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# RATIONALE OF THE STUDY OF MATHEMATICALLY PRECOCIOUS YOUTH (SMPY) DURING ITS FIRST FIVE YEARS OF PROMOTING EDUCATIONAL ACCELERATION

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**ABSTRACT**

*The Study of Mathematically Precocious Youth (SMPY) began officially at The Johns Hopkins University in September 1971 under a five-year grant from the Spencer Foundation. Its staff, headed by Professor (of psychology) Julian C. Stanley, seeks highly effective ways to facilitate the education of youths who reason extremely well mathematically. To do so, it is of course necessary first to identify such youths and understand them well. During SMPY's initial five years, much service was rendered to the mathematically talented in the State of Maryland, especially seventh and eighth graders in the Greater Baltimore area. This enabled the SMPY staff to develop and refine principles, techniques, and practices with which to improve the education of intellectually talented students there and elsewhere. SMPY's underlying rationale is not fully obvious from the two books that report its substantive achievements. Thus it seems desirable to state that rationale clearly so that its assumptions can be examined by all persons who consider using SMPY's practices. This chapter is the initial attempt to set forth explicitly the point of view guiding SMPY's activities.*

Results of the first year of the Study of Mathematically Precocious Youth at The Johns Hopkins University were reported in a book entitled *Mathematical Talent: Discovery, Description, and Development* (Stanley, Keating, and Fox 1974). Findings during the following three years are contained in a larger book entitled *Intellectual Talent: Research and Development* (Keating 1976). In this paper I shall not attempt to summarize the twenty-seven chapters of those two books, but instead shall present the rationale of the study as it has been worked out by me in close collaboration with a number of associates, especially Lynn H. Fox, Daniel P. Keating, Susanne A. Denham, Linda K. Greenstein, William C. George, Cecilia H. Solano, and Sanford J. Cohn. The reader will see how our extreme emphasis on educational acceleration has greatly helped many youths who were *eager* to move ahead academically.

## WHY MATHEMATICAL REASONING IS THE INITIAL BASIS FOR IDENTIFICATION

The Study of Mathematically Precocious Youth (abbreviated SMPY) began informally at The Johns Hopkins University during the summer of 1968 when Doris K. Lidtke, an instructor in computer science, called my attention to a twelve-year-old boy just out of the seventh grade who was doing remarkable things in the computer laboratory. It started slowly and without a name.

Emphasis on the mathematical and physical sciences began early, however. Persons often ask us why we chose mathematical reasoning ability rather than something else, or even why we decided to concentrate on one type of talent rather than studying all sorts. We wanted to steer a careful course between excessive specialism and overly broad coverage.

Sharply limited resources made this decision inevitable. Even for the first two years after the study was funded by the Spencer Foundation in 1971 it did not have a single full-time worker, and after that there was just one. During the 1976–77 academic year our entire regular staff consisted of William C. George, the full-timer; Cecilia H. Solano, a fourth-year doctoral student in psychology who worked ten hours per week on the study; Sanford J. Cohn, a second-year doctoral student in psychology who worked twenty hours weekly; me, who devoted to it as much time as being a professor of psychology with unreduced teaching responsibilities permitted; the administrative secretary, Lois Sandhofer; and a part-time secretary, Laura Thommen. Small wonder that we did not also select initially for other talents such as verbal reasoning ability, athletic prowess, musical talent, and leadership potential! No matter how hard we might work (and we do indeed put in long hours), relatively little could be done by us for that varied a group.

In response, however, to persistent inquiries about verbal reasoning ability after SMPY was funded, we encouraged JHU psychology professors Catherine J. Garvey, Robert T. Hogan, and Roger A. Webb to obtain from a philanthropic foundation a five-year grant (1972–77) with which to pioneer in that area. For reports of their work see Hogan et al. (1977), Viernstein et al. (1977), McGinn (1976), Viernstein and Hogan (1975), and Webb (1974).

Given that we must specialize, it seemed sensible to choose an ability closely related to major subjects in the academic curricula of public and private schools in the United States. Because we planned to help intellectually talented youths improve their education, it appeared wise to start at as early a grade and age level as the developing of the chosen ability permitted. In order to capitalize on the precocious development of this ability by greatly accelerating school progress in the subject-matter area concerned, it was necessary to choose school subjects much more highly dependent for their mastery on manifest intellectual talent than on chronological age and the associated life experiences. These considerations led to our choosing mathematical reasoning as the ability and the best of the standard courses in mathematics, the mathematical sciences, and the physical sciences as the subjects on which to focus directly. We did not want to develop curricula in mathematics, but instead to help mathematically talented boys and girls use their abilities more effectively in the various academic areas.

We were aided in this choice by more than just armchair considerations. Great precocity in mathematics and the physical sciences is documented by such writers as Harvey C. Lehman (1953), Catharine M. Cox (1926) in the second volume of Terman's *Genetic Studies of Genius* series,[1] Eric Temple Bell (1937), and Edna Kramer (1974). The only clear competitor was musical composition, where the almost unbelievably early accomplishments of Saint-Saëns, Mozart, and Mendelssohn are well known (see Schonberg 1970). This does not articulate well with school curricula, however, nor do we have the knowledge or facilities to nurture young composers. We eliminated chess because it is not an academic discipline.

Two who helped begin the study (Lynn H. Fox and I) had been teachers of high school mathematics, and I of chemistry and general science also. My undergraduate major had been physical science, and much of my graduate and postdoctoral work has been in statistics at three

[1]It is well for the reader to keep in mind the nature of these five volumes, the years in which they appeared, and the fact that their publisher (the Stanford University Press) has kept the whole series in print for more than half a century. References are as follows: Terman (1925), Cox (1926), Burks, Jensen, and Terman (1930), and Terman and Oden (1947, 1959). They have been extended by Oden (1968) and by chapter 3 in this volume. Further analyses of the 1972 follow-up survey are being conducted by Robert R. Sears (1977) and Lee J. Cronbach.

universities. As a Fellow of the American Statistical Association and of the American Association for the Advancement of Science, I felt competent to help students make decisions in the areas of mathematics and science, aided of course by consultation and collaboration with high school and college teachers, supervisors, and administrators.

Also, I had a master's degree in educational and vocational counseling and guidance and much background in evaluation and testing. These proved invaluable.

My interest in intellectually gifted youths began at the University of Georgia during the summer of 1938, after my first year of teaching in high school (see Stanley 1976*a*, pp. 6–9). It smouldered from then on, coming to the level of publication occasionally (e.g., Stanley 1954*a*, *b*; 1958; 1959*a*, *b*). Not until 1969, however, did I begin helping intellectually talented youngsters systematically (Stanley 1974, pp. 12–14; 1976*a*, p. 9).

It is interesting to note here as an aside that the SMPY staff has had little difficulty in planning closely with top-flight mathematicians and scientists, but has met with distrust from some mathematics supervisors and teachers who do not understand how university psychologists could know anything about their subjects. There is an element of defensiveness in this, of course, because we have prodded school personnel to do much more for mathematically highly talented students than is usually done.

Thus we settled upon mathematical reasoning ability developed to a high level at an early age as the basis for initial selection of students to be studied considerably more and helped to develop fast and well in mathematics and related subjects. We did this for logical, empirical, and personal reasons. Somewhat more of our rationale can be gleaned from Stanley (1954*a*, *b*; 1958; 1959*a*, *b*; 1974; 1976*a*–*f*).

We would not have begun this kind of project had we not agreed fully with Thomas Gray ("Elegy Written in a Country Churchyard," 1751, line 53) that

> Full many a gem of purest ray serene
> The dark unfathomed caves of ocean bear;
> Full many a flower is born to blush unseen,
> And waste its sweetness on the desert air.

## WHY SAT-M SCORE IS THE INITIAL CRITERION

We wanted to find youths who at an early age (mostly twelve or thirteen) were already able to reason extremely well with simple mathematical facts, students who even before taking or completing the first year of algebra would reason mathematically much better than the average male twelfth grader does. We gave applicants for the talent-search contest plenty of practice materials for the forthcoming test so that they would be on essentially the same footing with respect to opportunity to score well. Because reasoning mathematically involves reasoning with some mathematics, however elementary, this was essential in order to smooth out at least partially their differences in mathematical training and outside-of-school experiences. We did not want scores to depend much on rote knowledge of mathematical concepts or on computational ability, as the usual test of mathematical "aptitude" does, because we surmised that these could be taught readily and quickly to students whose mathematical reasoning ability is splendid. It seemed to us likely that the reasoning test would predict success in later mathematics, at least through advanced calculus and linear algebra, far better than items measuring rote memory and computational speed and accuracy would.

Thus we needed a mathematical reasoning test difficult enough that the average participant in our contest would score on it halfway between a chance score and a perfect score. For example, if there were sixty items and scores were "corrected for chance," we wanted the mean score of our examinees, a highly able group, to be about 30. Also, the test should have enough "ceiling"—be difficult enough for even the ablest entrants into the contest—so that virtually no scores of 60 would occur.

In addition to the considerations of reasoning content and appropriate difficulty, we wanted a professionally prepared, carefully standardized, reliable test for which several well-guarded ("secure") forms existed and for which well-known, meaningful interpretations of scores were available. High scores on the test should command immediate attention and respect at both the high school and college levels, because they could be compared with scores on the same test earned by superior high school seniors.

These considerations led to pilot studies of the mathematical part of the College Entrance Examination Board's Scholastic Aptitude Test (SAT-M).[2] Our first examinee, an obviously brilliant thirteen-year-old eighth grader, scored 669, which was then the 96th percentile of a random sample of male twelfth graders. On the verbal part of SAT, abbreviated SAT-V, he scored 590, the 93rd percentile of the same norm group. The next thirteen-year-old eighth grader on whom we tried the test scored 716 on M and 608 on V. Others scored similarly, some even higher. None scored near the perfect score of 60 right on M or 90 right on V. It seemed likely, then, that SAT-M would be excellent for identifying the level of mathematical reasoning ability we sought among seventh and eighth graders. SAT-V could be used with the high scorers on SAT-M to assess verbal reasoning ability, which seemed likely to be more closely related to

[2]For its history and rationale, see Downey (1961).

speed of thinking and of taking tests than is SAT-M. As has been shown in several publications, especially Stanley, Keating, and Fox (1974) and Keating (1976), for the students we tested SAT-M and SAT-V did indeed prove suitable in both content and difficulty. The mean on each was appropriately between the chance- and perfect-score levels. The highest scores were never perfect. Only an occasional examinee scored as high as 55 out of 60 on SAT-M. A twelve-year-old did score 58, and a thirteen-year-old scored 59, but these were the extreme exceptions among some 3,000 youths tested.

More importantly, SAT-M and SAT-V proved to have great value for predicting which students would be able to accelerate their mathematical education radically. Of course, motivational factors—especially, willingness to do difficult homework well—proved crucial within the high-scoring group, but without considerable ability of the SAT-M and SAT-V types students could not race ahead successfully in mathematics and related areas.

We have learned that the SAT-M score scale is valid right up to the top-reported score, 800, *if* the criteria themselves have enough "ceiling" for the group. For instance, in the usual eighth- or ninth-grade algebra I class, variation in this ability would probably make little difference in apparent success of students at SAT-M levels 500, 600, 700, or 800, because all of these exceed the mathematical-reasoning demands of the course. Paying attention and bothering to do homework and tests carefully are probably better determiners of grades among these high-scorers than are differences of the order of even 100 to 300 points. Put a 500-scorer into a fast-paced, homogeneously grouped 700-level algebra III class, however, and he is unlikely to be able to keep up at all. In general, most reports that a test of appropriate difficulty loses its validity at some point short of the top of its score scale are actually commentaries on the lack of ceiling of the criterion, rather than intrinsic dropping off of validity of the predictor. This seems especially true when both the predictor and the criterion variables are ability-test scores.

We realize that a factor analysis of SAT-M scores would show several factors, perhaps somewhat different for our youths than for the usual older examinees (see Pruzek and Coffman 1966). Because the criteria we use are also factorially heterogeneous, however, this is probably at least as much an asset as a liability.

The setting and rules of the mathematics talent searches tended to attract interested, mathematically able students who liked keen competition. The entrants were probably about the upper 1½ percent of their age group in mathematical reasoning ability (i.e., the top 1 in about 67). It would be foolish to administer the SAT-M to twelve- to thirteen-year-old students much less able than that, and even more unwise to test them with SAT-V, because SAT is designed for above-average eleventh and twelfth graders.

SAT-V proved rather difficult for some of the seventh graders who scored extremely high on SAT-M. Verbal reasoning ability seems more closely related to age than mathematical reasoning ability is and also more closely related to the verbal ability of the child's parents and their socioeconomic level. Nevertheless, splendid mathematical reasoners who were seventh or eighth graders seldom scored lower on SAT-V than the average twelfth grader does. For example, in the first mathematics and science talent search the 35 top boys out of the 265 male entrants averaged the 95th percentile of a random sample of high school seniors on M and the 87th on V. Of course, that type of regression (here, .4 of a standard deviation) is to be expected in any group chosen on one variable and then examined on another variable not perfectly correlated with it.

It would be rare, indeed, for a person to have excellent mathematical *reasoning* ability and yet be inferior to average thinkers in verbal *reasoning* ability. SMPY does not seek mere calculating freaks (Barlow 1952). Though its participants are not chosen explicitly for high IQ, virtually none of them have average or below-average IQs.

Most persons who upon entering their teens already reason extremely well mathematically, as indicated by a high score on SAT-M, will not become "pure" mathematicians. Far less than half of them will even major in mathematics as college undergraduates. Instead, most of the boys and some of the girls will specialize in the physical sciences (especially physics), engineering, computer science, mathematical statistics, operations research, economics, and other areas in which a good grasp of mathematics is essential. Some will go into medicine because of the prestige and financial compensation it usually offers, even though few persons holding M.D. degrees can make much use of great talent for mathematics. Medicine and law seem more likely choices for girls than for boys, because even yet the former tend to shy away from mathematics, engineering, and the physical sciences. A large percentage of the boys will probably work toward Ph.D. degrees.

Whenever one uses a test and has a fixed point above which the examinee is considered "successful" and below which he is considered "unsuccessful," the issue of false positives and false negatives arises. Some students will have a good day and equal or exceed the criterion, whereas others will have a bad day and drop below it. On another occasion the former would have failed and the latter have succeeded. SMPY guards against false positives by retesting at a later date with an extremely heavy battery of difficult tests all those persons who attained the criterion—e.g., SAT-M score of 640 or more during the second or third talent searches. The initially lucky scorer will be detected easily. Thus, for the retested group positive errors of measurement (see Stanley 1971) are not much of a problem, nor is the inevitably somewhat-less-than-perfect validity of SAT-M itself.

There will, however, be some youths inappropriately consigned to the below-640 group. A score of 630 represents only a point or two less, out of the possible 60 points, than a score of 640 does. The 10-point difference between 630 and 640 is only about one third of a standard error of measurement. Obviously, small fluctuations in score at this level will make the difference between being identified as an excellent enough mathematical reasoner to warrant being studied considerably more and helped a great deal and being consigned to the less mathematically brilliant group. This problem is unavoidable, no matter what score criterion is used. The 640 was chosen because it screened in just about as many students (about 7 percent of those who entered the contest) as it was feasible to test further and work with closely. Also, it was only about 20 points below the average SAT-M score as eleventh and twelfth graders of Johns Hopkins's freshmen, an impressive figure indeed for seventh and eighth graders.

There are several justifications for not worrying inordinately about the false negatives:

1. If seventh graders, they were eligible to enter the contest again the next year as eighth graders and were encouraged to do so. This worked, however, only for seventh graders tested in the March 1972 and January 1973 (i.e., the first and second) contests, because the January 1974 contest was the last of the initial series. (The contest resumed, with seventh graders only, in the fall of 1976.)

2. SMPY offered a great deal of help to all contestants who scored 420 or more, and most of them did.

3. It was unlikely that a student who scored as low as 630 would with better luck have exceeded 700, so probably few of the false-negative eighth graders would have been among the very highest scorers.

4. Relatively few students scored in the 610–630 range.

5. Nearly all of the students entering the contests would later, as eleventh graders or earlier, take both the Preliminary SAT and the SAT and recalibrate their levels of mathematical aptitude.

6. The SAT-M scores from the SMPY contest did not "count" anything for school or other purposes. Most such scores made the student look good and gave his parents and teachers evidence with which to argue that special provisions in mathematics for him/her were desirable. For example, 420 on SAT-M exceeds the score earned by approximately 57

percent of male eleventh and twelfth graders. To be that apt three to five years early is impressive.

## SMPY FOCUSES ITS EFFORTS

SMPY is developmental and longitudinal but not retrospective. Its staff identifies at the seventh- or eighth-grade level students who are already superior reasoners mathematically and observes their development (while trying to influence it) over the ensuing years. Its staff does not have the time or interest to delve deeply into the "whys" of their precocity. While not wholly without interest to us, questions such as "Is mathematical talent mainly inherited?" are largely outside SMPY's scope. We are concerned mostly with capitalizing on the high-level reasoning ability and the motivation to use it that can be found among youths twelve or thirteen years old. It is already-evident ability we seek, rather than some presumed underlying potential that has not yet become manifest. We leave it to others to study the origins of such ability, the effects of nature and nurture on it during the early years, the failure of mathematical ability to arise in what are otherwise bright children, and the treatment of "underachievers." These are important topics, but strenuous efforts to help the vastly neglected hordes of well-motivated mathematically apt youths who are caught in the interest-killing traps of routine mathematics classrooms leave us little time for them.

We are, however, greatly interested in the nature of mathematical talent as it develops and unfolds, especially from age twelve or so onward. We do care, too, how intellectual prodigies of the past have turned out (e.g., Wiener 1953 and Montour 1976*a, b*). Some books that we have found helpful are Bell (1937), Krutetskii (1976), and Skemp (1971). Also, see Fox (1976*c*).

## WHY IDENTIFICATION USUALLY BEGINS AT THE JUNIOR HIGH SCHOOL LEVEL

Elementary mathematics is, from the standpoint of the learner, heavily an algorithmic and deductive system, though for those who create it there are usually strong intuitive and aesthetic elements. Unlike understanding philosophy or great novels such as Tolstoy's *War and Peace*, personal experience outside the classroom and maturation closely tied to chronological age are not essential for learning mathematics well. Certain types of reasoning ability necessary for mastering subjects such as high school algebra develop at vastly different ages. A precocious ten-year-old may be superior in this respect to most adults. To him or her, mathematics and related subjects such as computer science may be seen as interesting games, little related to the real world of experience.

A startling example will illustrate this. At age ten one of SMPY's participants made the highest grade in a state college introduction-to-computer-science course, competing with seven of our exceptionally able older students and twelve adults. Before his eleventh birthday he completed at Johns Hopkins most of a second-level computer course on which he earned a final grade of A. At age eleven he earned, by examination, credit for two semesters of the calculus at Johns Hopkins. This is no ordinary boy, of course. His Stanford–Binet IQ at age eight was 190, and he had been in our special fast-mathematics classes for two years. Even he is not the most precocious youth we have discovered.[3] Furthermore, at age twelve to thirteen, when the typical child is in the seventh or eighth grade, there are quite a few students able to forge through all of precalculus mathematics *far* quicker than schools ordinarily permit them to do.

The first year of algebra usually causes serious problems for youths who are among the ablest few percent of their classmates in mathematical reasoning ability. Regardless of how advanced their ability is, seldom are they permitted to take this subject before the eighth or ninth grade. Then, no matter how much algebra I the student can already do or how quickly he or she could learn the material and go on to second-year algebra, the student is usually lockstepped into approximately 180 forty-five- or fifty-minute daily periods throughout the school year. Mathematically highly precocious youths need vastly less exposure to what is for them an extremely easy subject. This is especially true when the student has already had one or more years of "modern" mathematics that may have included much algebra covertly. Several examples from our experience will illustrate the mathematically talented youth's dilemma.

A twelve-year-old seventh grader who scored extremely high in one of SMPY's annual contests asked permission to join his junior high school's eighth-grade algebra I class in February but was refused on the grounds that he already had missed more than half the course. He insisted on being given a standardized test covering the first year of the subject. On this he made a perfect score, 40 right in forty minutes, which is two

[3]Even more psychometrically precocious was the boy of Chinese background who at age ten years one month scored 600 on SAT-V and 680 on SAT-M, and a year later scored 710V and 750M. SMPY's youngest college graduate thus far is Eric Robert Jablow, born 24 March 1962, who received his B.S. degree in mathematics *summa cum laude* from Brooklyn College in June of 1977. In the fall of 1977 he became a doctoral student in mathematics at Princeton University.

points above the 99.5th percentile of national norms for ninth-grade students who have been in this type of class all year. Upon seeing this achievement, the teacher agreed with the boy that he was indeed ready to join the class! Instead, he took a college mathematics course that summer and easily earned a final grade of A.

At the end of the sixth grade a student took second-year algebra in summer school without having had first-year algebra; his final grade was A. By the end of the eighth grade he had earned credit by examination for two semesters of college calculus. A year later he had completed third-semester calculus by correspondence from a major university, earning A as his final grade.

A student learned two and one-half years of algebra well by being tutored while in the fifth and sixth grades. He continued, by means of tutoring, with a high-level course in geometry. His tutor in geometry was a sixteen-year-old freshman at Johns Hopkins who enrolled for honors advanced calculus (final grade, A) and other subjects that most nineteen-year-olds would find extremely difficult. He, too, condensed his mathematics radically.

Several girls have accelerated their progress in mathematics considerably, though not as much as the boys discussed above. One of them graduated from high school a year early while being one of the best students in SMPY's second high-level college calculus class.

Many other such examples could be given (e.g., see Stanley 1974, 1976 *a-f*) to show that the usual high school pace in algebra I to III, geometry, trigonometry, analytic geometry, and the calculus is far from optimum for boys and girls who reason extremely well mathematically. Algebra I is a particularly virulent culprit, because being incarcerated in it for a whole year gives the apt student no really appropriate way to behave. He or she can daydream, be excessively meticulous in order to get perfect grades, harass the teacher, show off knowledge arrogantly in the class, or be truant. There is, however, no *suitable* way to while away the class hours when one already knows much of the material and can learn the rest almost instantaneously as it is first presented. Boredom, frustration, and habits of gross inattention are almost sure to result.

We are amazed that even more youths do not sustain obvious academic injury, and we suspect that the damage is greater than it seems. At least, it appears uncomfortably likely that motivation for mathematics may suffer appreciably in all but those few students devoted to the subject. After such snail-pacing in high school precalculus and calculus—often, five and one-half years or more—the number of top minds still excited by mathematics may be few.

The remedy for this unfortunate situation is conceptually simple but seldom employed. It consists of the regular and appropriate use of tests.

First, those students with great mathematical reasoning ability are found. Then various tests of achievement in mathematics are administered to them. This enables mathematics teachers to determine what a particular talented student does not yet know and arrange for him or her to learn those points, and those only, fast but well.

Seldom, though, does the teacher of beginning algebra use an achievement test during the first week of class to locate the students who might, with a little individual help, move into second-year algebra right away. Also, not nearly enough use is made of the mathematics scores from the achievement batteries that most schools administer. Those tests are not difficult enough to differentiate adequately among the top several percent of the group, but at least they do single out potentially exceptionally able youths.

In special classes where students are grouped homogeneously according to high mathematical reasoning ability, SMPY has found that first-year algebra can be mastered in from nine to twenty two-hour weekly periods—and, as noted above, some exceptionally able students do not need even that much. Details about this are contained in Fox (1974*a*, 1976*b*) and Stanley (1976*b*). Other precalculus courses and the calculus can also be learned quickly by mathematically apt youths, as George and Denham (1976), George (1976), and Stanley (1976*b*) document rather fully.

To go beyond first-year algebra, youths need certain better-developed mental qualities, especially excellent reasoning ability and Piagetian formal-operations status. SMPY's testing and experience with special instructional programs and the studies by Keating (1975) and Keating and Schaefer (1975) indicate that the intellectually top 1 or 2 percent of students as low as the fifth grade probably already have these abilities well enough developed to learn algebra II and other precalculus courses well. Speed of learning them is dependent on level of ability, quality of instruction and pacing, stimulation by classmates or tutor, and the mysterious ingredient called motivation that makes the student willing (or, ideally, eager) to do a great deal of homework excellently between classes.

For these reasons SMPY conducted its three annual mathematics talent searches among seventh and eighth graders, but also did special work among sixth graders and a few students even younger than that. Students whose mathematical reasoning abilities proved to be superb were encouraged to move fast through the high school mathematics sequence, beginning with algebra I or skipping it and ending *soon* with calculus so well learned that college credit for it could be obtained. Somewhat less able entrants were given less drastic suggestions, but nevertheless encouraged to speed up their progress in mathematics and science. Experience of several years has shown that youths able and eager to move ahead can do so readily if they and their parents are resolute and persistent in their search for suitable ways.

Tentative physiological evidence concerning the suitability of the age period twelve to thirteen for accelerating educational progress was suggested rather recently by Epstein (1974*a*, *b*). He found spurts in both brain development and mental age, one of them at chronological ages ten to twelve. Mental age seemed to grow especially slowly during the years twelve to fourteen and then to spurt again for the final time at fourteen to sixteen. Thus junior high school students (grades seven to nine) may be on a mental plateau. We do not know, however, whether his findings characterize precocious youths, who might spurt at different times than average students do. It seems congruent with our experience to postulate that by age twelve some youths already have great learning potential that seems to accelerate to the point that by age fourteen to sixteen they are fully ready to succeed in a selective college. We have not noticed any tendency for SMPY participants to have merely reached a rather high level of ability early and to remain there. Obviously, though, the developmental curve for a given ability might differ greatly from one person to another, depending on genetically programmed potential, environmental stimulation, and the interaction of these two.

## WHY NOT CONDUCT A CONTROLLED EXPERIMENT?

Because experimentation is a strong force in psychology and in my own background (e.g., Campbell and Stanley 1966; Stanley 1973), we were tempted to set SMPY up as a rigorously controlled experiment. Upon reflection, however, we came to believe that there were cogent reasons for not doing so. Some of those considerations were the following:

1. We were rather sure that the smorgasbord of accelerative educational opportunities we planned to offer the "experimental" subjects in the study were much more likely to help than to harm them. Therefore, it would be inadvisable to withhold such opportunities from a portion of the subjects (probably half of them) who in a controlled experiment would be assigned randomly to a "control" group.

2. There were not likely to be enough extremely high scorers to make the numbers in both the experimental and the control group sufficiently large to yield statistically powerful or precise comparisons between groups and subgroups. It seemed more sensible to take the *N* ablest subjects and mass the experimental efforts on them.

3. The procedures, principles, and techniques that SMPY planned to develop would be disseminated widely by the press and in speeches, letters, articles, books, and newsletters, so withholding knowledge of opportunities from a control group of subjects would be impossible. The control group would be substantially exposed to influences designed only for the experimental group, and that type of contamination would greatly weaken or even nullify the experiment.

4. By not having a control group from which certain presumably beneficial opportunities and information were withheld, it is possible to keep the study completely on an above-board basis, with no need to deceive anyone about anything. This openness is important in gaining the confidence of the students, their parents and teachers, and the general public.

5. Certain comparisons could be made by matching and other quasi-experimental procedures. Fox (1976*b*) did this in her study of sex differences in mathematical aptitude and achievement, as have other SMPY researchers in trying to determine how well a certain special procedure worked.

SMPY plans to use a completely controlled experimental design in its attempt to increase interest in chemistry among mathematically talented youths, but not to deceive either group about the nature of the study. Members of the control group will get equivalent educational stimulation, though not in chemistry. The staff of SMPY is not at all sure in advance that the chemistry "treatment" will be effective, so it seems reasonable to withhold it from some of the ablest youths (with their knowledge and consent) while giving them the same amount of attention in certain other areas. Of course, despite SMPY's best efforts, this experiment will be contaminated somewhat by knowledge of its nature and by whatever spillover of chemistry influence from the experimental to the control group that may occur, but if the experimental variables are not potent enough to triumph over these, they are probably not of great practical value. Careful attention to the sources of invalidity spelled out by Campbell and Stanley (1966) will help keep the experiment as unbiased as possible. Experimentation with humans in important, relevant "field" situations is seldom as easy or neat as experimentation under laboratory conditions can be. Often, however, it yields more important, albeit perhaps somewhat equivocal, information.

6. A great deal of SMPY's analysis of the results of its programs depends heavily on case-study clinical methods, using all known information about each individual with as much insight as can be mustered on the basis of considerable experience with many mathematically precocious youths (see Hudson 1975). Burt (1975, p. 138) states this point especially clearly:

> With human beings, when the problem is primarily psychological, statistical studies of populations should always be supplemented by case studies of individuals: early histories will often shed further light on the origin and development of this or that peculiarity. Tests should be supplemented by what Binet called the *méthode clinique*, and interpreted by introspective observations, designed to verify the tacit assumption that they really do test what they are intended to assess. After all, each child is a complex and conscious organism, nor a mere unit in a statistical sample.

Fortunately, many of SMPY's procedures yield results so different from the usual ones that the effects are obvious. For instance, it is almost preposterous to suggest that if SMPY had not found a certain youth when he was an over-age sixth grader and helped him in many ways to move ahead educationally fast and well he would, nevertheless, have been graduated from a major university at barely seventeen years of age. The youngest recipient of a bachelor's degree in 1971 at Johns Hopkins was nineteen years ten months old (Eisenberg 1977). Two years later, under SMPY's influence, the youngest was seventeen years seven months old, and three months later he had completed a master's degree also. Now seventeen-year-old graduates are frequent. Similar strong observations could be made about most of SMPY's programs, such as the effects of the fast-math classes (Fox 1974*b*; George and Denham 1976; Stanley 1976*b*).

## THREE SEQUENTIAL ASPECTS OF SMPY: D³

The first book-length report about SMPY's initial work (Stanley, Keating, and Fox 1974) was entitled *Mathematical Talent: Discovery, Description, and Development*. To emphasize the three D's, we sometimes abbreviate that title, pseudo-mathematically, as MT:D³. Discovery is the identification phase during which the talent is found. Description is the study phase during which the most talented students are tested further and otherwise studied a great deal. This leads to the prime reason for SMPY, the development phase. During it the youths who were found and studied are continually helped, facilitated, and encouraged. Each is offered a smorgasbord of educational possibilities (see Fox 1974*a*, 1976*a*; Stanley 1976*a*) from which to choose whatever combination, including nothing, that best suits the individual. Some splendid mathematical reasoners try almost everything at breakneck speed, whereas others do little special. SMPY offers as much educational and vocational counseling and guidance as its resources permit, both via memoranda and its newsletter—ITYB, the *Intellectually Talented Youth Bulletin*—and individually as requested.

Most studies of intellectually gifted children are heavy on description but light on educational facilitation. From the start the SMPY staff has been determined to intervene strongly on behalf of the able youths it found. Thus discovery and description were seen as necessary steps leading to strong emphasis on accelerating educational development, particularly in mathematics and related subjects.

## WHY ACCELERATION RATHER THAN ENRICHMENT IS STRESSED

There were both logical and empirical reasons why we chose to emphasize educational acceleration rather than enrichment. Some of them are implied above, such as that mathematically highly apt students can move through the standard mathematics curriculum much faster and better than they usually do. Fears expressed by teachers or parents about their missing important concepts or techniques because of the speed are usually groundless and, indeed, often merely a rationalization for inaction. Such students are likely to doze through the 5 percent they do not know when it is camouflaged by the 95 percent they already know, because under these circumstances there is no incentive for them to be alert. SMPY has evidence (see Fox 1974*b*, George and Denham 1976; Stanley 1976*b*) that students who reason extremely well mathematically learn first-year algebra considerably better in a few two-hour periods with their intellectual peers than they do in regular all-year classes.

There seem to be four main kinds of educational enrichment: busy work, irrelevant academic, cultural, and relevant academic. In our opinion, for reasons to be stated below, only the third (cultural) is well suited to mathematically highly precocious youths; it does not, however, meet their needs in mathematics itself or in the other usual academic subjects.

*Busy work* is a well-known way for some teachers to keep their brightest students occupied while the class goes on with its regular work. In a common form it consists of having them do a great deal more of the subject in which they are already superb, but at the same level as the class they have surpassed. One of our eighth graders, whose Stanford–Binet IQ as a kindergartner was 187, was asked by his algebra teacher to work every problem in the book, rather than just the alternate problems that the rest of the class was assigned. He already knew algebra I rather well and therefore needed to work few problems, so he resented this burdensome chore. The busy work proved to be a powerful motivator, however, because after that year he took all of his mathematics at the college level. First, though, during the second semester of the eighth grade and while he was still twelve years old this precocious youth took the regular introductory course in computer science at Johns Hopkins and earned a final grade of A. During the summer, still twelve until July, he took a course in college algebra and trigonometry at Johns Hopkins, earning a B. From then on for two academic years and two more summers he took college mathematics through the calculus and linear algebra and two years of college chemistry, with all A's. At age 15 1/6 years he entered Johns Hopkins as a full-time student with 30 percent of the sophomore year completed. During his first year at Hopkins he earned eight A's and one B on difficult courses, majoring in electrical engineering. Thus in a rather perverse sense his teacher had done him a great favor, but without his having been discovered by SMPY, he would probably have been forced to sit a whole year in each of numerous high-school mathematics courses far below his capabilities.

In May 1976 this remarkable young man completed his junior year at Johns Hopkins with an impressive record in both his studies and research. On his sixteenth birthday, July 10, 1975, he had begun work for the summer with General Electric. During the summer of 1976, while still sixteen, he was a full-time researcher at the Bell Telephone Laboratories. He is scheduled to receive a baccalaureate from Johns Hopkins a couple of months before his eighteenth birthday—that is, four years ahead of the usual age-in-grade progression—and continue on to earn a Ph.D. degree in electrical engineering by age twenty or twenty-one. Radical educational acceleration is certainly paying off well for him—academically, professionally, and personally. In March 1977 he was awarded a three-year National Science Foundation graduate fellowship to study electrical engineering at the Massachusetts Institute of Technology.

One of his classmates (who skipped grades seven, nine, ten, twelve, and thirteen) completed his baccalaureate work at Johns Hopkins in December 1976, a few days after his seventeenth birthday, with a major in quantitative studies and considerable work in political science, economics, and astronomy. He plans to start work toward the M.B.A. and Ph.D. in economics at the University of Chicago while still seventeen.

Another of their quite bright classmates received his bachelor's degree in electrical engineering while still 17 2/3 years old, and a physics major reached only 18 1/2. Both of these were elected to Phi Beta Kappa, and both won three-year National Science Foundation fellowships.

*Irrelevant academic enrichment* consists of not determining precisely what types of advanced stimulation the brilliant student needs, such as faster-paced mathematics for the mathematically precocious, but instead offering all high-IQ youths a special academic course such as high-level social studies or essentially nonacademic work such as games (e.g., chess) or creative training largely divorced from subject matter. Of course, while this may be splendid that year for those whose major interest is touched on, it does not assuage the mental hunger of the mathematically oriented. (See Stanley 1954*a*, 1958, 1959*a*.) Also, if the enrichment is academic, special efforts need to be made to alter later courses, or else the enriched students may be more bored than ever in subsequent years.

*Cultural enrichment* consists of providing certain "cultural" experiences that go beyond the usual school curriculum and therefore do not promote later boredom. Examples are music appreciation, performing arts, and foreign languages such as Latin and Greek (see Mill 1924 and Packe 1954). Early experiences with speaking modern foreign languages and learning about foreign cultures can also fit this pattern and may be a type of stimulation that parents and teachers of high-IQ youths should provide from the early years. These do not, however, meet the specialized academic needs of the intellectually talented.

This may be the place to decry what we at SMPY perceive to be vast overemphasis on the Stanford–Binet or Wechsler-type overall IQ in planning academic experiences for brilliant children. If one takes a group of students who all have exactly the same Stanford–Binet IQ (say, 140), one does not have a group homogeneous with respect to such special abilities as mathematical reasoning. The IQ is a global composite, perhaps the best *single* index of general learning rate. One can, however, earn a certain IQ in a variety of ways, e.g., by being high on memory but much lower on reasoning, or vice versa. *It is illogical and inefficient to group students for instruction in mathematics mainly on the basis of overall mental age or IQ.* Often this is done and then the students who lag behind in the class are accused of not being well motivated, when in fact they simply do not have as high aptitude for learning mathematics as some in the class who have the same IQ. These considerations also apply to other academic subjects, such as history or English literature.

It is difficult to form a group of students really homogeneous for instruction in a given subject even when one uses all the psychometric and other knowledge about them that can be gathered. To rely primarily on the IQ for this purpose, as quite a few city and state programs for intellectually talented youths do, seems curious. An obvious corollary is that students should be grouped for instruction separately for each subject and that these groupings should be subject to change from year to year. Probably administrative or political convenience is the cause of undue reliance on a single grouping measure such as IQ. Now that computer scheduling is available, however, this justification for an ineffective process is weakened.

The fourth and last type of enrichment is what we term *relevant academic*. It is likely to be both the best short-term method and one of the worst long-term ones. Suppose, for instance, that an excellent, forwardlooking school system provides a splendid modern mathematics curriculum for the upper 10 percent of its students from kindergarten through the seventh grade, and then in the eighth grade these students begin a regular algebra I course. How bored and frustrated they are almost sure to be! It is not educationally or psychologically sound to dump these highly enriched students into the mainstream, and yet that kind of situation often occurs. Only if the kindergarten through twelfth-grade curriculum is considered can this failure of articulation be prevented. Even then, a superb thirteen-year mathematics program without strong provisions for college credit would merely defer the boredom and frustration until the college years.

For the preceding logical reasons we feel strongly that any kind of enrichment except perhaps the cultural sort will, without acceleration, tend to harm the brilliant student. Also, there is excellent support for acceleration in the professional literature. Wiener (1953, 1956), Fefferman (Montour 1976*b*), Bardeen (Young 1972), Wolf (Keating 1976, see index; Montour 1976*a*), Watson (1968), and others have benefited greatly from it professionally. Norbert Wiener had his baccalaureate at fourteen and his Ph.D. degree at eighteen. Charles Louis Fefferman had his baccalaureate at seventeen and his doctorate at barely twenty; by age twenty-two he was a full professor of mathematics at the University of Chicago. Five years later he was the first winner of the National Science Foundation $150,000 Waterman Award.

John Bardeen, twice a Nobel Laureate in physics, completed high school at age fifteen. Merrill Kenneth Wolf, now a prominent neuroanatomist and talented musician, was graduated from Yale University shortly after becoming fourteen years old. James Watson had his Ph.D. degree at age twenty-three and had earned a Nobel prize before he became twenty-five. These examples could go on and on. Counterexamples, such as the ill-fated William James Sidis (Montour 1975, 1977), who was graduated from Harvard College at age sixteen but failed badly thereafter, are rare.

Lehman (1953), a psychologist, teamed up with a specialist in each of various fields to study the ages at which their greatest creative contributions were made by eminent scientists, scholars, and prodigies of other kinds. The typical age at which eminent mathematicians and physical scientists made their most highly rated achievements was lower than the average age at which the Ph.D. degree in those fields is awarded in the United States. Many brilliant young men and women are still students when according to logic and history they should be more independent researchers.

Terman and Oden (1947, pp. 264–66) found that the typical member of Terman's gifted group was graduated from high school about a year early. They advocated a moderate amount of acceleration for gifted youths. Hollingworth (1942), who worked with even abler children than the average of Terman's group, recommended considerable acceleration for them.

The University of Chicago's extensive experience with early entrance and fast progress in college during the 1930s showed that this was indeed a feasible approach for certain students. After this program was largely abandoned because of financial and other reasons, the Fund for the Advancement of Education (1953, 1957) set up studies at a number of colleges and universities to admit well-qualified students at the end of the tenth or eleventh grade. These were judged to be markedly successful.

Hobson (1963) and Worcester (1956) showed that, when properly arranged, early entrance to public school was beneficial. It seems to me especially unfortunate that their work is not well known to most educational administrators, because its scope, practicality, and clarity make the findings hard to ignore.

The most comprehensive study of educational acceleration was the splendid monograph by Pressey (1949). Anyone who can read it carefully and still oppose such acceleration certainly has the courage of his or her preconvictions. Pressey, Hobson, Worcester, and others reveal that opposition to acceleration is founded on emotionalized prejudices rather than facts. (Also, see Friedenberg 1966.) We do not know of a single careful study of actual accelerants that has shown acceleration not to be beneficial, though armchair articles against it abound (see Daurio 1977).

In SMPY's experience, the eagerness of the brilliant student himself or herself to move ahead rapidly seems crucial. If the youth is reluctant to take a particular accelerative path, such as going into algebra II early without bothering with algebra I, taking a college course, or skipping a grade, probably he or she should not be urged to do so. Unfortunately, many boys and girls are not allowed by their teachers, guidance counselors, principals, or even sometimes their parents to make a calm, rational decision about such matters. They may get so much bad advice that they give up in confusion. Many are simply forbidden to use a particular method of acceleration. It takes an unusually strong-willed youth to buck this adult obfuscation and tyranny.

From its inception SMPY has tried to communicate directly with the youths themselves, rather than through their parents. Reports of the results of the testing competition have gone to them, even including discussion of percentile ranks on national norms and the like. We have also written letters to them in response to their queries or their parents'. In the few instances where we have deviated from this policy—chiefly, with quite young boys and girls who came to our attention by way of their parents rather than through the formal talent search—the youngster's motivation has seemed to suffer. We believe that contacts of the facilitating agency such as SMPY should be mainly through the youth, even though he or she may be only nine or ten years old. After all, a child that age whose Stanford–Binet IQ is 170 or more (and SMPY seldom deals with any that young unless they are that bright) has a mental age of at least fifteen years. He or she will be as able to understand our communications as many parents are. We want the youths to take charge of their own academic planning early and to use their parents and us as means for implementing their own decisions. Some parents object to this approach, of course, because they want to keep their children dependent, but if communication from the beginning is with the student, such friction between SMPY and the parents will not usually be great.

In summary, the SMPY staff believes that offering each splendid mathematical reasoner a varied assortment of accelerative possibilities and letting him or her choose an optimum combination of these to suit the individual's situation is far superior to so-called special academic enrichment. Of course, we would be pleased to see individual courses and curricula improved and special accelerative classes set up by school systems for their intellectually talented students.

## SELF-PACING AS INAPPROPRIATE NEOENRICHMENT, VERSUS GROUP PACING

When we propose accelerative opportunities for mathematically highly talented youths, the school is likely to counter by offering to let them proceed "at their own pace." In practice this usually means still sitting in the too-slow class, such as first-year algebra, while working ahead in the book and perhaps into algebra II. Common sense and observation tell us that this is not likely to work well for most students, no matter how able. Any student that autonomous and well motivated would probably have little use for school. Our model is definitely not self-pacing, whether in the crude way described above or by means of programmed instructional materials, except for an occasional highly unusual student.

We have found that stimulation by one's intellectual peers within a homogeneously grouped class which is fast-paced by the teacher produces astoundingly good results for about half of the students enrolled. Skeptics should read about some of SMPY's fast-mathematics classes: Wolfson I (Fox 1974*b*; Stanley 1976*b*); Wolfson II (George and Denham 1976); and McCoart calculus (Stanley 1976*b*).

Our model is somewhere between the high-ability athletic team that stimulates its members to great achievement against an opposing team, and individual competition such as tennis singles or running the hundred-yard dash. The difference between SMPY's special fast-mathematics classes and athletic events is that the mathematically precocious youths have an opponent against which all of them can win and be stars—namely, national norms on standardized achievement tests. Though they pace each other fast, and students who proceed too slowly may have to leave the group, the SMPY students are not competing directly with one another or with any other team except the anonymous national one.

Programmed instructional materials are almost sure to contain too many steps, and too small ones, for mathematically extremely apt students, who will therefore tend to be bored and frustrated by them. Also, such materials do not usually lend themselves to group-paced stimulation. Most of our precocious youths do not perform well against an abstract standard such as number of chapters or frames completed, just as a track man does not usually run well alone or a tennis player perform his or her best against a weak opponent. Most of our students who have tried self-pacing or correspondence-study courses move far less swiftly and well than they do in special fast-mathematics classes. Therefore, we consider the group-pacing feature essential for most persons (cf. Macken et al. 1976).

## EMPHASIS ON COUNSELING AND TUTORING THE INDIVIDUAL

All of SMPY's efforts are directed toward helping each youth use his/her mathematical and other abilities best for the ultimate benefit of the person—and, we assume, thereby for society itself. The smorgasbord of accelerative educational possibilities that SMPY develops, tries out, and refines is meant to be adapted flexibly to each student. No one program, in mathematics or other educational areas, could possibly serve many of this highly able group well.

This approach makes the "description" (i.e., the study) phase of SMPY follow crucially from the "discovery" (i.e., identification) phase and lead naturally to the "development" (i.e., facilitation) efforts. Without intensive study of the aptitudes, achievement, interests, values, and attitudes of the youths who scored quite high on SAT-M, appropriate counseling would not be possible. Such study continues, of course, during the entire period that the youths are being helped and followed, but a massive initial assessment program helps begin the counseling process. (See Stanley, Keating, and Fox 1974; Keating 1976.)

Part of this studying is done via diagnostic testing and the ensuing specific teaching of just those points not yet known by the student. For example, many seventh- or eighth-grade youths who reason extremely well mathematically can score high on a standardized test of knowledge of first-year high school algebra even though they have not yet studied a school subject entitled "Algebra I." If, for example, such a student can answer correctly thirty out of forty items on Form A of Educational Testing Service's Cooperative Mathematics Algebra I Test in the forty-minute time limit, he has scored better than 89 percent of a random national sample of ninth graders did after studying algebra I for a whole school year. Then the youth is handed back the test booklet, told which ten items he missed, and asked to try them again. If he still misses, say, six items, they are examined carefully and he is helped by a tutor to learn quickly those points that he does not know. After suitable instruction on *just those points* and on any other points in the test about which he was unsure (e.g., items guessed right), he takes Form B of the test under standard conditions and his success is studied. In this way an able youth can often go on to algebra II within a few hours, rather than wasting nearly all of a long, tedious 180-period school year on algebra I. He already knows most of the material of the first course or can learn almost any not-yet-known point almost instantaneously. This type of diagnostic testing and teaching of superior mathematical reasoners makes so much sense that we cannot understand why it is tried so seldom. SMPY has formalized the procedure into a day-long "algebra tutorial clinic."

As a valuable part of its smorgasbord, SMPY has begun to develop into expert tutors mathematically talented youths who are not much older than the persons they tutor. This one-to-one relationship, modeled on the tutorial system of Oxford and Cambridge universities rather than the remedial tutoring arrangement more common in the United States, is proving to be the fastest and best way to move the typical quite young, mathematically highly apt youth ahead fast and well in mathematics.

For example, a seventh grader who scored 720 on SAT-M was tutored by a brilliant eleventh grader less than two years older than he through algebra I to III and geometry easily on Saturday mornings during eight months of the school year. The tutored youth then entered the ongoing Wolfson II fast-math class that summer and was its best student in trigonometry. He skipped the eighth grade and at barely fourteen years of age received by examination credit for two semesters of college calculus. As a tenth grader he made A's on both calculus III and differential equations. At fifteen he took complex-variable theory in the Johns Hopkins summer session and made a final grade of A. Besides all that, he had completed college courses in oceanography and computer science! After the eleventh grade, two years accelerated, he will enter college with sophomore standing or more at the ancient age of 16½ years. Think how much boredom this extremely able, well-motivated young man would undoubtedly have suffered had his mother not "discovered" SMPY when he was beginning the seventh grade.

## ARTICULATION WITH THE SCHOOLS

SMPY is not a curriculum-development project. We decided early not to attempt altering the best of the standard school courses and textbooks. That in itself would be a multimillion-dollar project. Fortunately, in the wake of Russia's Sputnik I from 1957 until recently many programs such as SMSG mathematics, BSCS biology, and PSSC physics were carried out on a comprehensive scale by specialists. Elements of these have been incorporated into most high school courses and textbooks. It would be unnecessary and presumptuous of SMPY to engage in curriculum construction.

Thus we work within the better school mathematics curricula, usually in the conventional order of algebra I to II, geometry, college algebra and trigonometry, analytic geometry, and calculus. The special mathematics classes move through these extremely rapidly at a high level of rigor, abstraction, and proof, using standard textbooks. (For calculus a college textbook is used.) Creativity in these courses is promoted by the subject matter itself, the creative skills of the teacher, and the influence of able classmates, rather than by training for so-called creativity itself. We do not deny that such training can probably be useful for some students in certain courses or grades, but for our purposes the direct approach to creative performance in mathematics itself seemed preferable. Actually, until even the brightest students get into mathematics of at least number-theory or advanced-calculus level, much of their learning is algorithmic—how to perform processes and why these processes work. Originating proofs and derivations can be encouraged early, but for quite a while most students will be kept rather busy trying to understand the algorithms and proofs that the instructor and the textbook introduce, rather than devising their own.

A caution is in order here: Before a young student abandons pre-algebra mathematics, including arithmetic, for algebra (which, if he or she is able enough, may be easy), diagnostic testing should be done to discover specifically what this particular student does not yet know about arithmetic concepts and computation so that this material can be taught fast and well on an individual basis. This point has been mentioned earlier in another context; it is especially relevant when, for example, a nine-year-old enters a fast-mathematics class such as the one described by Fox (1974*b*).

Our early rejection of curriculum revision as a goal of SMPY has enabled us to save schools considerable time and money and still not upset their sequences of courses. If, for instance, a student learned all of precalculus mathematics well in one of our special classes while still a seventh or eighth grader, the next stage would simply be finding a high school (or college) calculus course for him or her. Most senior high schools are cooperative about this. The greatest problem occurs in the three-year junior high schools (grades 7 to 9), some of which offer algebra I and II, whereas others offer algebra I and plane geometry. Few provide courses in both algebra I and II and geometry, so the student who completes both years of algebra or algebra I and geometry while a seventh or eighth grader may be left without any mathematics to take for a year or two unless a senior high school is nearby. Some friction between certain junior high schools and SMPY has resulted because of this, but sincere efforts by both parties reduced it.

Our initial purpose was to try out procedures that would augment the usual work of the schools. SMPY was meant to be prototypal, producing exportable principles, techniques, and programs that public and private schools could adopt and adapt for their own uses. We were not going into business as an educational agency except to develop, try out, and improve whatever special procedures mathematically highly gifted youths seemed to need. We did not want to criticize the schools' performance of their usual functions, but merely to offer them ways to meet the highly special needs of a relatively small but extremely important group of their students. Thus articulation of our methods with theirs was important from the start.

Being aware of the vast and often cumbersome bureaucracy of educational systems, however, we did not want to get enmeshed in prolonged deliberations with supervisory personnel of city and country school systems. We planned to work with the youths themselves, and, through them, with their parents. As noted above, our communications are addressed directly to the students. As we said somewhat facetiously, the students are free to share our memoranda and letters with their parents, who in turn might share them with teachers, counselors, and principals if they wish to do so. Usually, we send an extra copy of each memorandum, to make that easy. We believe that this is the desirable way for us to proceed, because more change can be effected quickly for particular individuals at the child–parent–teacher–counselor–principal level than by trying to institutionalize innovations in a school system. Also, such innovations, even if finally adopted, tend to differ from the original model in what we would consider unfortunate ways. We want to develop our own innovations with minimum demands on the schools and then offer *them* for adoption throughout the country, not just in the Baltimore area.

We departed from this plan with one school system that contacted us early and expressed interest in cooperating. This resulted in many long high-level meetings that took much of our limited time and did not seem productive enough. Supervisory personnel may be quite cautious about proposed innovations, preferring to express their concerns and reservations about them rather than to take positive action. Such talk often serves mainly to delay or fend off the innovation.

This is not to say that school systems cannot be led or forced to change curricular policies. Often they can, especially if a sizable group of determined, well-informed parents whose mathematically highly talented children attend the schools concentrate on attaining specific objectives. Outsiders such as SMPY have far less political leverage, but by working directly with students and their parents they can help initiate pressure for needed policies and programs.

Excellent private schools can often provide well for students who are somewhat above average, e.g., those with IQs of 120 to 140. For youths with IQs much above 140 or so, however, the small size of most private schools and their social nature (usually more intimate than that of public schools) may make them less flexible in dealing with extremely gifted youths than public schools can be. Especially, faculty members of many private schools are even more opposed to educational acceleration than most public school teachers are.

In any event, private schools are no automatic panacea for the intellectually extremely talented. Parents who expect *any* school to provide optimally for their 160- to 225-IQ child without much help from them simply do not understand the extreme nature of such brightness. In an important sense, an IQ of 160 is the mirror image of an IQ of 40, because both deviate 60 points from the average IQ of people in general. A child with an IQ of 160 is about as bright as a child with an IQ of 40 is dull. Both need much special attention if they are to utilize their respective abilities effectively. A great deal of the thinking and planning for a brilliant child must come from its parents or other interested persons bent on supplementing the efforts of the school.

SMPY is not primarily a service project. It is meant to be prototypal —that is, to develop principles, techniques, and practices that can be used widely to improve the mathematical and other education of youths who reason extremely well mathematically.

## BENEFITS TO STUDENTS

The benefits to SMPY's participants are numerous. Among them are the following:

1. Increased zest for learning and life, reduced boredom in school, and therefore a better attitude toward education and other activities.
2. Enhanced feelings of self-worth and accomplishment.

3. Reduction of egotism and arrogance. At first this may seem counterintuitive, but repeatedly we have observed that SMPY students who compete with their intellectual peers in rigorous settings such as special fast-mathematics classes tend to develop more realistic understanding of their ability. These youths learn that, compared with national norms on standardized tests, they are superb, but less spectacular relative to each other. In regular mathematics classes the typical SMPY participant earns such good grades with little effort that the temptation to feel superior is strong. For example, the 190-IQ boy who by age eleven had done so well in two college computer-science courses and on the Advanced Placement Program examination in college calculus seems far less egotistical than he was before entering one of our special precalculus classes at age ten. In the SMPY courses he had to work hard to maintain an average rank, whereas as an accelerated sixth grader he was vastly overqualified for all his regular subjects.

4. Becoming far better prepared educationally than they otherwise would be, especially in mathematics, which is basic to many disciplines.

5. Better preparation for the most selective colleges and improved chance of being admitted to them. For example, in the fall of 1975 four of the students whom SMPY had helped entered Harvard or Radcliffe Colleges, two of them two years early each and one of those as a highly prestigious National Scholar.

6. Getting into college, graduate school, and a profession earlier, thus having more time and energy for creative pursuits.

7. Increased opportunities to explore more specialties and hobbies.

8. More time to explore various careers before marriage.

9. Less cost. Most accelerative procedures save the student and/or the parents money. Even skipping the last year of junior high school and going into senior high school a year early eliminates a year that the student must be supported at home. Eight credits earned by means of a $32 Advanced Placement Program examination in calculus were worth $1000 of tuition at Johns Hopkins in the fall of 1977, and such costs tend to rise almost every year. Graduating from college in three years rather than four saves about one-fourth of all costs and can lead to paid full-time employment a year earlier than otherwise.

10. Being an unusually well-prepared, advanced entrant to college often brings the student to the attention of professors who help him or her get started on important research early. This, in turn, usually leads to better graduate-school opportunities, including improved financial support there. For example, five of SMPY's six radically accelerated youths who were graduated from college in 1977 at ages fifteen to eighteen won National Science Foundation three-year graduate fellowships.

11. Ultimately, we hope, considerably greater success in life, both professionally and personally.

## BENEFITS TO SOCIETY

Presumably, whatever helps a sizable group of talented individuals use their abilities better should also benefit the larger society. It is easy to see that a number of the points made above about benefits to SMPY participants themselves fall into this category. Below we shall list a few other, somewhat related gains that society itself can expect from the three D's of SMPY and similar programs.

1. Students superbly prepared to major in the mathematical sciences, physical sciences, quantitative social sciences, and other areas where mathematical talent and keen analytical ability are essential or helpful.
2. More years of professional contribution and effective adulthood.
3. Happier, more effective citizens who will understand better how to educate their own children.
4. Reduced cost of education. The types of policies and activities that SMPY espouses save school systems and colleges money, rather than increasing educational expenditures. When a student who already knows first-year algebra is moved into algebra II, room for another pupil is created in the algebra I class, or the teacher can probably work more effectively with the lesser number because a potential distracter and irritant has been removed. When a student skips an entire school grade, the cost of educating him or her that year is saved. If four and one-half years of precalculus mathematics can be learned in a year, a great saving is likely to ensue. Passing introductory college calculus by examination increases room in the class and enriches the next mathematics course by moving an able, well-motivated student directly into it. Students who go through selective colleges in three years rather than the usual four enable those schools to handle more students.

Of course, it would be naive to assume that special policies and provisions for mathematically highly talented youths do not require any extra efforts. Of course they do, but the more effectively the facilitators of these students work, the greater the savings that can accrue to the school system, above and beyond their salaries and other expenses. Much of the identification, study, and implementation can be done by regular personnel in the mathematics supervisor's office. Even if in a strict cost-accounting sense the mathematically precocious were to cost a little extra, it would be an almost negligible amount relative to the expenditures for other types of special education within most school systems.

An often overlooked factor reducing the cost of working with intellectually gifted youths is the tremendous output that one gets for inputs which take little time. A few instructional minutes spent with a brilliant youth can produce amazing results. This contrasts sharply with the much greater amount of time that one must devote to a slow learner in order to get even moderate gains. Similarly, counseling SMPY participants and their parents by memorandum, telephone, letter, or case conference does not usually require a great deal of time but often produces striking changes in their education.

An added advantage is that most intellectually precocious youths have bright parents who can and will read counseling information before asking questions, thereby saving the advisers considerable time.

The two sentences with which I ended the first chapter of the first volume of SMPY's *Studies of Intellectual Precocity* (this is the third) seem appropriate here: "Expensive curricular adjustments are made, quite justifiably, for slow learners. It is past time that fast learners get the much less costly 'special education' they deserve" (Stanley, 1974, p. 19).

## SCARCE RESOURCES AND ELITISM

But even after the above points some readers may still feel that any special attention to mathematically highly precocious youths is an unwarranted and unnecessary diversion of scarce special resources. Won't the talented boy or girl get along rather well with the regular resources of the school? Don't elective courses such as algebra I, offered specially in the eighth grade of some school systems, and the considerable array of honors-type subjects in senior high school (calculus being a strong example) take care of the needs of the gifted satisfactorily? Why provide more for those who already have so much? Isn't that elitism and therefore contrary to the American way of life? One could argue endlessly about the philosophical content of these questions. Empirically, however, the answer is clear: many of the youths in the top few percent of their age mates with respect to mathematical reasoning ability can learn mathematics and related subjects faster and better than the curricula of most schools permit. If held to the age-grade lockstep, a large percentage of them will develop poor work habits and lose interest in the area. Even those who do not would usually benefit from better opportunities.

An example, not highly unusual for SMPY, may serve to illustrate the point that quite a few students lag undesirably far behind their capabilities in the usual school setting. We discovered a certain young man at the end of the summer after he had completed the seventh grade of a public junior high school. Standardized testing showed that without actually having had an algebra course he already had almost perfect knowledge of the first year of that subject. In September he entered our first fast-mathematics class, which had begun in June and had covered algebra I quickly during the summer (see Fox 1974*b*, Student No. 1; Stanley 1976*b*, app. 7.2). By the next August—that is, in about fifty two-hour Saturday-morning classes—he had completed algebra II and III, geometry, trigonometry, and analytic geometry well. That fall, as a ninth grader, he entered a selective independent school in the Baltimore area. It took considerable effort by us to convince the calculus teacher that he should be allowed in that twelfth-grade subject. As the year wore on he became one of the very best students in the class. At age 14 he took the higher-level (BC) national calculus examination of the Advanced Placement Program and made a grade of 4 (meaning that he was "well qualified" for two semesters of college credit). Only a few of the twelfth graders at that excellent school did as well. While a tenth grader at a public senior high school he took a two-semester course in *advanced* calculus at a state college and made A's. Besides that, he has taken several other college courses and made excellent grades. In the fall of 1976 he entered Johns Hopkins as a sophomore after completing the eleventh grade.

If we had not intervened, it is extremely likely that this boy would have been required to take algebra I (which he did not need) as an eighth grader, algebra II as a ninth grader, and plane geometry as a tenth grader. He could have done splendidly on these with virtually no effort, but probably without any zest, either. From his case and many others one sees that a laissez-faire policy for education of the mathematically talented is misguided and harmful to them. Perhaps "genius will out," but much of the superior talent with which SMPY deals is unlikely to do so if unaided. Valuable time and energy will be squandered in the usual too slowly paced courses.

## RELATIONSHIP TO TERMAN'S LONGITUDINAL STUDY

SMPY owes a heavy debt to Terman's five *Genetic Studies of Genius* volumes and Oden's (1968) monograph. They provided many of the ideas and cautions that undergirded SMPY's initial efforts. It is natural, then, that there should be a number of similarities. Because of the half-century that intervened between the start of Terman's study in 1921 and SMPY's official beginning in 1971, however, it is natural, too, that there should be substantial differences. Some of the similarities, most of which have already been implied in this paper, are the following:

1. Both studies sought approximately the ablest 1 in 200 youths. For some purposes SMPY dipped down to the top 15 in 1,000, and for others went up to the ablest 1 in 1,000 or more. Terman also had special subgroups, though not below IQ 135.

2. Participants in both studies were chosen via standardized tests.

3. Both studies were conducted state-wide, California for Terman and Maryland for SMPY, over a several-year period.

4. Both are longitudinal. Terman's group, born on the average in 1910, is still being followed up. SMPY's first three groups, born as early as 1955 (but chiefly from 1958 to 1961), are meant to be followed until at least the end of this century.

5. Both sexes are involved.

6. No quota was set for representation of any sex or other group.

7. Identification was only the first step. After being found, students were studied extensively.

8. Results of both studies are reported in books, articles, and speeches. Terman's (1925) first book appeared four years after he began. SMPY's first one came out in three (Stanley, Keating, and Fox 1974).

9. Both studies were based in departments of psychology. This may seem somewhat ironic; many of the prime considerations in both belong to the area called educational psychology, which in recent years has involved the gifted all too little. Also, mathematics educators in most universities seem far more interested in curriculum development and textbooks for the average and somewhat-above-average student than for facilitation of the mathematically highly talented. We have detected more interest among some heads of mathematics departments in senior high schools and some college teachers of mathematics.

Certain differences between the studies are indicated above. Others are as follows:

1. SMPY tries to help its participants greatly educationally, rather than just observing their natural progress over the years. We intervene on their behalf vigorously, often, and in varied ways.

2. SMPY's initial screening is by a difficult mathematical reasoning test, rather than an intelligence test. Tests that yield IQs are not used for its later testing, either, though sometimes intelligence-test information is furnished us through the parents. But few of our prime group of about 200 students would have Stanford-Binet IQs much less than 140, and two of them reached 212.

3. We are working rather intensively with about 250 youths, whereas Terman started with more than 1,500. About 1,800 more of SMPY's students are getting considerable counseling and suggestions from us, though. This secondary group represents approximately the upper 1.5 percent of the age group with respect to mathematical aptitude.

4. Nearly all of SMPY's participants entered the difficult test competitions of a mathematical talent search sponsored by SMPY at Johns Hopkins. Thus there is probably a strong volunteering bias that makes our youths somewhat more academically aggressive and self-confident than were quite a few of Terman's. Also, a majority of them are definitely oriented toward academic subjects that involve considerable mathematics.

5. Most of our participants were eleven to thirteen years old and in the seventh or eighth grade when first tested. Terman's ranged across all the school grades.

6. Because of SMPY's initial selection procedure, emphasizing mathematical reasoning ability, most of the high scorers in the contest also score well on other reasoning tests, both nonverbal and verbal.

7. In various ways, including a printed newsletter appearing 10 times per year,[4] we encourage SMPY participants to accelerate their educational progress, particularly in the mathematical and physical sciences. SMPY has devised and tried out many special programs for its students. Terman's study was not meant to be interventional.

## TALENT VERSUS GENIUS

Many persons seem hostile toward intellectually talented youths, perhaps a little less so toward those splendid in mathematics than toward the verbally precocious. This contrasts sharply with their generally favorable attitudes toward prodigies in music and athletics. Friedenberg (1966) and Stanley (1974), among others, have discussed how deep-seated this prejudice is. Expressions such as the following abound in literature back to Shakespeare's time: "Early ripe, early rot"; "So wise so young, they say, do never live long"; "For precocity some great price is always demanded sooner or later in life"; and "Their productions . . . bear the marks of precocity and premature decay" (Stanley 1974, pp. 1-2).

We noted earlier that one disguise for dislike of the intellectually talented is to argue that they need no special help; it is assumed that they will succeed well educationally without it. Another tactic we have noticed is the comparison of a highly able youth with Gauss, Euler, Fermat, Galois, Pascal, Newton, or (especially) Einstein, a sort of *reductio ad absurdum* denigration of talent by asserting that it is not the rarest genius. Terman encountered a great deal of this. Some reviewers criticized him because in his frontier-state sample, identified in a short while, he did not discover someone who later became a worthy successor to the greatest

[4]It is called *ITYB*, the *Intellectually Talented Youth Bulletin*.

musicians, artists, and writers. [Some insight into problems of defining and predicting genius may be obtained from Albert (1975) and Bell (1937).]

Obviously, in the State of Maryland during a three-year period we do not expect to have located or helped to produce a Nobel Laureate, much less a successor to Gauss. To have in the sample someone even of the caliber of Norbert Wiener (1953, 1956) is perhaps more than we can reasonably expect. On his sixteenth birthday, however, one young man already through the sophomore year of college began important research in electrical engineering. Another, at age nineteen, did original research in mathematics. At age seventeen another solved an important problem in computer science. Because SMPY's participants were identified young recently, only nine had been graduated from college by June 1977. Achievements of participants will be studied for at least the next twenty years.

On the other hand, we do believe that SMPY is helping a number of exceptionally able young men and women to go far beyond what they would probably have done without our intervention. That is sufficient for us: strong enhancement of talent, rather than the creation of genius. We might have been able to help a lonely, awkward person such as Wiener use his great talents better at an earlier age, and probably Einstein would have scored quite high in a contest like ours had he deigned to enter it, but those two men are examples of persons who somehow achieved magnificently anyway. If one has already thrown a coin and it has landed with the "head" side up, what is the probability of *that* occurrence? This is a foolish question, of course, but no sillier than reasoning from the success of Einstein and Wiener that great intellectual talent will lead inevitably to success. Those country churchyards chronicled by Thomas Gray hold their share of "mute, inglorious" Wieners and Einsteins as well as of Miltons. We suspect that many classrooms also serve as premature tombs for mathematical talent.

## A STRONG BOND

SMPY's top 200 participants differ considerably in most personal characteristics except age. Some are tall and others are short. Some are introverted and others are extroverted. Some are much better verbal reasoners than others. Some are males and others are females. In fact, they probably differ at least as much from each other as do youths their age who are only average mathematically. These students have one important thing in common, however: they entered a challenging mathematical-aptitude competition and scored extremely well on a difficult mathematical reasoning test designed to be used with above-average students three to five years older than they. This is a powerful commonality that reminds me of the famous lines from Rudyard Kipling's "The Ballad of East and West":

> Oh, East is East, and West is West, and never the
> twain shall meet,
> Till Earth and Sky stand presently at God's great
> Judgment Seat;
> But there is neither East nor West, Border, nor
> Breed, nor Birth,
> When two strong men stand face to face, though
> they come from the ends of the earth!

Read Kipling's male-chauvinistic "two strong men" as "mathematically highly precocious youths" and you have a summing up of the rationale for SMPY. We believe that mathematical talent does transcend sex, circumstance, and nationality and mandates special educational treatment of mathematical prodigies with respect to their area(s) of great talent. We consider accelerative procedures crucial because—to paraphrase Robert Browning—"a mathematically precocious youth's reach should exceed his/her grasp, or what's an educational system for?" We at SMPY will continue helping to extend both the reach and the grasp of youths who reason extremely well mathematically.

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