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FACTORIAL STUDIES OF INTELLIGENCE

BY L. L. THURSTONE
AND
THELMA GWINN THURSTONE

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FACTORIAL STUDIES OF INTELLIGENCE

By

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CHAPTER I

INTRODUCTION

PURPOSE OF THIS STUDY

THE purpose of this factorial investigation can be described in relation to the factorial studies that we have made in the past few years. The first large study involved fifty-six psychological tests that were given to a population of volunteer subjects among college students. That study revealed a number of primary abilities, some of which were clearly defined by the configuration of test vectors, while others were indicated by the configuration but less sharply outlined. All these factors have been studied in subsequent test batteries in which each primary factor has been represented by new tests specially designed to feature the primary factors in the purest possible form. The object has been, of course, to construct tests that are heavily saturated with one primary factor and in which the secondary factors are minimized. This is the purification of tests by reducing their complexity. The complexity of a test is the number of primaries that are significantly involved in it.

From the beginning of our work in this field we have frequently raised the question whether the primary factors could be isolated and appraised for younger subjects. In order to contribute to this problem and to investigate further the nature of the primary mental abilities, we have made several studies on Chicago high-school Seniors. One of these studies was made at the Lane Technical High School with new tests intended to identify the perceptual speed factor. That factor appeared again in the new test battery, but its psychological nature was not satisfactorily determined. Another study was made at the Hyde Park High School, with special interest in a new set of tests for induction. The inductive factor was again identified in the old and in the new tests that were specially constructed for this factor, but the tests for induction did not have so high a validity as the tests for the more definitely identified primary factors.

An experimental edition of tests for seven primary mental abilities was made available in response to a rather general interest in the problem of isolating mental abilities. We made the stipulation that these

tests were not to be distributed as service tests, since they were experimental, and the first edition of the test forms was so designated. A number of improvements have been made in some of the tests since the experimental edition was printed, so that the first edition will soon be revised.

The present study was arranged to determine whether primary mental abilities could be identified among younger subjects—in this case, eighth-grade children—and also to determine whether our original interpretation of the word-fluency factor *W* could be sustained. For this purpose we devised a number of new tests which were thought to be well saturated with this factor, and it will be seen in the subsequent analysis that our first interpretation of this factor is well supported by the findings.

RECENT RESEARCH ON THE PRIMARY MENTAL ABILITIES

We shall review here the present status of each of the primary mental abilities that have been isolated by multiple-factor studies of large test batteries. It should be stated at the outset that we claim no priority for the discovery of primary mental abilities, which have long been known as special abilities without being isolated factorially in the configurations of large test batteries. An example is the verbal factor *V*, which is certainly not surprising in view of the fact that psychologists have long been accustomed to deal with verbal intelligence as one of the most important categories in the study of individual differences.

The verbal factor *V* is one of the clearest of the primary mental abilities. It can be expected in any of the tests involving verbal comprehension—for example, tests of vocabulary, opposites and synonyms, the completion tests, and the various reading-comprehension tests. It is also involved in such verbal-comprehension tests as proverbs, absurdities, and, to some extent, in syllogistic tests and in statement problems in arithmetic where verbal comprehension is significantly involved.

It might be well to give a warning here about judging the factorial composition of a test merely by looking at its content and without considering the subjects whose individual differences are to be discriminated by the test. For example, a vocabulary test might be a good measure of the verbal factor *V* among school children, but the same test might fail to measure this factor if it were given to a population of college graduates. If the test were given by the time-limit method, there would be individual differences in any population, but they might not signify the same factor in the two populations. In the population of school

children the test might appraise the verbal factor *V*, whereas, in the population of college graduates the test might appraise some perceptual speed factor, or some other nonverbal factor, since it could be supposed that all of the older subjects would know the words that are differentiating for the children. A psychological test does not have a fixed factorial composition; the factorial composition is, of course, dependent on the subjects. This is another way of stating the familiar principle that the validity of a test is not a fixed attribute of the test. It is a function of the criterion (the factors) and the population for which it is intended.

The word-fluency factor *W* is also one of the most clearly defined primary mental abilities. It is involved whenever the subject is asked to think of isolated words at a rapid rate. It is for this reason that we have called it a "word-fluency factor." It can be expected in such tests as anagrams; rhyming; producing words with a given initial letter, prefix, or suffix; or writing words in a given category, as boys' names or things to eat and drink. Any task in which the verbally fluent person has an advantage should involve this factor, which is clearly distinct from the verbal-comprehension factor. These two verbal factors are, however, correlated. Whether the correlation is in some way intrinsic is a question that cannot yet be answered. It may be that we do not yet have any tests in which one of these factors is present while the other is absent, or it may be due to the existence of two associated but distinct parameters in the processes involved in verbal intelligence. Tests of the sort that we have found for this factor have also been used by some investigators as tests of temperamental qualities. It is not unlikely that the word-fluency factor is indicative of some temperamental traits in addition to its cognitive implications. If such double interpretations can be sustained, they will serve to break down the conventional differentiation between intellectual and temperamental parameters, which are probably much more interwoven than we have supposed.

One of the most interesting findings in the present study is the distinction between the verbal-comprehension factor *V* and the word-fluency factor *W*, which can be illustrated by two of the tests in the present battery. Both of these tests involved synonyms, but the test procedures were different. One of these tests, Vocabulary (45), was an ordinary recognition form of vocabulary test in which the subject merely checked the response word which had the same meaning as the given stimulus word. This test had high saturation (.68) on the verbal-comprehension factor, which accounts for half of the variance of the test. Its saturation on the word-fluency factor vanishes (.015). This is as we

should expect, because the response words are printed in the test so that the subject needs only to check the correct word. The test involves the understanding of the given words, but the subject is not required to supply words. In order to feature the word-fluency factor, we included a free-recall form of synonyms test. The test gives a list of common adjectives which are easily understood by all of the subjects. They were asked to *write* three synonyms for each given word. Here it was not a question of whether the subjects understood the given words, since they were all ordinary adjectives. The task was to supply quickly three synonyms for each given word. This test had a high saturation (.51) on the word-fluency factor W, but its saturation on the verbal-comprehension factor V vanished ($-.005$). It should be noted that both of these tests called for synonyms; they differed in that one of the tests was given in recognition form while the other test was given in recall form with simpler words. A third test, Same or Opposite (51), gave a similar result. It was given in the recognition form with words of some degree of difficulty and, hence, involved the verbal-comprehension factor (.62), but its saturation on the word-fluency factor vanished (.03).

The space factor S is another of the clearly defined primary mental abilities. It seems to be involved in any task in which the subject manipulates an object imaginally in two or in three dimensions. This ability is involved in many mechanical tasks and in the understanding of mechanical drawings, often called the "reading of blueprints." Such material cannot be used generally in psychological tests intended for schools because mechanical drawing and related arts involve training in particular drafting conventions and in conventionalized forms. We have, therefore, tried to incorporate the imaginal manipulation of objects in tasks that are so simple and easily comprehended that anyone without training in the mechanic arts will at least understand what he is expected to do. The best tests for this factor are those we have called "Cards," "Figures," and "Flags," which all involve the manipulation of a simple object in two or three dimensions. This factor should also be involved in such subjects as descriptive geometry and in solid geometry, but we have not yet investigated this possibility experimentally. It is known that some students of descriptive geometry find the subject very easy, so that they need not consult their textbooks after they once understand the general plan for solving a problem in shades and shadows, for example, while other students have great difficulty in visualizing such problems and are dependent on the routine textbook

solutions. According to our interpretation, these individual differences should involve the space factor S to a marked degree. This hypothesis should be investigated experimentally.

Another primary factor that is clearly defined is the number factor N. It is involved in simple arithmetical tasks. This factor can be expected in any test in which the subject actually does simple arithmetical work, but it is not found in a test simply because it contains numbers. A simple cancellation test with numbers probably will not involve the number factor; but if the subject is asked to check every number that is larger than the adjacent numbers, this factor can be expected. Arithmetical reasoning tests with statement problems have been found to involve the number factor to some degree, as well as other factors, such as the verbal and the inductive. The best tests for the number factor are the simple numerical tasks. Two of the number tests have as high validity as the tests for the two verbal factors and the space factor, and the simple number tasks have been consistent in revealing the number factor in all studies in which such tests have been included. Further work remains to be done in determining the nature of the processes that underlie numerical thinking. For example, quantitative thinking can be nonnumerical, and in some subjects it might possibly even be nonspatial, although that seems less probable. The existence of the number factor in any given task is rather easily predicted whenever the subject is asked to do simple numerical work, but the psychological nature of this primary factor is not so clear as the other primaries. The number factor can be appraised in psychological tests just as it is appraised in educational achievement tests, even though the fundamental nature of numerical thinking is not yet understood.

The memorizing factor M is one of the clearly defined factors, although the tests for it do not have validities so high as the tests for the verbal and the space factors. The memorizing factor M is to be expected in any test in which the subject profits by ability to memorize anything quickly. It is involved in rote memory for words, numbers, paired associates, and the memorizing of names. The factor transcends the immediate nature of the content; the same memory factor has been found in tests with verbal, numerical, and spatial content. This factor seems to be quite distinct from the other primary mental abilities in that the correlations between the memory factor and the other primaries have been found to be rather uniformly low.

The memory factor has been found also with lower but probably significant saturations in some tests that were not designed as tests for

rote memory. In every such case we have found that the subject was aided by the memorizing of notation or code that was involved in the test. For example, in the Classification test (9) the subject is asked to classify each name in a given list into one of four categories. By memorizing the eight class limits he can proceed faster than by consulting the code for each name. A small component was found in the memory factor. In Figure Naming (20) the subject is to designate each figure in the test into one of four categories. Again the memorizing of the notation facilitates the performance.

A large factorial study is now in progress involving twenty-four different kinds of memory tests which were combined with tests for the other primaries. The analysis of the results has not been completed, but the indications are that several retentive primaries will be found in addition to the rote-memorizing factor that we have denoted *M*. For example, there seems to be indication that the memorizing of temporal sequence, as in digit span, letter span, Knox cube, and serial-learning, involves a retentive ability that is different from the rote memorizing of paired associates. The factorial results of the large battery of memory tests will be reported in a later publication.

The inductive factor *I* has been found in several factorial studies, but the tests for this factor do not have validities so high as we should desire. The factor is involved in tasks that require the subject to discover a rule or principle that covers the material of the test. It has been found in the well-known number series tests and appears in similar tests constructed with letter series. The deciphering of code also involves the inductive factor. The inductive factor has appeared in tests of varied content, including verbal, spatial, and numerical tasks, so that the factor seems to transcend the immediate nature of the content. Although we have not succeeded, so far, in finding tests with high validities for this factor, the existence of the factor seems to be fairly clear. The inductive factor can be appraised by using a combination of several tests, each of which has appreciable saturation on the factor, until single tests are found with higher validities. It is psychologically interesting that the inductive factor has higher correlation with the second-order general factor for the eighth-grade children than any of the other primaries.

The deductive factor *D* has been indicated in several studies, but it has not always appeared where it might have been expected. This factor should, therefore, be regarded as tentative and subject to reinter-

pretation if it can be found in clearer form in repeated studies. In revising the experimental test battery for the primary mental abilities we shall omit this factor because it has not been sustained in repeated studies. Further study of the tests in which it has been indicated may give some new interpretation for the primary factors involved, which should be tested with specially designed tests. It seems clear now that our first interpretation of this factor was erroneous.

The perceptual-speed factor P has been one of the most troublesome of the primaries. Its existence has been clearly indicated, and it has appeared in all of the test batteries that have been analyzed so far. The difficulty with this factor is that we have not been able to locate clearly its bounding hyperplane. To do this, we must find tests which have practically zero saturation on the factor and others in which the saturation is appreciable. Another study of this factor is now being made with individual laboratory tests in an effort to identify it more clearly in the configuration of the test battery. The difficulty with the perceptual-speed factor may be due to our testing methods. The group tests with time-limit procedures may introduce the perceptual-speed factor in so many of the tests that we have no base from which to measure it, with few tests in which this factor is entirely absent. We feel reasonably sure that a primary factor exists that involves perception and speed, but our interpretations cannot be checked with assurance so long as the bounding plane for this primary factor is unstable. The experimental work now in progress may throw light on this factor.

It will be seen from this brief review of the present status of our work on the primary mental abilities that six of them seem to be clearly defined, some better than others, and that two of them are not clearly defined. These two factors are the deductive factor and the perceptual-speed factor. The primary mental abilities that we consider clearly indicated by repeated studies are: (1) the verbal-comprehension factor V, (2) the word-fluency factor W, (3) the space factor S, (4) the number factor N, (5) the memorizing factor M, and (6) the inductive or reasoning factor I, which has also been denoted R. The test validities are highest for the first three of these factors, namely, V, W, and S. Even though there is still some question about the nature of the inductive factor I, it seems worth while to include it in a practical test battery because of the fact that it has the highest correlation with the second-order general factor, which may be the much debated "general-intellective factor" of Spearman.

DIFFERENTIAL DESCRIPTION OF MENTAL ENDOWMENT

For many years psychologists have been accustomed to the problems of special abilities and disabilities. These are, in fact, the principal concern of school psychologists who deal with children who cannot read, with children who have a blind spot for numbers, or with children who do one thing remarkably well and other things poorly. It seems strange that, with all this experience in differential psychology, we have clung so long to the practice of summarizing a child's mental endowment by a single index, such as the mental age, the intelligence quotient, the percentile rank in general intelligence, and other single average measures. An average index of mental endowment should be useful for many educational purposes, but it should not be regarded as more than the average of several tests. Two children with the same mental age can be entirely different persons, as is well known. There is nothing wrong about using a mental age or an intelligence quotient if it is understood as an average of several tests. The error that is frequently made is that the intelligence quotient is sanctified by the assumption that it measures some basic functional unity, when it is known to be nothing more than a composite of many functional unities.

The present investigation seems to give justification for dividing the composite measure of mental endowment into separate functions or processes so that each child can be described by a *mental profile* instead of by a single index of general intelligence. The general index can be easily obtained again by merely taking the average of the abilities represented in the profile, and these separate abilities may be weighted in any way that seems desirable. It is our purpose to make available, as soon as possible, a battery of psychological tests so designed that there will be three tests for each of six primary mental abilities. Each set of three tests will be called a "composite test," and there will be one composite for each of the primary mental abilities. The tests for each primary are self-contained, so that each primary factor may be appraised independently of the others. The result is a profile for each child with six indices instead of one, but a single average index of mental endowment can be easily obtained by taking the average of the six measures on the profile.

In presenting for general use a differential psychological examination which appraises the mental endowment of children, it should not be assumed that there is anything final about six primary factors. No one knows how many primary mental abilities there may be. We know

about one memory factor now, but several new memory factors may be found. The tests that are prepared for general use will be improved and extended to more factors as a result of further investigation, but we must be sure that new factors are not added simply because new factorial names can be added to the list. Every new factor that is added to the profile must be shown factorially to be distinct from the factors already found and to be a functional unity with some stability in repeated experiments. It should not be assumed that the primary mental abilities are elemental and indivisible. For example, the primary ability that has been called "verbal comprehension" is almost certainly not an indivisible element of any kind. But in a wide variety of psychological examinations it behaves as *a functional unity that is strongly present in some tests and almost completely absent in many others*. This is the fundamental idea of a primary mental ability.

As each new primary is isolated from the whole field of possible psychological examinations, the judgment must be made whether it is socially, vocationally, or educationally significant. If it is considered to be of some importance and usefulness, it should be added to the portfolio of the school psychologist. If it is considered to be of limited significance, it should not be included in general psychological examinations even though it may be of considerable theoretical and scientific interest.

The psychological examination that is now being prepared for general use has been arranged from the material of the selected test battery that is described in the second part of this monograph. The general examination will contain the tests listed in *Table 1* of *Chapter III*, except for the three tests for the perceptual factor, which has not been stable enough to justify general use in mental profiles. In presenting this study and the general psychological examination which is to follow, the authors hope that future factorial studies will reveal many other important primary abilities, so that the mental profiles of students may eventually be adequate for appraising their educational and vocational potentialities. In such a program the present study can be only a starting-point. Psychologists have been describing mental endowment by a single intelligence index, but we must work toward the differential description of mental endowment with a profile of ever increasing fundamental traits.

The authors wish to acknowledge the contributions of an exceptionally competent staff of research assistants, including Mr. Ledyard R.

Tucker, who has been responsible for the factorial computations and who has developed new machine methods of factorial computing; Mrs. Katharine Van Steenberg, Mr. Willis C. Schaefer, Mr. Clyde H. Coombs, Mr. Harold P. Bechtoldt, and Mr. Albert L. Hunsicker, who assisted with examining and computing; Miss Dorothy C. Adkins and Miss Luzelle Denton, who took much responsibility in constructing and editing the psychological tests; and Miss Dorothy Case, for a large share of the editing of tests and manuscripts and the secretarial work of these projects. The University of Chicago Press has been generous with competent advice and assistance in the preparation of these studies for publication. We appreciate especially the assistance of Miss Mary D. Alexander of the editorial staff. We wish to acknowledge the sustained financial assistance of the Social Science Research Committee of the University of Chicago on these factorial investigations. We are grateful, also, for a grant from the Carnegie Foundation of New York, which has enabled us to complete these studies in a relatively short time. The Foundation does not, of course, assume any editorial responsibility for the publications that issue from their research grants. One of the important parts of this investigation has been the participation of the Chicago schools; all of the experiments described in this monograph were made in the public schools of Chicago. To Superintendent William H. Johnson and to the director of the Bureau of Child Study, Dr. Grace Munson, we wish to express our appreciation for the facilities that have been made available for us. Interest and co-operation could not be better than that which we have enjoyed in the Chicago schools during these investigations. We appreciate also the sponsorship and the interest of the American Council on Education and its Committee on Measurement and Guidance.

CONSTRUCTION OF THE TESTS

Bearing in mind that the purpose of the present investigation was to determine whether primary mental abilities could be isolated for children at the fourteen-year age-level, the construction of the tests consisted essentially in the adaptation for the younger children of tests previously used with college and high-school students. In some of the tests little or no alteration was necessary, while for some tests it was considered advisable to revise vocabulary and other aspects of the tests to suit the younger age-level. A number of new tests were added to those selected from previous experimental batteries. Sixty tests constituted the battery. The editorial work of constructing the new test

battery was done by Dorothy C. Adkins, Luzelle Denton, and Thelma Gwinn Thurstone.

When the tests had been designed and printed, they were given in a trial form to children in Grades VIIA and VIIIA in several schools for the purpose of determining appropriate time limits. Groups of from fifty to one hundred children in these two grades were used for the purpose of standardizing procedures and, especially, for setting time limits.

ADMINISTRATION OF THE TESTS

Fifteen Chicago elementary schools were selected by Miss Minnie L. Fallon, assistant superintendent in charge of elementary education, and by Dr. Grace E. Munson, director of the Bureau of Child Study.

Table 1

School	No. of Children Tested	Principal	Adjustment Teacher
Altgeld	89	George White	Bernice M. Grannon
Gage Park	80	Anna B. Van Nice	Margaret Shevlin
Gallistel	77	C. H. Dowman	Eileen Leach
Hookway	46	Margie C. E. Doyle	Thelma S. Hicks
Kozminski	85	Mary D. Mulroy	Mary C. Minnahan
Lewis-Champlin	91	Anna R. Jordan	Berenice McDermott
Mann	93	Gretta M. Brown	Julia Berger
McKay	73	Mary R. Hanlon	Marguerite B. Flynn
Oglesby	86	Harry F. Yates	Harriet E. Wall
Parkside	88	Julia McInerney	Esther Nelson
Ray	75	Ray A. Bixler	Reba Hilburn
Scott	78	Ruth L. Whitaker	Helen R. Israel
Wadsworth	68	Mary L. Patrick	Etta Kew
Warren	35	Marie A. McCahey	Margaret Carey
West Pullman	90	William H. Spurgin	Mabel Daniels
	1,154		

At a meeting of the principals of these schools and the district superintendents, the purpose and the plan of the study were explained. This study could not have been completed without the genuine interest and assistance of these principals, who co-operated in revising class schedules and arranging for the tests. *Table 1* lists the schools at which the tests were given, together with the names of the principals and adjustment teachers. We have also recorded the number of children in each school who took the tests.

The tests in the main investigation were administered in the schools by the adjustment teachers. These adjustment teachers had had special training in testing procedures by the Bureau of Child Study and

Table 2

Schedule of Sixty Tests

I. Monday, November 28:	Identical Pictures
	Four-Letter Words
	Multiplication
	Pursuit
	Proverbs
	Association
II. Tuesday, November 29:	Dot Counting II
	First and Last Letters
	Absurdities
	Verbal Enumeration
	Addition
	Word Checking
III. Wednesday, November 30:	Dot Counting III
	Suffixes
	Scattered *X*'s
	Classification
	Directions
	Mazes
IV. Thursday, December 1:	Identical Numbers
	Prefixes
	Three-Higher
	A B C
	Pedigrees
	Anagrams
V. Friday, December 2:	High Number
	Number Patterns
	Geometrical Forms
	Dot Counting I
	Letter Series
VI. Monday, December 5:	Picture Naming
	Figures
	Secret Writing
	First Names
VII. Tuesday, December 6:	Dot Patterns
	Rhyming Words
	Flags
	Incomplete Words
	Word-Number Recall
VIII. Wednesday, December 7:	Completion
	Arithmetic
	Cards
	Backward Writing
	Letter Grouping

Table 2—Continued

IX. Thursday, December 8:	First Letters Figure Naming Word Puzzles Figure Recognition Same or Opposite Reasoning
X. Friday, December 9:	Faces Synonyms Disarranged Sentences Figure Grouping Reading Vocabulary—pages 3 and 4 Sentences—pages 5 and 6 Paragraphs—pages 13, 14, and 15 Paragraphs—pages 18 and 19
XI.	Paragraph Recall Digit Span

had also had considerable experience in giving psychological and educational tests. Special instructions in the procedures for these tests were given to the adjustment teachers, who were also furnished written instructions for each day's testing program.

Eleven hundred and fifty-four children participated in this study. The complete battery of sixty tests was given in eleven one-hour sessions to the VIIIB sections in each school. In *Table 2* we have listed the tests that were given at each of the eleven sessions with the date of each session.

CHAPTER II

FACTORIAL ANALYSIS OF THE SIXTY TESTS

THE STATISTICAL PROCEDURES

ALL the tests were scored for the number of right responses and for the number of wrong responses. The scoring was independently checked. Frequency distributions were made for each school, and these were combined into a total frequency distribution for the whole population of 1,154 for all schools. A separate report was given to each school, containing the test scores of each child, a summary of his scores in eight groups of tests, and explanatory notes about the nature of the tests. This work was done by clerks assigned from the Works Progress Administration. We wish to express our appreciation to the Chicago public schools for arranging for this assistance.

In addition to the sixty tests, we added three variables: chronological age, mental age, and sex. The latter test data were available in school records. They were determined by the Kuhlmann-Anderson tests, which had been given previously to the same children. Therefore, the battery to be analyzed factorially contained sixty-three variables. The time limits and scoring formulas are listed in *Table 1*.

The total population in this study consisted of 1,154 eighth-grade children. When all the records had been assembled, it was found that 710 of these subjects had complete records for all of the sixty variables. We decided to base our correlations on this population of complete records rather than to use the large population with varying number of cases for the correlation coefficients. For convenience of handling with the tabulating-machine methods,[1] the raw scores were transmuted into single-digit scores from which the Pearson product-moment correlation

[1] During the past year we have been using new machine methods for factorial computations. These methods have been developed by Mr. Ledyard Tucker, and they will be described in a monograph which is now in preparation. The new methods include the use of tabulating machines with punched cards for determining product-moment correlation coefficients which are produced on a printing tabulator, the use of similar equipment for plotting the rotational diagrams, and the computations of the centroid method of factoring.

A matrix multiplying machine with a capacity of fifteen columns was built by the International Business Machines Corporation and is now in use in our laboratory. It is essentially a modified form of electric scoring machine. This instrument will also be described in the forthcoming monograph.

Table 1

The Battery of Sixty-three Variables

Code No.	Test	Time Limits (Minutes)		Score	Code No.	Test	Time Limits (Minutes)		Score
		Fore-exercise	Test Proper				Fore-exercise	Test Proper	
1....	A B C	10	5	2R–W	33....	Letter Grouping	7	3	3R–W
2....	Absurdities	3	3	R–W	34....	Letter Series	8	6	R
3....	Addition	3	6	R–W	35....	Mazes I	3	3	R
4....	Anagrams	3	5	R	36....	Mazes II	.....	5	R
5....	Arithmetic	5	7	4R–W	37....	Multiplication	2	5	R–W
6....	Association	3	4	R	38....	Number Patterns	7	5	R
7....	Backward Writing (Mirror Reading)	4	4	3R–W	39....	Paragraph Recall	.....	Read twice	R
8....	Cards	6	6	R–W	40....	Pedigrees	4	3	R
9....	Classification	5	4	3R–W	41....	Picture Naming	2	4	R
10....	Completion	5	5	4R–W	42....	Prefixes	3	4	R
11....	Digit Span	.....	1 per sec.	R	43....	Proverbs	4	6	4R–W
					44....	Pursuit	3	3	R–W
12....	Directions	5	3	R	45....	Reading: Voc.—pp. 3, 4	.....	4	4R–W
13....	Disarranged Sentences	3	3	R	46....	Reading: Sen.—pp. 5, 6	.....	4	4R–W
14....	Dot Counting I	3	5	R					
15....	Dot Counting II	4	6	R	47....	Reading: Par.—pp. 13–15	.....	6	4R–W
16....	Dot Counting III	3	6	R					
17....	Dot Patterns	3	4	3R–W	48....	Reading: Par.—pp. 18, 19	.....	6	R–W
18....	Faces	2	3	2R–W					
19....	Figure Grouping	3	3	4R–W	49....	Reasoning	3	5	R–W
20....	Figure Naming	2	1	R	50....	Rhyming Words	3	5	R_W*
21....	Figure Recognition	3–2–4	4	R–W	51....	Same or Opposite	2	4	3R–W
22....	Figures	7	7	R–W	52....	Scattered X's	3	5	R
23....	First and Last Letters	3	5	R	53....	Secret Writing	12	5	2R–W
					54....	Suffixes	3	4	R
24....	First Letters	2	4	R	55....	Synonyms	3	7	R_W*
25....	First Names	3–2–7	5	R	56....	Three-Higher	4	5	R–W
26....	Flags	5	3	R–W	57....	Verbal Enumeration	4	7	R
27....	Four-Letter Words	3	4	R	58....	Word Checking	3	3	R–W
28....	Geometrical Forms	3	3	R	59....	Word-Number Recall	3–2–6	5	R
29....	High Number	4	5	R–W	60....	Word Puzzles	3	8	R
30....	Identical Numbers	3	4	R	61....	Age	.....	.....	.....
31....	Identical Pictures	3	6	R–W	62....	Sex	.....	.....	.....
32....	Incomplete Words	5	5	R	63....	Mental Age	.....	.....	.....

* Rhyming Words was scored by the weighted sum of acceptable responses. Each response in the first two columns was given a weight of 1; each response in the third and fourth columns, a weight of 2.

Synonyms was also scored as a weighted sum. Each response in the first column was given unit weight; each response in the second column, a weight of 2; and each response in the third column, a weight of 3.

In both of these tests the examiners were asked to be generous in accepting responses.

Table 2

Distribution of Tenth-Factor Residuals

From	To	*f*	From	To	*f*
.11	.12	1	.01	.02	273
.10	.11	1	.00	.01	355
.09	.10	0	−.01	.00	364
.08	.09	2	−.02	−.01	285
.07	.08	3	−.03	−.02	186
.06	.07	12	−.04	−.03	103
.05	.06	12	−.05	−.04	48
.04	.05	33	−.06	−.05	12
.03	.04	90	−.07	−.06	3
.02	.03	165	−.08	−.07	5

coefficients were computed. With sixty-three variables there were 1,953 Pearson correlation coefficients. They are given in *Table 1* of the *Appendix*.

The correlation matrix of *Table 1* was factored by the centroid method on the tabulating machines by means of punched cards. The centroid matrix with ten factors is shown in *Table 2* of the *Appendix*. This

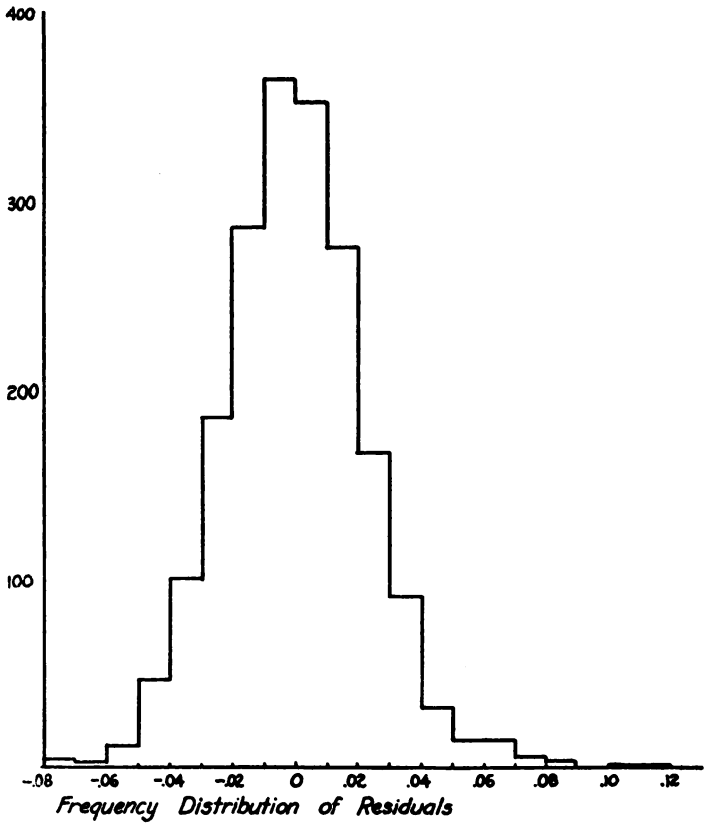


FIGURE 1

table also contains a list of the communalities, which may be seen to vary considerably. For example, Digit Span (11) has very little in common with the rest of the battery, while Reading Vocabulary (45) has a large part of its variance in common with the rest of the tests. The tenth-factor residuals are shown in *Table 2* of this chapter and in *Figure 1*. It may be seen from the frequency distribution of residuals that there is only a small proportion of residuals greater than .03.

Successive rotations were made by the method of extended vectors. The diagrams for determining the rotations were obtained on a printing

tabulator from punched cards. This procedure reduces very considerably the computational labor involved in each rotation. The result of eighteen successive rotational approximations to simple structure gave the rotated oblique factorial matrix V shown in *Table 3* of the *Appendix*. The transformation from the centroid matrix to the rotated matrix V is given by the matrix Λ of *Table 3*. The columns of Λ are the direction

Table 3

Transformation Matrix Λ

	N	*W*	*S*	*P*	*V*	*M*	*I*	X_1	X_2	X_3
I	.105	.202	.165	.223	.252	.134	.184	.132	.080	.187
II	−.163	.322	−.382	−.130	.365	−.103	.054	−.320	−.212	.237
III	.025	−.555	.348	−.175	.407	.013	.135	−.222	.175	−.062
IV	−.338	.638	.359	.007	−.262	−.286	−.066	−.419	.172	−.156
V	−.389	−.200	−.109	.185	.485	−.307	−.287	.208	.465	.492
VI	.364	.197	.148	−.578	.070	−.277	−.410	.554	.055	−.170
VII	.717	−.082	−.081	.294	−.285	−.193	.113	−.476	.409	.155
VIII	.161	−.175	.387	.545	.281	−.049	−.729	−.217	−.321	.211
IX	.107	.137	−.292	−.087	.263	.659	−.324	−.118	.596	−.193
X	.106	−.075	−.550	.379	.316	−.494	.203	.133	−.209	−.710

Table 4

The Matrix $\Lambda'\Lambda$ *Showing Cosines of Angular Separations of the Reference Vectors of* Λ

	N	*W*	*S*	*P*	*V*	*M*	*I*	X_1	X_2	X_3
N	.999	.....	.....	.....	.....	.....	.....	.....	.....	.....
W	−.192	1.000	.....	.....	.....	.....	.....	.....	.....	.....
S	−.022	−.063	1.001	.....	.....	.....	.....	.....	.....	.....
P	.085	−.206	−.074	1.000	.....	.....	.....	.....	.....	.....
V	−.195	−.321	−.212	.151	1.000	.....	.....	.....	.....	.....
M	.018	−.037	.032	−.186	−.033	.999	.....	.....	.....	.....
I	−.051	−.027	−.305	−.065	−.288	−.058	1.000	.....	.....	.....
X_1	−.052	−.102	−.108	−.372	.161	−.091	−.113	1.000	.....	.....
X_2	.112	−.002	−.042	−.116	.082	.260	−.096	−.128	1.000	.....
X_3	−.172	−.106	.300	.118	.116	.124	−.240	−.155	.175	1.000

cosines of the reference vectors which define the bounding hyperplanes. The angular separations between the reference vectors are shown in the matrix $\Lambda'\Lambda$ of *Table 4*.

In order to facilitate the interpretation of the primary factors revealed in the simple structure of *Table 3* of the *Appendix*, the tests have been rearranged in *Table 5*. *Table 3* gives the factorial composition of the tests arranged according to code number, which was also the alphabetical order by name of test. It shows the projection of each test vec-

Table 5

Summary of Factor Matrix for Sixty Variables

	N	*W*	*S*	*V*	*M*	*I*	*P*	X_1	X_2	X_3
Factor *N:*										
37. Multiplication	.46						.24	.26		
3. Addition	.44							.32		
56. Three-Higher	.41						.21			
5. Arithmetic	.36			.20		.22				
9. Classification	.27		.20		.26					
49. Reasoning	.26									
Factor *W:*										
42. Prefixes		.61								
24. First Letter		.59								.20
23. First and Last Letters		.58								
54. Suffixes		.58								
27. Four-Letter Words		.53								
50. Rhyming Words		.52								.21
55. Synonyms		.51								.20
4. Anagrams		.41					.23			
6. Association		.39								.28
60. Word Puzzles		.39					.37			
Factor *S:*										
8. Cards			.68							
22. Figures			.62							
26. Flags			.52							
29. High Number	.23		.25							
Factor *V:*										
45. Reading Vocabulary				.68						.22
46. Reading Sentences				.64						.27
51. Same or Opposite				.62						.25
10. Completion				.55						.20
39. Paragraph Recall				.55						
47. Reading Paragraphs				.53						.31
48. Reading Paragraphs				.45						.24
43. Proverbs				.40						
2. Absurdities				.35						
13. Disarranged Sentences				.28			.27			.27
Factor *M:*										
21. Figure Recognition					.40					
25. First Names					.40					
59. Word-Number Recall					.33					
11. Digit Span					(.19)					
Factor *I:*										
34. Letter Series						.49				
40. Pedigrees					.27	.41				
33. Letter Grouping						.37				
53. Secret Writing			.20		.25	.37				
63. Mental Age						.35				
12. Directions				.29		.30				
38. Number Patterns						.26	.24			
61. Chronological Age						−.22				−.20

Table 5—Continued

	N	*W*	*S*	*V*	*M*	*I*	*P*	X_1	X_2	X_3
Factor *P*:										
30. Identical Numbers	.22						.44			
7. Mirror Reading		.20					.40			
31. Identical Pictures							.36			
18. Faces			.27				.32			
32. Incomplete Words		.29					.32			.30
52. Scattered *X*'s							.27			
19. Figure Grouping							.25			
Factor X_1:										
14. Dot Counting I								.69		
16. Dot Counting III								.70		
15. Dot Counting II								.54		
17. Dot Patterns	.27							.31		
1. A B C								.25		
Factor X_2:										
35. Mazes I			.28						.60	
36. Mazes II			.33						.54	
44. Pursuit			.20					.25	.29	
28. Geometrical Forms			.22			.21			.23	
Factor X_3:										
57. Verbal Enumeration				.38			.42			.48
58. Word Checking				.32						.36
41. Picture Naming									.27	.35
20. Figure Naming					.28		.23			.28

tor on each of the ten reference vectors. In *Table 5* the same data have been rearranged so that the tests are in groups according to the factor of maximum saturation. For example, the first group of tests in *Table 5* shows all of the tests which have their maximum saturation in the first column of the factor matrix *V*. The entries in *Table 5* are recorded to two decimals, since that is sufficient for inspectional purposes and interpretation; and the small values in the matrix *V* have been omitted to avoid confusion in the interpretation. All projections smaller than .20 were omitted from this table. It will be recalled that a projection of .20 means only 4 per cent of the total variance of a test. This dividing-line is arbitrary, but it has been found useful. Different dividing-lines can be used with the data of *Table 3*, which shows all of the projections.

INTERPRETATION OF THE FACTORS

Inspection of the first group of tests in *Table 5* shows the same tests which have previously been associated in the identification of the number factor *N*. All the tests in this group are numerical except Reasoning (49), which contains only syllogisms involving quantitative comparisons. Since there is a low saturation of .26 for the Reasoning test on the

factor N, one might raise a psychological question concerning the number factor and its possible relation to nonnumerical quantitative comparisons.

The arithmetical reasoning test (5) has repeatedly shown a verbal component. This is not surprising in view of the fact that the test is composed entirely of statement problems. There is also indication of a slight inductive component in this test. This component in arithmetical reasoning has been found in previous factorial studies; it probably depends on the degree of novelty of the arithmetical problems and on the age-level of the subjects.

The same table shows slight components also of space and memory in the Classification test (9). The appearance of a memory component in this test is not surprising. If the subject can remember the eight limits that are involved in the test, his progress is much more rapid than if he must refer to the table of class limits for each item. The small space component may be associated with the visual imagery of the scale which may be used by some subjects. All three of these components are too small to be considered more than suggestive. The improvement of tests in the direction of reducing complexity involves the elimination of those features in a test which produce small saturations on the secondary factors. Such improvement should increase the saturation on the factor which is to be featured by the test.

There are ten tests which have their highest saturations in the common factor W. Several of these tests have been previously identified for the word-fluency factor W; some of them are new. They were designed to feature, as far as possible, the ability of the subject to produce words, as distinguished from his ability to understand them when they are given to him in the test. Each of these tests has some restriction, such as the writing of four-letter words, of words that begin and end with specified letters, of words that have given suffixes or prefixes, etc. All the tests of this type showed appreciable saturation on the factor W. It is of interest to note that none of these tests have any significant saturation on the verbal-comprehension factor V, which was identified by another group of tests. With all of these verbal tests there appeared a third verbal factor, denoted X_3, the interpretation of which is in considerable doubt. All of the tests which have any saturation on the factor X_3 were verbal in character, but the factor is clearly distinct from the verbal-comprehension factor V and the word-fluency factor W. If some hypothesis can be found in terms of these tests as to the possible nature of the new verbal factor, then tests can be constructed which

feature its postulated characteristics so that it may be added eventually to the list of known cognitive factors. The evidence here presented merely indicates the presence of a third verbal factor whose nature can only be surmised in terms of the tests which contain it and the tests from which it is absent. The authors will be grateful for suggested interpretations of the new verbal factor.

In previous studies we have called attention to the fact that the word-fluency factor *W* may be responsible for the strange lapses in amnesic aphasia. Further study should be made of this factor with aphasic patients.

The next column of *Table 5* contains a group of four tests whose highest saturations are on the space factor *S*. The factor is identified by three tests which have been found previously to be heavily saturated with this factor. They are Cards (8), Figures (22), and Flags (26). The test High Numbers (29), requires that the subject mark every number which is higher than the adjacent numbers in the row. This test has a low saturation on the number factor, which is not surprising, and a low saturation on the space factor. If the latter is significant, it might indicate that the children used visual imagery in judging the relative magnitudes of the numbers. Both of these saturations are low, so that they are only suggestive. It is significant that a test does not have a high saturation on the number factor simply because it contains a lot of numbers. Superficial inspection of a test often leads to erroneous factorial interpretation. It seems to be essential, and it seems also to be psychologically reasonable, that we must judge introspectively the psychological processes that are involved in the principal task of a test. To do column addition or multiplication involves the number factor, but to judge which numbers in a row are larger than the adjacent ones seems to involve relatively little of this factor, as far as we can judge by these findings. Factorial results of this kind should eventually throw light on the nature of psychological processes. The space factor has been found repeatedly, and its general nature seems to be sufficiently evident so that we can make fairly dependable predictions as to which tests are going to be heavily saturated with it. The space factor is found in tests which require that the subject manipulate an object imaginally in two or three dimensions. It is quite distinct from perceptual processes which require only the perception of detail in a flat surface and which do not require the imaginal movement of an object in two or in three dimensions. The psychological question still remains whether the space factor is related somehow to kinesthetic imagery.

The next column contains a group of ten tests with maximum saturation in the verbal-comprehension factor V. This factor has also been identified in previous studies, and its differentiation here from the word-fluency factor W is very clear, since both of these factors are represented by a large number of tests in the present battery. In fact, one of the principal objectives in the present battery was to determine whether these two verbal factors would separate clearly, as indicated by the previous interpretations. The whole study would be justified in this finding alone, since the separation between the two factors is very definite. The third verbal factor X_3, whose nature is as yet unknown, is present to a lesser extent in many of the tests in both of the V and W groups.

The test battery contained four memory tests which show maximum saturation on the memory factor M. None of these tests have high communalities. The Digit Span test differs from the other memory tests in that it requires the retention of a sequence, and this may constitute a separate retentive factor. A large factorial study of memory has just been completed which may throw light on the existence of several distinct retentive factors in addition to the rote-memorizing factor which we have called M. Although Word-Number Recall (59) contains numbers and words to be combined as paired associates, the test does not show saturation on either the number factor or the word factor. This finding agrees with our interpretation of these factors: that the rote memorizing of the pairs does not require numerical manipulation or the free flow of words. Since a test performance in rote memorizing depends very largely on the degree of interest and attention that can be obtained from the subjects, it may be more difficult to construct tests with high saturations on the rote-memory factor M than in other tests in which the participation of the subjects is easier to obtain. This may account for the relatively low communalities of the memorizing tests, but it is also possible that the underlying processes which these tests have in common are not represented in sufficiently pure form to make them conspicuous in the factorial analysis. The latter interpretation would call for a refinement in our hypotheses concerning the nature of the rote-memory factor M.

There are eight variables in the group whose principal loadings are in the inductive factor I. Several of these tests have been used before, and they have shown a common factor that has been interpreted as the ability to extract a principle or rule that is common to the material of the problem or test. This interpretation of the factor transcends the

immediate nature of the content, and the same interpretation is applicable here. Mental age as determined by the Kuhlmann-Anderson tests had its only appreciable saturation on the inductive factor. Since the mental age obtained by any of the well-known tests represents necessarily a composite of abilities, we should not expect to find it with a high loading on any of the primary factors. Its variance can be expected to spread over a number of abilities. The tests which Spearman has designed as the best measures of his general factor seem to be inductive in character. The loadings with chronological age are negative. This is due to the fact that the brighter children reach the eighth grade sooner, on the average, than the less gifted children. Within the eighth-grade population we find, therefore, a negative correlation between the chronological age and test performance.

There are seven tests in the group whose highest loadings are in the perceptual factor P. The existence of the perceptual factor seems quite certain because it appears in all the test batteries, but we are not satisfied with present attempts to interpret its nature. Individual experimental work in the laboratory may determine whether it is essentially the ability to find quickly the detail of visual perceptual material. This interpretation does seem to fit the tests which have appreciable saturation on the perceptual factor, but it may be only an approximation to a more satisfactory interpretation.

The next group of tests in *Table 5* have their highest saturation on the factor which was here denoted X_1, but we have no definite interpretation to propose for it. Three of the tests in the battery involved the counting of dots, which were arranged in different ways. In one of them (14) the dots were arranged in a row with frequent blank spaces that were intended to encourage grouping of the spots in counting them. The question was raised whether this performance would show a saturation on the number factor N, but such was not the case. In the second of the dot-counting tests (15), the spots were arranged irregularly in a square, and the subject was asked to count them quickly by any suitable grouping, or singly, if he so preferred. In the third test of this set (16) the spots were arranged in groups of two, three, four, or five, and their spatial arrangement was varied. It was thought that the perception of the geometrical pattern might be perceived as symbolic of numerical quantity by the children without explicit counting of each spot. All these tests have a factor in common; but since the three dot-counting tests are practically isolated from the rest of the battery and without any saturation on the number factor, we have very little to suggest the

nature of the factor. It is, no doubt, the sort of function that would ordinarily be lost in the specific variance of the tests if only one of these dot-counting tests had been included in the battery. The test A B C (1) was designed to represent a form of chain association which might be typical of the number tests. The test did not show any saturation on the number factor; however, it did show some saturation in common with the dot-counting tests. Introspection shows a similarity between the test called A B C and the dot-counting tests, but there is not enough evidence to indicate the nature of the factor. We have referred to the dot-counting group as a triplet, meaning a set of three tests that are very similar and, hence, not sufficiently diversified to indicate the unique character of the principal factor that they have in common.

The next group contains four tests with principal saturation in the factor of column X_2. The two highest loadings are in the two maze tests, (35) and (36). The two tests in this group which are less conspicuous are Pursuit (44) and Geometrical Forms (28). This group is not large enough or sufficiently diversified in character to justify any confident interpretation of the nature of this factor, but it may be surmised that the factor of column X_2 represents the visual pursuit that is evidently common to the four tests. Whether this is the correct interpretation could be determined by a separate experiment in which the pursuit characteristic might be incorporated in a variety of tasks. As far as the present battery is concerned, this factor is little more than a doublet for the maze tests with a suggestion concerning the factor in the two other tests of this group. Note that all of the tests for X_2 have some saturation on the space factor.

A POSSIBLE SECOND-ORDER GENERAL FACTOR

The correlations between the ten primary factors are shown in *Table 6*. Our main interest centers on the seven primary factors that can be given interpretation and, especially, on the first six of these factors for which the interpretation is rather more definite. Among the high correlations we note that the number factor is correlated with the two verbal factors. The word-fluency factor has high correlation with the verbal-comprehension factor and with induction. The space factor has some association with the verbal-comprehension factor and with induction. The rote-memory factor seems to be independent of the other factors. These correlations are higher than the correlations between primary factors for adults.

Because of the psychological interest in the correlations of the pri-

mary mental abilities, we have made a separate analysis of the correlations for those factors which seem to have reasonably certain inter-

Table 6

Matrix R_p *Showing the Correlations of Primary Abilities*

	N	*W*	*S*	*P*	*V*	*M*	*I*	X_1	X_2	X_3
N	1.000									
W	.330	1.000								
S	.124	.270	1.000							
P	− .058	.242	.208	1.000						
V	.348	.422	.380	− .061	1.000					
M	.028	.130	.080	.237	.078	1.000				
I	.237	.318	.426	.170	.422	.103	1.000			
X_1	.058	.227	.191	.480	− .055	.160	.234	1.000		
X_2	− .165	− .019	.118	.197	− .102	− .184	.066	.178	1.000	
X_3	.225	.102	− .234	− .106	− .031	− .080	.138	.096	− .181	1.000

Table 7

Correlational Matrix for Six Primary Abilities

	N	*W*	*S*	*V*	*M*	*I*
N	1.000					
W	.330	1.000				
S	.124	.270	1.000			
V	.348	.422	.380	1.000		
M	.028	.130	.080	.078	1.000	
I	.237	.318	.426	.422	.103	1.000
g_2*	.399	.615	.502	.715	.143	.630

* Second-order general factor.

Table 8

Residuals for Second-Order General Factor in Table 7

	N	*W*	*S*	*V*	*M*	*I*
N	.841					
W	.085	.622				
S	− .076	− .039	.748			
V	.063	− .018	.021	.489		
M	− .029	.042	.008	− .024	.980	
I	− .014	− .069	.110	− .028	.013	.603

pretation. These correlations are shown separately in *Table 7*. If these six primary mental abilities are correlated because of some general intellective factor, then the rank of the correlation matrix of *Table 7*

should be 1. Upon further examination, this actually proves to be the case. In making the best-fitting single-factor interpretation for the correlations of the primary abilities, we have used one of Spearman's formulas.[2] The single factor loadings are shown in *Table 7*, and the residuals in *Table 8*. It will be seen that the residuals are quite small and that the single factor accounts for most of the correlations between the primary factors in *Table 7*.

The single factor loadings show that the verbal factor has the highest loading and the rote-memory factor the lowest loading on the common general factor in the primary abilities. This general factor is what we have called a "second-order general factor." It makes its appearance, not as a separate factor, but as a factor inherent in the primaries and their correlations. If further studies of the primary mental abilities of children should reveal this general factor, it will sustain Spearman's contention that there exists a general intellective factor. Instead of depending on the averages or centroids of arbitrary test batteries for its determination, the present method should enable us to identify it uniquely. We have not been able to find in these data a general factor that is distinct from the primary factors, but the second-order general factor should be of as much psychological interest as the more frequently postulated, independent general factor of Spearman. Our findings seem to support Spearman's claim for a general intellective factor, but he has been so critical of our work on the primary mental abilities that it is uncertain whether he would accept our support for a general intellective factor. We have not found any occasion to take sides as regards the existence of a general intellective factor. Our factorial methods are adequate for finding such a factor, either as a factor independent of the primaries or as a factor operating through correlated primaries. We have reported on primary mental abilities in adults, which seem to show only low positive correlations except for the two verbal factors. Here we find higher correlations among the primary factors for eighth-grade children. It is now an interesting question to determine whether the correlations among primary abilities of still younger children will reveal, perhaps even more strongly, a second-order general factor.

[2] See L. L. Thurstone, *The Vectors of Mind* (Chicago: University of Chicago Press, 1935), p. 146.

CHAPTER III

FACTORIAL ANALYSIS OF THE TWENTY-ONE TESTS

THE SELECTED BATTERY

THE analysis of the larger battery of sixty tests revealed essentially the same set of primary factors which had been found in previous factorial studies. Six of the factors seem to have sufficient stability for the several age-levels that have been investigated to justify an extension of the tests for these factors into practical test work in the schools. In making this extension we have been obliged to consider carefully the difference between research on the nature of the primary factors and the construction of tests for practical use. Several of the primary factors are not yet sufficiently clear as regards psychological interpretation to justify an attempt to appraise them generally among school children. The primary factors that do seem to be clear enough for such purposes are the following: verbal comprehension V, word fluency W, number N, space S, rote memory M, and induction or reasoning I. We shall use the notation R for reasoning to denote the inductive factor. The factors which in several studies are not yet sufficiently clear for general application are the perceptual factor P and the deductive factor D, which was not identified in the present study. The existence of a perceptual factor seems quite certain, but its nature is, as yet, obscure. The deductive factor has been negligible in several factor studies, and it should, therefore, be regarded as tentative. The six factors listed above have been identified clearly in every study in which the nature of the tests led us to expect them. The inductive factor needs further study to clarify its nature.

In adapting the tests for practical use in the schools for the appraisal of six primary mental abilities, we must recognize that the new test program to be described here has for its object the production of a profile for each child, as distinguished from the description of a child's mental endowment in terms of a single intelligence index. For many educational purposes it is still of value to appraise a child's mental endowment roughly by a single measure, but the composite nature of such single indices must be recognized.

The factorial matrix of *Table 5* in *Chapter II* was inspected to find the

three best tests for each of the six primary factors to be included in the selected battery. In making the selection of tests for each primary factor we considered not only the factorial saturations of the tests, which is, of course, the most important consideration, but also the availability of parallel forms that may be needed in case the tests should come into general use. Ease of administration and ease in understanding of the instructions are also important considerations. The selection that was

Table 1

Selected List of Twenty-one Tests

Code No.	Factor	Test	Time Limits (Minutes)		Score
			Fore-exercise	Test Proper	
1....	*P*	Identical Numbers	3	5	*R*
2....	*P*	Faces	2	3	$2R-W$
3....	*P*	Mirror Reading	4	4	*R*
4....	*M*	First Names	2–1–6	5	*R*
5....	*M*	Figure Recognition	1–1–2	4	*R*
6....	*M*	Word-Number	3–1–6	5	*R*
7....	*V*	Sentences	3	4	*R*
8....	*V*	Vocabulary	3	4	*R*
9....	*V*	Completion	3	8	*R*
10....	*W*	First Letters	2	5	*R*
11....	*W*	Four-Letter Words	3	4	*R*
12....	*W*	Suffixes	3	4	*R*
13....	*S*	Flags	12	5	$R-W$
14....	*S*	Figures	7	7	$R-W$
15....	*S*	Cards	6	6	$R-W$
16....	*N*	Addition	3	6	$R-W$
17....	*N*	Multiplication	3	5	$R-W$
18....	*N*	Three-Higher	5	4	$R-W$
19....	*R*	Letter Series	6	6	*R*
20....	*R*	Pedigrees	5	4	*R*
21....	*R*	Letter Grouping	7	4	*R*

determined upon is shown in *Table 1*. In this table are also listed the code numbers of these tests for the smaller battery of twenty-one tests, the time limits for the fore-exercises and tests, and the scoring formulas.

The three tests for each primary factor were printed in a separate booklet, and the material was so arranged that the three tests for any factor could be given easily within a 40-minute school period. The main purpose of the larger test battery was to determine whether the primary factors could be found for eighth-grade children, but the purpose of the present battery was to produce a practical, useful test bat-

tery. For this reason, the selected tests were edited and revised so that they could be used for either hand scoring or machine scoring. The word-fluency tests constitute an exception in that they do not seem to be suitable for machine scoring in any of the tests now known to be saturated with this factor. Figure Recognition (5) was replaced by a revised form of this type of test that was used in another investigation of the memory factor. First Names (4) was set up for machine scoring instead of hand scoring, and the time limit was reduced from 7 to 6 minutes. Word-Number (6) was also revised for machine scoring. Mirror Reading (3) was increased in length. Faces (2) was revised by eliminating several items. Identical Numbers (1) was increased from 4 to 6 minutes. Addition (16) was changed from items with six two-digit numbers to four two-digit numbers. Multiplication (17) was unchanged. Three-Higher (18) was revised by changing the time limit from 5 to 4 minutes. New tests were constructed for vocabulary and for reading. The new verbal test, Vocabulary (8), consisted of synonyms instead of a mixture of both synonyms and opposites. The reading test, Sentences (7), was essentially similar to the Chicago Reading Test II. Completion (9) was revised as to selection of items. No changes were made in the word-fluency tests. Cards (15) was doubled in length. Figures (14) was reduced in length with revisions of the items. The form of the Flags test (13) was changed. Pedigrees (20) was made longer with a longer time limit. Letter Series (19) and Letter Grouping (21) were lengthened.

In order to check the factorial analysis at the present age-level, we arranged to give the selected list of twenty-one tests to a second population of eighth-grade children. In December, 1939, the tests of the selected battery were given to Grades VIIA, VIIIB, VIIIA and in three elementary schools in Chicago, as follows: 209 subjects from Kozminski School, in five sections; 173 subjects from Lewis-Champlin School, in four sections; and 134 subjects from Parkside School, in three sections. The resulting data were factored independently of the larger battery of sixty tests. There were 437 subjects in this population who took all of the twenty-one tests. This population was used for the new factor analysis.

FACTORIAL ANALYSIS OF THE SELECTED BATTERY

The scores of the 437 eighth-grade subjects were reduced to two-digit scores and recorded on tabulating-machine cards. Pearson product-moment correlation coefficients were determined. The correlation ma-

trix (*Table 4* of the *Appendix*) was factored by the centroid method on the tabulating machines, and the resulting factorial matrix to eight factors is shown in *Table 5* of the *Appendix*. Since the battery was selected to contain seven primary factors, it was to be expected that eight factors would include the primaries and one residual factor. The table also shows the communalities of the tests, which are in general higher than in the previous battery because of the fact that

Table 2

Distribution of Residuals

(*Root Mean Square Residual* = .017)

From	To	*f*	From	To	*f*
.04	.05	1	−.01	.00	58
.03	.04	7	−.02	−.01	32
.02	.03	17	−.03	−.02	17
.01	.02	25	−.04	−.03	7
.00	.01	45	−.05	−.04	1

Table 3

The Transformation Matrix Λ *Showing Direction Cosines of the Reference Vectors*

	P	*N*	*W*	*V*	*S*	*M*	*R*	Residual
I.....	.15	.21	.17	.20	.22	.12	.14	.01
II.....	−.23	−.48	.23	.31	−.52	.21	.24	−.01
III.....	−.03	−.43	−.47	.44	.54	.33	−.11	−.02
IV.....	−.04	.32	−.69	.52	−.32	−.34	.24	−.16
V.....	−.38	−.16	.40	.50	.43	−.65	−.26	−.04
VI.....	−.02	−.59	.15	−.33	.07	−.44	.76	−.31
VII.....	−.88	.17	−.19	−.20	.31	.18	.41	.16
VIII.....	.12	−.19	−.02	.04	−.07	−.25	.21	.92

the best tests for the primary factors were selected for the present battery. In *Table 2* of this chapter we have the frequency distribution of eighth-factor residuals, which are seen to be vanishingly small.

Table 6 of the *Appendix* shows the rotated oblique factorial matrix for the primary factors and one residual factor, the mean principal component of the present system. The interpretation of this table is a principal object of the factorial analysis. The transformation matrix Λ by which the co-ordinate axes of F_c are rotated to the axes of V is given in *Table 3*. The columns of Λ show the direction cosines of the reference axes in terms of the orthogonal centroid reference frame.

The matrix product of *Table 4* shows the cosines of the angular separations of the reference vectors of Λ.

The direction cosines of the primary trait vectors are shown in the columns of H' in *Table 5*, and the correlations of the primary traits are

Table 4

Matrix $\Lambda'\Lambda$ *Showing Cosines of Angular Separations of the Reference Vectors*

	P	*N*	*W*	*V*	*S*	*M*	*R*	Residual
P	1.01	.....	.....	.....	.....	.....	.....	.....
N	.04	1.00	.....	.....	.....	.....	.....	.....
W	.02	− .27	1.00	.....	.....	.....	.....	.....
V	− .08	− .06	− .27	1.00	.....	.....	.....	.....
S	− .30	− .08	.01	.08	1.00	.....	.....	.....
M	.04	.12	− .21	− .17	− .03	.99	.....	.....
R	− .29	− .34	− .11	− .28	− .18	− .19	1.00	.....
Residual	.00	.01	.01	− .01	− .01	.01	.00	1.00

Table 5

Matrix H′ *Showing Direction Cosines of Primary Vectors*

	P	*N*	*W*	*V*	*S*	*M*	*R*	Residual
I	.59	.71	.71	.73	.57	.52	.82	.00
II	−.28	−.29	.33	.43	−.57	.41	.13	−.02
III	.11	−.47	−.39	.21	.44	.29	−.06	.00
IV	.01	.26	−.40	.34	−.24	−.32	.20	−.15
V	−.30	−.04	.26	.28	.19	−.48	−.21	−.03
VI	.20	−.27	−.01	−.18	.20	−.28	.41	−.30
VII	−.64	.20	−.06	−.13	.15	.17	.19	.16
VIII	.16	−.11	−.07	.03	.01	−.20	.12	.93

Table 6

Matrix R_p *Showing the Correlations between Primary Factors*

	P	*N*	*W*	*V*	*S*	*M*	*R*	Residual
P	1.000	.242	.211	.290	.420	.163	.471	.001
N	.243	1.000	.466	.385	.256	.186	.540	− .013
W	.211	.466	1.000	.511	.174	.391	.480	− .015
V	.290	.385	.512	1.000	.167	.393	.548	− .001
S	.420	.256	.174	.167	1.000	.149	.386	.006
M	.162	.187	.390	.393	.149	1.000	.390	− .012
R	.472	.541	.480	.548	.386	.389	1.000	− .006
Residual	.001	− .013	− .015	− .001	.006	− .012	− .006	1.000

shown in R_p of *Table 6*. It may be seen that the primary factors are correlated. With only three tests to define each of the primary factors the correlations of the primaries are probably not stable. This can be seen by comparing the correlations between primaries in *Table 6* for

the selected test battery and the corresponding correlations of *Table 6* in *Chapter II* for the larger battery. The correlations between primaries have been found in previous factorial studies to be unstable, with a few exceptions. The correlation between V and W, for example, is always appreciable. It seems clear that the exact location of each primary trait vector in relation to the other primaries is difficult to determine unless there is a very large number of test vectors to define each hyperplane with considerable certainty. With this difficulty it is a fortunate circumstance that when a test has a high saturation on a primary trait, the projection of the test on the primary vector is altered only slightly, with considerable displacement of the primary vector. This circumstance has the result of stabilizing the test validities even with some variation in the possible location of the primary trait vectors.

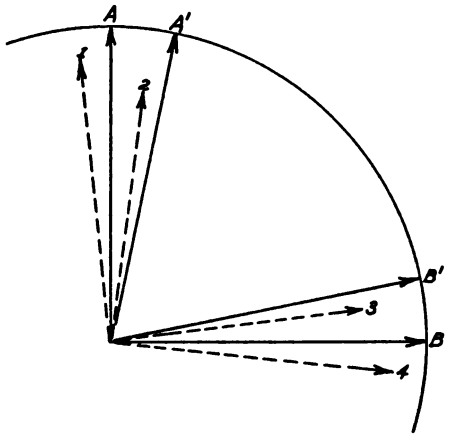


FIGURE 1

The principle that is involved here can be illustrated by the diagram of *Figure 1*. Here we have drawn two orthogonal vectors, A and B, to represent two primary traits with a correlation of zero. In this case, the correlation between the traits is the cosine of the angular separation. Two tests for the trait A are numbered 1 and 2, and two tests for trait B are numbered 3 and 4. The validities of tests 1 and 2 are .90 and .80 for the criterion A, as shown in *Table 7;* the validities of tests 3 and 4 are .80 and .90 for the criterion B. Now suppose that we shift the primary trait vectors to the positions A' and B' so that the correlation of the primary trait criteria is +.40, as shown in *Figure 1*. The validities of .90 and .80 now become .86 and .80, respectively, as shown in the table and in the figure. The validities of the tests have then changed only slightly, although the two criteria have changed correlation from zero to .40. The reason for this circumstance in which the validities are altered only slightly while the correlation of the criteria has changed markedly is to be found in the fact that the cosine of a small angle changes very slowly while the cosine of a larger angle, near 90°, changes very rapidly for the same angular increments.

Because of the correlations of the primary factors, it is to be expected that a test which has a satisfactory validity with one primary will also

be correlated with other primaries. This is evident in *Table 8*, which contains the correlations of each test with each primary factor. The tests were selected so that there should be three tests for each primary factor. Each of the tests has its maximum validity for the primary fac-

Table 7

Hypothetical Example Where the Correlation AB *Is Zero and the Correlation* A'B' *Is .40*

(Tests 1 and 2 Measure the Criterion A *or* A', *While Tests 3 and 4 Measure the Criterion* B *or* B')

		A	*A'*	*B*	*B'*
Tests for *A* or *A'*	1..	.90	.86	−.10	.08
	2..	.80	.80	.10	.26
Tests for *B* or *B'*	3..	.10	.26	.80	.80
	4..	−.10	.08	.90	.86

Table 8

The Correlations R_{jp} *between the Tests and the Primaries*

	P	*N*	*W*	*V*	*S*	*M*	*R*	Residual
1	.54	.56	.31	.25	.22	.07	.38	.07
2	.70	.42	.25	.33	.52	.22	.50	−.14
3	.59	.42	.47	.36	.36	.24	.53	.12
4	.15	.28	.36	.35	.10	.68	.41	.01
5	.36	.09	.22	.23	.27	.43	.33	−.18
6	.15	.23	.28	.27	.13	.67	.28	.08
7	.29	.39	.46	.90	.11	.30	.58	.06
8	.26	.40	.54	.92	.13	.40	.53	.04
9	.31	.37	.47	.88	.31	.36	.52	−.12
10	.31	.39	.84	.47	.18	.34	.44	−.10
11	.20	.37	.80	.41	.23	.31	.42	−.07
12	.18	.34	.68	.50	.09	.32	.33	.09
13	.31	.28	.19	.16	.78	.13	.34	−.07
14	.35	.13	.09	.08	.84	.08	.26	.08
15	.40	.17	.11	.15	.83	.14	.30	.14
16	.22	.79	.36	.30	.26	.18	.42	−.04
17	.21	.84	.40	.31	.18	.18	.46	.00
18	.28	.67	.38	.39	.42	.18	.56	−.13
19	.36	.48	.44	.48	.31	.35	.82	.02
20	.37	.37	.38	.57	.25	.38	.75	−.02
21	.40	.48	.47	.40	.31	.25	.72	.06

tor for which it was selected. So far, it has not been possible to find tests which are pure measures of one factor. Every test has some correlation with more than one factor, and this condition will necessarily obtain wherever the primary factors are themselves correlated. A simple

example is the differentiation between height and weight as correlated primaries. Any acceptable measure of weight must have a correlation with height as long as the two criteria are correlated.

Table 9

Projections of the Composite Vectors on Centroid Axes

	I	II	III	IV	V	VI	VII	VIII	h^2
P_c	.74	−.28	−.10	.03	−.16	.05	−.31	.08	.7675
N_c	.70	−.27	−.35	.23	−.04	−.17	.21	−.13	.8298
W_c	.69	.30	−.29	−.33	.24	−.02	−.11	−.06	.8328
V_c	.73	.40	.18	.33	.26	−.14	−.10	.04	.9330
S_c	.51	−.53	.42	−.25	.17	.17	.13	.05	.8571
M_c	.51	.27	.13	−.19	−.39	−.26	.18	−.10	.6481
R_c	.78	.16	−.07	.17	−.15	.31	.14	.12	.8204

Table 10

Projections of Composites on Reference Vectors

	P	*N*	*W*	*V*	*S*	*M*	*R*	Residual
P_c	.52	.27	.09	.00	.08	−.01	.02	.02
N_c	−.01	.67	−.02	.01	.08	.00	.07	−.05
W_c	.06	−.01	.67	.08	.02	.01	−.01	−.01
V_c	.00	.01	.01	.72	.01	−.02	.05	.00
S_c	.02	−.01	.01	.00	.82	.00	−.01	.04
M_c	.01	.13	−.01	−.01	−.02	.65	.03	.06
R_c	.02	.01	.04	.06	.01	.00	.55	.02

Table 11

Table of Validities

	P	*N*	*W*	*V*	*S*	*M*	*R*	Residual
P_c	.77	.58	.43	.40	.46	.23	.59	.01
N_c	.28	.90	.44	.39	.33	.21	.57	−.06
W_c	.27	.43	.91	.54	.20	.39	.47	−.03
V_c	.31	.41	.52	.97	.19	.38	.58	.00
S_c	.40	.22	.15	.15	.92	.13	.34	.06
M_c	.17	.31	.37	.36	.14	.79	.41	.05
R_c	.45	.52	.51	.57	.34	.38	.90	.02

The three tests of each group were combined with approximately equal weighting into what we have called a "composite." The score on each combination of three tests was called a "composite score." Since each test is represented by a test vector in the common-factor space,

the combinations into groups of three can also be represented as composite vectors in the same space. The centroid projections of each composite are shown in the rows of *Table 9*. The projections of the composites on the reference vectors are listed in *Table 10*, and the validities of the composites for the primaries in *Table 11*. Here it will be seen that each composite score has its highest correlation with the primary trait for which the composite was selected.

Table 12

Factorial Matrix for Selected Test Battery

Code No.	Name of Test	Factors							
		P	*N*	*W*	*V*	*S*	*M*	*R*	Residual
2.....	Faces	.45	.....	.....	.....	.20	.....	.....	.....
1.....	Identical Numbers	.42	.40	.....	.....	.....	.....	.....	.....
3.....	Mirror Reading	.36	.....	.....	.....	.....	.....	.....	.....
16.....	Addition	.....	.64	.....	.....	.....	.....	.....	.....
17.....	Multiplication	.....	.67	.....	.....	.....	.....	.....	.....
18.....	Three-Higher	.....	.38	.....	.....	.20	.....	.....	.....
10.....	First Letters	.....	.....	.63	.....	.....	.....	.....	.....
11.....	Four-Letter Words	.....	.....	.61	.....	.....	.....	.....	.....
12.....	Suffixes	.....	.....	.45	.....	.....	.....	.....	.....
9.....	Completion	.....	.....	.....	.67	.....	.....	.....	.....
8.....	Vocabulary	.....	.....	.....	.66	.....	.....	.....	.....
7.....	Sentences	.....	.....	.....	.66	.....	.....	.....	.....
14.....	Figures	.....	.....	.....	.....	.76	.....	.....	.....
15.....	Cards	.....	.....	.....	.....	.72	.....	.....	.....
13.....	Flags	.....	.....	.....	.....	.68	.....	.....	.....
6.....	Word-Number	.....	.....	.....	.....	.....	.58	.....	.....
4.....	First Names	.....	.....	.....	.....	.....	.53	.....	.....
5.....	Figure Recognition	.20	.....	.....	.....	.....	.31	.....	.....
19.....	Letter Series	.....	.....	.....	.....	.....	.....	.53	.....
20.....	Pedigrees	.....	.....	.....	.22	.....	.....	.44	.....
21.....	Letter Grouping	.....	.....	.....	.....	.....	.....	.42	.....

INTERPRETATION OF THE FACTORIAL MATRIX

In *Table 12* we have summarized the data of *Table 6* of the *Appendix* so as to show more clearly the grouping of the tests as regards the primaries. As in *Table 5* of *Chapter II*, we have omitted all entries less than .20, so as to show more clearly the nature of the simple structure in this battery of tests.

The first group of three tests has maximum saturation in the primary represented by the first column of *Table 12*. This is the perceptual factor which has been found in previous test batteries. It should be noted that, although this factor is found in different test batteries, the factor

is not very strongly represented in the best tests for this factor. The maximum loadings are less than .50. Although we identify this factor as the same perceptual factor which has been found previously in the same tests and in similar tests, the factor is not stable enough to justify the general application of these tests for appraisal of perceptual speed. The experimental test battery for adults which was made available for research purposes included this factor, but we do not regard it as sufficiently well defined at present to include it in a practical school program of tests. The nature of this factor is an interesting psychological problem to be investigated further.

The next group of three tests has its highest projections on the number factor N. This factor is so clearly present in all test batteries containing number tests that we feel justified to include it in a selected test program for use in the schools.

The third group of tests represents the word-fluency factor W, which shows validity correlations high enough to include the factor in the school test program.

The fourth group of tests represents the verbal factor V, which has also been clearly identified in previous studies. In the practical application of these tests, it will be of interest to study further the differences between word fluency W and verbal comprehension V.

The next group of tests represents the space factor S. These tests and the space factor have been so definitely identified in successive studies that we can include this factor in the selected test program.

The last group of tests represents the inductive factor I, which we tried to distinguish from the deductive factor D in our earlier studies. Subsequent studies have not sustained in any satisfactory way the identification of the deductive factor, but the inductive factor has reappeared in several studies. The inductive factor is being called "reasoning R" in the present analysis. The projections of three tests on this reference axis are not so high as we should desire, only one of them being higher than .50; but it was found that this factor has the highest saturation on the second-order general factor in the correlated primaries. For this reason, this group of three reasoning tests has been retained so as to improve the appraisal of the second-order general factor, which may perhaps be interpreted as Spearman's general factor or as a maturation factor.

The correlations of the individual tests with the several primary factors are shown in *Table 8*. Since the primaries are correlated, it is to be expected that each test will have positive correlation with several of the

primaries. The comparison of the primaries in this regard can be made by noting the number of high correlations in this table. It will then be seen that the inductive or reasoning factor *R* has the largest number of significant correlations. The most independent primary factor seems to be the memory factor *M*, which has the smallest number of significant correlations with the separate tests.

A SECOND-ORDER GENERAL FACTOR

In making the analysis for a second-order general factor we have retained only the six primary factors that were considered stable enough

Table 13

Correlations between Six Primaries

	N	*W*	*V*	*S*	*M*	*R*
N	1.000	.....	.....	.....	.....	.....
W	.466	1.000	.....	.....	.....	.....
V	.385	.512	1.000	.....	.....	.....
S	.256	.174	.167	1.000	.....	.....
M	.187	.390	.393	.149	1.000	.....
R	.540	.480	.548	.386	.389	1.000
g	.603	.686	.676	.339	.474	.843

First-Factor Residuals

	N	*W*	*V*	*S*	*M*	*R*
N	.636	.052	−.023	.052	−.099	.032
W	.052	.529	.048	−.059	.065	−.098
V	−.023	.048	.543	−.062	.073	−.022
S	.052	−.059	−.062	.885	−.012	.100
M	−.099	.065	.073	−.012	.775	−.011
R	.032	−.098	−.022	.100	−.011	.289

to justify inclusion in a practical test battery. In *Table 13* we have summarized the correlations of the six primary factors of *Table 6* after eliminating the perceptual factor, which has some uncertainty, and the residual factor, which was the mean principal component in the present case. The matrix of *Table 13* is nearly of rank 1. The first factor residuals are shown at the bottom of the same table, and it will be seen that the off-diagonal entries are quite small. This finding is interesting in view of the fact that the intercorrelations of the primary factors are often unstable. Under the correlation matrix we have listed the correlation of each primary factor with the second-order general factor. It

will be seen that the inductive or reasoning factor *R* has the highest correlation with the second-order general factor, namely, .843. Space and memory have lower correlations with the general factor, namely, .339 and .474, but these are still significant. It is not likely that these correlations will be stable in successive studies of different populations, but it may be significant that the inductive or reasoning factor has the highest correlation with the second-order general factor. This finding raises the interesting question whether a unique general factor can be determined. Its interpretation here would be that the primary mental abilities are correlated by a general factor which operates through each of the primaries. Each of the primary factors can be regarded as a composite of an independent primary factor and a general factor which it shares with other primary factors. The psychological interpretation of the general factor must be only tentative at the present time. We must await much data from further investigation to determine whether the second-order general factor is sustained in repeated experiments, whether it can be found also for adult subjects, and what psychological or physiological interpretation may be given to it.

SIMPLE STRUCTURE IN THE SELECTED TEST BATTERY

The configurations by which the bounding hyperplanes were determined in this test battery are shown in *Figures 2* and *3*. Since there were seven dimensions in addition to the residual factor, we have shown here the complete set of twenty-one diagrams by which the reference axes were located. The location of each reference axis is determined by inspection of these diagrams. The simple structure shown here is remarkable for its clarity. Most of the diagrams show a concentration of points at or near the origin and a grouping of three tests along each of the two axes of each diagram. The simple structure in the present battery is sharp, with only one common factor conspicuously present in each test, so that the structure could have been determined by inspection for clusters. Each group of three tests would show a cluster. Further slight adjustments might be made in the locations of the bounding-planes by these diagrams, but they would not be sufficient to cause any appreciable change in the higher saturations.

THE PROBLEM OF INVARIANCE IN THE FACTOR LOADINGS

In the present study we are dealing with two test batteries that were given to eighth-grade children in the same city. The two factorial analyses were made independently. This situation should enable us to

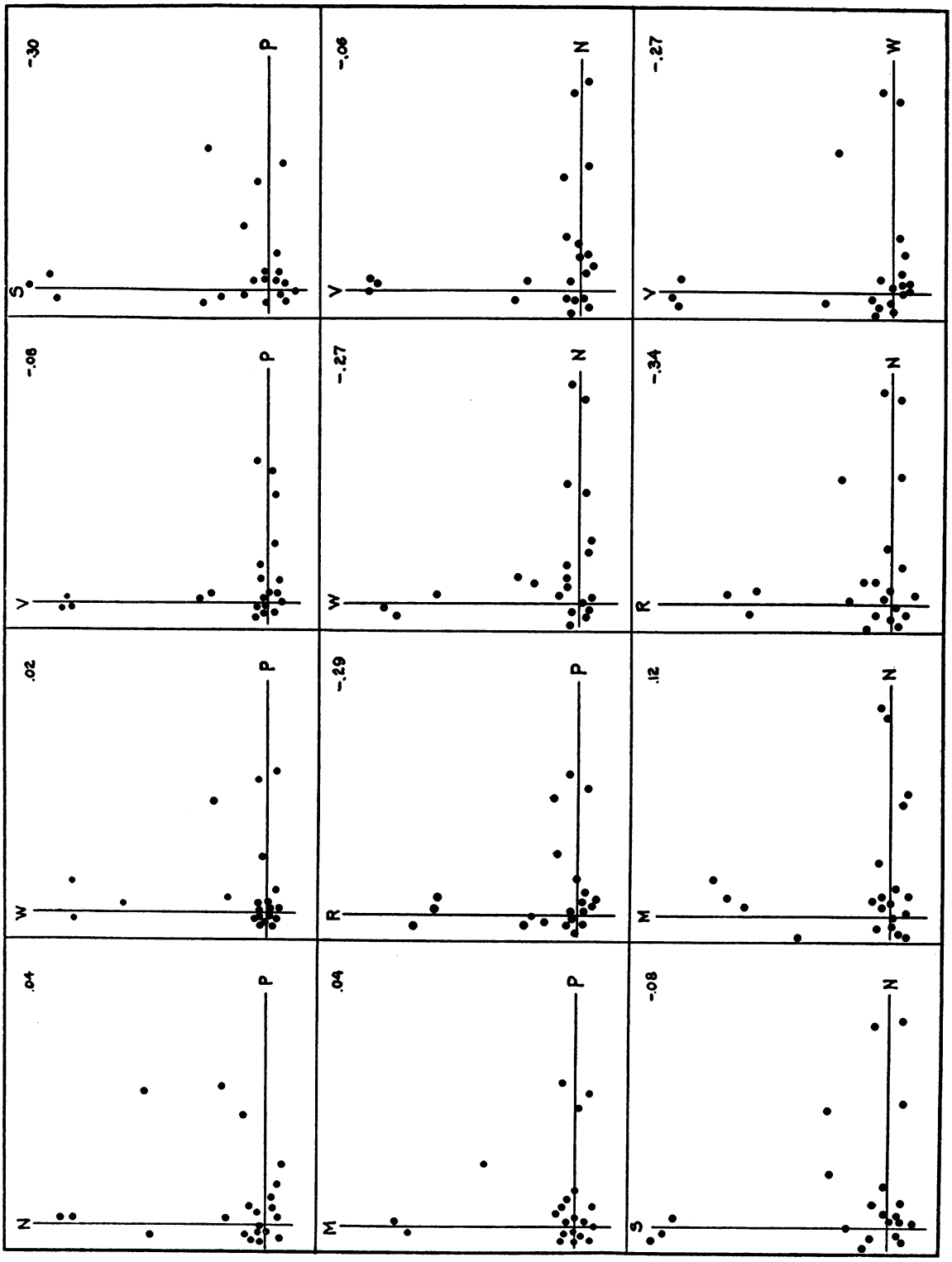


FIGURE 2

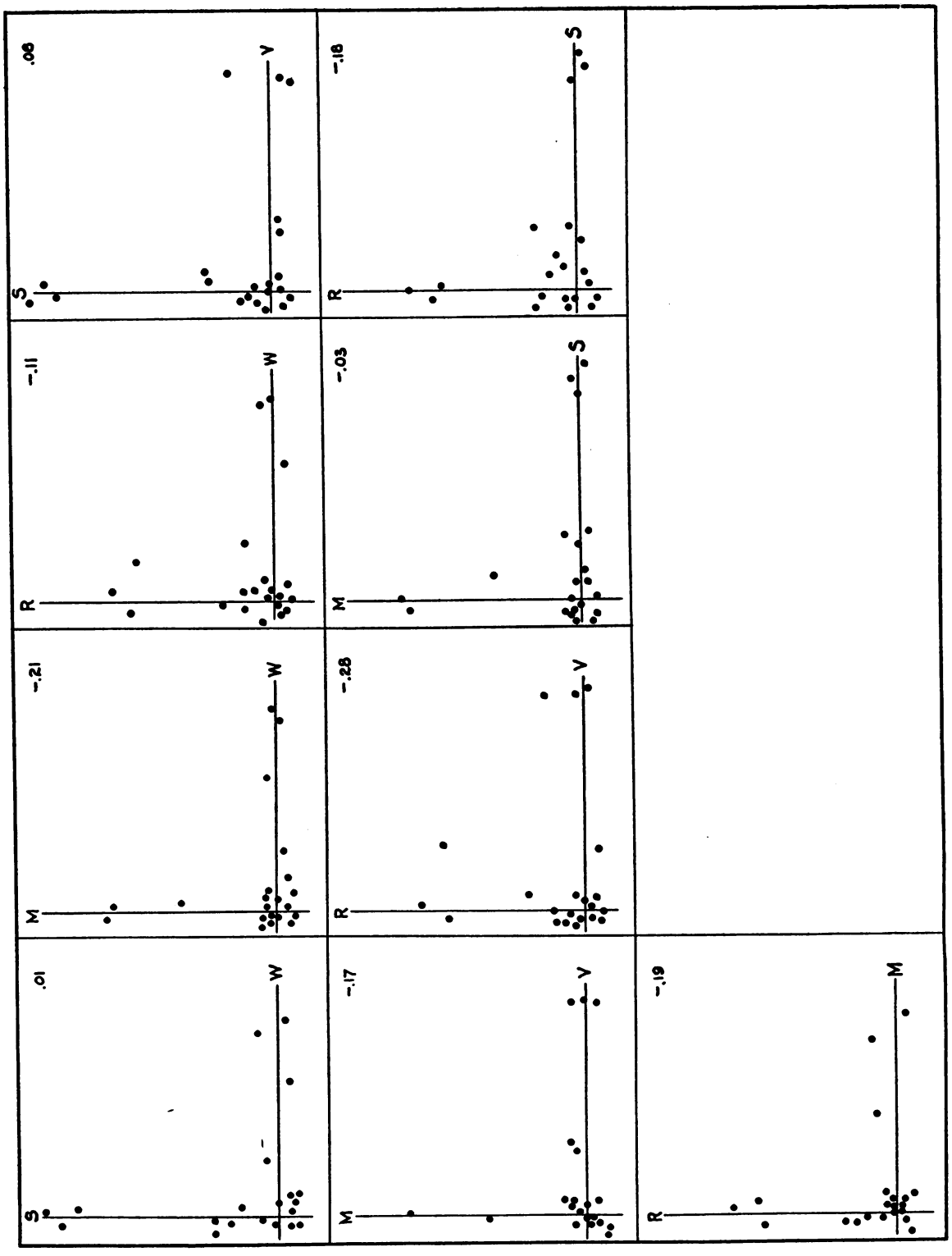


FIGURE 3

make a comparison of the factor loadings for the tests which were common in order to throw some light on the question whether factor loadings are invariant. The present study was not set up for this particular problem, but the comparison can be made under some limitations. The twenty-one tests in the selected battery were essentially the same as the corresponding tests in the larger first battery, so that we should expect the factorial compositions to be practically the same in the two batteries. If this problem were to be investigated directly, the tests should, of course, be exactly the same both as to items and as to time limits in the two batteries; but in the present case this condition can only be approximated. The larger group of children, from fifteen schools, and the smaller group of children, from three schools, were comparable as to age and grade; but it seems quite certain that the smaller group had, in general, a higher mean test performance and that they were less heterogeneous, so that the dispersions of test scores were somewhat smaller than those of the larger group. It cannot be maintained that the two groups were comparable in the sense of being random samples from the same general population, but their differences should not have been large. The groups were sufficiently similar, and the two test batteries were sufficiently similar as to the overlapping tests, so that a comparison of the factorial compositions should be of some interest.

The entries of the factor matrix V (*Appendix, Table 6*) were compared with the entries in the corresponding factor matrix V (*Appendix, Table 3*) for the corresponding tests. The paired values have been plotted, as shown in *Figure 4*. Inspection of this plot shows that there is high correlation between the factor loadings v_{jp} in the two test batteries. In this plot the pairs of projections on the reference axes have been combined for all the axes. Similar plots could be made separately for each of the reference axes. For each of the factors we find in this plot that the tests which had high loadings in one battery had high loadings also in the other battery. The loadings near the origin of the range of $\pm .20$ may be regarded as of less significance because they represent a variance of less than 4 per cent of the total test variance.

A straight line was drawn through the origin by inspection so as to represent the relation between the two sets of factor loadings, and it will be seen that the slope is slightly greater than unity. This means that the factor loadings were higher, in general, for the selected test battery of twenty-one tests than for the larger battery. Several conditions may contribute to this result. The selected tests had been edited and improved so that they should represent at least slightly better concen-

tration on the primaries which they were intended to measure. A difference in the heterogeneity of the two groups would also result in a slope differing from unity, since the standard scores of the subjects in the tests and in the primaries are determined in part by the dispersion of ability in the group.

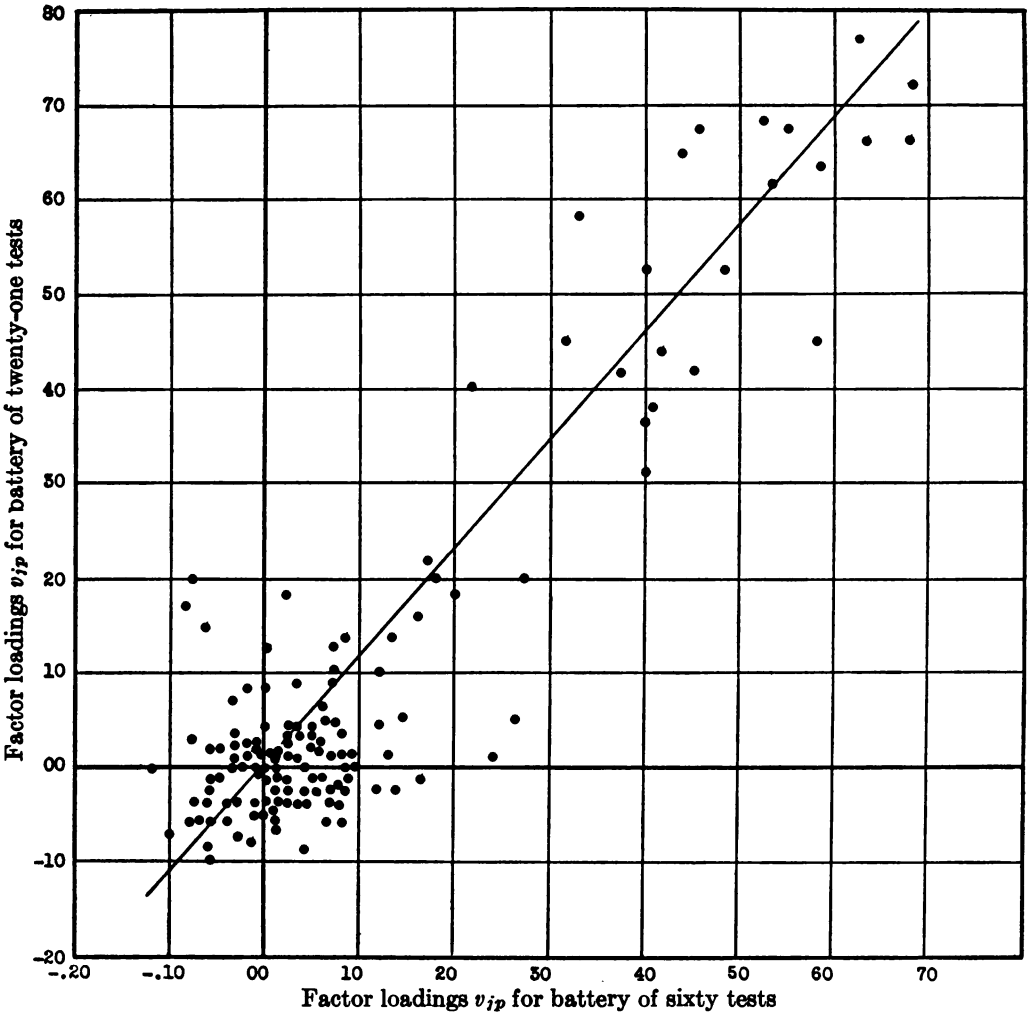


FIGURE 4

The subject of invariance of factorial composition should be considered under at least two cases, namely, the conditions for invariance in the same population and the conditions for invariance in different populations. These two cases are distinct, and they involve different considerations. The simpler case, which should be the easier to demon-

strate, is that in which two batteries are assembled from the tests that have been given to one population. If the two batteries have some overlapping tests, the factorial compositions of these overlapping tests should be invariant in the independent factor analyses for the two batteries. This is a fundamental requirement of any satisfactory factorial method. This condition cannot be obtained unless the factors that characterize the overlapping tests are common factors in both of the test batteries, and it is also necessary that the two batteries be large enough so that each of the common factors in the overlapping tests is clearly defined by tests which contain the factor and by tests in which the factor is absent. Ambiguous cases could be set up in which a factor in one of the overlapping tests is inadequately defined by one of the batteries. Such a situation might lead to highly correlated primaries or to ambiguities of interpretation of the primaries. These ambiguities or errors of interpretation could be clarified and corrected by subsequent factorial analyses with augmented test batteries. No matter how clear a primary factor appears to be in a factorial analysis, it will be accepted as a psychological finding only when it has been identified in repeated studies with different test batteries on different populations. The requirement about invariance of factorial composition was set up as one of the objectives to be attained in the development of a satisfactory factorial method. It means that we should not use a factorial method which is known to alter the factorial composition of a test when it is merely moved from one battery to another.

The second case concerns the possibility of invariance of factorial composition of a test when it is given to different populations. This is not a requirement of factorial method because it is to be expected, by psychological considerations, that the factorial composition of a test will change as it is given to populations that differ in age and schooling. Although this is not the case which has been the subject of controversy, it is interesting to consider the conditions under which invariance might be obtained even for different populations. Let us consider two populations that are comparable as to age and schooling, so that essentially the same abilities are brought into use in doing a given task T. Let us also consider a simple case in three dimensions, so that it can be easily represented graphically, as in *Figure 5*. The test T in the first diagram of the figure is shown to be a function of the primaries A and B but not C. The test performance T can be determined, therefore, as a linear combination of the scores in the pure primaries A and B with no effect of the primary C, whose loading should be zero. The loading for B should

be higher than that for A, since the test T is nearer B than A. Let us suppose that the loading for B is twice that of A. If the first diagram represents a right spherical triangle, the three primaries should have zero intercorrelations.

Consider, now, the second triangle, which represents the same three primaries for another population. They are denoted A', B', and C' in the second triangle; and the angular separation between A' and B' is smaller, to correspond to the fact that these primaries are assumed to be correlated in the second population. If the test T involves the same primary abilities in the second population but not C, we should expect the test vector T to be in the plane $A'B'$ as shown, and we should expect it to be closer to B' than to A', as before. We should even expect the

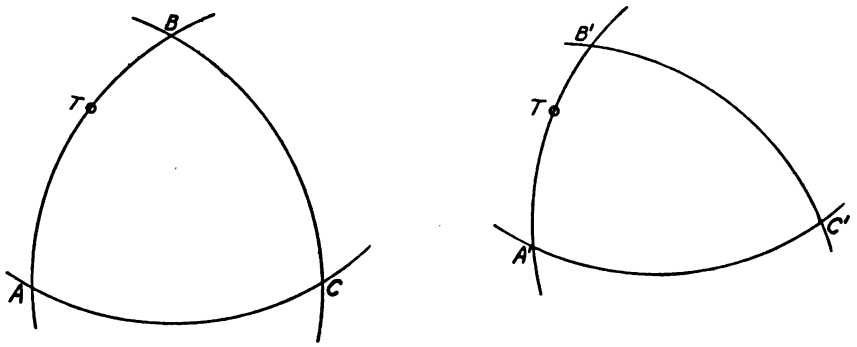


FIGURE 5

test vector T in this case to be a linear function of the two primary vectors A' and B' with the same numerical relation, so that the coefficient for B' should be about twice that of A'. Here we should expect an invariance in the regression coefficients s on x, even though the test has been given to two different populations, but we must not ignore certain limitations in making comparisons for two different populations.

First among these limitations in making comparisons of factorial results for different populations is the assumption that the two populations use essentially the same abilities in doing the task T. The regression coefficients should remain essentially invariant under changes of obliqueness of the primaries in the two populations, but the invariance will be disturbed by marked changes in the dispersions of the abilities in the two populations. Consider an extreme case. Suppose that the dispersion in ability B' were reduced to insignificance in the second population. This might be brought about by selecting the second popu-

lation so as to be uniform as to this primary. Then there would be no individual differences in this ability, and it would not appear as a dimension. The second triangle would collapse into the two-dimensional figure $A'C'$, and the test vector T would move into the position A'. Its communality would be reduced, and the regression coefficients would no longer remain invariant. The development of factorial theory will, no doubt, be extended to cover various conditions of transfer from one population to another, but we must not forget that statistical considerations do not cover the fact that two populations may differ in their ways of doing a task T either because they use the abilities that seem effective or because they have been taught to use them differently. The important case for the controversial principle of simple structure is the first case which concerns invariance in factorial composition when a test is moved from one battery to another for the same population, with the reservation that the common factors of the test must be clearly defined in both batteries.

CHAPTER IV

DESCRIPTION OF THE TESTS

WE SHALL describe here the nature of each of the tests without reproducing them in full. For most of the tests we have reproduced only the instructions and fore-exercise, which were timed separately. Their purpose was to familiarize the subject with the nature of the test and to give him some preliminary practice material. This section of each test was followed by the test proper, which was also given a separate time limit. The complete tests have been made available through the American Documentation Institute, in Washington, D.C., in the form of bibliofilm copies.[1] Those who want to work in detail with the actual test material can obtain the complete photographic copies, which show not only all the items but also the typographical arrangement of all the tests. For most purposes the present summary of the nature of each test will probably suffice. In the interpretation of the factors it is essential to have access to the detail of the tests, since the name of a test is often inadequate for its psychological interpretation.

For each of the following tests we have indicated in parentheses the code number by which it is identified in some of the tables, the verbatim instructions and examples, and a statement of the number of items in the test proper.

A B C (1)

Only three letters, *A*, *B*, and *C*, are used in this test. They are combined in various ways, but there are only two rules. The rules are:

1. A combination of any two different letters is equal to the third letter.
   EXAMPLES: $A\,B = C$, $A\,C = B$, $B\,C = A$,
   $B\,A = C$, $C\,A = B$, $C\,B = A$.
2. If a letter is followed by itself, the combination is equal to that letter.
   EXAMPLES: $A\,A = A$, $B\,B = B$, $C\,C = C$.

The example below has been worked out according to these rules.

$$A\,B\,A = B$$

[1] This set of tests is filed as an Auxiliary Publication of the American Documentation Institute, Care of Science Service, 2101 Constitution Avenue, Washington, D.C. It is available in microfilm form as Document No. 1434, at a cost of $2.00.

Here is the way you solve the problem. Combine the first two letters. The combination *A B* = *C*. Then combine this *C* with the next letter, which is *A*. The combination *C A* = *B*.

Work the following examples and write the answers in the blanks.

C B B = _____ *C A C* = _____

You should have written *C* in the first blank and *A* in the second blank.

Here is a longer example which has been worked out.

C A B C = *A*.

The first two letters, *C A*, equal *B*. This letter, combined with the next, gives *B B*, which is equal to *B*. This letter, combined with the next, gives *B C* = *A*.

Find the answer to the following example and write it in the blank.

B C A C = _____

You should have written *B*.

Here are more problems for you to practice on. Write the answers in the blanks.

B C C A C = _____ *B A B B A* = _____ *A C B C A* = _____

The test contained sixty items.

ABSURDITIES (2)

For each of the following sentences mark *F* if the sentence is foolish. Mark *S* if it is sensible.

S	F	
=	=	Mrs. Smith has had no children, and I understand that the same was true of her mother.
S	F	
=	=	While the businessman was eating his lunch, he was interrupted by a long-distance telephone call.
S	F	
=	=	I have three brothers—Paul, Ernest, and myself.

The first sentence is foolish, so you should have marked *F*. The second sentence is sensible, so you should have marked *S*. You should have marked *F* for the third sentence.

The test contained twenty-six statements.

ADDITION (3)

Below are two columns of numbers which have been added. Add the numbers for yourself to see if the sums are correct.

16	42
38	61
45	83
99	176
R ▬	R =
W =	W ▬

The first sum is right, so the *R* below it is marked. The second sum is wrong, so the *W* is marked.

Check the sums of the columns below. If a sum is right, mark the *R* immediately under the sum. If a sum is wrong, mark the *W* below the sum.

17	35	63
84	28	17
29	61	89
140	124	169
R =	R =	R =
W =	W =	W =

The test contained fifty-six columns of six two-place numbers.

ANAGRAMS (4)

Look at the word below:

COMFORTABLE

Using the letters of this word, you can build many smaller words.

There is only one rule: *In any one word do not use a letter more times than it appears in the given word.* For example, the word BATTLE would not be right because it has two *T*'s and there is only one *T* in COMFORTABLE.

Sample words have been written in the first few lines. Write as many more words as you can, using only the letters given. Go ahead.

The test instructions were as follows:

Write as many words as you can, using the letters in the word below:

ABBREVIATION

ARITHMETIC (5)

In this test you will be given some problems in arithmetic. After each problem there are five answers, but only one of them is the correct answer. Solve each problem and mark the correct answer. The following problem is an example.

How many pencils can you buy for 50 cents at the rate of 2 for 5 cents?

10 = 25 = 125 = 100 = 20 ▬

The correct answer is 20, and therefore this answer has been marked.

Mark the correct answer to the following problem:

If James had 4 times as much money as George, he would have $16. How much money has George?

$4.00 = $8.00 = $12.00 = $16.00 = $64.00 =

You should have marked $4.00.

Mark the answers to the following problems:

In 5 days Harry has saved a dollar. What has his average daily saving been?

20¢ = 22½¢ = 25¢ = 30¢ = 40¢ =

A watch lost 1 minute 18 seconds in 39 days. How many seconds did it lose per day?

2 = 2½ = 5 = 7½ = 10 =

John sold 4 magazines at 5 cents each. He kept one-half the money, and with the other one-half he bought papers at 2 cents each. How many did he buy?

3 == 4 == 5 == 6 == 10 ==

The test contained twenty arithmetical statement problems.

ASSOCIATION (6)

Write as many words as you can that are names of things to eat or drink.

MIRROR READING (7)

Look at the two words below.

cat

tac

The first word is *cat*. The second is also *cat*, but it is printed backward.

Below are two lines of words. In the first line the words are printed forward. In the second line the same words are printed backward.

aunt	skip	dump	found	bind
tnua	piks	pmud	dnuof	dnib

The first word in each column below is printed forward. Below it are four words printed backward. One of the four words printed backward is the same as the word at the top of the column. The word which is the same as the first word is marked.

flag	town
gorf	nrut
talf	enot
galf	spot
sraef	nwot

In each column of words below, mark the word printed backward which is the same as the first word.

lamp	book	purse	horse	most
kool	dnob	esrup	hcnum	erom
ekal	tneb	ladep	yenob	taem
pmal	knab	esabp	egdab	tsom
snel	koob	yellup	esrob	klim

The test contains fifty columns of four words.

CARDS (8)

Here is a picture of a card. It looks like an *L*, and it has a hole in one end.

The two cards below are alike. You can slide one around on the page to fit the other exactly.

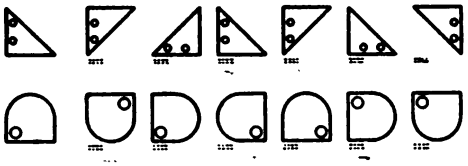
Now look at the next two cards. They are different. You cannot make them fit exactly by sliding them around on the page.

Here are more cards. Some of the cards are marked. The cards which are like the first card in this row are marked.

Below is another row of cards. Mark all the cards which are like the first card in the row.

You should have marked the second and third cards. They are like the first card.

Here are some more cards for you to mark. In each row mark every card that is like the first card in the row.



The test contained twenty rows of seven figures.

CLASSIFICATION (9)

Below is a list of the names of ten students and their grades in arithmetic and spelling. You are to classify the students in four groups as follows:

Group A: Students whose spelling grades are above 20 and whose arithmetic grades are above 50

Group B: Students whose spelling grades are above 20 and whose arithmetic grades are below 50

Group C: Students whose spelling grades are below 20 and whose arithmetic grades are above 50

Group D: Students whose spelling grades are below 20 and whose arithmetic grades are below 50

There are no grades of 20 or 50.

Write a letter in the last column of the table below to indicate the correct group for each student. The first three names have been marked. Check them to see that they are right.

Name	Arithmetic	Spelling	Group
John	53	27	*A*
Albert	62	15	*C*
Mary	38	11	*D*
Helen	56	21	.....
Elizabeth	29	28	.....
Richard	33	17	.....
Margaret	72	10	.....
Anne	55	19	.....
Paul	36	31	.....
Ralph	51	25	.....

The test contained fifty-six names to be classified.

COMPLETION (10)

The following sentence has a word missing at the place indicated by the parentheses. You are to think of the word that best completes the meaning of the sentence. The number in parentheses is the number of letters in the missing word.

A (4) is a contest of speed.....

B= F= M= P= R━

The missing word is *race*. The number in the parentheses is the number of letters in the missing word. The letter *R* has been marked because it is the first letter in the missing word.

Do the following example:

A (9) is a place or building for athletic exercises.....

C= D= G= H= T=

You should have marked *G* because it is the first letter in the missing word *gymnasium*. This word has nine letters and it completes the sentence.

Do the following examples in the same way:

A (5) is an organized company of singers in church service....

B= C= D= F= G=

The thin cutting part of an instrument, as of a knife or sword, is called its (5).....

A= B= E= H= W=

A mark made with a hot iron, as to indicate ownership, quality, etc., is called a (5).

B= L= P= S= V=

An (7) is the commander in chief of a navy.....

U= O= I= E= A=

A (3) is a small or portable bed, as of canvas stretched on a frame.....

T= N= H= C= B=

The test contained sixty definitions.

DIGIT SPAN (11)

Instructions to the Examiner

See that the students print their names and the name of their school on the answer sheets. Then read the following directions to the students:

"I am going to read aloud some numbers. Listen carefully so that you will remember them. While I am reading the numbers, my right hand will be raised, like this:" (*Examiner raises right hand.*) "At the end of each series of numbers, I will lower my hand. That will be the signal for you to write the numbers in the same order in which they were read. Do not write anything until I lower my hand."

"At the beginning of each series, I shall give you the number of the series to indicate the corresponding row on the answer sheet. Be sure to write each series of numbers in the proper place on the answer sheet. If you don't remember all the numbers, write as many as you can."

"Here is the first practice problem. Are you ready?" (*Examiner raises right hand and reads the following numbers at the rate of one per second:*)

4–1–2–7

(*Examiner lowers right hand, pauses for students to write, and then says:*) "When I lowered my hand, you should have written 4–1–2–7 on the first line under 'Practice Problems.'"

"Here is the second practice problem:" (*Examiner proceeds as before.*)

5–8–3–1

"When I lowered my hand, you should have written 5–8–3–1 on the second line under 'Practice Problems.'"

The test contained twenty-one series of digits ranging from five to eleven in span.

DIRECTIONS (12)

Read the following sentence and do just what it tells you to do.

If Washington, D.C., is the capital of the United States, write *W* here _____.

You should have written *W* in the blank.

Do just what you are told to do in the three sentences below:

If the word *cat* has three letters, write 4 here _____.

If 12 is less than 8, make a ring around 8. If *William* is a boy's name, write *girl* here _____.

You should have written 4 in the first blank. You should not have made a ring around 8. You should have written *girl* in the last blank.

Here is some material for you to practice on:

If battleships are heavier than canoes, write *no* here _____.

Circle the first letter *e* in the sentence you are now reading.

If New York is larger than Boston, add 8 and 6 and put a wrong answer here _____; but if New York is smaller than Boston, put the right answer here _____.

Write *Easter* here _____ and a boy's name here _____.

The test contained a longer section with fifteen blanks.

DISARRANGED SENTENCES (13)

The words below make a sentence if they are arranged in proper order.

drank milk she

The sentence would be

She drank milk.

Here is another disarranged sentence. You can make a sentence with these words if you put them in the proper order.

was late he school for

The sentence is: *He was late for school.* Notice that two words have been underlined. They are the first and last words in the correct sentence.

Think of a sentence using the words below. Then mark the first and last words in that sentence.

ball boys play

You should have underlined *boys* and *ball.* The sentence is: *Boys play ball.*

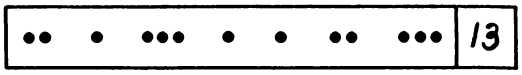
In each row below think of a sentence using the words in the row. Then underline the first and last words in that sentence.

floor	she	the	swept		
twelve	the	struck	clock		
balls	cats	like	play	to	with

The test contained twenty disarranged sentences.

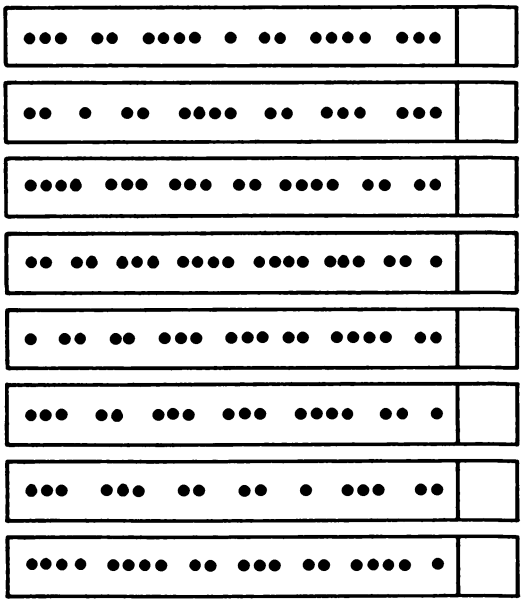
DOT COUNTING I (14)

Count the number of dots in the row below.



The number of dots in the row is 13, so 13 is written in the space at the end of the row.

Here are some more problems for you to practice on. Count the dots in each row and write the number in the space at the end of the row. Work as fast as you can.

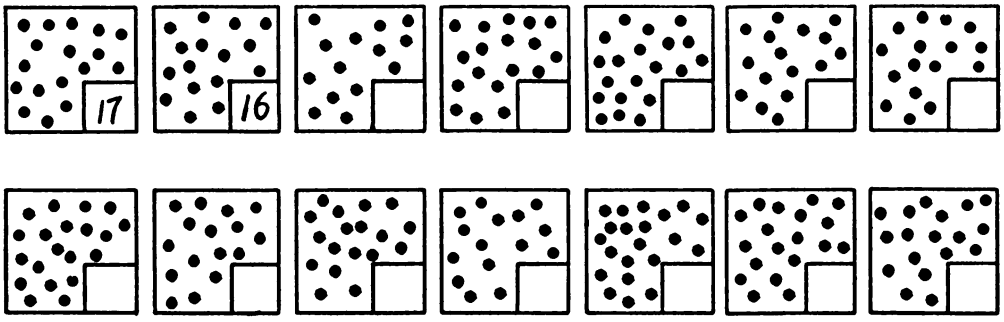


The test contained seventy-six rows of dots.

DOT COUNTING II (15)

Count the dots in the first square below. Do not draw any lines or make any marks in the square. The first square has 14 dots, so 14 is written in the corner. The second square has 16 dots, so 16 is written in the corner.

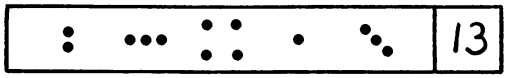
Here are some more problems for you to practice on. The first two have already been answered for you. Count the dots yourself to be sure the answers are right. Do not make any marks except to write the answers in the corners. Work as fast as you can.



The test contained eighty-four squares of dots.

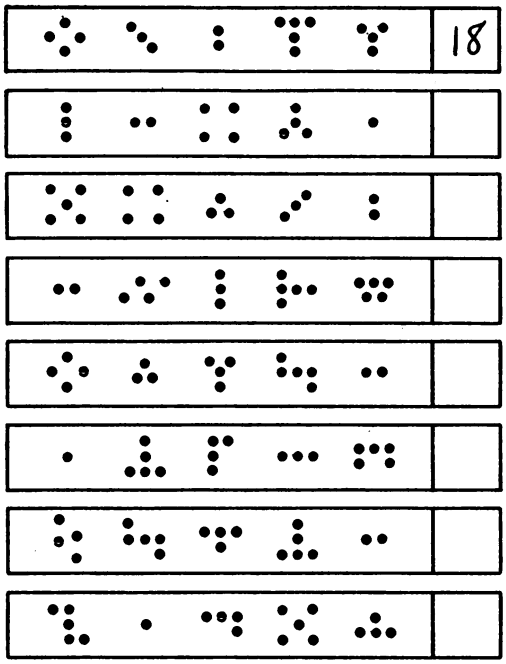
DOT COUNTING III (16)

Count the dots in the row below.



There are 13 dots, so 13 is written in the space at the end of the row.

Here are some problems for you to practice on. The first one has been answered for you. Count the dots yourself to see if 18 is the right number. Find the number of dots in each row and write the number in the space at the end of the row. Work as fast as you can.



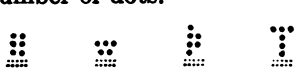
The test contained sixty-eight rows of dots.

DOT PATTERNS (17)

Look at the four groups of dots in the row below.

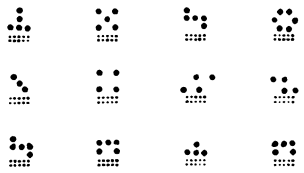
Three of the groups have the same number of dots. One group has a different number of dots. That group is marked.

In the row below, three of the groups have the same number of dots. Mark the one group which has a different number of dots.



You should have marked the second group. It is the only group which does not have 6 dots.

In each row below, mark the group which has a different number of dots.



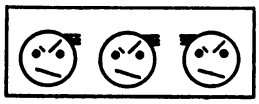
The test contained sixty rows of dot patterns.

FACES (18)

Here is a row of faces. One face is different from the others. The face that is different is marked.

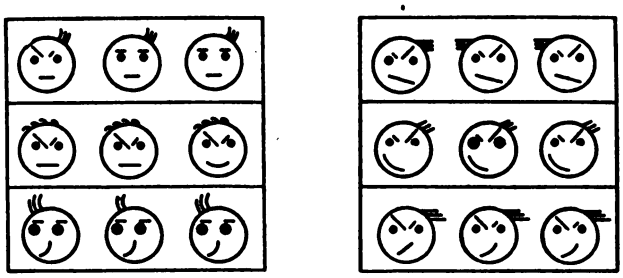
Look closely to be sure that you see why the middle face is marked. The mouth is the part that is different.

Here is another row of faces. Look at them and mark the one that is different.



You should have marked the last face.

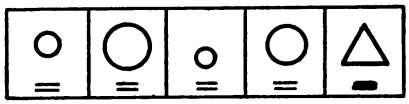
Here are more pictures for you to practice on. In each row mark the face which is different from the others.



The test contains sixty rows of faces.

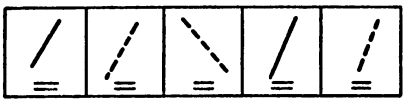
FIGURE GROUPING (19)

Look at the five drawings below. Four of the drawings are alike in some way. One drawing is different. It is marked.



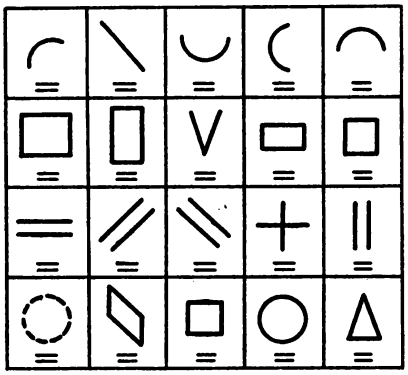
Four of the drawings are circles. One drawing is not a circle. It is marked because it is different from the others.

Look at the next row of drawings. Four of the drawings are alike in some way. One drawing is different. Mark the drawing which is different.



You should have marked the third drawing. All the other lines slope down to the left, but the third slopes down to the right.

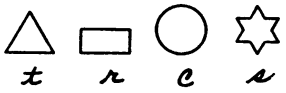
Below are four sets of drawings for you to practice on. In each set, mark the drawing which is different from the other four.



The test contained thirty sets of drawings.

FIGURE NAMING (20)

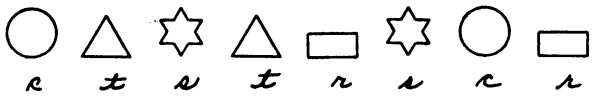
The figures below are a triangle, a rectangle, a circle, and a star.



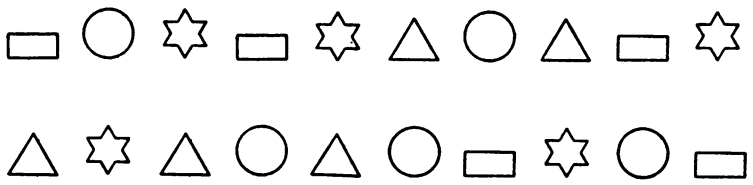
Each figure is marked with the first letter of its name.

The *triangle* is marked *t*.
The *rectangle* is marked *r*.
The *circle* is marked *c*.
The *star* is marked *s*.

In the row below, the correct letter is written under each figure.



Write the correct letter under each figure below. Work as fast as you can.

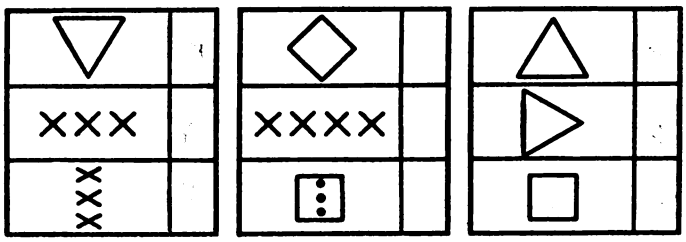


The test contained one hundred and forty-four figures.

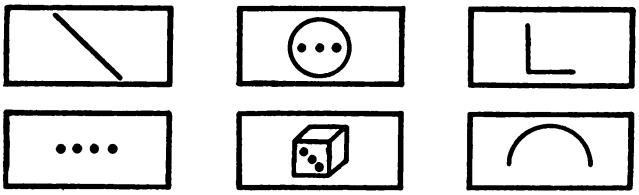
FIGURE RECOGNITION (21)

Study the figures below so that you can recognize them when you see them again.

In the list below put a check mark (√) after each of the figures that were listed above.



In a similar manner study the list below so that you can check these figures when you see them again on the next page.



In the test proper twenty figures were shown, and these were to be checked in a longer list of sixty figures presented on a separate sheet.

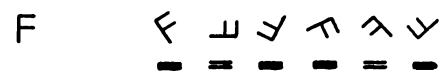
FIGURES (22)

Look at the row of figures below. The first figure is like the letter *F* which is right side up. All the other figures are like the first, but they have been turned in different directions.

Satisfy yourself that all of these figures look like the first one if they are turned right side up.

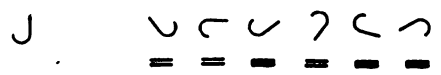
Now look at the next row of figures. The first one looks like an *F*. But none of the other figures would look like an *F* even if they were turned right side up. They are all made backward.

Some of the figures in the next row are like the first figure. Some are made backward. The figures like the first figure are marked.



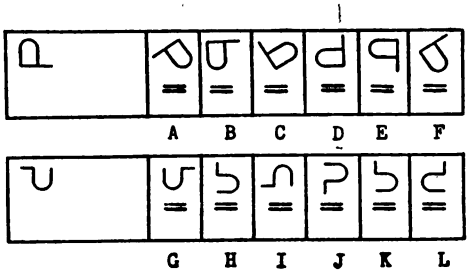
Notice that all the figures like the first figure are marked.

Some of the figures in the next row are like the first figure. Some are made backward. The figures like the first figure are marked.



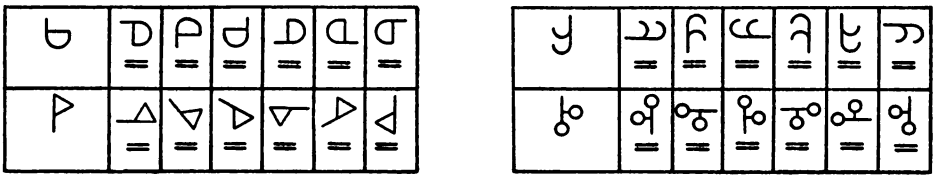
Notice that all the figures like the first figure are marked.

In each row of figures below, mark every figure which is like the first figure in the row. Do not mark the figures which are made backward.



In the first row you should have marked figures *A* and *D*. In the second row you should have marked figures *J* and *L*.

In each row below mark every figure which is like the first figure in the row. Go right ahead. Do not wait for any signal.



The test proper contained sixty rows of seven figures.

FIRST AND LAST LETTERS (23)

Each word in the list below begins with *S* and ends with *K*.

soak stick streak squeak

On the blanks below write several words which begin with *M* and end with *T*. The words may be as long or as short as you like; the number of letters does not matter. Go ahead.

The instructions for the test were as follows:

Write as many words as you can which begin with *T* and end with *E*. The length of the words does not matter.

FIRST LETTERS (24)

Look at the words below. Each word begins with *D*.

doll dinner daisy doughnut

On the blanks below, write several words which begin with *P*. One word you might write is *pretty*. Go ahead and write more words which begin with *P*.

The instructions for the test were as follows:

Write as many words as you can which begin with *S*.

FIRST NAMES (25)

In each row below is written a name. You are to learn the names so well that when the last name is given you can write the first name. On the next page the last names are listed in a different order. You will be asked to write the first names.

If writing helps you to remember, you may copy the first and last names on the blanks below. Study silently until you are told to stop. Begin studying now. Do not wait for any signal.

First Name	*Last Name*	*First Name*	*Last Name*
Mary	Brown		
John	Davis		
Ruth	Preston		
Fred	Smith		

In the first row the correct first name has been written. Write the correct first names in the other blanks.

First Name	*Last Name*
Ruth	Preston
	Brown
	Smith
	Davis

The test consisted in memorizing twenty first names which were to be associated with given surnames.

FLAGS (26)

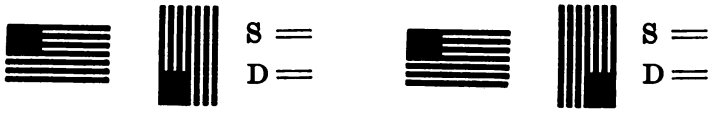
Here are two pictures of a flag. These two pictures of the flag are the same. You can slide one picture around to fit exactly on the other picture.

S is marked to show that the pictures are the *same.*

The next two pictures of the flag are different. You cannot slide the pictures around to make them fit exactly.

D is marked to show that the pictures are *different.*

Here are some pictures for you to mark. Try to fit the pictures together by sliding them around flat on the paper. If the two pictures of the flag are the same, mark *S.* If the two pictures are different, mark *D.*



You should have marked *S* for the first pair and *D* for the second pair.

The test contained forty-eight items.

FOUR-LETTER WORDS (27)

Each of the words in the list below has four letters and begins with *B.*

bear bone bald bent

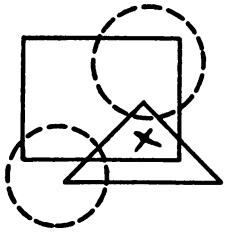
On the blanks below write several four-letter words which begin with *M.*

The instructions for the test were as follows:

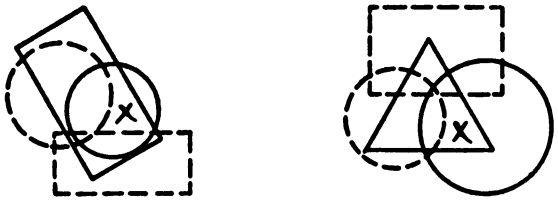
Write as many words as you can which have four letters and begin with *C.*

GEOMETRICAL FORMS (28)

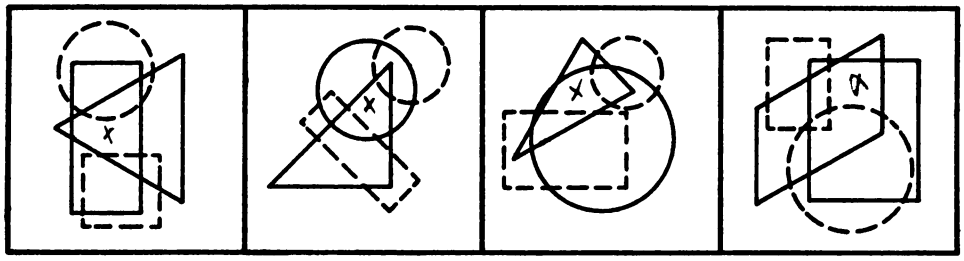
Look at the drawing below. Two figures in the drawing are made in solid lines, and two figures are made in dotted lines. The *X* is inside both solid-line figures and outside both dotted-line figures.



Here are two more drawings. Check each of them to see that the *X* is inside the two solid-line figures and outside the two dotted-line figures.



Here are some problems for you to practice on. In each drawing, make an *X* inside the two solid-line figures and outside the two dotted-line figures.



The test contained sixty diagrams.

HIGH NUMBERS (29)

In the row of numbers below, 13 is marked because it is higher than the numbers on either side of it. The number 70 is marked for the same reason. It is a larger number than the numbers on either side of it.

81 12 **13** 9 7 14 18 **70** 62 11 15

In the next row of numbers mark every number that is larger than the numbers on either side of it.

62 5 9 **88** 74 19 **52** 31 62 **69** 41 31

You should have marked 88, 52, and 69.

Here are more problems for you to practice on. In each row, mark every number that is larger than the numbers just before it and just after it. Work as fast as you can.

42 19 31 38 53 19 22 60 68 74 73 82 55 43 12

15 92 88 89 94 96 58 44 27 18 20 15 13 61 60

58 54 27 34 25 16 6 75 66 93 98 90 54 53 20

24 28 44 35 39 52 70 61 83 88 94 95 80 82 74

73 76 91 95 93 90 12 8 9 4 32 27 29 40 32

The test contained thirty lines.

IDENTICAL NUMBERS (30)

The number at the top of the first column of figures is 634. A mark has been made under each 634 in the column. In the second column a mark has been made under the 876 because 876 is the number at the top of that column. In the third column the two 795's have been marked because 795 is the number at the top of the third column.

The number at the top of each of the other columns is repeated one or more times in that column. Find those numbers as quickly as possible and put a mark under each of them. Go right ahead.

634	876	795	423	279	374
693	643	583	837	363	282
850	328	795	115	643	663
634	932	189	423	279	539
513	879	342	528	375	314
398	375	795	969	470	475
696	470	896	274	887	576
634	697	247	423	699	374
574	876	319	627	291	850
628	294	468	423	983	677
634	982	543	962	585	846

The test contained thirty columns of numbers.

IDENTICAL PICTURES (31)

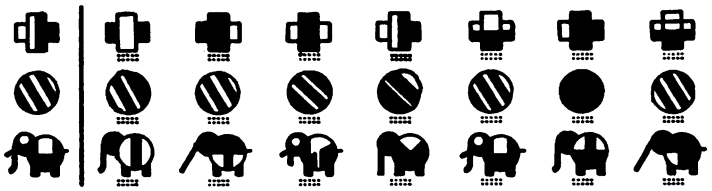
All the pictures in the row below are somewhat alike, but not exactly. Some of the pictures are exactly like the first one in the row. Look at the pictures and find those that are *exactly* like the first one in the row.

Three pictures are exactly like the first one. They have been marked.

Now you are to mark the pictures in the next row youself. Look carefully at all the pictures and mark each one that is exactly like the first one in the row.

You should have marked two pictures, the second and the next to the last.

There are three more problems on this page. Go ahead and mark them for practice.



The test contained sixty rows of pictures.

INCOMPLETE WORDS (32)

A letter is missing in the word below.

h____art

If *e* is written in the blank, the word is *heart*. Write an *e* in the blank.

Two letters are missing in the word below.

co____fortab____e

If *m* is written in the first blank and *l* in the second blank, the word is *comfortable*. Write *m* and *l* in the blanks.

Write a letter in each blank below to complete each word.

l____mp

de____k

wi____dow

You should have written *a* in the first blank to make the word *lamp*, *s* in the second blank to make *desk*, and *n* in the third blank to make *window*.

Practice on the words below. If you cannot get a word, do not spend too much time on it. Go on to the next word.

f____sh	an____m____l
bre____d	s____h____ol
bul____e	f____m____us

The test contained sixty items.

LETTER GROUPING (33)

Look at the groups of letters below.

AABC ACAD ACFH AACG

Three of the groups have two *A*'s. The group which does not have two *A*'s is marked.

Here is another problem. Three of the groups are alike in some way. Can you find three groups which are alike?

XVRM ABCD MNOP EFGH

In three of the groups the letters are arranged in alphabetical order. The first group is not in alphabetical order. Mark it to show that it is different.

Three of the groups in the next row are alike in some way. Mark the group that is different.

KABC KEFG LOPQ KUVW

Three of the groups start with *K*. You should have marked the third group, which is different.

Here is another problem. Mark the group that is different.

ACDE ILMN LNOP QSTU

Three of the groups omit only one letter. You should have marked the second group, which is different.

Here are more problems for you to work. In each row three of the groups are alike in some way. Mark the group that is different. Go right ahead.

AAAB	AAAM	AAAR	AATV
DCBA	HGFE	MRVX	PONM
RSTT	LMNL	FGHF	BCDB
ABCE	FGHJ	KLMO	RSTW

The test contained twenty lines.

LETTER SERIES (34)

Read the row of letters below.

a b a b a b a b ____

The next letter in this series would be *a*. Write the letter *a* in the blank at the right.

Now read the next row of letters and decide what the next letter should be. Write that letter in the blank.

c a d a e a f a ____

You should have written the letter *g*.

Now read the series of letters below and fill in each blank with a letter.

c d c d c d ____
a a b b c c d d ____
a b x c d x e f x g h x ____

You should have written *c*, *e*, and *i*.

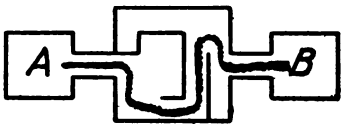
Now work the following problems for practice. Write the correct letter in each blank.

a a a b b b c c c d d ____
a x b y a x b y a x b ____
a b m c d m e f m g h m ____
r s r t r u r v r w r x r ____
a b c d a b c e a b c f a b c ____

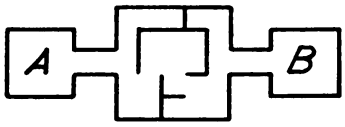
The test contained twenty-five series.

MAZES I AND II (35, 36)

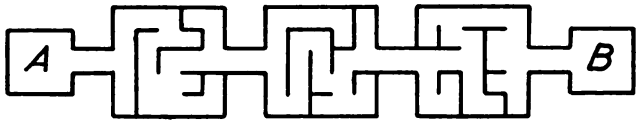
Look at the following drawing. A pencil line has been drawn from the letter *A* to the letter *B*. Notice that this pencil line does not cross any printed line.

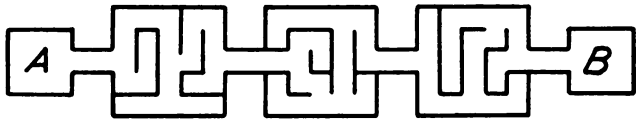


Now look at the next drawing. Make a line with your pencil from *A* to *B* without crossing a line.



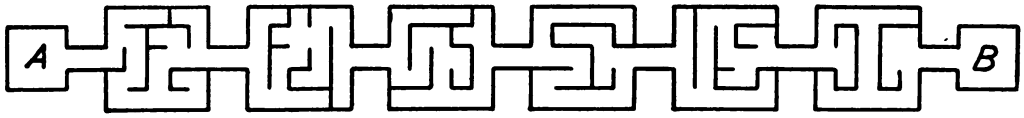
Practice on the drawings below. In each drawing make a line from *A* to *B* without crossing a line.





MAZES I

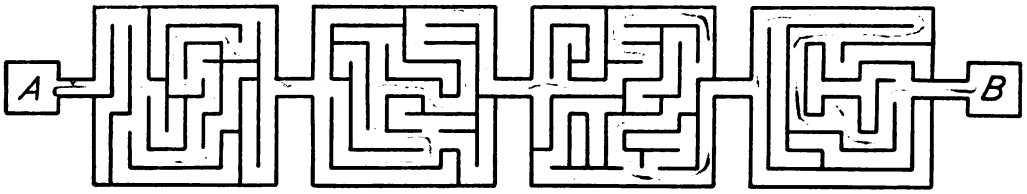
In each drawing make a line from *A* to *B* without crossing a line.



This test (Mazes I) contained nine mazes.

Mazes II

In each drawing make a line from *A* to *B* without crossing a line.



This test (Mazes II) contained six mazes.

MULTIPLICATION (37)

Below are two multiplication problems which have been worked out. Multiply the numbers for yourself to see if the products are correct.

64	39
7	4
448	166
R —	R =
W =	W —

The first answer is right, so the *R* below it is marked. The second answer is wrong, so the *W* is marked.

Check the answers in the problems which are worked out below. If a multiplication is right, mark the *R* immediately under the answer. If the multiplication is wrong, mark the *W* under the answer.

57	46	29
6	8	7
342	358	193
R =	R =	R =
W =	W =	W =

The test contained seventy problems.

NUMBER PATTERNS (38)

Here is a square with the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9. The numbers go across the square in each row.

1	2	3
4	5	6
7	8	9

The square below is larger, and some of the numbers are used twice. Read the numbers in the order: 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7.

1	2	3	4
5	6	7	8
9	1	2	3
4	5	6	7

Look at the next square. Begin reading with 1 and read in the order: 1, 2, 3, 4, 5, etc.

1	5	9	4
2	6	1	5
3	7	2	6
4	8	3	7

Here is another square. Read these numbers in order beginning with 1.

1	2	3	4
8	7	6	5
9	1	2	3
7	6	5	4

A number is left out of the next square. The missing number is 3. The 3 at the side of the square is marked to show that the missing number is 3.

1	2		4
5	6	7	8
9	1	2	3
4	5	6	7

1 6
2 7
3 8
4 9
5

Here is another square with a number left out. It is one of the numbers at the side of the square. Mark the number that is left out.

1	5	9	4
2		1	5
3	7	2	6
4	8	3	7

1 6
2 7
3 8
4 9
5

You should have marked the 6. The number 6 is missing.

Here are two more squares for you to try. A number has been left out of each square. Mark the number left out of each square.

1	8	9	7
2		1	6
3	6	2	5
4	5	3	4

1 6
2 7
3 8
4 9
5

4	3	2	1
8	7	6	5
	2	1	9
7	6	5	4

1 6
2 7
3 8
4 9
5

You should have marked 7 and 3.

Here is another square. Begin with the 3 and read: 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, etc.

3	4	5	6
7	8	9	1
2	3	4	5
6	7	8	9

A number is left out of the next square. Mark the missing number.

4	8	3	
5	9	4	8
6	1	5	9
7	2	6	1

1 6
2 7
3 8
4 9
5

You should have marked 7. The number 7 is missing.

Here are two more squares for you to try. Mark the number left out of each square.

5	6	7	8
9	1	2	3
4		6	7
8	9	1	2

1 6
2 7
3 8
4 9
5

7	2	6	1
8	3	7	2
9	4	8	
1	5	9	4

1 6
2 7
3 8
4 9
5

You should have marked 5 and 3.

Practice on the squares below. Mark the missing number at the right of each square. Go right ahead.

7	8	9	1
5	4	3	2
6	7	8	9
4	3	2	

1 6
2 7
3 8
4 9
5

6	7	8	9
5	4	3	2
7	8	9	
6	5	4	3

1 6
2 7
3 8
4 9
5

The test contained thirty squares.

PARAGRAPH RECALL (39)

Instructions to the Examiner

Say to the students:

"I shall read aloud a short paragraph. Then I shall read it a second time. Listen carefully and remember as much of it as you can. The same paragraph is printed on

an answer sheet, but some of the words are missing. When I have finished reading, the proctors will give you your answer sheets. Then you will fill in the blanks with the proper words. Only one word goes in each blank. If you are not certain that you remember the exact word, write the word that you think goes in the blank. Ready? Listen carefully."

Then read the following passage:

> From her appearance, one could never have guessed that she had spent the entire afternoon in the kitchen. She had prepared a lovely dinner for six guests, and every detail was perfect.

Then say: "Listen carefully and I will read it again." Read the passage again.

Then have the proctors distribute answer sheets, have students fill in the blanks, and have them print their names and the name of their school.

Then say:

"Now I shall read a longer passage. I shall read it twice. Then, when the signal is given, you will turn the page and fill in the blanks with the proper words. If you are not certain that you remember the exact word, fill in the blank anyway. Only one word goes in each blank. Are you ready for the passage? Listen carefully."

Then read the following passage:

> Even in early times the Phoenicians were a mighty nation. Their strength and wealth came, principally, from two sources: trade and colonies. They claimed territory both where there was a market for their own goods and where they could bargain for commodities they did not possess. Their cities were busy centers of exchange both for their own products and for imported products. Much of their commerce was carried on with countries in the interior of Asia, and safe passage was guaranteed to their caravans by agreement with the rulers of the countries through which they had to travel. The Phoenicians furnished woolen materials to the entire ancient world, and they manufactured so much glass that they are often credited with inventing it. They established many trading stations on the shores of the Mediterranean, and these centers influenced the direction and spread of early Asiatic culture. They also settled along the coast of Asia Minor. There they exchanged manufactured wares for slaves, skins, and wool. They also worked the mines and collected snails, from which they made purple dye. At that time the Greeks were a primitive people, but they were quick to adopt these new products, and thus to benefit from a civilization which had been in existence for centuries.

Then say: "Listen carefully, and I will read it again." Read the passage again.

Then say: "Turn the page and fill in the blanks," and start timing.

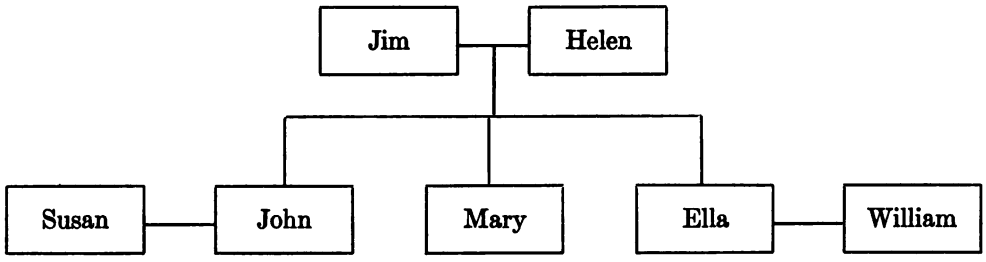
THE TEST

Fill in as many of the blanks below as you can.

Even in early times the Phoenicians were a _____ nation. Their strength and _____ came, principally, from two sources: _____ and _____. They claimed territory both where there was a _____ for their own goods and where they could _____ for commodities they did not possess. Their _____ were busy centers of exchange both for their own products and for _____ products. Much of their commerce was carried on with countries in the interior of Asia, and _____ passage was guaranteed to their _____ by agreement with the _____ of the countries through which they had to travel. The Phoenicians furnished _____ materials to the entire _____ world, and they manufactured so much _____ that they are often credited with _____ it. They established many trading stations on the shores of the _____, and these centers influenced the _____ and spread of early Asiatic _____. They also settled along the coast of Asia Minor. There they exchanged manufactured wares for _____, skins, and wool. They also worked the _____ and collected _____, from which they made _____ dye. At that time the _____ were a primitive people, but they were quick to adopt these new products, and thus to benefit from a _____ which had been in existence for _____.

PEDIGREES (40)

Look at this chart.



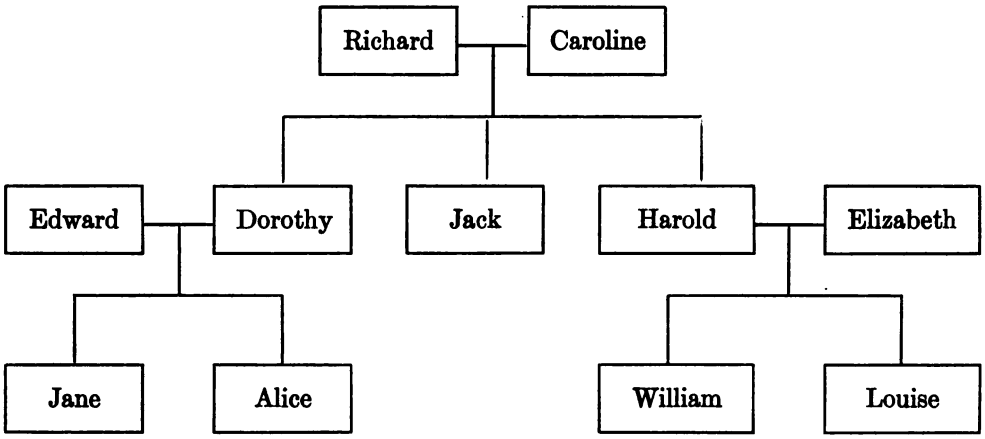
This chart tells you that Jim and Helen were married and had three children, John Mary, and Ella. John married a girl named Susan, and Ella married a man named William.

Now answer these questions by consulting the chart.

Mary's brother is _____.
How many children did Helen have? _____.
How many brothers-in-law does Mary have? _____.
How many brothers-in-law does Ella have? _____.
Jim's daughter-in-law is _____.
William's mother-in-law is _____.
How many daughters has Jim? _____.
Helen's husband is _____.
Susan married _____.
Ella's sister-in-law is _____.

The Test

Consult the chart for the answers to the questions below.



Harold married _____.
Does Alice have a brother? _____.
William's aunt is _____.
How many grandchildren does Richard have? _____.
Jack is Louise's _____.
Jane's cousins are _____ and _____.
Elizabeth's sister-in-law is _____.
How many children has Harold? _____.
Jack's nephew is _____.
How many nieces has Jack? _____.

Harold's brother is _____.
Caroline's grandson is _____.
Dorothy's niece is _____.
Edward's nephew is _____.
Does Harold have two nieces? _____.
Who is Alice's aunt? _____.
Louise's cousins are _____ and _____.
Richard's daughter is _____.
How many uncles has Jane? _____.
Jack's brother-in-law is _____.

PICTURE NAMING (41)

Below is a picture of a house. Under the picture is written *h*, because *h* is the first letter of the word house.

Under each picture below, write the first letter of the word it represents.

The test contained one hundred and forty-seven pictures.

PREFIXES (42)

Look at the words in the following list. Each of them begins with *in*.

independent infant inhale invest

Write in the blanks below several words beginning with *ex*. Go ahead.

The instructions for the test were as follows:

Write as many words as you can which begin with *con*.

PROVERBS (43)

Read carefully the five proverbs below. Notice that four of the proverbs have almost the same meaning. One proverb has a meaning different from the other four. Mark the proverb which is different.

= Where there is honey there are bees.
= There is no rain without a cloud.
= Shallow brooks are noisy.
= No result without a cause.
= Where there is smoke there is fire.

You should have marked the third proverb, the one about shallow brooks.

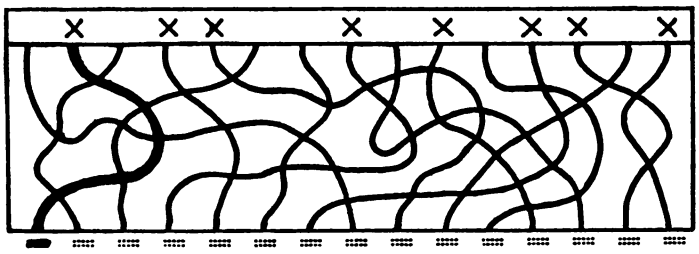
Read carefully the next group of five proverbs. Mark the proverb which is different in meaning from the other four.

= Put not all your crocks on one shelf.
= The fox with one hole is soon caught.
= Better to have a second string to your bow.
= Little pitchers have big ears.
= Don't put all your eggs in one basket.

The test contained twenty groups of proverbs.

PURSUIT (44)

Some of the lines in the following drawing are labeled *X*. The first *X*-line is a heavy line. The other end of the heavy *X*-line is marked.



Now trace the second *X*-line. Mark the space at the end of the line.

Follow each *X*-line and mark the space at the end of it. Mark only the *X*-lines. Do not trace or mark lines not labeled *X*. Mark the *X*-lines in the above drawing now.

The test contained twelve diagrams.

READING TESTS (45), (46), (47), AND (48)

The Reading tests were taken from the Chicago Reading Tests, prepared for the Chicago schools by M. D. Engelhardt and Thelma Gwinn Thurstone. Four scores were used, as follows:

Vocabulary (45), pp. 3 and 4, a synonyms test
Sentences (46), pp. 5 and 6, a sentence-completion test
Paragraphs (47), pp. 13–15, a reading-comprehension test
Paragraphs (48), pp. 18 and 19, a reading-comprehension test

REASONING (49)

Read the two sentences at the left below. They are called "given facts." At the right is a "conclusion." If you are told that *A* is larger than *B* and that *B* is larger than *C*, then you can conclude that *A* is larger than *C*. The word *larger* has been written in the blank.

Given Facts		*Conclusion*
A is larger than *B*		
B is larger than *C*	therefore	*A* is *larger* than *C*

Read the two "given facts" below. A "conclusion" is given at the right. One word is left out of the "conclusion." Decide what word is left out and write it in the blank.

Given Facts		*Conclusion*
P is longer than *Q*		
R is shorter than *Q*	therefore	*P* is _____ than *R*

You should have written *longer* in the blank.

Read the "given facts" below and fill in the blank in the "conclusion."

Given Facts		*Conclusion*
M is larger than *O*		
O is larger than *T*	therefore	*T* is _____ than *M*

You should have written *smaller* in the blank.

Here are more problems for you to practice on. Read the "given facts" and fill in the blanks in the "conclusions."

Given Facts		*Conclusions*
M is younger than *N*		
K is older than *N*	therefore	*K* is _____ than *M*
R is richer than *S*		
T is poorer than *S*	therefore	*T* is _____ than *R*
H is thicker than *Z*		
Z is thicker than *L*	therefore	*H* is _____ than *L*

The test contained thirty syllogisms.

RHYMING WORDS (50)

The first word in the row below is *whip*. Look at the other four words and think how they sound.

whip tip ship lip dip

The four words *tip*, *ship*, *lip*, and *dip* rhyme with *whip*.

The first word in the next row is *there*. The other four words rhyme with *there*.

there air bear where hare

In each of the rows below, write four words which rhyme with the given word. If you cannot think of four words, write as many as you can and then go to the next row.

snail _____ _____ _____ _____

game _____ _____ _____ _____

The test consisted of rhyming twenty words.

SAME OR OPPOSITE (51)

The first word in the following line is *many*.

many ill few down sour

One of the other words means either the same as or the opposite of *many*. The word *few* has been marked because it is the opposite of *many*.

The first word in the following line is *ancient*. Mark one of the other words that means the same as or the opposite of *ancient*.

ancient	dry	long	happy	old

You should have marked *old* because it means the same as *ancient*.

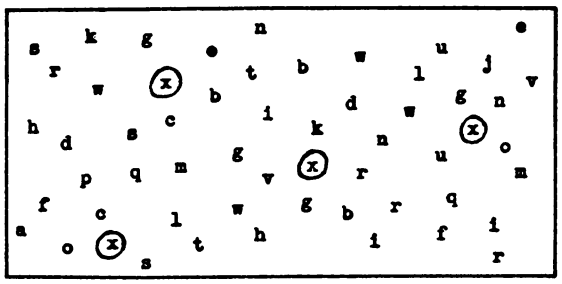
In each of the following lines mark the word that means the same as or the opposite of the first word.

deep	blue	shallow	tense	watery
awkward	clumsy	loyal	passive	young
hot	dry	cooked	red	cold

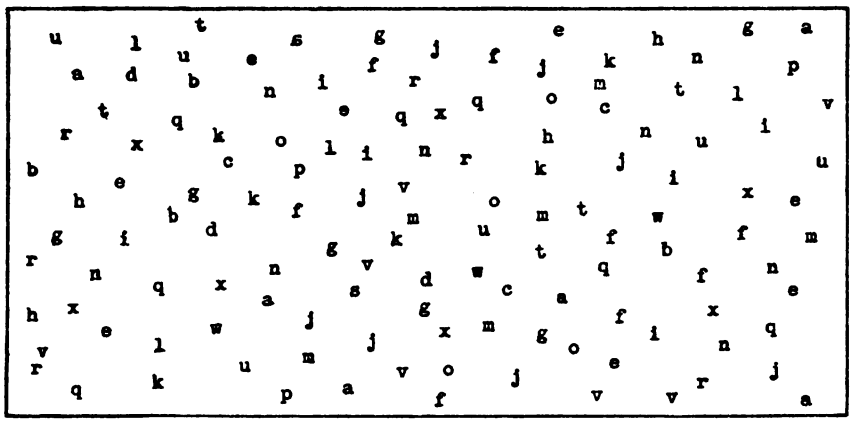
The test contained fifty items.

SCATTERED *X*'S (52)

Look at the letters below. A ring has been made around each letter *x*.



There are seven *x*'s in the group of letters below. Make a ring around each letter *x*. Go ahead.



When the signal is given (not yet), turn the page and mark more *x*'s. There are seven *x*'s on each page of this test. Do not spend too much time on any page. If you cannot find all seven *x*'s on a page quickly, go on to the next page.

The test contained seven pages of scattered letters.

SECRET WRITING (53)

In the first column below, "Words," are three words: *saw*, *sat*, and *was*. In the second column, "Secret Writing," the same words are given in a secret writing or code. Each number stands for a letter. You are to find the letter that corresponds to each number. The words are not in the same order in the first two columns. In the last column, "Translation," you are to write the words in the same order as in the secret writing.

Words	*Secret Writing*	*Translation*
saw	3 8 6	___ ___ ___
sat	5 8 3	___ ___ ___
was	3 8 5	___ ___ ___

There are several ways to solve this problem. Here is one way:

Look closely at the three words in the first column. Notice that two of the words begin with the same letter. The words *saw* and *sat* begin with *s*. The number which occurs at the beginning of two words is 3. Therefore 3 stands for *s*. Write *s* in each of the three blanks corresponding to the 3's.

The other word begins with *w*, so 5 must stand for *w*. Write *w* in each of the blanks corresponding to the 5's.

The middle letter of each word in the first column is *a*. The middle number of each word in the secret writing is 8. This tells you that 8 stands for *a*. Write *a* in each middle blank in the third column.

The only word which is not complete now is *sat*, so you know that 6 must correspond to *t*. Write *t* in the last blank of the first word.

The words in the last column should be in the order: *sat, was, saw.*

Here is another sample problem. The secret writing is different. Solve the problem to find out which letter each number stands for. Write the words in the correct places in the third column.

Words	*Secret Writing*	*Translation*
bet	8 0 9	___ ___ ___
rat	5 2 8	___ ___ ___
cab	4 2 9	___ ___ ___

Did you notice that two of the words end in *t*? The number which occurs twice as a last letter is 9. Then 9 must stand for *t*. Write *t* in the last column in the two blanks corresponding to the 9's.

Now notice also that *rat* and *cab* have the same middle letter, *a*. The number which occurs twice in the middle of a word is 2. Write *a* in the blanks corresponding to the 2's.

Now finish the word *rat*. The other word which ends in *t* is *bet*. Write it. Then the second word in the translation must be *cab*, and that checks because 8 is *b* in *bet*. The three words in the last column are *bet, cab*, and *rat.*

Here is another problem. Find the letters which correspond to the numbers and write them in the third column. It will help you to get started if you notice that there are three *a*'s in the words. Find the number which occurs three times and write *a*'s in the corresponding blanks in the third column. Finish the solution of the problem.

Words	*Secret Writing*	*Translation*
are	8 5 1	___ ___ ___
oar	2 5 3	___ ___ ___
saw	5 3 9	___ ___ ___

You should have written the words in the order: *saw, oar, are.*

Here is another problem for you to try. Notice that the letter *g* occurs only once in the three words. Find the number that occurs only once, and you will see the rest of the solution easily.

Words	*Secret Writing*	*Translation*
pig	4 2 7	___ ___ ___
pit	4 2 9	___ ___ ___
tip	7 2 4	___ ___ ___

You should have written the words in the third column in the order: *pit, pig, tip.*

Here are two more problems for you to practice on. Translate the words in secret writing and write the words in the correct places in the third column.

Words	*Secret Writing*	*Translation*	*Words*	*Secret Writing*	*Translation*
man	2 4 6	___ ___ ___	run	2 3 9	___ ___ ___
tan	8 3 2	___ ___ ___	art	2 4 9	___ ___ ___
met	8 4 6	___ ___ ___	ran	3 2 8	___ ___ ___

The test contained ten problems.

SUFFIXES (54)

Look at the words in the following list. Each of them ends with *able.*

capable valuable comfortable hospitable

Write in the blanks below several words ending with *ent.* Go ahead.

The instructions for the test were as follows:

Write as many words as you can which end with *tion.*

SYNONYMS (55)

The first word in the row below is *cold.*

cold cool wintry chilly

The other three words are *cool, wintry,* and *chilly.* They mean almost the same as *cold.*

The first word in the next row is *ancient.* The other three words in the row mean almost the same as *ancient.*

ancient old antique aged

In each of the rows below write three words which mean almost the same as the given word. If you cannot think of three words, write as many as you can and then go to the next row.

bad _____ _____ _____

little _____ _____ _____

The test contained eighteen words.

THREE-HIGHER (56)

In the row of numbers below, 10 is marked because it is 3 more than the number 7, which is just before it. The number 8 is also marked because it is 3 more than the number just before it.

5 7 **10** 12 14 11 3 5 **8** 12

Here is another row of numbers. Mark every number that is exactly 3 more than the number just before it.

4 11 14 10 9 12 16 8 10 3 15 18 9

You should have marked 14, 12, and 18.

Here are more problems for you to practice on. In each row mark every number that is exactly 3 more than the number just before it. Work as fast as you can.

3	7	10	14	11	9	12	13	16	8	2	1
5	9	11	14	8	11	7	10	12	5	9	12
9	6	9	2	5	8	15	16	21	19	22	18
13	15	19	24	23	26	18	14	11	13	19	12
7	10	12	14	28	23	16	15	18	13	16	5
15	19	21	26	29	22	25	5	8	7	11	4

The test contained thirty rows of numbers.

VERBAL ENUMERATION (57)

The heading of the first column of words below is "Color." Look at the words in this column. Some of the words are names of *colors*. Notice that a mark has been made under each color. No other words in the column are marked.

The heading of the second column of words is "Fruit." Look at the words in this column. Some of the words are names of *fruits*. A mark has been made under each of them.

The heading of the third column is "Food." Some of the words are names of *foods*. Find those words as quickly as possible and put a mark under each of them.

Look at the other column headings and then mark the words in the same way. Go right ahead.

Color	Fruit	Food	Metal	Bird
motor	coast	tower	greet	sled
paper	pear	bread	place	cedar
green	column	cover	allow	robin
class	apple	thread	tender	pedal
poetry	planet	search	answer	gear
dwell	author	near	copper	wren
purple	noise	potato	family	misty
second	banana	crutch	jingle	thrush
liquid	quarter	motor	silver	branch
yellow	apricot	hose	foreign	ridge
switch	quality	meat	part	oriole
deal	minute	claim	brake	blouse
brown	freeze	cart	swim	driver

The test contained thirty columns of words.

WORD CHECKING (58)

In this test you are to mark every word that means something which does not grow and which is smaller than a football.

Look at the words below.

horse	match
chair	boat
pencil	rat

Two of the words, *pencil* and *match*, are marked. A pencil and a match do not grow and they are smaller than a football. A chair and a boat do not grow, but they are larger than a football. A rat is smaller than a football, but it grows. A horse grows, and it is larger than a football. *Only things which do not grow and which are smaller than a football are marked.*

In the list below, mark every word that means something which does not grow and which is smaller than a football.

bird	chimney
cigar	cow
planet	watch

You should have marked *cigar* and *watch*. They do not grow and are smaller than a football.

Here are some more words for you to practice on. Mark every word that means something which does not grow and is smaller than a football.

bug	saxophone	flea
auto	eagle	door
pen	bed	stamp
typewriter	clam	minnow

The test contained one hundred and forty-four words to be checked.

WORD-NUMBER RECALL (59)

Each object in the list below has a number. The number of *box* is 66, the number of *chair* is 21, and so on. You are to remember the number of each object. On the next page, the names of the objects are listed in a different order. You will be asked to write the number of each object.

If writing helps you to remember, you may copy the pairs of words and numbers on the blanks below. Study silently until you are told to stop. Begin studying now. Do not wait for any signal.

Object	*Number*	*Object*	*Number*	*Object*	*Number*
box	66				
chair	21				
fan	92				
lamp	77				

Do not turn back this page.

In the first row the correct number has been written.

Write the number of each of the other objects. Go right ahead.

Object	*Number*
chair	*21*
lamp	
box	
fan	

The test proper contains fifteen word-number combinations.

WORD PUZZLES (60)

Rearrange the letters on each of the following lines to spell the name of an *animal*. In the first line the letters *ebar* can be arranged to spell *bear*, which is written in the blank space. In the next line the letters *odg* spell *dog*, which is written in the blank space. In the same way the letters *atc* spell *cat*.

Animals

ebar bear

odg dog

atc cat

Rearrange the letters on each of the following lines to spell the name of a *boy*. The first two names have already been written for you. Write the third.

Boys' Names

lpau Paul

rcla Carl

honj _____

Rearrange the letters on each of the following lines to spell the name of a *bird*. Go right ahead. Do not wait for any signal.

Birds

uckd _____

cowr _____

wahk _____

The test contained seventy-two disarranged words, in eight categories.

APPENDIX

Table 1

Product-Moment Correlations for Sixty-three Variables
(With Decimal Points Eliminated)

	1	2	3	4	5	6	7	8	9	10	11	12	13
1		190	291	233	285	234	304	205	277	305	196	347	263
2	190		033	132	284	197	177	185	163	464	141	390	320
3	291	033		267	264	257	232	126	229	234	194	276	221
4	233	132	267		212	399	418	132	205	363	215	339	333
5	285	284	264	212		286	231	269	363	462	254	465	248
6	234	197	257	399	286		307	075	153	424	208	442	383
7	304	177	232	418	231	307		276	252	340	221	311	378
8	205	185	126	132	269	075	276		294	206	068	175	149
9	277	163	229	205	363	153	252	294		292	196	307	212
10	305	464	234	363	462	424	340	206	292		225	599	546
11	196	141	194	215	254	208	221	068	196	225		230	200
12	347	390	276	339	465	442	311	175	307	599	230		460
13	263	320	221	333	248	383	378	149	212	546	200	460	
14	282	016	383	158	080	202	227	054	138	084	156	185	164
15	273	029	271	151	094	186	270	162	172	068	166	191	203
16	243	011	332	122	−029	184	251	066	092	030	145	144	176
17	352	102	320	240	240	213	372	242	305	138	085	247	191
18	316	198	152	302	129	256	489	376	285	255	132	221	309
19	129	146	060	138	119	133	314	295	176	148	060	189	192
20	239	074	216	228	068	288	275	072	230	192	114	198	280
21	217	144	065	140	170	116	170	196	179	150	155	148	128
22	236	228	097	133	257	062	213	621	270	223	130	155	140
23	183	170	195	476	205	416	338	110	127	313	150	284	282
24	185	130	248	494	171	536	385	106	136	348	187	316	367
25	184	144	211	285	180	312	250	004	187	309	190	267	279
26	197	265	085	147	313	127	271	623	281	272	085	229	205
27	194	170	207	483	254	413	310	115	138	374	203	302	301
28	224	238	027	130	248	080	270	384	276	238	115	235	195
29	324	216	220	231	332	194	314	390	334	315	153	295	267
30	282	047	284	315	034	219	431	114	232	184	126	221	237
31	246	151	054	211	090	173	379	296	257	186	073	206	229
32	208	147	238	399	175	446	488	092	134	337	168	330	404
33	314	204	234	330	243	326	377	198	251	335	207	367	357
34	310	291	207	304	395	303	378	236	304	397	253	439	400
35	144	148	−008	133	102	086	232	286	192	142	089	173	115
36	142	129	−028	152	158	042	207	344	183	126	077	141	086
37	325	077	499	317	208	321	307	023	222	286	160	328	281
38	201	132	160	268	092	210	294	186	180	202	114	236	254
39	258	388	195	202	444	286	220	156	251	625	153	476	384
40	310	399	126	283	403	312	307	232	288	472	187	420	349
41	204	112	137	182	010	377	318	030	083	236	115	243	333
42	126	120	165	374	148	418	289	032	079	280	154	247	296
43	197	340	086	214	389	182	200	197	296	520	166	426	264
44	232	095	081	117	143	115	283	315	188	138	126	152	157
45	239	420	159	210	430	311	209	158	227	718	192	497	454
46	278	394	184	251	420	333	206	189	219	681	176	511	455
47	205	342	191	233	360	285	177	144	198	568	152	458	395
48	227	334	177	160	350	235	158	107	201	494	124	397	304
49	212	239	162	160	354	141	165	191	296	342	140	342	214
50	217	261	193	423	222	484	404	102	154	458	260	409	448
51	255	397	168	266	365	344	240	151	231	672	187	486	497
52	052	−066	036	131	−030	−008	234	123	048	−064	−001	−008	−006
53	311	177	165	260	329	258	331	329	373	324	198	342	256
54	154	181	225	367	181	396	337	059	111	311	200	277	354
55	081	133	125	275	107	451	184	012	015	310	188	270	298
56	332	212	394	301	454	279	364	307	335	381	154	411	341
57	258	314	217	319	230	430	385	142	222	547	133	490	541
58	270	331	209	240	340	337	301	219	257	485	211	437	440
59	082	001	108	118	005	061	138	036	112	021	065	−023	041
60	243	183	279	533	236	461	526	107	180	400	174	400	472
61	−180	−158	−126	−180	−258	−210	−157	−003	−172	−335	−154	−345	−260
62	−067	034	010	−149	297	−207	−112	178	098	130	003	016	−088
63	320	356	262	387	476	385	409	314	366	552	249	543	504

Table 1—Continued

	14	15	16	17	18	19	20	21	22	23	24	25	26
1.....	282	273	243	352	316	129	239	217	236	183	185	184	197
2.....	016	029	011	102	198	146	074	144	228	170	130	144	265
3.....	383	271	332	320	152	060	216	065	097	195	248	211	085
4.....	158	151	122	240	302	138	228	140	133	476	494	285	147
5.....	080	094	−029	240	129	119	068	170	257	205	171	180	313
6.....	202	186	184	213	256	133	288	116	062	416	536	312	127
7.....	227	270	251	372	489	314	275	170	213	338	385	250	271
8.....	054	162	066	242	376	295	072	196	621	110	106	004	623
9.....	138	172	092	305	285	176	230	179	270	127	136	187	281
10.....	084	068	030	138	255	148	192	150	223	313	348	309	272
11.....	156	166	145	085	132	060	114	155	130	150	187	190	085
12.....	185	191	144	247	221	189	198	148	155	284	316	267	229
13.....	164	203	176	191	309	192	280	128	140	282	367	279	205
14.....		503	699	474	272	100	280	062	027	156	210	133	088
15.....	503		598	443	353	228	324	082	102	119	179	132	171
16.....	699	598		418	313	176	290	074	038	097	211	160	096
17.....	474	443	418		440	246	271	107	187	225	228	126	282
18.....	272	353	313	440		408	289	176	319	288	264	183	337
19.....	100	228	176	246	408		127	−004	168	125	141	053	227
20.....	280	324	290	271	289	127		137	016	116	241	246	006
21.....	062	082	074	107	176	−004	137		175	093	098	266	141
22.....	027	102	038	187	319	168	016	175		109	120	−003	510
23.....	156	119	097	225	288	125	116	093	109		520	171	147
24.....	210	179	211	228	264	141	241	098	120	520		290	143
25.....	133	132	160	126	183	053	246	266	−003	171	290		039
26.....	088	171	096	282	337	227	006	141	510	147	143	039	
27.....	138	092	070	189	226	061	115	115	172	514	473	230	188
28.....	030	167	039	202	267	267	103	116	322	107	060	037	354
29.....	223	302	189	378	406	288	213	106	292	165	168	127	365
30.....	355	388	318	454	393	204	386	104	044	251	264	170	086
31.....	137	294	189	332	516	324	225	174	209	159	120	104	265
32.....	283	253	273	313	322	171	288	004	078	350	446	282	116
33.....	244	244	203	332	370	202	205	142	148	287	273	268	227
34.....	174	243	165	271	324	201	138	226	238	273	273	301	309
35.....	088	302	153	234	330	330	149	115	206	160	090	−065	261
36.....	010	236	053	233	267	246	058	086	256	111	062	−123	329
37.....	416	296	333	396	235	122	335	053	007	239	310	259	083
38.....	171	278	174	277	313	223	180	107	102	169	174	148	211
39.....	097	014	046	137	161	082	099	182	170	178	160	260	218
40.....	053	146	081	213	276	175	168	231	221	216	214	304	277
41.....	281	289	316	294	266	224	415	058	−040	204	329	200	056
42.....	101	095	124	128	141	076	118	061	078	461	506	148	133
43.....	001	−014	−030	078	161	002	−006	176	253	244	191	190	280
44.....	202	455	273	346	400	292	167	144	218	125	104	−009	322
45.....	016	−030	−024	042	160	038	075	162	211	237	243	246	231
46.....	080	024	017	072	179	047	081	172	268	212	226	242	242
47.....	014	−020	−022	050	129	−047	060	171	207	206	208	259	159
48.....	056	−002	034	030	118	013	087	134	212	116	183	207	148
49.....	000	024	−037	157	129	053	151	137	224	108	101	163	234
50.....	165	108	146	173	209	083	171	133	102	465	563	264	178
51.....	078	035	030	078	194	099	148	162	178	245	290	281	195
52.....	084	137	118	141	146	124	100	015	063	078	072	−010	094
53.....	090	240	128	313	351	248	244	172	291	185	158	251	328
54.....	155	115	134	189	187	032	158	116	080	479	530	252	122
55.....	075	051	104	039	125	107	107	028	041	391	490	125	090
56.....	276	268	210	416	309	243	180	110	292	200	256	212	328
57.....	197	234	196	284	341	168	321	110	094	266	329	284	144
58.....	158	136	154	282	294	164	219	188	236	197	342	236	227
59.....	102	120	114	173	114	059	122	130	009	060	075	294	024
60.....	233	201	224	314	303	146	275	121	122	455	565	328	174
61.....	−050	−088	−054	−026	−090	−060	−140	−091	−018	−076	−142	−217	−085
62.....	−169	−147	−227	−039	−116	004	−169	−082	193	−089	−164	−258	172
63.....	139	190	100	290	303	225	187	124	261	339	338	276	311

Table 1—Continued

	27	28	29	30	31	32	33	34	35	36	37	38	39
1.....	194	224	324	282	246	208	314	310	144	142	325	201	258
2.....	170	238	216	047	151	147	204	291	148	129	077	132	388
3.....	207	027	220	284	054	238	234	207	−008	−028	499	160	195
4.....	483	130	231	315	211	399	330	304	133	152	317	268	202
5.....	254	248	332	034	090	175	243	395	102	158	208	092	444
6.....	413	080	194	219	173	446	326	303	086	042	321	210	286
7.....	310	270	314	431	379	488	377	378	232	207	307	294	220
8.....	115	384	390	114	296	092	198	236	286	344	023	186	156
9.....	138	276	334	232	257	134	251	304	192	183	222	180	251
10.....	374	238	315	184	186	337	335	397	142	126	286	202	625
11.....	203	115	153	126	073	168	207	253	089	077	160	114	153
12.....	302	235	295	221	206	330	367	439	173	141	328	236	476
13.....	301	195	267	237	229	404	357	400	115	086	281	254	384
14.....	138	030	223	355	137	283	244	174	088	010	416	171	097
15.....	092	167	302	388	294	253	244	243	302	236	296	278	014
16.....	070	039	189	318	189	273	203	165	153	053	333	174	046
17.....	189	202	378	454	332	313	332	271	234	233	396	277	137
18.....	226	267	406	393	516	322	370	324	330	267	235	313	161
19.....	061	267	288	204	324	171	202	201	330	246	122	223	082
20.....	115	103	213	386	225	288	205	138	149	058	335	180	099
21.....	115	116	106	104	174	004	142	226	115	086	053	107	182
22.....	172	322	292	044	209	078	148	238	206	256	007	102	170
23.....	514	107	165	251	159	350	287	273	160	111	239	169	178
24.....	473	060	168	264	120	446	273	273	090	062	310	174	160
25.....	230	037	127	170	104	282	268	301	−065	−123	259	148	260
26.....	188	354	365	086	265	116	227	309	261	329	083	211	218
27.....		065	219	178	103	372	264	261	054	066	221	134	261
28.....	065		319	098	307	060	167	297	318	324	−052	223	168
29.....	219	319		278	314	225	325	322	306	304	222	255	251
30.....	178	098	278		397	368	286	154	214	151	467	299	054
31.....	103	307	314	397		185	269	278	296	237	145	286	103
32.....	372	060	225	368	185		357	275	084	039	388	192	206
33.....	264	167	325	286	269	357		437	105	077	295	294	237
34.....	261	297	322	154	278	275	437		122	126	203	321	308
35.....	054	318	306	214	296	084	105	122		684	039	169	077
36.....	066	324	304	151	237	039	077	126	684		−042	180	066
37.....	221	−052	222	467	145	388	295	203	039	−042		242	183
38.....	134	223	255	299	286	192	294	321	169	180	242		112
39.....	261	168	251	054	103	206	237	308	077	066	183	112	
40.....	257	289	344	116	268	262	341	473	093	114	126	218	385
41.....	136	081	146	361	243	369	243	117	239	091	311	232	108
42.....	408	035	132	192	072	375	244	188	096	057	177	136	180
43.....	336	226	201	003	121	108	245	335	101	099	063	072	431
44.....	086	346	262	309	450	089	139	203	437	386	122	216	109
45.....	304	169	175	−008	078	257	226	335	043	064	133	048	624
46.....	291	177	225	026	110	272	187	342	020	061	172	090	623
47.....	227	104	166	021	075	260	197	307	−039	001	157	034	547
48.....	170	101	143	−031	015	196	158	243	−028	−018	157	018	495
49.....	122	210	256	083	122	110	182	293	050	146	181	113	335
50.....	474	126	180	216	144	461	339	310	121	080	237	157	284
51.....	306	171	216	126	140	292	258	326	059	040	210	112	550
52.....	101	102	100	304	192	132	046	003	146	235	059	100	−062
53.....	198	328	385	238	352	241	396	473	216	244	163	296	239
54.....	425	042	122	216	114	415	263	220	052	−006	262	099	221
55.....	321	076	104	097	061	281	200	107	116	089	127	146	232
56.....	230	261	499	291	230	348	334	376	170	200	436	274	324
57.....	274	131	277	407	350	494	291	303	190	131	356	224	418
58.....	275	203	296	173	227	347	279	313	164	108	230	152	420
59.....	078	037	089	120	017	057	072	098	077	014	125	127	039
60.....	445	096	246	369	224	583	388	383	092	101	353	268	266
61.....	−126	−098	−143	−075	−072	−243	−212	−299	005	−030	−146	−085	−325
62.....	−074	151	060	−143	−084	−207	−222	−151	169	295	−108	−144	170
63.....	391	296	402	233	278	372	416	490	168	193	302	280	427

Table 1—Continued

	40	41	42	43	44	45	46	47	48	49	50	51	52
1.....	310	204	126	197	232	239	278	205	227	212	217	255	052
2.....	399	112	120	340	095	420	394	342	334	239	261	397	−066
3.....	126	137	165	086	081	159	184	191	177	162	193	168	036
4.....	283	182	374	214	117	210	251	233	160	160	423	266	131
5.....	403	010	148	389	143	430	420	360	350	354	222	365	−030
6.....	312	377	418	182	115	311	333	285	235	141	484	344	−008
7.....	307	318	289	200	283	209	206	177	158	165	404	240	234
8.....	232	030	032	197	315	158	189	144	107	191	102	151	123
9.....	288	083	079	296	188	227	219	198	201	296	154	231	048
10.....	472	236	280	520	138	718	681	568	494	342	458	672	−064
11.....	187	115	154	166	126	192	176	152	124	140	260	187	−001
12.....	420	243	247	426	152	497	511	458	397	342	409	486	−008
13.....	349	333	296	264	157	454	455	395	304	214	448	497	−006
14.....	053	281	101	001	202	016	080	014	056	000	165	078	084
15.....	146	289	095	−014	455	−030	024	−020	−002	024	108	035	137
16.....	081	316	124	−030	273	−024	017	−022	034	−037	146	030	118
17.....	213	294	128	078	346	042	072	050	030	157	173	078	141
18.....	276	266	141	161	400	160	179	129	118	129	209	194	146
19.....	175	224	076	002	292	038	047	−047	013	053	083	099	124
20.....	168	415	118	−006	167	075	081	060	087	151	171	148	100
21.....	231	058	061	176	144	162	172	171	134	137	133	162	015
22.....	221	−040	078	253	218	211	268	207	212	224	102	178	063
23.....	216	204	461	244	125	237	212	206	116	108	465	245	078
24.....	214	329	506	191	104	243	226	208	183	101	563	290	072
25.....	304	200	148	190	−009	246	242	259	207	163	264	281	−010
26.....	277	056	133	280	322	231	242	159	148	234	178	195	094
27.....	257	136	408	336	086	304	291	227	170	122	474	306	101
28.....	289	081	035	226	346	169	177	104	101	210	126	171	102
29.....	344	146	132	201	262	175	225	166	143	256	180	216	100
30.....	116	361	192	003	309	−008	026	021	−031	083	216	126	304
31.....	268	243	072	121	450	078	110	075	015	122	144	140	192
32.....	262	369	375	108	089	257	272	260	196	110	461	292	132
33.....	341	243	244	245	139	226	187	197	158	182	339	258	046
34.....	473	117	188	335	203	335	342	307	243	293	310	326	003
35.....	093	239	096	101	437	043	020	−039	−028	050	121	059	146
36.....	114	091	057	099	386	064	061	001	−018	146	080	040	235
37.....	126	311	177	063	122	133	172	157	157	181	237	210	059
38.....	218	232	136	072	216	048	090	034	018	113	157	112	100
39.....	385	108	180	431	109	624	623	547	495	335	284	550	−062
40.....		147	165	384	142	396	418	367	337	339	317	396	−018
41.....	147		248	053	181	066	080	044	015	−038	358	147	074
42.....	165	248		172	086	227	189	184	115	099	447	241	061
43.....	384	053	172		078	551	492	439	364	253	301	476	−089
44.....	142	181	086	078		063	071	015	−001	076	104	106	132
45.....	396	066	227	551	063		769	671	553	315	383	730	−161
46.....	418	080	189	492	071	769		681	567	310	346	661	−138
47.....	367	044	184	439	015	671	681		512	324	342	579	−152
48.....	337	015	115	364	−001	553	567	512		286	207	520	−104
49.....	339	−038	099	253	076	315	310	324	286		177	258	−063
50.....	317	358	447	301	104	383	346	342	207	177		402	043
51.....	396	147	241	476	106	730	661	579	520	258	402		−063
52.....	−018	074	061	−089	132	−161	−138	−152	−104	−063	043	−063	
53.....	459	111	103	221	257	212	241	220	200	311	176	223	120
54.....	213	192	554	219	083	296	219	256	164	121	487	311	067
55.....	192	302	445	169	092	239	221	185	122	051	462	277	018
56.....	313	175	176	269	181	281	331	258	240	303	238	310	090
57.....	387	432	210	236	243	439	479	435	319	196	448	500	072
58.....	349	273	161	290	166	456	478	428	354	178	346	473	−011
59.....	100	068	007	000	026	−052	−050	−051	−074	008	034	−050	051
60.....	307	263	387	214	094	306	279	257	199	169	516	343	168
61.....	−278	−116	−079	−220	−074	−342	−314	−306	−228	−153	−233	−308	070
62.....	−160	−231	−114	083	088	127	106	069	039	152	−126	025	013
63.....	508	176	231	431	191	464	469	413	360	330	409	450	056

Table 1—Continued

	53	54	55	56	57	58	59	60	61	62	63
1.....	311	154	081	332	258	270	082	243	−180	−067	320
2.....	177	181	133	212	314	331	001	183	−158	034	356
3.....	165	225	125	394	217	209	108	279	−126	010	262
4.....	260	367	275	301	319	240	118	533	−180	−149	387
5.....	329	181	107	454	230	340	005	236	−258	297	476
6.....	258	396	451	279	430	337	061	461	−210	−207	385
7.....	331	337	184	364	385	301	138	526	−157	−112	409
8.....	329	059	012	307	142	219	036	107	−003	178	314
9.....	373	111	015	335	222	257	112	180	−172	098	366
10.....	324	311	310	381	547	485	021	400	−335	130	552
11.....	198	200	188	154	133	211	065	174	−154	003	249
12.....	342	277	270	411	490	437	−023	400	−345	016	543
13.....	256	354	298	341	541	440	041	472	−260	−088	504
14.....	090	155	075	276	197	158	102	233	−050	−169	139
15.....	240	115	051	268	234	136	120	201	−088	−147	190
16.....	128	134	104	210	196	154	114	224	−054	−227	100
17.....	313	189	039	416	284	282	173	314	−026	−039	290
18.....	351	187	125	309	341	294	114	303	−090	−116	303
19.....	248	032	107	243	168	164	059	146	−060	004	225
20.....	244	158	107	180	321	219	122	275	−140	−169	187
21.....	172	116	028	110	110	188	130	121	−091	−082	124
22.....	291	080	041	292	094	236	009	122	−018	193	261
23.....	185	479	391	200	266	197	060	455	−076	−089	339
24.....	158	530	490	256	329	342	075	565	−142	−164	338
25.....	251	252	125	212	284	236	294	328	−217	−258	276
26.....	328	122	090	328	144	227	024	174	−085	172	311
27.....	198	425	321	230	274	275	078	445	−126	−074	391
28.....	328	042	076	261	131	203	037	096	−098	151	296
29.....	385	122	104	499	277	296	089	246	−143	060	402
30.....	238	216	097	291	407	173	120	369	−075	−143	233
31.....	352	114	061	230	350	227	017	224	−072	−084	278
32.....	241	415	281	348	494	347	057	583	−243	−207	372
33.....	396	263	200	334	291	279	072	388	−212	−222	416
34.....	473	220	107	376	303	313	098	383	−299	−151	490
35.....	216	052	116	170	190	164	077	092	005	169	168
36.....	244	−006	089	200	131	108	014	101	−030	295	193
37.....	163	262	127	436	356	230	125	353	−146	−108	302
38.....	296	099	146	274	224	152	127	268	−085	−144	280
39.....	239	221	232	324	418	420	039	266	−325	170	427
40.....	459	213	192	313	387	349	100	307	−278	−160	508
41.....	111	192	302	175	432	273	068	263	−116	−231	176
42.....	103	554	445	176	210	161	007	387	−079	−114	231
43.....	221	219	169	269	236	290	000	214	−220	083	431
44.....	257	083	092	181	243	166	026	094	−074	088	191
45.....	212	296	239	281	439	456	−052	306	−342	127	464
46.....	241	219	221	331	479	478	−050	279	−314	106	469
47.....	220	256	185	258	435	428	−051	257	−306	069	413
48.....	200	164	122	240	319	354	−074	199	−228	039	360
49.....	311	121	051	303	196	178	008	169	−153	152	330
50.....	176	487	462	238	448	346	034	516	−233	−126	409
51.....	223	311	277	310	500	473	−050	343	−308	025	450
52.....	120	067	018	090	072	−011	051	168	070	013	056
53.....		129	080	330	277	260	153	291	−277	−080	449
54.....	129		395	221	291	228	094	505	−165	−143	318
55.....	080	395		137	266	215	018	305	−088	−038	244
56.....	330	221	137		364	386	062	379	−207	108	429
57.....	277	291	266	364		557	015	426	−297	−077	471
58.....	260	228	215	386	557		002	336	−259	019	432
59.....	153	094	018	062	015	002		081	−045	−096	044
60.....	291	505	305	379	426	336	081		−227	−208	468
61.....	−277	−165	−088	−207	−297	−259	−045	−227		050	−302
62.....	−080	−143	−038	108	−077	019	−096	−208	050		020
63.....	449	318	244	429	471	432	044	468	−302	020	

Table 2

Centroid Matrix F_c for Ten Factors

	I	II	III	IV	V	VI	VII	VIII	IX	X	h^2
1.....	.50	−.15	.05	−.19	−.08	.07	−.06	−.05	.06	.06	.3357
2.....	.43	.19	.27	.01	.09	−.07	−.10	−.06	.02	.03	.3219
3.....	.41	−.08	−.18	−.28	−.18	.33	.12	.10	−.07	.06	.4595
4.....	.56	.10	−.26	.17	−.17	−.04	.07	.06	.06	.08	.4691
5.....	.50	.14	.36	−.07	−.23	.20	.19	−.08	.05	.07	.5469
6.....	.58	.23	−.28	.04	.03	.04	.09	−.13	−.05	−.14	.5189
7.....	.63	−.16	−.16	.14	−.02	−.19	−.02	.13	.04	.10	.5331
8.....	.40	−.35	.39	.30	−.14	.05	−.09	.18	−.20	−.15	.6497
9.....	.46	−.16	.22	−.10	−.18	.01	.10	.04	.13	−.06	.3602
10.....	.70	.37	.23	−.06	.14	.03	.08	.06	.10	.12	.7383
11.....	.34	.04	−.02	−.04	−.08	.05	−.05	−.15	.13	−.05	.1725
12.....	.66	.22	.14	−.12	.06	.03	.17	−.16	−.05	.08	.5859
13.....	.62	.20	−.05	−.05	.15	−.12	.04	.04	−.09	.04	.4792
14.....	.37	−.32	−.33	−.33	.08	.33	−.17	−.09	−.15	.10	.6419
15.....	.41	−.46	−.20	−.18	.15	.13	−.12	−.21	−.09	.02	.5585
16.....	.36	−.38	−.35	−.27	.18	.23	−.26	−.14	−.18	−.03	.6752
17.....	.51	−.43	−.12	−.11	−.07	.10	.05	.03	−.05	.10	.5023
18.....	.57	−.36	−.01	.11	.07	−.15	−.15	.10	−.07	.08	.5379
19.....	.33	−.30	.06	.15	.13	−.15	.09	−.07	−.10	.09	.2955
20.....	.39	−.19	−.23	−.23	.12	−.10	.10	.08	.15	−.17	.3862
21.....	.28	−.03	.10	−.08	−.14	−.05	−.24	.03	.24	−.15	.2564
22.....	.37	−.20	.38	.26	−.17	.13	−.18	.19	−.16	−.16	.5544
23.....	.49	.17	−.28	.34	−.11	.11	−.02	−.05	.07	.14	.5146
24.....	.55	.21	−.40	.25	−.05	.11	.03	.02	−.04	−.13	.6035
25.....	.41	.15	−.17	−.26	−.19	−.20	−.12	.08	.13	−.15	.4234
26.....	.45	−.23	.35	.27	−.13	.10	−.10	.04	−.16	−.08	.5213
27.....	.50	.25	−.21	.28	−.17	.11	−.04	.02	.04	.13	.4965
28.....	.37	−.22	.35	.19	.02	−.11	−.03	−.13	.07	.06	.3827
29.....	.53	−.27	.18	.02	−.09	.02	.14	−.03	−.07	.04	.4221
30.....	.46	−.35	−.33	−.08	.08	−.07	.13	.18	.14	.16	.5552
31.....	.44	−.35	.06	.10	.12	−.27	−.07	.04	.03	.13	.4413
32.....	.57	.10	−.37	.02	.07	−.08	.09	.11	−.18	−.04	.5377
33.....	.55	−.04	−.09	−.05	−.16	−.14	.02	−.16	−.12	.08	.4067
34.....	.60	.03	.12	−.09	−.23	−.18	−.08	−.23	−.12	.08	.5488
35.....	.33	−.41	.18	.32	.32	.05	.12	−.19	.19	−.10	.6133
36.....	.29	−.37	.28	.37	.21	.07	.21	−.13	.14	−.06	.5695
37.....	.47	−.10	−.31	−.33	−.09	.18	.22	.15	−.03	.11	.5603
38.....	.40	−.22	−.07	−.01	−.05	−.18	.07	−.10	−.04	.11	.2769
39.....	.54	.33	.30	−.17	.11	.10	.02	.08	.10	.06	.5619
40.....	.58	.15	.22	−.07	−.15	−.25	−.08	−.19	.02	−.06	.5437
41.....	.41	−.11	−.34	−.06	.33	−.09	.10	−.10	.05	−.15	.4614
42.....	.42	.23	−.33	.30	−.04	.18	.03	−.08	.08	−.07	.4808
43.....	.46	.30	.32	.03	−.10	.09	−.15	−.05	.13	.11	.4770
44.....	.38	−.42	.11	.13	.24	.04	−.12	−.09	.09	.07	.4445
45.....	.56	.51	.34	−.09	.16	.13	−.13	.12	.04	.07	.7777
46.....	.57	.44	.34	−.15	.16	.12	−.12	.15	−.08	.05	.7424
47.....	.49	.47	.27	−.16	.08	.08	−.09	.17	−.06	−.06	.6165
48.....	.42	.36	.27	−.18	.06	.10	−.08	.12	−.08	−.04	.4537
49.....	.37	.08	.29	−.10	−.18	.05	.13	.04	.09	−.06	.3025
50.....	.59	.31	−.26	.20	.05	.04	−.03	−.07	.06	−.08	.5717
51.....	.59	.43	.21	−.10	.20	.04	−.09	.12	.04	.09	.6609
52.....	.12	−.28	−.14	.19	.03	−.10	.07	.13	.09	.08	.1957
53.....	.55	−.15	.19	−.04	−.19	−.24	.06	−.13	−.04	−.11	.4906
54.....	.49	.27	−.34	.23	−.10	.12	−.07	.05	.08	−.05	.5222
55.....	.38	.26	−.25	.26	.13	.14	.07	−.18	.05	−.13	.4353
56.....	.60	−.13	.10	−.10	−.15	.12	.25	.08	−.16	.11	.5404
57.....	.65	.14	−.05	−.13	.37	−.17	.13	.17	−.05	−.04	.6772
58.....	.59	.15	.12	−.10	.16	−.03	.04	.12	−.11	−.10	.4596
59.....	.14	−.14	−.15	−.10	−.20	−.10	−.06	.02	.18	−.10	.1681
60.....	.64	.15	−.34	.12	−.09	−.11	.10	.13	−.09	.08	.6237
61.....	−.35	−.21	−.09	.19	−.03	.12	−.06	.10	.03	.05	.2431
62.....	−.07	−.02	.45	.16	.08	.37	.28	.16	.16	.09	.5144
63.....	.70	.15	.14	.02	−.11	−.08	.13	−.08	−.10	.12	.5987

Table 3

Rotated Factorial Matrix V *for Sixty-three Variables*

Test	*N*	*W*	*S*	*V*	*M*	*I*	*P*	X_1	X_2	X_3
1	.161	−.049	.043	.145	.167	.120	.038	.245	.029	−.041
2	−.119	−.008	.038	.350	.051	.160	.029	−.026	.065	.110
3	.437	.038	.020	.004	.001	−.043	.026	.320	−.133	−.021
4	.105	.410	−.007	−.020	.060	.088	.229	−.072	−.045	−.004
5	.361	−.013	.084	.203	.058	.215	−.094	−.036	.084	−.079
6	.029	.393	−.039	.003	.044	.146	.038	.049	.050	.278
7	.001	.197	.119	.060	.092	.070	.396	−.008	.014	.036
8	.007	−.059	.684	−.035	−.011	.000	.046	−.041	−.019	.013
9	.273	−.099	.201	.085	.256	.088	.062	−.007	.127	.001
10	.086	.060	−.057	.549	.013	.089	.169	−.053	.122	.198
11	.040	.145	−.029	.058	.191	.119	−.097	.099	.090	.004
12	.164	.045	−.077	.285	−.022	.299	.084	.035	.114	.155
13	−.024	.106	−.027	.275	−.025	.140	.274	.003	−.005	.272
14	.141	−.005	−.042	.006	−.054	−.015	−.044	.694	−.113	−.039
15	.035	−.041	.056	−.059	.033	.112	.003	.540	.102	.000
16	−.036	.005	.036	−.038	−.006	−.005	−.059	.696	−.064	.077
17	.267	−.029	.157	−.063	.043	.061	.192	.307	.025	−.081
18	−.091	.011	.265	.081	.040	.067	.323	.147	.012	.019
19	−.019	−.039	.153	−.032	−.090	.179	.245	.003	.159	.008
20	.143	−.034	.019	.026	.285	−.078	.226	.124	.182	.278
21	−.057	.017	.115	.142	.399	−.028	−.081	.061	.031	.020
22	−.026	−.003	.624	.045	.016	−.053	−.059	−.002	−.085	.009
23	−.015	.579	−.046	−.009	−.066	.066	.068	−.001	−.012	−.103
24	−.005	.590	.071	−.084	−.009	−.013	.068	.009	−.036	.204
25	.028	.083	−.066	.094	.404	.070	.120	.007	−.139	.184
26	−.002	.026	.524	.017	−.025	.091	−.035	.018	.005	−.035
27	.011	.528	−.023	.052	−.051	.057	.068	−.032	−.101	−.082
28	−.057	−.064	.215	.117	.075	.211	.055	−.053	.232	−.094
29	.232	−.064	.249	.007	.026	.192	.140	.054	.111	−.026
30	.222	.048	−.023	.010	.090	−.097	.445	.161	.118	.005
31	−.101	−.077	.151	.101	.072	.119	.360	.029	.127	−.016
32	.030	.287	.025	−.002	−.065	.052	.325	.036	−.106	.298
33	.074	.125	−.019	−.044	.064	.368	.154	.065	−.083	−.012
34	.017	.042	.004	.062	.133	.488	.062	.046	−.117	−.051
35	−.043	.042	.277	.012	.046	.000	.005	.043	.596	.065
36	.055	.021	.332	−.017	−.021	.019	.021	−.074	.539	.004
37	.460	.021	−.086	.018	.017	−.039	.240	.263	−.083	.053
38	.075	.015	.008	−.066	.054	.257	.236	.041	.045	−.046
39	.106	−.068	−.024	.546	.058	.020	.050	.018	.083	.182
40	−.059	.027	.029	.174	.268	.406	.036	−.092	.010	.096
41	−.043	.141	−.058	.003	.095	.007	.192	.157	.269	.350
42	−.012	.605	−.013	−.062	−.010	−.015	−.056	.005	.070	.078
43	−.021	.099	.002	.403	.095	.134	−.104	−.008	.002	−.069
44	−.095	−.044	.198	.127	.026	.005	.074	.253	.294	−.049
45	−.063	.015	−.008	.679	−.021	−.021	.003	.019	−.020	.223
46	−.036	−.067	.054	.639	−.066	.002	.042	.062	−.088	.265
47	−.016	−.028	.065	.530	.019	−.020	.017	−.013	−.114	.313
48	.015	−.079	.068	.453	.005	.008	−.015	.040	−.104	.244
49	.258	−.079	.134	.162	.177	.094	−.012	−.098	.072	.043
50	−.102	.518	−.037	.110	.048	.058	.042	−.003	.059	.212
51	−.064	.030	−.053	.615	−.013	.001	.122	.023	.000	.246
52	.032	.085	.103	−.106	.007	−.092	.266	−.054	.109	−.069
53	.093	−.057	.202	−.036	.251	.367	.121	−.088	.066	.077
54	−.035	.577	.000	.023	.054	−.057	.028	.009	−.064	.091
55	−.083	.509	−.042	−.005	−.040	.035	−.091	.001	.188	.203
56	.409	−.051	.179	.050	−.075	.161	.208	.073	−.029	.005
57	−.007	−.031	.016	.381	−.004	−.009	.421	−.009	.131	.483
58	.027	−.033	.154	.316	.016	.038	.186	.016	.025	.361
59	.078	.057	.011	−.116	.328	.003	.035	.031	−.004	−.032
60	.080	.390	−.009	.005	−.035	.105	.369	−.073	−.139	.136
61	−.032	.050	.088	−.189	−.103	−.220	−.055	.032	−.023	−.204
62	.310	−.147	.199	.217	−.175	−.257	−.100	−.123	.302	−.112
63	.142	.112	.036	.176	−.016	.349	.187	−.079	−.009	.052

Table 4

Product-Moment Correlation Coefficients for the Selected Test Battery

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1.....		.461	.430	.129	.156	.129	.261	.247	.204	.297	.239	.231	.181	.143	.221	.434	.497	.369	.292	.259	.348
2.....	.461		.505	.209	.338	.175	.297	.264	.400	.256	.221	.183	.416	.402	.424	.339	.334	.433	.378	.398	.401
3.....	.430	.505		.263	.224	.209	.343	.349	.332	.447	.372	.350	.279	.291	.298	.317	.354	.347	.418	.385	.460
4.....	.129	.209	.263		.280	.478	.295	.364	.282	.286	.299	.311	.103	.046	.061	.196	.268	.261	.320	.381	.285
5.....	.156	.338	.224	.280		.292	.151	.242	.234	.243	.184	.122	.227	.183	.252	.121	.040	.155	.290	.254	.217
6.....	.129	.175	.209	.478	.292		.234	.260	.251	.240	.217	.236	.100	.078	.166	.216	.203	.140	.249	.254	.192
7.....	.261	.297	.343	.295	.151	.234		.829	.768	.419	.356	.407	.108	.033	.108	.298	.309	.351	.492	.555	.425
8.....	.247	.264	.349	.364	.242	.260	.829		.775	.472	.415	.482	.115	.061	.125	.323	.347	.369	.468	.525	.381
9.....	.204	.400	.332	.282	.234	.251	.768	.775		.428	.354	.433	.272	.205	.238	.296	.271	.385	.446	.523	.396
10.....	.297	.256	.447	.286	.243	.240	.419	.472	.428		.654	.557	.176	.092	.127	.293	.348	.319	.391	.355	.398
11.....	.239	.221	.372	.299	.184	.217	.356	.415	.354	.654		.514	.192	.165	.144	.261	.308	.334	.367	.323	.381
12.....	.231	.183	.350	.311	.122	.236	.407	.482	.433	.557	.514		.100	.009	.066	.233	.254	.293	.305	.319	.303
13.....	.181	.416	.279	.103	.227	.100	.108	.115	.272	.176	.192	.100		.636	.626	.249	.183	.369	.271	.185	.279
14.....	.143	.402	.291	.046	.183	.078	.033	.061	.205	.092	.165	.009	.636		.709	.138	.091	.254	.180	.147	.191
15.....	.221	.424	.298	.061	.252	.166	.108	.125	.238	.127	.144	.066	.626	.709		.190	.103	.291	.225	.179	.245
16.....	.434	.339	.317	.196	.121	.216	.298	.323	.296	.293	.261	.233	.249	.138	.190		.654	.527	.399	.262	.356
17.....	.497	.334	.354	.268	.040	.203	.309	.347	.271	.348	.308	.254	.183	.091	.103	.654		.541	.407	.296	.394
18.....	.369	.433	.347	.261	.155	.140	.351	.369	.385	.319	.334	.293	.369	.254	.291	.527	.541		.471	.437	.429
19.....	.292	.378	.418	.320	.290	.249	.492	.468	.446	.391	.367	.305	.271	.180	.225	.399	.407	.471		.613	.610
20.....	.259	.398	.385	.381	.254	.254	.555	.525	.523	.355	.323	.319	.185	.147	.179	.262	.296	.437	.613		.496
21.....	.348	.401	.460	.285	.217	.192	.425	.381	.396	.398	.381	.303	.279	.191	.245	.356	.394	.429	.610	.496	

Table 5

Centroid Matrix F_c *for Twenty-one Tests*

	I	II	III	IV	V	VI	VII	VIII	h^2
1	.51	−.27	−.26	.11	−.13	−.09	−.25	.10	.5102
2	.62	−.31	.12	.06	−.16	.12	−.27	−.07	.6163
3	.63	−.09	−.10	−.09	−.09	.08	−.21	.17	.5106
4	.47	.28	.08	−.14	−.33	−.17	.17	−.10	.5020
5	.38	.06	.22	−.14	−.23	.10	−.11	−.17	.3199
6	.40	.19	.15	−.19	−.34	−.28	.14	−.07	.4732
7	.66	.40	.11	.39	.22	−.10	−.09	.13	.8432
8	.69	.43	.12	.27	.23	−.19	−.09	.06	.8490
9	.70	.28	.26	.27	.28	−.10	−.10	−.08	.8137
10	.63	.26	−.29	−.31	.19	.04	−.17	−.10	.7213
11	.59	.22	−.26	−.34	.23	.07	−.03	−.09	.6465
12	.53	.29	−.19	−.19	.20	−.15	−.09	.04	.5094
13	.48	−.44	.29	−.18	.16	.16	.16	−.08	.6237
14	.40	−.50	.41	−.25	.17	.18	.11	.08	.7204
15	.46	−.47	.42	−.22	.12	.12	.08	.13	.7094
16	.57	−.26	−.32	.19	−.05	−.23	.17	−.12	.6297
17	.58	−.21	−.42	.21	−.08	−.24	.18	−.08	.7038
18	.64	−.21	−.14	.18	.04	.03	.18	−.13	.5575
19	.69	.13	−.08	.15	−.15	.29	.19	.10	.6746
20	.65	.23	.06	.21	−.13	.23	.08	.07	.6042
21	.65	.04	−.16	.07	−.10	.26	.09	.14	.5599

Table 6

Rotated Factor Matrix V

	P	*N*	*W*	*V*	*S*	*M*	*R*	Residual
1	.42	.40	.05	−.02	−.07	−.06	−.06	.08
2	.45	.17	−.06	.04	.20	.05	.02	−.12
3	.36	.09	.19	−.02	.05	−.01	.09	.12
4	−.02	.09	.02	.00	−.05	.53	.10	.02
5	.20	−.10	.02	−.02	.10	.31	.07	−.17
6	.02	.13	−.03	.00	.01	.58	−.04	.04
7	.00	.01	−.03	.66	−.08	−.05	.13	.07
8	−.01	.02	.05	.66	−.04	.02	.02	.05
9	−.01	.00	−.01	.67	.15	.00	−.01	−.11
10	.12	−.03	.63	.03	−.02	.00	−.00	−.08
11	−.02	−.05	.61	−.01	.08	−.01	.04	−.05
12	.04	.03	.45	.18	−.03	.03	−.08	.10
13	−.04	.05	.03	−.01	.68	.00	.01	−.07
14	.02	−.06	.01	−.02	.76	−.02	−.02	.07
15	.07	−.03	−.03	.03	.72	.02	−.03	.13
16	.01	.64	−.02	.01	.05	.01	−.02	−.03
17	.01	.67	.01	−.03	−.05	.02	.02	.01
18	−.05	.38	−.01	.06	.20	−.05	.16	−.12
19	−.03	.03	.03	.02	.00	.02	.53	.02
20	.02	−.05	−.03	.22	−.03	.05	.44	−.02
21	.06	.06	.13	−.04	.01	−.06	.42	.06

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