score varies from state to state and is greatest in those states where the highest-scoring students reside.
3. Acceleration by the groups is discussed further later in this chapter.
4. Six of the ten students in Group A attended Johns Hopkins.
5. High verbal ability was found to be important for success in a fast-paced mathematics class (Fox 1974; George & Denham 1976).

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APPENDIX 4.1: Supplementary Questionnaire Used to Update the Educational Progress of All the Students Eligible for the Wolfson I Class

THE JOHNS HOPKINS UNIVERSITY • BALTIMORE, MARYLAND 21218

STUDY OF MATHEMATICALLY PRECOCIOUS YOUTH (SMPY)

Please reply care of: DEPARTMENT OF PSYCHOLOGY

PROFESSOR JULIAN C. STANLEY, Director of SMPY
Ms. LOIS S. SANDHOFER, B.A., Administrative Assistant
127 Ames Hall, (301) 338-7087

Mr. WILLIAM C. GEORGE, Ed.M., Associate Director
125 Ames Hall, (301) 338-8144

Ms. CAMILLA P. BENBOW, M.A., Assistant Director
126 Ames Hall, (301) 338-7086

QUESTIONNAIRE FOR EVALUATING SMPY'S FAST-PACED MATHEMATICS PROGRAM

Please fill out carefully and completely all of the questionnaire below that applies to you. Please print or type all answers and send the fully completed questionnaire as soon as possible to the address on the letterhead. All information will be kept strictly confidential; you will not be publicly identified with the information herein in any way.

NAME: ____
First   Middle   Last   (Maiden if applicable)

Permanent Address: ______________________________ Telephone: (   ) ________
Street   City   State   Zip   (Area Code)

Temporary Address if different from above: ______________________________
Street

______________________________ Telephone (   ) ________
City   State   Zip   (Area Code)

1. Are you currently employed full-time? (Circle one.)   Yes   No

   If yes, please supply the following information about your present and past post-high school occupations in chronological order.

	Type of Occupation	Duties Involved	Employer	Dates of Employment
1)				
2)				
3)				

   If you need more space, please continue on a separate sheet.

2. Please check the box that applies to you with regard to your attendance at an institution of higher education (including technical school).

   ___ I am currently a full-time undergraduate student.

   ___ I have graduated from college and am not furthering my education at the present time.

   ___ I have graduated from college and am furthering my education on a part-time basis.

   ___ I have graduated from college and am (or will be this fall) furthering my education on a full-time basis.

___ I am currently a part-time undergraduate student after having attended full-time.

___ I am a part-time undergraduate student.

___ I am not currently enrolled as a student but was previously.

___ I am not and have not been enrolled as a student in an institution of higher education. (Go to question 3.)

a. Which school are you currently, will you, or were you attending? (Do not list schools from which you have transferred.) ________________

b. Dates of attendance: ________________

c. If you have graduated, please indicate the date of graduation: ________
Month/Year

From which school if different from above? ________________

d. What is or was your undergraduate major? ________________

e. If you have switched majors in college, please list the previous one(s) in chronological order. ________________

f. If you are furthering your education beyond college, please name the planned field. ________________

g. Please list the titles of the mathematics course(s) you have already taken as an undergraduate (including your grade in this course(s) and the semester(s) of attendance.) If you prefer, send us a copy of your transcript.

Mathematics Course	Final Course Grade	Semester(s) of Attendance (Include semester and date)
1.		
2.		
3.		
4.		
5.		
6.		

If you have taken more mathematics, please continue on a separate sheet.

h. Please list the title(s) of the science course(s) (including engineering and computer science) you have already taken as an undergraduate in college, your final grade in these courses, and the semester(s) of attendance. If you prefer, send us a copy of your transcript

Science Course	Final Course Grade	Semester(s) of Attendance (Include Semester and Date)
1.		
2.		
3.		
4.		
5.		
6.		

If you have taken more science, please continue on a separate sheet.

i. Please list any awards, honors, or scholarships you may have won as an undergraduate or graduate of college (Phi Beta Kappa, etc.). ____________

____

3. Please list the college-level mathematics courses (if any) that you are planning to take in the future. ____

4. Please describe your career goal (i.e., professor of mathematics or a practicing pediatrician). ____

5. Have you been accelerated in your educational progress (circle one)? Yes No

a. If no, do you wish you would have been? Yes No (Circle one.)

b. If yes to Question 5, please circle the letter of the sentences that are applicable to you and then complete them.

1) I skipped the following grades: ________________

2) I took ____ Advanced Placement Program (APP) examinations for which I
(Number)
received ______ credits of advanced placement in college.
(Number)

3) I was accelerated in subject matter placement in ____ different subjects.
(Number)

4) I took college courses on a part-time basis as a secondary school student, for which I received ______ credits of advanced standing in college.

5) I finished college in ___ years, rather than 4.

6) I received my master's degree concurrently with my bachelor's.
(Circle one.) Yes No

7) Other (Departmental examination, etc. Please specify.): ______________

____

c. If you were to reconsider your decision to accelerate, which one of the following would best describe your thoughts (check the most appropriate box)?

☐ I would not accelerate my education at all.

☐ I would accelerate my education somewhat but not as much as I have done.

☐ I would accelerate my education to the degree which I have already done.

☐ I would accelerate my education somewhat more than what I have already done.

☐ I would accelerate my education much more.

6. How important do you feel mathematics will be or is in your career? (Circle one.)

Very Fairly Slightly Not very Not at all

7. If you have taken the Graduate Record Examinations (GRE), please supply the following: GRE Quantitative Score___ GRE Verbal Score ___
Advanced test score ____ in ________________________ area.

I hereby certify that I have read over my responses carefully and thoroughly. They are as complete and accurate as I can make them.

Please return this questionnaire to:

____
Signature

Ms. Camilla Benbow
SMPY, Dept. of Psychology
The Johns Hopkins University
Baltimore, Maryland 21218

APPENDIX 4.2: Educational and Occupational Status of the Students Eligible for the Wolfson I Class as of June, 1980

Student (Sex) (Date of Birth)	Undergraduate Institution	Year Graduated	Major	Graduate Institution	Field or Degree	Employer
Fast Group (N = 10)						
M 4/10/63	Johns Hopkins University	Senior	Mathematical Sciences			
M 8/9/60	Johns Hopkins University	B.Engr.S., 79	Mechanics and Material Sciences	Part-time, Carnegie-Mellon	Master's in Mech. Eng.	Electrical/Mechanical Engineer, Westinghouse
F 8/6/60	Princeton University	Junior	Architecture			
M 5/5/60	Johns Hopkins University	B.A., 79	Electrical Engineering, Biomedical Engineering	Drexel Institute	Master's, Elec. Engr., 12/81	Electrical Engineer, AAI Corporation
F 5/2/60	University of Virginia	Senior	Russian			
M 4/12/60	Johns Hopkins University	B.A., 78	Mathematical Sciences	University of California, Santa Barbara	Computer Science, Master's, 81	Westinghouse
M 12/4/59	Johns Hopkins University	B.A., 1/77	Quantitative Studies	University of Chicago, Business	M.B.A., 1979 Ph.D. in Finance, 12/81	
M 10/29/59	Johns Hopkins University	B.A., 80 M.S.Engr., 80	Mathematical Sciences	Carnegie-Mellon	Computer Science	
F 2/1/59	University of Michigan	B.S.Engr., 80	Computer Engineering			Assoc. Engineer, JHU Applied Physics Lab.
M 9/18/58	University of Steubenville	B.A., 79	Mathematics			Computer Programmer, Mellon Bank NA

Student (Sex) (Date of Birth)	Undergraduate Institution	Year Graduated	Major	Graduate Institution	Field or Degree	Employer
			Slow Group ($N = 6$)			
F 11/12/60	Virginia Polytechnic Institute	Withdrew, 5/79[a]	Forestry			Paid by CETA to take accounting and office skill courses
M 9/20/60	Johns Hopkins University	B.A., 80	Economics	University of Pennsylvania	Law	
F 5/30/60	College of Wooster	B.A., 80	Religion	Yale University	Social Work	
F 5/20/60	University of Pennsylvania	Junior	Computer Science, Engineering			
M 5/6/60	University of Delaware	Junior	Civil Engineering			
F 1/5/60	Virginia Polytechnic Institute	Junior	Civil Engineering			

[a] Later entered The Bryant Institute, Tulsa, Oklahoma; graduated in data processing, 9/82.

Student (Sex) (Date of Birth)	Undergraduate Institution	Year Graduated	Major	Graduate Institution	Field or Degree	Employer
			Group That Dropped Wolfson I ($N = 7$)			
M 10/5/61	Did not attend[a]					
M 1/2/61	University of Maryland, College Park	Junior	Electrical Engineering			
M 10/12/60	University of Delaware	Junior	Chemistry			
M 9/20/60	University of Virginia	Junior	Economics			
M 4/11/60	University of Delaware	Junior	Accounting			
F 2/29/60	James Madison University	Junior	English			
F 1/30/60	University of Richmond	Junior	Political Science, Sociology			

[a] Later entered and then withdrew from the University of Tampa, Florida.

Student (Sex) (Date of Birth)	Undergraduate Institution	Year Graduated	Major	Graduate Institution	Field or Degree	Employer
Group That Never Enrolled in Wolfson I ($N = 10$)						
M 12/28/60	Wharton School, University of Pennsylvania	Junior	Accounting and Finance			
M 10/8/60	Youngstown State	Withdrew				
F 8/8/60	University of Denver	Junior	Biology			
M 6/9/58	Harvard	Sophomore	Biochemistry			
F 5/9/60	University of Chicago	Sophomore	Mathematics			
F 4/2/60	Yale University	Junior	Biology			
M 2/26/60	Towson State University	Junior	Accounting, Computer Science			
M 2/21/60	Maryland Institute College of Art	Junior	Graphic Design			
M 2/6/60	Georgia Institute of Technology	Junior	Mechanical Engineering			
F 1/6/60	Western Maryland College	Junior	English and German			

APPENDIX 4.3: Evaluation of the Wolfson II Class

In evaluating the long-term effects of SMPY's fast-paced mathematics classes, we followed up the students who were eligible for and participated in Wolfson I, the results of which were discussed in this chapter, and also the students eligible for and participating in the second class of this kind conducted by SMPY (Wolfson II). The initial selection procedures and results for this class can be found in George and Denham (1976). The students in the Wolfson II class were somewhat older (mainly end-of-year eighth-graders) than those in Wolfson I. Furthermore, to be eligible for the class the students had to have scored at least 500 on SAT-M, 400 on SAT-V, and above a combined score criterion on two standardized tests of knowledge of algebra I. Of the ninety-two students eligible for the class, thirty-three participated. All but two students began to study algebra II in June, 1973.

George and Denham (1976) discussed the success of this class. In summary, twenty-three students of the thirty-three (the fast group) mastered algebra II and III and plane geometry. Among the twenty-three, fifteen students also mastered trigonometry, and fourteen analytic geometry. As a result, twelve students were able to enter a calculus class in the fall of 1974, 120 class hours after the start of algebra II. An additional five students in Wolfson II (but in its slow group) successfully mastered algebra II and plane geometry. One person among them also completed algebra III. Five youths dropped the class before completing the study of algebra II.

The longitudinal evaluation of Wolfson I and II resulted in quite similar findings. The most successful students in both classes (i.e., the fast groups) achieved much more in high school and became more accelerated than the other students eligible for or participating in the classes. Furthermore, the students completing the program in the slow groups were more successful than the students who dropped the classes or never enrolled.

It was found that the students in the fast group were somewhat abler than the other students on the SAT in high school but not on the College Board's achievement tests. It was of special interest to take note of the students' performance on the College Board's Math Level I and the more difficult Math Level II achievement tests. On both tests most of the students earned nearly the top score possible. Thus we can conclude with confidence that learning mathematics at a rapid and accelerative pace is not detrimental to long-term achievement or learning. The opposite seems to be true, because many more of the successful students in the Wolfson classes than students not in the classes took calculus in high school, took the AP examinations in calculus, and in college took more courses in mathematics. In addition, those students showed more interest in mathematics.

There was one major difference between the Wolfson I and II classes. This was in terms of acceleration. The successful students in Wolfson I were much further ahead educationally (course-work and acceleration) than the successful students in Wolfson II, although both fast groups were much further ahead of the other groups. This difference might be related to the age difference between the two classes. Members of Wolfson II were older than members of Wolfson I when they began the class. Perhaps by the time a bright student reaches the end of the eighth grade he or she has lost some motivation because of the extra time spent in a classroom not geared to his or her intellectual level. Thus it seems more beneficial or necessary to find and educationally stimulate students earlier than the eighth grade.

Finally, members of both classes felt that their association with SMPY had been of considerable help to them, and they viewed their acceleration as benefiting them positively.

The fast-paced mathematics classes did have long-term educational benefits. Since the students involved also had positive feelings toward their experiences in class and with SMPY, we conclude that this is one excellent way of catering to the differential needs of the intellectually talented.