ADVANCED PLACEMENT ORIENTED CALCULUS FOR HIGH SCHOOL STUDENTS

KAREN MEZYNSKI
Johns Hopkins University*

JULIAN C. STANLEY
Johns Hopkins University

In recent years considerable efforts have been made to develop educational programs suitable for high-ability students. As part of these, the Study of Mathematically Precocious Youth (SMPY) at Johns Hopkins University has been concerned with facilitating the education of youths who reason exceptionally well mathematically (Stanley, 1977). One successful SMPY project was the development of fast-paced classes in mathematics. Detailed descriptions of some of the earlier classes are given by Fox (1974), George (1976), George and Denham (1976), and Stanley (1976).

Rationale

In this paper two supplementary calculus classes sponsored by SMPY are described. The first of these (Class I) provided, for many of its students, a continuation of the instruction received in previous fast-paced precalculus courses. A brief, preliminary report on Class I was given by Stanley (1976, pp. 146–150). The second class (Class II) was intended to replicate and extend the findings of the first; the students in Class II were older and had more background in regular mathematics classes but less in fast-paced mathematics instruction than did Class I.

Both classes were designed to maximize the students' performance on the College Entrance Examination Board's Advanced Placement (AP) examination in calculus, higher level (BC). The AP level BC syllabus was chosen to provide coverage of all topics normally taught in two semesters of college calculus at a selective college or university. Although many high schools offer a level AB course (equivalent to one semester of college calculus), fewer high schools have enough demand to offer the more rigorous level–BC-oriented course. Some schools do not offer any calculus course that will prepare students well for either level of the AP examination. Quite a few do not offer calculus at all. SMPY's calculus course was intended to supplement the students' high school instruction by covering more topics faster and in greater depth.

Class I was held during the 1974–75 academic year; Class II followed one year later. Both classes met at least 2 hours each Saturday for 30 weeks.

November 1980   347

Students were required to be concurrently enrolled in a high school calculus course. Thus, the Saturday meetings were additional. SMPY offered no direct credit for participation. There were, however, several worthwhile reasons such as the following for attending. First, the students were expected to learn college-level calculus well and thereby improve their scores on the AP examination (as compared with taking only the high school class). Second, the breadth and depth of the calculus topics to be taught were greater than most high school classes offered; it seemed likely that students participating in SMPY's course would finish calculus with excellent preparation for higher-level mathematics courses. The third benefit was the intellectual challenge the class provided; the students were well motivated and clearly enjoyed mathematics. Some of these students (particularly in Class II) had never before been exposed to a mathematics classroom situation where they were sufficiently challenged. For many of them, most school work had always been easy.

Procedure

All aspects of Classes I and II were kept as similar as possible. The course was taught by a professor of mathematics from a nearby college. Aside from his considerable general college teaching experience, this instructor also had previously taught one of SMPY's fast-paced algebra courses. During the year, he covered all chapters of Leithold's (1972) textbook. In addition, official AP aids were used throughout the course.

Homework problems were assigned each week, to be completed by the next meeting. Homework was collected and graded by the instructor or his teaching assistant. The students were expected to turn in their thoroughly prepared assignments regularly. Homework was regarded as being especially important, since students could easily fall behind the quick class pace if they did not consolidate the lessons well.

In-class examinations were given six times during the year, at approximately 1-month intervals; they were the same ones given to the instructor's college classes in Calculus I, II, and III. The tests ranged from 60 to 80 minutes in allotted time. Homework assignments and test grades served to monitor each individual's progress and also provided important feedback. Some standardized testing was used in each class as well to compare the class members' knowledge with national performance levels.

Subjects

Although Classes I and II were identical in course structure and goals, there were important differences between the student populations with respect to age, high school grade, means of selection, and previous mathematics training. Fifteen students, all of them male, enrolled in Class I; 11 had attended SMPY fast-paced mathematics classes. Two students dropped out fairly early, but the remaining 13 completed the course. The members of Class I were, on the whole, quite young. Five had been accelerated in

school 1 to 3 years. At the start of the class, students ranged in age from an 11-year-old ninth grader to three high school seniors who were almost 17; the mean age was 14.9 years. Most of the Class I students were in the 10th grade.

SMPY organized Class II to examine the effects of supplementary instruction on an older and somewhat less select (but nonetheless mathematically talented) group of students. Approximately 60 students were invited by mail to participate in Class II. Some had been tested previously by SMPY through its talent searches (see George, 1979a), and others were high scorers in a mathematics contest for 11th graders sponsored by Johns Hopkins University's mathematics department. In addition, information letters were sent to high school mathematics teachers in the Baltimore area, asking them to contact eligible students. Eligibility requirements were based on SAT or PSAT scores. Minimum scores were 600 on the mathematics section and 500 on the verbal section, or, alternatively, a mathematics score of at least 600 and a combined score of at least 1100. (One exception was made. That student performed well and remained in the class until the end; he earned a 4 on the level BC examination.)

Eighteen boys and five girls enrolled in Class II; only two had been in SMPY's precalculus classes. Five students were in the 11th grade and 18 were in the 12th grade. The class members' SAT scores ranged from 610 to 800 on the mathematics section, with a mean score of 709, and from 360 to 790 on the verbal section, with a mean of 571.

The attrition from Class II was much higher than from Class I, perhaps because the students were not as carefully screened in the second class. Of the 23 initial enrollees, 11 (48%) dropped out of the course, most of them quickly. The high school grade was not a highly differential factor in attrition: 9 of the seniors (50%) and 2 of the juniors (40%) dropped out. Ability, as measured by SAT scores, did not differentiate the students who completed the course from those who dropped out. The average mathematics score was 696 for the dropouts and 722 for the students who did not drop. The average verbal score was 584 for the students who dropped out and 565 for those who remained. Three girls (60%) and eight boys (44%) dropped the class. All the students who dropped out of the course probably had enough ability to remain in the class, but they lacked the interest or the motivation to do the supplementary work. Both Classes I and II challenged the students much more than their school mathematics classes; perhaps most of the Class I students accepted the challenge more readily due to their previous fast-mathematics experiences. Also, the course was time-consuming; homework preparation was expected to take at least 5 hours each week. The older Class II students may have had other obligations (such as part-time jobs) that made it difficult for them to devote the necessary amount of time.

Standardized Testing Prior to the AP Examination

Class I was administered Forms A and B of the Educational Testing

Service's (1964) Cooperative Mathematics Tests in calculus (CMTC). Form B was given February 1, and Form A was given in May, shortly before the AP examination. On Form B of the CMTC, 11 of the 13 students scored above the 99th percentile on the national college norms for students who have completed two semesters of calculus. The other two students scored at the 98th and 94th percentiles. According to the more stringent high school norms, only two students scored below the 94th percentile; one was at the 88th and the other at the 76th percentile. On Form A, every member of Class I scored above the 99th percentile of the national college norms, and all the test scores were at the 94th percentile or better for the high school norms.

Class II was administered Form B of CMTC in early May. Nine of the 12 students scored above the 99th percentile of national college norms, whereas two scored at the 98th and one at the 92nd percentile. On the stricter high school norms, nine scores were at or above the 94th percentile; the lowest score was at the 82nd percentile.

In April, a practice AP level BC calculus examination was given to Class II. The May 1973 test was used; students took both parts I and II under standard 3-hour testing conditions. The instructor followed AP guidelines in grading the test. Out of a total possible 210 points, raw scores on the practice test ranged from 78 to 169. The mean was 117.5, with a standard deviation of 30.7. The average change in score from the practice test to the May 1976 test was a gain of 10.8 points; the change in scores ranged from -10 to 65 points. On the whole, achievement on the practice test predicted performance on the May examination well; the correlation between scores on the two tests was .75 ($p < .01$).

Results

The AP examination in level BC calculus consists of a multiple-choice section and a free-response section, each usually requiring 90 minutes of testing time. The student's grade is based on an equal weighting of the sections. Responses are scored on a total possible 210 points, and then grades are reported to examinees on a 5-point college-oriented scale. A "5" indicates that the student is extremely well qualified to receive credit for two semesters of college calculus. A "4" means well qualified (also receiving two semesters' credit); a "3" means qualified (giving one or two semesters' credit, depending on the particular school—many prestigious colleges and universities give two semesters' credit for a "3"); a "2" means possibly qualified; and "1" indicates no recommendation of credit.

The results of the AP test for Class I are given in Table 1. Nine of the 13 students received the highest obtainable grade, 5. This represented approximately 0.7% of all 5s awarded nationally on the BC calculus examination in May of 1975. (Twenty-four percent of the 7112 students who took the level BC calculus examination in May 1979 scored a 5.) Three students received a grade of 4, and one student received a 3.

Class II's results are shown in Table 2. One-third of the 12-member class received 5s, and another third received 4s. One-fourth made 3s, and one student scored a 2.

Table 1
Age, Grade, and Test Information for Class I

Raw score on AP calculus test[a]	Scores on 1–5 scale	Percentile rank of AP score	Age to nearest month as of 1 Sept. 1974 (years, months)	High school grade
179	5	97	16, 11	12
175	5	96	16, 11	12
166	5	93	16, 10	12
166	5	93	15, 01	10
160	5	90	15, 06	10
157	5	89	15, 06	11
150	5	84	15, 02	10
146	5	81	14, 01	10
139	5	75	14, 09	10
133	4	70	11, 02	9
130	4	67	15, 00	10
114	4	52	13, 04	9
111	3	48	13, 11	10

Mean = 148.2
SD = 21.0

[a] The AP calculus level BC examination was taken in May 1975. It is scored on a raw-score scale from 0–210 points, but only 1-to-5 (5 is highest) grades on a scale are reported. SMPY thanks the Advanced Placement Program office, which furnished us the raw scores for research purposes.

Table 2
Age, Grade, and Test Information for Class II

Raw score on AP calculus test[a]	Scores on 1–5 scale[b]	Percentile rank of AP score[c]	Age as of 1 Sept. 1975 (years, months)	High school grade
192	5	99	16, 07	12
178	5	97	16, 00	11
145	5	81	17, 02	12
139	5	75	17, 04	12
127	4	66	15, 00	11
126	4	65	17, 03	12
123	4	60	17, 04	12
117	4	53	15, 09	11
111	3	48	17, 07	12
109	3	47	17, 06	12
95	3	40	15, 06	12
79	2	20	17, 03	12

Mean = 128.4
SD = 32.0

[a] The AP calculus level BC examination was taken in May 1976. It is scored on a scale from 0–210 points. Raw scores were obtained by SMPY from the Advanced Placement Program office for research purposes.
[b] Scores are reported to students on a 1–5 scale, where 5 is highest and 1 is lowest.
[c] Approximate percentiles were interpolated from percentile information on the May 1975 AP level BC calculus examination.

Discussion

Several criteria can be used to judge the success of Classes I and II. To evaluate whether the supplemental course helped improve students' per-

formance on the AP examination compared with what they would have achieved with only their high school courses, the scores are compared with (a) the scores of other able Baltimore-area students who did not take the course and (b) the scores of students who enrolled but then dropped out of the course.

SMPY was able to learn the scores of 13 students who attended an excellent nonpublic nonparochial high school in Baltimore and took the BC level calculus examination in May of 1975 (i.e., at the same time as the 13 students in Class I). This school has a fine mathematics department and is one of the few Baltimore high schools to offer a calculus course that prepares a considerable percentage of its calculus students for the BC level test. Of those 13 students who took the test, one received a 5, six received 4s, three made 3s, and three received 2s. One of the students in the class who earned a 4 also participated in Class I (he was the 11-year-old ninth grader).

Two boys did not complete Class I. One of them earned a low 3 (raw score of 88) on the BC level test. The other student took only the AB level test and scored a 3.

Eleven students had dropped from Class II. Six of them took the BC level test; each scored a 4. Another student took the AB level test and received a 4. Three other students did not take either level of the calculus test, and the one remaining student could not be contacted.

The mean scores for Class I, Class II, and the 13 high school students mentioned above were compared. For Class I, the mean AP grade was 4.62, with standard deviation 0.65. Class II's mean grade was 3.92, and the standard deviation was 1.00. The mean grade for the high school class was 3.39, with standard deviation 0.96. A one-way analysis of variance determined a significant difference among the means: $F(2, 35) = 15.60$ ($p < .01$). Tukey's HSD test for multiple comparisons (see Kirk, 1968) was performed. The critical value for a significant difference ($p < .05$) between means was 0.548. Using this criterion, Class I was judged superior to Class II and the high school class. The difference of 0.532 between the means of Class II and the high school class was nearly significant. This information suggests that the supplemental instruction did help students improve their scores on the test, especially for Class I. The classes apparently helped students to raise their AP test scores by one or more grades; that is, a score of 2 without attending the supplemental class would tend to become at least a 3 if the student did attend the class, a 3 would become a 4, and 4 would become a 5 with the extra background the course provided.

Another aspect of evaluation in a supplemental course of this type is the cost-effectiveness. The costs for Classes I and II were the same. Thirty classes at $5 tuition per meeting, plus a $14 textbook and the AP examination fee (then $29), totaled $193. Tuition would have been lower had more students enrolled and persisted. Due to the experimental nature of the course, SMPY absorbed any costs above what the students contributed. An investment of approximately $200, plus transportation, for a potential eight

credits of college calculus is much less than a student would pay at a private college. The cost of those credits at a selective university could run $1333 or more (e.g., 8/30 of $5000 tuition). By earning the credits while still in high school, the students save time and money when entering college. At the end of the course, nearly all members of both classes were well prepared for a third semester of calculus. Follow-up information indicated that most of the students went on to selective colleges and have been highly successful in their advanced mathematics courses.

The Role of the High School Course

Most of those enrolled in Classes I and II were attending level AB calculus courses in their high schools; less than one-third of them were enrolled in level BC courses. SMPY insisted that all students in the supplemental classes attend their regular high school classes also. There were three reasons for the requirement. First, the pace of the supplemental class was so fast that students might become discouraged quickly. Until they adapted to the intense, challenging Saturday morning class, the high school course provided security; the students could see they were doing well in their high school class and would be more likely to withstand the initial frustrations of the higher-level supplemental course. Second, the high school course provided daily review and repetition necessary to obtain a good foundation in basic concepts and to develop skill in problem solving. Finally, in the event that a student found the demands of the supplemental class too great, he or she would still have a calculus class to attend.

Classes I and II covered the material so rapidly that by mid-December all the students were far ahead of their high school courses. In many cases, they found themselves contributing more to their high school classes than they received. Once the students had adjusted to the level of the Saturday classes, the high school course was relied on less for learning new material than for review and practice.

Establishing Supplemental Calculus Classes

Supplemental classes can help meet the educational needs of the ablest mathematics students. The mechanics of establishing a fast-paced mathematics class are given in George (1979b) and Bartkovich and George (1980), but some general guidelines are as follows:

1. Student selection. SAT or PSAT scores can be used to establish eligibility. Students must demonstrate high motivation as well as mathematics ability. Previous exposure to fast-paced instruction is desirable but not essential.

2. High school calculus. Concurrent attendance of the high school class is important, especially for the first semester. After several months, high school class attendance could possibly be made optional, provided that performance in the supplemental class remains excellent.

3. Class size. An instructor and a well-qualified assistant can begin with 20–30 pupils. Several high schools could pool their most highly qualified students.

4. Instructor. The instructor must be able to conduct the course at a rapid rate, gearing lessons to the best students in the class. College-level teaching experience is desirable.

5. Homework and attendance. Homework is an extremely important requirement; students must be willing to spend several hours each week doing assignments well. Problem sets need not be lengthy, but they must be challenging and help the students assimilate course material. Regular class attendance is essential.

6. Textbook and materials. In the AP Course Description published annually by the Advanced Placement Program of the College Entrance Examination Board, a variety of calculus textbooks are recommended. AP practice materials, also mentioned, should be used throughout the course.

Conclusion

The classes described here were successful in preparing students for the AP examination in level BC calculus. Class I members, who had previous fast-paced instruction, were probably abler and performed better than did Class II members. Students in both classes received higher scores than high school students without the supplemental instruction. The major benefit of the program is to provide a high-level background in first-year college calculus to students who would ordinarily not have that opportunity in high school. Mathematically talented youths are one of our nation's greatest resources for the future. As educators and scientists, we can help them fulfill their potential by meeting their intellectual needs effectively.

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