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SEX DIFFERENCES IN THE DEVELOPMENT OF PRECOCIOUS MATHEMATICAL TALENT

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ABSTRACT

In 1972 the Study of Mathematically Precocious Youth (SMPY) began its search to identify highly able mathematical reasoners. With some variations in the target population and the selection procedures, the talent searches have continued to the present. This chapter reviews the results of the 1972, 1973, 1974, 1976, 1978, and 1979 talent searches, with particular emphasis on sex differences. Follow-up data available on the 1972, 1973, and 1974 participants are analyzed, particularly as they relate to sex-role identity and willingness to accelerate. Attempts to foster precocious achievement in mathematics by means of special, accelerated classes for mixed-sex and same-sex groups are described.

Our knowledge of precocious mathematical ability and achievement in childhood and adolescence typically has been gleaned from retrospective study of the lives of eminent persons. Several famous scientists, mathematicians, and philosophers such as Pascal, Leibnitz, and Gauss, who dealt with quantitative topics, were reported to have been mathematically precocious children (Cox 1926). Since far fewer women than men have achieved eminence in mathematics, it is not surprising that there are few reports of genius and childhood precocity among women (Bell 1937; Cox 1926; McCurdy 1957; Stanley 1974; Stern 1971). There has been no evidence, however, to suggest whether precocious development is indeed more rare among females than it is among males or simply less visible.

Perhaps because of their assumed rarity, cases of precocious intellectual development and educational achievement have not been well-researched. Not even the monumental longitudinal study of intellectual giftedness by Terman (1925; see also Terman and Oden 1947, 1959) provides information concerning precocious mathematical talent and achievement among children designated as gifted by measures of global intelligence.

An ongoing study of mathematical precocity at The Johns Hopkins University offers some interesting insight into the question of sex differences in mathematical precocity. First, it provides information concerning the existence of precocious mathematical reasoning ability among adolescents, and second, it explores the question of how precocious achievement in mathematics can be fostered.

PRECOCIOUS MATHEMATICAL REASONING ABILITY IN ADOLESCENTS

The Study of Mathematically Precocious Youth (SMPY) began in the fall of 1971 to search for junior-high-school-age students who were precocious in mathematical reasoning ability, as evidenced by very high scores on the Scholastic Aptitude Test Mathematics (SAT-M). In order to discover these talented students, SMPY conducted a talent search in each of the following years: 1972, 1973, 1974, 1976, 1978, and 1979. The rationale for the use of difficult pre-college-level tests to discover precocity is discussed in Keating 1974, 1976; Solano 1979; Stanley 1977; and Stanley, Keating, and Fox 1974. The results of each year of testing are summarized in the following sections.

The 1972 Contest

In March 1972, seventh-, eighth-, and young-in-grade ninth-grade students in the Greater Baltimore area who had scored at or above the 95th percentile on the numerical-concepts subtest of an in-grade standardized achievement test such as the Iowa Tests of Basic Skills were invited to participate in a contest. Three hundred ninety-six students (223 boys and 173 girls) accepted the challenge and took the SAT-M.

The results of the testing were startling. Twenty-two boys (about 10 percent) scored 660-790. This was better than the average Hopkins student scored as an eleventh or twelfth grader. Clearly, there are many mathematically precocious boys. The highest score for a girl, however, was 600. Although 44 percent of the contestants were girls, 19 percent of the boys scored higher than the highest-scoring girl. When the data were analyzed by grade, only 7.8 percent of the seventh-grade boys outperformed the highest-scoring seventh-grade girl, but 27.1 percent of the eighth-grade males scored higher than the highest-scoring eighth-grade girl.

The mean scores for boys and girls, by grade, are shown in table 7.1. Since the number of young-in-grade ninth graders was small, their scores

TABLE 7.1. Summary of gender-based differences in mathematical reasoning ability (SAT-M)

Talent Search	Grade	Sex	N	Mean SAT	S.D. SAT	t-test (df)	p <	Effect[c]	Percentage of males who outscored highest-scoring female
March 1972	7	Male	90	460	104	2.68	.005	0.42	7.8[d]
		Female	77	423	75	(165)			
	8[a]	Male	133	528	105	5.32	.001	0.71	27.1[d]
		Female	96	458	88	(227)			
January-February 1973	7	Male	135	495	85	5.14	.001	0.70	3.0[d]
		Female	88	440	66	(221)			
	8[a]	Male	286	551	85	5.18	.001	0.52	9.8[d]
		Female	158	511	63	(442)			
January 1974	7	Male	372	473	85	4.92	.001	0.42	3.0[d]
		Female	222	440	68	(592)			
	8[a]	Male	556	540	82	7.05	.001	0.47	2.2[d]
		Female	369	503	72	(923)			
December 1976	7[b]	Male	507	458	88	6.77	.001	0.46	2.0
		Female	366	422	65	(871)			
January 1978	7[b]	Male	1,549	448	87	11.55	.001	0.44	0.1[e]
		Female	1,249	413	71	(2,796)			
January 1979	7[b]	Male	2,046	436	87	15.61	.001	0.52	0.0[f]
		Female	1,628	404	77	(3,672)			

SOURCE: Study of Mathematically Precocious Youth (SMPY).

NOTE: The authors wish to thank Julian C. Stanley, director of SMPY, for his suggestions concerning the format of this table.

[a] Accelerated ninth graders and a few accelerated tenth graders were included in this category.

[b] Persons of seventh-grade age who were in higher grades were included in this category.

[c] Effect = $\left(\sqrt{\frac{1}{n} + \frac{1}{n}}\right)t.$

[d] These percentages are based on grouped frequency data. The actual percentages are probably higher than those indicated.

[e] One girl scored 760 on SAT-M. The second-highest-scoring girl earned a 640. If a score of 640 is taken as the basis of differentiation, 2.5 percent of the boys earned a higher score than the second-highest-scoring girl did.

[f] The highest-scoring boy earned a 790 on the SAT-M, the highest-scoring girl a 740. A second girl earned a 730. The third-highest-scoring girl earned a 670. If 670 is taken as the basis of differentiation, 0.8 percent of the boys earned a higher score than the third-highest-scoring girl did.

are reported with those of the eighth graders. The highest mean score for any group was five hundred twenty-eight for eighth- and ninth-grade boys. Seventh-grade boys had a mean score of 460, followed by eighth- and ninth-grade girls and seventh-grade girls whose mean scores were 458 and 423, respectively. Sex differences in scores on SAT-M were statistically significant at very stringent levels ($p < .005$ and $p < .001$ for seventh graders and eighth graders, respectively).

The 1973 Contest

In the winter of 1973 a second talent search was conducted. This time students were considered eligible for the contest if they had scored at or above the ninety-eighth percentile on an in-grade numerical-concepts subtest of a standardized test such as the Iowa Tests of Basic Skills. Wider publicity helped to increase the number of students who participated. There were 667 students in the contest (421 boys and 246 girls). The percentage of girls, however, had dropped from 44 percent in 1972 to 37 percent in 1973. This decrease in participation by girls may have been due in part to the fact that there were actually two contests in 1973, one for mathematics in January and one in the verbal area in February.[1] Students in both contests took the SAT-M and the SAT-V. Students were told they could enroll for either contest and be eligible for prizes in both. The total number of students in both contests was 953. There were 537 boys (56 percent) and 416 girls (44 percent).

The highest SAT-M score for a girl in the 1973 contests was 650, while two boys (one a seventh grader) attained scores of 800 (Stanley 1973). Seven percent of the boys in the 1973 contests scored 660 or more. No girl did. The mean scores on the SAT-M, by sex, grade, and contest entered, are shown in table 7.1. Note that only 3 percent of the seventh-grade boys outscored seventh-grade girls, while 9.8 percent of the eighth-grade boys did better than eighth-grade girls. For both grades the sex differences once again reached very stringent levels of statistical significance ($p < .001$).

The 1974 Contest

In January 1974 a third talent search for mathematics was held. Students throughout the entire state of Maryland who had scored at or above the ninety-eighth percentile on the numerical-concepts subtest of a standardized

[1] In 1972 the Study of Verbally Gifted Youth (SVGY) was begun at The Johns Hopkins University. Thus in the winter of 1973 there were two contests: SMPY held its contest in January, and SVGY held its in February. The SAT-M and the SAT-V were given at both contests. Students were told to register for the January contest if they were primarily interested in mathematics and for the February contest if they were interested primarily in the verbal area. Students were eligible for prizes in both contests, however.

achievement test were eligible for the contest. Testing was conducted in four centers across the state (The Johns Hopkins University, the University of Maryland at College Park, Salisbury State College, and Frostburg State College).

A total of 1,519 students took the SAT-M. Thirty-nine percent of the participants were girls (591). Sixty-one students scored 660 or above. Seven of those students were girls. One girl scored 700. The highest score earned by a boy was 760. In 1974, 3 percent of the seventh-grade boys scored higher than the highest-scoring seventh-grade girl, while 2.2 percent of the eighth-grade boys outperformed eighth-grade girls. Mean SAT-M scores in 1974 are shown in table 7.1. The pattern of mean scores in 1974 was similar to that of 1973. Within each grade group, there were statistically significant sex differences ($p < .001$) in favor of the boys.

The 1976 Contest

After a hiatus of nearly two years, SMPY held its fourth talent search in December 1976. Students were eligible to participate in that search if they were in the seventh grade or of seventh-grade age but in a higher grade and if they lived in Maryland or in the bordering regions of a state that shared a common boundary with Maryland (Delaware, Pennsylvania, Virginia, West Virginia, and the District of Columbia). Forty-two percent of the 873 participants in the 1976 talent search were girls. Mean scores by gender are shown in table 7.1. Only 2 percent of the boys scored higher than the highest-scoring girl, but while the highest-scoring male earned a 730, the highest-scoring female scored 610 (a difference of 120 points). Once again the sex differences reached stringent levels of statistical significance ($p < .001$).

The 1978 Contest

January 1978 saw a major change in SMPY's talent-search strategy. Seventh graders or students of seventh-grade age but in a higher grade were eligible if they lived in Maryland or in any part of a state bordering Maryland (Delaware, Pennsylvania, Virginia, West Virginia, and the District of Columbia). In order to accommodate participants from such a broad geographical region, SMPY arranged for the Educational Testing Service (ETS) to provide the study with its own code numbers for use during the regular January 1978 administration of the SAT. Students took the test at local testing centers, and their scores were reported to SMPY.

Of the total 2,798 participants in the 1978 talent search, 44.6 percent were girls. Boys continued to outperform the girls, 0.1 percent of the boys scoring higher than the highest-scoring girl. One girl scored 760 on the SAT-M; the girl scoring next highest earned a 640. If a score of 640 is taken as the basis of differentiation, 2.5 percent of the boys earned a higher score than the girl scoring second highest. Differences by gender reached stringent levels of statistical significance ($p < .001$).

The 1979 Contest

The national administration of the January 1979 SAT served also as SMPY's sixth talent search. Eligibility criteria were exactly the same as they had been the previous year. Forty-four percent of the participants (3,674) were girls. In this most recent search for precocious mathematical reasoners only one boy scored higher on the SAT-M than the highest-scoring girl. Similar to the pattern of the previous five contests, however, sex differences on mean scores were 32 points ($p < .001$).

Sex Differences

Boys and girls who participated in a voluntary mathematics contest (and who qualified for that contest on the basis of high scores on standardized tests of grade-level mathematics achievement) differed considerably with respect to performance on a difficult pre-college-level test of mathematical reasoning ability. Mean scores for boys in each of the six contests were at least 31 points higher than those for girls.

Thus as early as grades seven and eight, boys outperformed girls on difficult pre-college-level tests of mathematical reasoning ability, and the differences were particularly striking at the upper end of the distributions. In eight years of study SMPY has identified considerably more male than female highly precocious mathematical reasoners. The self-selection aspect of a contest may have contributed to the greater male participation, but this does not explain why the ratio of boys to girls who scored 660 or better on the SAT-M (14.4 to 1) was so much greater than the overall ratio of boys to girls in the contests (1.3 to 1).

Whether these apparent sex differences in mathematical aptitude are a result of biological differences or differential cultural reinforcement over time, or of a combination of the two, is not clear. One would expect to find a large gap at the upper end of the distribution of mathematical ability (as was found by SMPY) if the biological explanation of sex differences in mathematical ability, as suggested by Ellis Page in a previous volume in this series (Page 1976), is correct. At the present time, however, many researchers feel that there is too little known about the inheritance of specific abilities such as mathematical aptitude to justify such a conclusion (Astin 1974; Maccoby and Jacklin 1972).

The argument put forth by Fennema (in chapter 6 of this volume) that sex differences are a result of differential course-taking does not hold for this population. Many researchers believe, however, that the differences between the sexes in average performance on tests of specific abilities such as mathematics reflect differential cultural reinforcements over time that have shaped students' career and educational goals, interests, and achievements (Aiken 1970; Astin 1968*a*, 1968*b*, 1974; Hilton and Berglund 1974).

SMPY's study of the characteristics of mathematically precocious adolescents lends some support for the social explanation of sex differences at the higher levels of ability and achievement. Boys who scored 660 or more on the SAT-M had a stronger orientation toward investigative careers in mathematics and science and a greater theoretical-value orientation than their less mathematically precocious male and female peers (Fox 1973; Fox and Denham 1974). Many of the highly mathematically precocious boys reported studying mathematics and, sometimes, science textbooks systematically with the help of a parent or interested teacher, while others have worked informally with mathematical puzzles, games, and books. This extracurricular pursuit of knowledge appears to have been motivated by strong theoretical and investigative values and interests.

Even the most mathematically talented girls seem less eager than boys—particularly the most mathematically talented boys—to seek out special experiences related to mathematics and science. Girls tend to have values and interests that are more social than theoretical (Fox 1973; Fox and Denham 1974). Thus differential performance by the sexes on difficult pre-college-level tests of mathematical reasoning ability at grades seven and eight could be partially a result of differential exposure to and practice with mathematical problem-solving situations, which result from different interests and value orientations.

Girls may also receive less encouragement at home to consider scientific pursuits. In a small sample of gifted students studied by Astin (1974), boys' parents often had noticed their sons' interest in science at an early age. Parents of boys typically reported that they had discussed college careers in science, mathematics, medicine, and engineering with their son. These parents reported providing more scientific materials (such as toys, books, and games) for their child than did parents of girls. Very few parents of girls had noticed their daughters' interest in mathematics or science at an early age. The occupations that these parents had discussed with their daughters were more apt to be traditionally feminine ones, such as nursing and teaching. The girls' parents had given less thought to future educational plans than had the parents of boys.

The Initial Cohort from SMPY's First Follow-up Survey

In December 1976, SMPY surveyed participants from its first three talent searches who would have entered college (if they chose) by September 1976 if they had undertaken no educational acceleration. In this group were eighth graders from the 1972 contest, ninth graders from the 1973 search, and tenth graders from the 1974 contest, all of whom had scored at least 420 on the SAT-M (except for several 1972 science-contest participants, who were included via a separate eligibility criterion). Two hundred fourteen students were polled. Ninety-four percent of them (202) were located, and all but two boys responded to an extensive questionnaire concerning their educational activities up to that time and their plans for the future.

In order to assess the degree of educational acceleration each follow-up participant put to use, two variables were developed.[2] The first was an index of general educational acceleration, based on the student's birthdate and date of high-school graduation (or entrance full-time to college if high-

[2] In terms of data analysis, SMPY is interested ultimately in two measures of accelerative facilitation among its cohorts of identified, talented mathematical reasoners: (1) age at the time of receiving the bachelor's degree and (2) age at the time of earning the doctorate. Records of these age markers will depend upon two follow-up studies of these students, projected to be held in 1981-82 and 1986-87.

In the meantime several interim variables describing accelerative facilitation had to be devised. Since most of the intervention offered by SMPY to its participants is accelerative in nature, a general index of pace through the educational lock step was needed. In addition, a measure of the extent to which a student used educationally accelerative options in mathematics training was required.

In order to compute the general index (an age-acceleration variable), a student was said to be "right on time" in the typical American classroom if he or she had an eighteenth birthday on 1 July 1976 (plus or minus fifteen days); that is, he or she would turn eighteen years old during the calendar year when high-school graduation occurred. This is a stringent criterion because many states do not have 31 December deadlines for enrollment—sometimes the deadline is as early as 31 August.

If a student's eighteenth birthday came after 1 July 1976, he or she was said to have been "accelerated" by as many months as there were between the two dates. (If the difference in the fraction of a month was more than fifteen days, it was counted as a full month's difference.) Similarly, if a participant's birthday came before 1 July 1976, he or she was said to have been decelerated by as many months as there were between the two dates. This variable was named ACCAGE.

A measure of educational acceleration or deceleration, ACCAGE is a fairly precise general index of how fast one is progressing through the typical educational structures in our society. Using the number of grades skipped as one's general index would become confusing if a student additionally entered kindergarten or first grade early, and such a variable would provide less information than ACCAGE does.

By far the most often recommended and pursued accelerative activities from SMPY's smorgasbord of options are called collectively "subject-matter acceleration in mathematics." A number of alternative modes exist for accomplishing subject-matter acceleration in one's educational scheme, including: (1) starting the pre-calculus sequence early by taking algebra I before the eighth grade; (2) taking several high-school mathematics courses during a single year; (3) taking college mathematics courses while still in high school; (4) enrolling in fast-math classes; and (5) having a mentor use diagnostic and prescriptive teaching methods in mathematics. In order to account for the many subvariables making up a "subject-matter-acceleration-in-mathematics" variable, a rather elaborate point system was created. Each participant's sum of points then became the variable SBJACC. A detailed formulation of the variable SBJACC is provided in the following five steps: (1) Two points were credited for each grade before the eighth grade in which algebra I was taken. For example, if algebra I was taken in the sixth grade, the student was given four points. This subvariable was called YALG (2·[4 − ALGIG], where ALGIG was a code for the grade in which algebra I was taken [5 = grade 9; 4 = grade 8; 3 = grade 7; 2 = grade 6]. If algebra I was not taken, YALG was equal to zero). (2) Points were given for each high-school mathematics course taken in the pre-calculus sequence according to the following scheme: Two points were given for algebra I, algebra II, and plane geometry; one point was given for college algebra, trigonometry, analytic geometry, elementary functions, matrices, and analysis. (The number of such courses was tallied as the subvariable TOCALI.) Three points were given for each mathematics course beyond and school graduation was skipped). The second variable was a measure of the number and kinds of accelerative options the student used in studying mathematics, a subject-matter-acceleration variable of sorts.

In the case of both variables strong sex differences appeared that added considerable evidence to the instances cited earlier. Of particular note appears to be the relationship between level of mathematical reasoning ability (as measured at the time of the talent search) and the degree to which educational acceleration was applied subsequently in one's educational career.

Figure 7.1 demonstrates dramatically how differently accelerative techniques were used by boys and by girls. For the boys a strong positive relationship is shown between mathematical ability and the degree of general educational acceleration employed ($p < .001$). That is, on the average, the more able the boy was, the younger he tended to be when he graduated from high school (or when he entered college full-time if he skipped high-school graduation). No such relationship is evident for the girls. The sex differences in the degree to which mathematical ability is related to the general-educational-acceleration variable are highly significant ($p < .001$). Younger, more mathematically apt boys appear then to more frequently skip grades, enter kindergarten or first grade early, or in other ways speed up transit through the educational lock step than do girls.

With regard to using specific techniques of subject-matter acceleration in the study of mathematics, trends of sex differences similar to those

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including calculus I: calculus I calculus II, calculus III, advanced topics, and so on. (The number of these courses was tallied as POCALI.) Several mathematics courses that were considered irrelevant to the precalculus sequence, to the actual calculus courses, or to higher-level mathematics following advanced calculus courses each were assigned one point value. Courses tallied under the subvariable ENRICH (worth one point each) were logic, computer mathematics, business mathematics, and probability and statistics. Points earned for completion of high-school mathematics courses were tallied as MACOUR = TOCALI + 3·POCALI + ENRICH. (3) In some instances students took college equivalents of high-school mathematics courses. The same tally system was applied, but students who took college mathematics courses earned twice as many points. The college-mathematics-course variable was called CACOUR (2·TOCALI + 6·POCALI + 2·ENRICH). (4) If calculus I was completed in high school, two points were presented for each year less than the typical four-and-a-half-year span from algebra I through calculus I. This subvariable was called QUIK (4.5 − [CALIG − ALGIG] + 1, where [CALIG − ALGIG] is the span of years from algebra I through the pre-calculus sequence). This number was then subtracted from the number of grades it would take to traverse this five-year course sequence in age-grade lock step, that is, four and one half grades.

In cases in which calculus I was taken as a college course while the student was in high school or in which grade(s) had been skipped between algebra I and calculus I, another variable, SQUEEZ, was defined as the number of actual years from starting algebra I to completion of college calculus I. If a student did not take calculus I, both QUIK and SQUEEZ were equal to zero. A dummy variable, SPEED, was used to select the appropriate QUIK or SQUEEZ, depending on when calculus I was taken or whether relevant grades had been skipped (If SQUEEZ was less than QUIK, then the equation SPEED = SQUEEZ was used; otherwise SPEED was equal to QUIK). In either case, two points were presented for each year (as a dummy variable SQUIK) under the usual four-and-a-half-year span from algebra I through calculus I (SQUIK = 2·SPEED). (5) All of the points earned by a student for subject-matter acceleration in math were summed as SBJAAC = YALG + MACOUR + CACOUR + SQUIK.

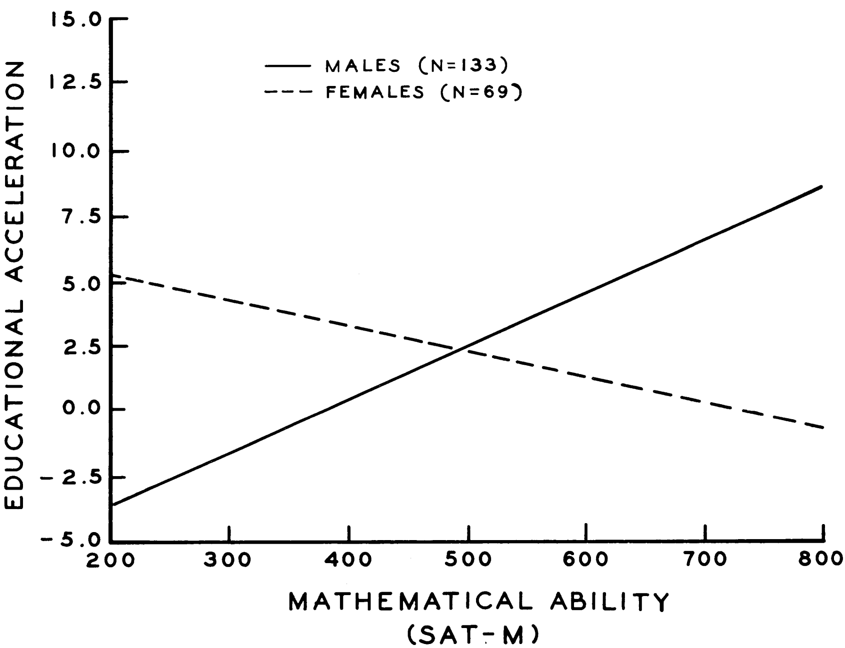


FIGURE 7.1. Comparison, by sex, of the regression lines for general educational acceleration (ACCAGE) on mathematical ability (SAT-M) among the initial cohort of SMPY's first follow-up survey. The *y*-axis represents months of acceleration.

shown for general educational acceleration are demonstrated in figure 7.2. Once again the more talented a boy was mathematically, the more he tended to take advantage of special, "fast-math" classes, college courses while still in high school, and the many other options for moving ahead in mathematics as rapidly as he could. Girls, on the other hand, manifested no such logical relationship between ability and the degree to which they chose to develop it. The difference between the two regression lines shown in figure 7.2 is statistically significant ($p < .01$).

The evidence presented in this section strongly suggests that in spite of whatever sex differences in mathematical reasoning ability appeared at the time of the talent search, girls tended to develop their abilities to a considerably lesser degree than boys. Thus not only are there sex differences in mathematical ability appearing at about age twelve, but there are sex differences as to how those abilities will be developed.

Sex-role Identity

An important factor in considering sex differences is sex-role identity, or the degree to which a person sees himself or herself as typical of a

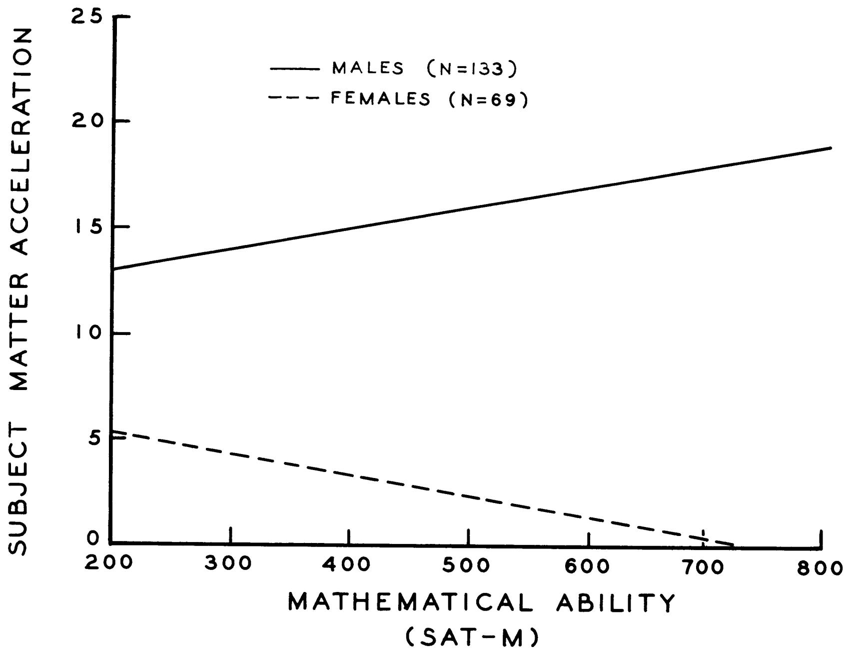


FIGURE 7.2. Comparison, by sex, of the regression lines for subject-matter acceleration in mathematics (SBJACC) on mathematical ability (SAT-M) among the initial cohort of SMPY's first follow-up survey. The y-axis represents points gathered on the subject-matter-acceleration variable (see n. 2, this chapter).

stereotypic male or female, regardless of the person's actual gender; it has received increasing attention among social psychologists (Bem 1974; Spence and Helmreich 1978). An important step in the development of this construct was a rejection of the view of sexual identity as a single dimension with masculinity at one end and feminity at the other. Instead, masculinity and feminity are conceived as independent dimensions. A boy, for example, could score high in masculinity and low in feminity, in which case he would be "same-sex typed." On the other hand, he could score high in feminity and low in masculinity, in which case he would be "cross-sex typed." He even could score relatively low in both dimensions ("undifferentiated") or high in both ("androgynous").

The Bem Sex-Role Inventory (BSRI) was administered to the top-scoring third of the 1976 talent-search participants (188 males and 90 females), who had been invited to take an extensive series of cognitive and affective tests so that SMPY could counsel them educationally. The BSRI yields a masculinity score and a feminity score. From those scores, Sanford Cohn has devised a technique by which the degree of sex-role differentia-

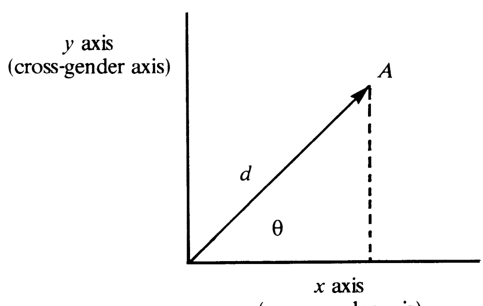
TABLE 7.2. Gender-based differences in sex-role differentiation and orientation for students in the top third of SMPY's 1976 talent search

	Males (N = 188)	Females (N = 90)	*t*-test ($df = 276$)	p <
Orientation (in radians)				
Mean	.702	.774	6.18	.000
S.D.	.093	.086		
Differentiation				
Mean	6.74	6.89	1.85	.065
S.D.	.65	.68		

tion and orientation can be determined.[3]

Table 7.2 summarizes the results of this study. Note that the higher one's sex-role-differentiation score, the more well defined one's sex-role identity is. The higher one's sex-role-orientation score, the more cross-sex-identified one is. In short, the girls were somewhat more differentiated in terms of sex-role identity than were the boys ($p < .07$). This is to be expected among students who are thirteen years old (on the average), since girls tend to mature earlier than boys. A more interesting observation, however, is that the girls in this group are more cross-sex-identified than the boys ($p < .001$); the girls in this group appeared to be significantly more masculinely sex-role identified than the boys were femininely sex-role identified. This finding is consistent with the fact that mathematics is

[3]Cohn suggests the following method as a measure of sex-role differentiation and orientation, using as its basis masculinity and femininity scores.



Sex-role differentiation is rather simple, consisting of finding the geometric sum of the masculinity and femininity scores. That geometric sum is graphically illustrated here as a vector (d) from the origin to point A. The length of this vector is a measure of how well differentiated one's sex-role identity is; the higher the value of d, the better differentiated one is.

Note that the X axis is always the same-gender axis, and the Y axis is always the cross-gender axis. For boys, then, the masculinity score would be plotted on the X axis, and the feminity score, on the Y axis (and vice versa for girls). The greater the angle θ is, then, the more cross-sex identified a person is. In terms of degrees, one might consider a range of θ from $0°$ to $30°$ as describing same-sex identified; a range from $31°$ to $60°$, androgenous; and a range from $61°$ to $90°$, cross-sex identified. Hence θ becomes a measure of *sex-role orientation.* In table 8.2, however, sex-role orientation (θ) is expressed in radians rather than in degrees.

considered in our culture to be a masculine pursuit.

Perhaps girls are being discouraged away from developing their mathematical talent in an attempt by their parents and educators to make them fit more closely to feminine stereotypes. In light of this possibility, it might be wise to offer effective role models to girls several years before they turn twelve years old in order to offset their more rapid rate of sex-role differentiation and give them greater security in pursuing the development of their precocious mathematical ability traditionally identified as masculine.

FOSTERING PRECOCIOUS ACHIEVEMENT

Although it is difficult to draw conclusions about the relative influences of biological and social factors upon the performance on measures of aptitude (for example, some would even argue the possibility that some of the differences in test performance are artifacts of biased test materials), there is clear evidence that precocious achievement in mathematics can be directly influenced by environmental factors. SMPY's attempts to foster acceleration in mathematics provide some insight into the dynamics of precocious achievement among bright adolescent boys and girls.

Through 1974 SMPY sponsored three experimental accelerated-mathematics classes on the Johns Hopkins campus and two classes in a public junior high school (the details of these classes are reported in depth in Fox 1974*a*, 1974*b*; George and Denham 1976; and Stanley 1976). A summary of the results of these five classes and their implications for understanding the differences between the sexes with respect to precocious achievement is presented in the following sections.

Class 1—boys and girls. In the summer of 1972 thirty end-of-the-year sixth graders (eighteen boys and twelve girls) were invited to a special, summer mathematics class that met two hours a week.[4] Fourteen boys (78 percent) and seven girls (58 percent) enrolled for the program. The initial success of the class in mastering algebra I with only eighteen hours of instruction was so great that the class continued to meet for two hours a week through the middle of the following summer. Of the twenty-one students who initially began the course, six boys (43 percent) and one girl (fourteen percent) completed their study of all the pre-calculus mathematics (algebra I, algebra II, algebra III, plane geometry, trigonometry, and analytic geometry). Six of the boys took calculus the following year in a senior high school.

Class 2—boys and girls. In the summer of 1973 eighty-five students (fifty-one boys and thirty-four girls) who had participated in the 1973 talent search and who had scored at least 500 on the SAT-M and 400 on the SAT-V were invited to a summer accelerated-mathematics class. Most of

[4]Thirty students were invited. One was an end-of-the-year third grader. Another was an end-of-the-year eighth grader. The remaining students were end-of-the-year sixth graders.

these students were eighth graders who had completed algebra I. Twenty-two boys (43 percent) and nine girls (29 percent) enrolled. Fourteen boys (64 percent) and none of the girls completed all the pre-calculus mathematics by the middle of the following summer, meeting only two hours a week during the school year and four hours a week during the second summer (George and Denham 1976).

Although these classes were highly successful in promoting precocious achievement in mathematics among boys, both were far less successful with girls. First, more boys than girls were eager to enroll in such a program. Second, girls who did enroll tended to drop out of the classes before their completion. Interviews with the girls indicated that one major reason for dropping out was a reluctance to become accelerated in their placement in school. Many of the girls seemed to fear being labeled as different from their friends by virtue of becoming somewhat accelerated. Girls also reported that the class meetings were dull, and some made references to the boys in the classes as "little creeps." The overall reaction to the classes by the girls was that it was socially unappealing and might have negative social consequences in school.

It has been reported that even very bright girls often select themselves out of advanced mathematics classes in high school (Haven 1972) and that few women ever pursue doctoral degrees in mathematics (in 1969, for example, only 7 percent of the doctoral degrees awarded in mathematics were earned by women [Bisconti and Astin 1973]). Until this present study, however, it was not known that bright girls in junior high school would be far more reluctant than boys to participate in special accelerated-mathematics programs and, especially, to persist in them.

Class 3—all girls. The results of testing values and interests of boys and girls in the 1973 contest suggested that even the most mathematically able girls were likely to prefer social activities to theoretical ones. In combination with the results of the first two accelerated-mathematics classes, this suggested that to interest girls in learning mathematics faster it would be important to consider the social aspects of a program.

Thus in the spring of 1973 an accelerated algebra I class was organized for seventh-grade girls who had been in the 1973 contest and who had scored at least 370 on the SAT-M (the average for female juniors in high school).[5] (The details of the program for girls are reported in chapter 10 of this volume.) In brief, the class was designed to appeal to the social interests of girls in a number of ways. It emphasized social cooperation rather than competition and was taught by a woman rather than by a man. Male and female scientists and mathematicians spoke to the girls about exciting

[5]Two girls who had not participated in the 1973 contest were later tested on the SAT-M and allowed to take the course. One of these girls scored 350 on the SAT-M. Since she had been eligible for the first class but had not enrolled, the decision was made to let her be in the all-girls class. Her score of 350 was considered to be an underestimate of her ability. The following year she scored 570 on a different form of the SAT-M.

careers in mathematics and science (such as operations research, health statistics, and social-science research) that deal with social problems as well as theoretical ones. This approach to an accelerated program was considerably more effective in recruiting girls. Of the thirty-four girls invited, twenty-six (76 percent) enrolled; eighteen girls (69 percent) completed the course. Not all the girls, however, chose to accelerate their mathematics in school the following year, and a few actually met with school resistance to their acceleration. Eleven did take algebra II the following year; ten of these (38 percent of the total female enrollees) were considered to have been successfully accelerated.

The emphasis on the social interests of girls was moderately effective in promoting greater achievement in mathematics for girls than the two mixed-sex, more theoretically oriented classes had. This approach, however, did not promote the same extent of acceleration for the girls that the other two programs did for the boys. Five of the girls from the all-girl class indicated some interest in becoming further accelerated in mathematics (by as much as two or three years) by the time they complete high school and enter college.

Classes 4 and 5—city public school. In the winter of 1974 Roland Park School, in Baltimore City, asked SMPY to set up in that school a fast-paced mathematics class based on the principles learned from the first three classes. Twelve boys and twelve girls in grades four through seven were selected to participate. On the basis of past experience, SMPY suggested that there be two fast-paced classes, one for boys, taught by a male college professor, and one for girls, taught by a female college professor. One boy and one girl dropped out of the program. Both classes made rapid progress through algebra I, meeting two hours a week for a total of thirty-seven hours the first year, and all who remained in the school the following year elected to continue in the fast-paced class to study algebra II. Although on the average the girls were somewhat less able than the boys, the two groups performed about equally well on a standardized algebra I test at the end of the first year. Both classes were considerably more successful in mastering algebra I than the class of eighth graders in a regular, full-year algebra I program (Stanley 1974).

The success of these two classes in fostering high achievement at an accelerated pace suggests that special programs of this type may be more successful for girls when they are conducted within the context of the regular school. Further research is needed to determine just how successful these programs can become for both boys and girls if they are implemented on a large scale within public schools or school systems. Whether sex segregation and female teachers as role models are actually crucial for the success of girls needs to be studied systematically within school settings.

These five classes represent prototypes that have been used, modified, and revised for students from the last three talent searches. With the establishment of the Intellectually Gifted Child Study Group (IGCSG) at Johns Hopkins, mathematically precocious girls have a specialized group with whom to consult. Although SMPY continues to sponsor fast-math classes for its highest-scoring students regardless of gender, it works predominantly with mathematically talented males.

CONCLUSIONS AND IMPLICATIONS

On the basis of SMPY's research on the mathematically precocious, it appears that males are more likely than females to perform at a very high level on pre-college-level tests of mathematical reasoning ability (at least in a voluntary contest situation). The sizable gap between the sexes on mean SAT-M scores and at the upper end of the distribution as early as grade seven suggests that there may be biological differences between the sexes with respect to mathematical aptitude. There are, however, strong indications that some of the apparent differences are related to environmental factors. Whether greater efforts to encourage and develop mathematical interests among women in childhood and adolescence could eliminate or reduce this sex difference at the higher levels of ability is not known.

Clearly it is much more difficult to foster precocious achievement and acceleration in mathematics among girls than among boys. In structuring learning environments to foster accelerated achievement among young women some attention to their social interests appears to increase their rate of participation and success. To date, however, SMPY has not effectively helped to accelerate any girl as far or as fast as most of the boys in its programs. This should not be interpreted as meaning that it is unprofitable to work with bright girls. Although mathematical precocity (both in measured ability and in achievement) is far more evident among young males, SMPY's efforts to foster greater achievement among very bright students suggests that girls can be helped to develop their quantitative potentials more fully.

Even if there are biological differences between the sexes that account for much of the differing degree of precocity between the sexes, it is still desirable to develop ways of fostering greater achievement among both men and women. It would appear, however, that our instructional strategies and classroom environments should be more systematically studied and regulated to avoid unnecessarily discouraging young women from developing their mathematical potentials to the fullest.

The fact that at the present time mathematical precocity appears to be not only less visible but rarer among female adolescents than among males can lead us to one of two approaches for future educational planning and development: First, we could concentrate all efforts to find and foster high-level achievement and talent in mathematics on boys, since they will be easier to find and to work with (which would be very much like what is occurring, perhaps unintentionally, in most schools today). The second approach would be to try to identify talented young women, as well as young men, but to modify or restructure instructional strategies for girls in order to optimize their chances for high-level achievement. The long-term benefits of this second approach could be quite gratifying.

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