Scaling of Atypical Knowledge Combinations in American Metropolitan Areas from 1836 to 2010

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abstract

Cities are epicenters for invention. Scaling analyses have verified the productivity of cities and demonstrate a superlinear relationship between cities' population size and invention performance. However, little is known about what kinds of inventions correlate with city size. Is the productivity of cities only limited to invention quantity? I shift the focus on the quality of idea creation by investigating how cities influence the art of knowledge combinations. Atypical combinations introduce novel and unexpected linkages between knowledge domains. They express creativity in inventions and are particularly important for technological breakthroughs. My study of 174 years of invention history in metroplitan areas in the US reveals a superlinear scaling of atypical combinations with population size. The observed scaling grows over time indicating a geographic shift toward cities since the early twentieth century. The productivity of large cities is thus not only restricted to quantity but also includes quality in invention processes.

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Introduction

It is well known that invention activities are spatially concentrated (Audretsch and Feldman 1996) and primarily an urban phenomenon (Bettencourt, Lobo, and Strumsky 2007). Empirically, scaling analyses demonstrate the predominance of cities and reveal a superlinear scaling of inventors and inventions with respect to city size. That is, a disproportionate number of inventors and inventions concentrate in large cities indicating increasing returns to urbanization (O'hUallichain 1999; O'hUallichain and Leslie 2005; Bettencourt, Lobo, and Strumsky 2007; Bettencourt et al. 2007).

The productivity of cities rests on the idea of inventions being the outcome of knowledge combinations. This requires people to interact as knowledge is distributed across individuals, organizations, and institutions. (Usher 1954; Nelson and Winter 1982; Utterback 1996; Hargadon 2003; Arthur 2009). Large cities provide more opportunities for knowledge combinations due to the concentration of critical requirements such as people, diversity, creativity, skills, infrastructure, and financial resources (Kuznets 1960; Jacobs 1969; Florida 2002; Glaeser 2011). The compactness of these factors in cities facilitates information flows among actors, stimulating knowledge combinations and in turn inventive outcomes (Bettencourt, Lobo, and Strumsky 2007). But how urban environments influence the art of knowledge combinations remains unexplored.

The large and diverse pool of existing knowledge provides large cities with more opportunities to explore atypical combinations than their nonurban counterparts. Atypical combinations introduce novel and unfamiliar linkages between less connected knowledge domains. They are an essential feature of creativity and a fundamental building block of high-impact science and technological breakthroughs (Schilling and Green 2011; Uzzi et al. 2013; Kim et al. 2016). The exclusive focus on invention quantity in existing scaling analyses, however, overlooks such differences in quality (O'hUallichain 1999; O'hUallichain and Leslie 2005; Bettencourt, Lobo, and Strumsky 2007; Bettencourt et al. 2007).

In this article, I address the lacuna in scaling analysis by studying knowledge combinations with respect to city size and particularly ask the following: How does urban knowledge diversity relate to knowledge combinations? How do atypical knowledge combinations scale with city size? Are cities more explorative because their diversity allows them to be?

Empirically, I rely on scaling analysis to study how knowledge combinations relate to cities' population size and technological diversity. Following Uzzi et al. (2013), I distinguish between atypical and typical knowledge combinations based on *z*-score measures to proxy knowledge exploration and exploitation, respectively. This empirical approach relies on historic patent data from 1836 to 2010, which enables me to study the geography of knowledge combinations over 174 years of US invention history (Petralia, Balland, and Rigby 2016). Studying almost two centuries allows me to reveal true long-term dynamics of knowledge combinations.

My main findings suggest that large cities increasingly concentrate atypical combinations and thus have become crucially important for knowledge exploration in the long run. I associate this development to the systematic relationship between knowledge diversity and city size. The knowledge diversity in large cities provides more opportunities for distinct knowledge combinations and to explore new combinations. Thus, large cities drive technological progress not only in quantitative but also in qualitative terms. The increasing concentration in large cities, however, reinforces a widening between urban centers and the rest of the country.

The article is organized as follows. The literature on the geography of invention and knowledge combinations is presented in the next section. I describe the data and empirical methods in the two sections following that. The results are presented and discussed in the penultimate section. The final section concludes the article.

Theoretical Underpinnings

The Geography of Invention

The notion of the death of distance has culminated in Friedman's (2005) claim of the *flat world*. This stream of research argues that technological change erodes the obstacles (e.g., physical barriers, travel time, sociocultural differences) that once limited the exchange of labor, goods, and knowledge (O'Brien 1992; Castells 1996; Cairncross 1997). In particular, innovation in telecommunication and computing technologies unfasten the mobility of production factors and detach economic activity from its territorial and socioeconomic context (O'Brien 1992; Castells 1996). Accordingly, technological progress spreads economic activities to every part of the world and enhances the global diffusion of knowledge. In such a scenario, location becomes less relevant, reducing the geographic concentration of economic activities of all kinds and eventually diminishing spatial inequalities over time.

Friedman's thesis has revitalized an active debate about the role of geography for economic activities (Christopherson, Garretsen, and Martin 2008; Florida, Gulden, and Mellander 2008; Rodríguez-Pose and Crescenzi 2008). The spatial distribution of the world economy doubts a flattening of the world, as economic activities and wealth are increasingly concentrated in space. More precisely, overwhelming empirical evidence is pointing in the exact opposite direction to what was proclaimed by Friedman and others. Scott (1993) and Saxenian (1994), for example, analyzed the prevailing concentration of certain industries (i.e. semiconductors and aerospace) in California and Massachusetts showing that geographic clustering is a common phenomenon. Most paradoxically, the digital industry—believed to be the driver that flattens the world—is itself highly clustered (Zook 2000). Beyond single case studies, it has been shown that economic activities, more generally, concentrate in specific locations and that the concentration tends to grow over time (Marshall 1890; Hall and Markusen 1985;

Ellison and Glaeser 1999; Dumais, Ellison, and Glaeser 2002; Ellison, Glaeser, and Kerr 2010). Geography therefore represents an important determinant in order to understand economic development and inequalities between cities.

Of all economic activities, the tendency toward spatial concentration is even stronger for invention activities. Spatial patterns of invention have been the subject in a growing body of empirical studies, showing that invention activities are not equally distributed across regions, but rather occur highly concentrated in space (Feldman 1994; O'hUallichain 1999; Acs, Anselin, and Varga 2002; Dumais, Ellison, and Glaeser 2002; Sonn and Storper 2008; Feldman and Kogler 2010; Castaldi and Los 2017). Most striking, the spatial concentration is relatively persistent and, more importantly, increases over time (Varga 1999; Co 2002; O'hUallichain and Leslie 2005; Sonn and Park 2011) challenging the *death of distance* argument. Since knowledge is a crucial source for economic growth (Lucas 1988; Romer 1990), regions more capable of creating new knowledge possess an economic advantage over less inventive regions (Feldman and Florida 1994).

The observed concentration is systematic, since a large body of empirical research suggests invention is primarily an urban phenomenon. In particular, the inventive performance of metropolitan areas grows disproportionately with population size, indicating increasing returns to urbanization (O'hUallichain 1999; Bettencourt et al. 2007; Bettencourt, Lobo, and Strumsky 2007). These findings indicate a spatial concentration of invention activities in larger metropolitan areas. Kuznets (1960) elaborated how a larger population size is associated with a greater productivity of new knowledge. However, not explicitly referring to Kuznets' work, more recent contributions rely on his thoughts about cities as centers for knowledge creation. Bettencourt et al. (2007) adopted a theoretical and methodological framework called scaling, which stems from biology (Schmidt-Nielsen 1984; West 1997) and quantifies the relation between size (e.g., body size, population size) and aggregated outcomes (e.g., metabolism rate, wealth, and inventions). The dependence of invention activity Y on population size N can be expressed as a scaling law of the following form (Bettencourt, Lobo, and Strumsky 2007)

$$Y = Y_0 N^{\beta} \tag{1}$$

where β is the scaling exponent, which falls into three broad categories revealing three different scaling mechanisms. First, β smaller than one expresses a sublinear relationship implying economies of scale. Second, a linear relationship is evident if β equals one. Third, if β is greater than one, the relation between population size and inventive performance of a city is superlinear, revealing increasing returns to urbanization, as reported, for example, in Bettencourt, Lobo, and Strumsky (2007). That is, if a city doubles its population size, it increases its inventive output more than twice as much. The empirically confirmed superlinear scaling of invention activities (Carlino, Chatterjee, and Hunt 2007; Arbesman, Kleinberg, and Strogatz 2009) reveals the dominant role of large metropolitan areas for invention, at least, in quantitative terms.

But why are cities so remarkably productive with respect to inventions? The literature on urban scaling attributes the productivity to two major interdependent factors: population size and knowledge diversity (Kuznets 1960; Jacobs 1969; Bettencourt, Lobo, and Strumsky 2007). Highly skilled and creative minds increasingly concentrate in urban areas stimulating creative processes such as invention activities. Inventors in cities thus have access to a larger and also more diverse pool of knowledge than inventors living outside of cities. This is crucially important, since inventions often build on the combination of existing knowledge (Usher 1954; Nelson and Winter 1982; Utterback 1996) and thus on interpersonal interactions that are facilitated by geographic proximity (Liben-Nowell et al. 2005). Urban environments provide more opportunities for knowledge exchange between actors and thus facilitate knowledge combinations (Bettencourt, Lobo, and Strumsky 2007). Nevertheless, existing scaling analyses do not ask how cities influence knowledge combinations. Therefore, they disregard qualitative differences of knowledge combinations and treat inventions as a homogeneous quantity (O'hUallichain 1999; O'hUallichain and Leslie 2005; Bettencourt et al. 2007; Bettencourt, Lobo, and Strumsky 2007; Carlino, Chatterjee, and Hunt 2007; Sonn and Park 2011). By analyzing and evaluating the novelty of knowledge combinations, I particularly shift the focus from quantity to quality and extend existing approaches. In the next section, I argue that knowledge combinations are heterogeneous and that cities concentrate essential factors, which affect knowledge combinations in their quality.

Geography of Knowledge Combinations

Knowledge combinations represent an important mechanism of idea creation (Usher 1954; Nelson and Winter 1982; Utterback 1996; Hargadon 2003; Arthur 2009). Inventions consist of multiple components that are put together in a novel way to fulfill a specific purpose. The components themselves are rarely completely new; rather, they typically represent existing pieces of knowledge (Arthur 2009). Crucially, the art of creatively combining different knowledge domains is one important source for different degrees of novelty across inventions (Ahuja and Lampert 2001).

Exploration and exploitation are two important search processes in research and development (R&D), which differ significantly in their underlying combinatorial characteristics (March 1991). Exploitation thereby refers to the reuse and refinement of existing combinations, whereas exploration describes the search for and development of new combinations. Exploring new combinations implies higher costs and risks than reusing proven combinations. Due to these characteristics, combinations identifying exploitation occur more frequently and hence represent *typical* combinations. In contrast, combinations resulting from exploration are rather rare and *atypical* among observed combinations. In line with previous studies, I rely on the terminology of atypical (typical) combinations as proxies for exploration (exploitation) (Schilling and Green 2011; Uzzi et al. 2013; Kim et al. 2016). Combinatorial characteristics are a strong predictor for the impact of inventions. It is the combination of previously disconnected components, i.e. exploration, that leads to novel ideas and high impact results (Fleming 2001; Dahlin and Behrens 2005; Schoenmakers and Duysters 2010; Schilling and Green 2011; Uzzi et al. 2013; Kim et al. 2016; Verhoeven, Bakker, and Veugelers 2016).

Consequently, spatial variance of exploration and exploitation will affect the regional outcome of invention quality. That is, places of knowledge exploration (i.e., regions that are more capable of combining knowledge in an explorative fashion) are more likely to produce atypical inventions. However, no study exists that seeks to identify such interregional variations. The literature on knowledge combinations is silent about possible geographic patterns and offers little insight into the geography of invention. Although the combinatorial character of knowledge is embedded in contemporary concepts of economic geography, for example, related variety (Frenken, van Oort, and Verburg 2007), differences between regions have not been taken into account to explain spatial inequalities. I therefore shift the focus explicitly to knowledge combinations as the research object in order to disentangle the geography of invention in qualitative terms. But why should places differ regarding the intensity of exploration and exploitation?

The literature on urbanization externalities suggests regional diversity playing a major role for knowledge combinations. The argument harkens back to Jacobs (1969), who described the benefits of large and diverse cities for socioeconomic interactions. Firms, for example, benefit from a cross-fertilization of ideas between industries, rather than being stuck in industry-internal thought patterns. Hence, diversity increases the likelihood of knowledge spillovers between heterogeneous actors (Bettencourt, Lobo, and West 2008; Arbesman, Kleinberg, and Strogatz 2009). Regions with large (knowledge) diversity, in particular, provide more opportunities for knowledge combinations than less diverse cities where such diversity is missing. Being located in diverse environments allows drawing from larger pools of distinct knowledge *pieces* (e.g., technologies, sectors, industries), which in turn increases the opportunities for atypical combinations.

The geographic nature of knowledge spillovers reinforces the importance of regional diversity for combination processes. An exhaustive literature demonstrates that knowledge, in general, does not travel easily over long geographic distances. More precisely, knowledge tends to stay in the same region where it was once created, although the effect diminishes as technologies mature (Jaffe 1989; Jaffe, Trajtenberg, and Henderson 1993; Anselin, Varga, and Acs 1997; Varga 2000). It is often argued that codified knowledge travels more easily than tacit knowledge, while tacit knowledge is more likely to adhere to specific places (von Hippel 1994; Maskell 1999; Gertler 2003). Yet, it is difficult to assess the difference between codified and tacit knowledge empirically. Balland and Rigby (2017) disentangled the two knowledge types by arguing that tacitness can at least partially be captured by the complexity of what is known. Their findings suggested that knowledge complexity limits the geographic distance of knowledge spillovers even more. However, in most instances, codified and tacit knowledge are complements, and, hence, the geographic stickiness of the latter will also reduce the mobility of the former (Cowan and Foray 1997). Accordingly, the local knowledge base represents a crucial determinant of regional knowledge combination processes. Consequently, more diverse cities have access to a larger variety of local knowledge, enabling them to realize more distinct combinations than less diverse cities.

Diversity is critically linked to urbanization. Larger cities, usually, host more different industries than smaller towns. Recently, Youn et al. (2016) analyzed how diversity of business activities relates to city size in US metropolitan areas. They found a linear relationship between city size and business diversity. In an earlier work, Mori, Nishikimi, and Smith (2008) observed a similar relationship between industrial activities and the population size of metropolitan areas in Japan. Clearly, this pattern is not limited to a single nation. The theoretical logic behind the observed linear scaling of population and diversity rests on the notion of the urban hierarchy (Christaller 1933). The central idea is that activities found in the largest cities include those located in the smallest towns, but not vice versa. Larger cities (i.e. central places) provide more sophisticated products, services, and technologies for their less populated surroundings. New York, for example, has a larger potential to explore new knowledge combinations than Branson, Missouri.

Regional diversity, however, is not sufficient to actually explore new combinations. It rather indicates the potential that could be explored. Importantly, exploration requires certain skills and actors to use the given potential, which are not equally distributed across space (Glaeser and Maré 2001; Florida 2002; Bettencourt et al. 2007; Combes, Duranton, and Gobillon 2008; Bacolod, Blum, and Strange 2009; Storper and Scott 2009; Lee, Sissons, and Jones 2016). Spatial wage disparities (i.e. the urban wage premium) indicate that people living in larger cities earn more then their nonurban counterparts (Weber 1899; Glaeser and Maré 2001). Combes, Duranton, and Gobillon (2008) attributed this observation to the spatial sorting of skills. Up to half of the wage disparities is explained by differences of the local workforce composition. Relatedly, Bettencourt et al. (2007) observed a superlinear scaling for both creative employment, as defined by Florida (2002), and R&D employment. That is, individuals with better qualifications for exploring and exploiting knowledge combinations tend to concentrate in larger and more densely populated cities. It follows that cities not only have the larger potential for atypical knowledge combinations but also have a higher capacity (due to the urban concentration of the skills and talents needed for this task) to exploit these potentials. Based on this, I expect atypical combinations to concentrate in large cities. I hypothesize this relationship as follows:

> Atypical and typical knowledge combinations scale superlinearly with city size. However, atypical combinations scale to a larger extent with city size than typical combinations.

Data

In line with previous studies, I rely on patent data to analyze invention activities as results of combinatorial search processes (Fleming 2001; Dahlin and Behrens 2005; Schoenmakers and Duysters 2010; Arts and Veugelers 2015; Kim et al. 2016). Patent data has some peculiarities, which affect the results. Patent activities are not equally distributed across firms, technologies, and sectors. Most importantly, the tendency to patent an invention is biased in favor of manufacturing activities (Griliches 1990). Thus, patents underestimate the inventive outcome in less manufacturing-intensive regions. Eventually, the decision to patent rests on strategic judgment. Not every invention results in a patent for various reasons, for example, information disclosure, the ease of circumventing patent claims, and application costs (Cohen, Nelson, and Walsh 2000). Acs, Anselin, and Varga (2002), however, found that patents are a reliable indicator for measuring invention activities at the regional level.

I draw the patent data from three different data sources. The first source is HistPat, which was recently generated by Petralia, Balland, and Rigby (2016) and is publicly available. This data set contains geographic information on patents from the *United States Patent and Trademark Office (USPTO)* ranging from 1836 to 1975. I complement HistPat by using the data set from Li et al. (2014), which covered the years 1975 to 2010 and contained geographic information as well. Third, I used the *Master Classification File* of the USPTO Bulk Storage System, which provides information on technology classes for the whole time span. The data sets were matched by using patent numbers as unique identifiers. With this data in hand, I was able to analyze the geography of knowledge combinations for granted US patents over the last 174 years.

Patent data reveal how knowledge is combined, as each invention is classified into at least one technology class. In many cases, one single invention is grouped into more than one class. This information has been used to study the knowledge combination process (Fleming 2001; Dahlin and Behrens 2005; Schoenmakers and Duysters 2010;

Kim et al. 2016). The underlying classification scheme is the *Cooperative Patent Classification (CPC)*. The CPC has been established to harmonize individual classification systems between the USPTO and the European Patent Office. Using the CPC thus allows for cross-country comparison of empirical results.

Scholars have long debated how to define a city theoretically and for the purposes of quantitative research (Arcaute et al. 2014; Louf and Barthelemy 2014). HistPat locates patents not to American cities, but rather to counties. This signifier of invention location does not suffice. The county level represents a narrow administrative boundary; it does not take into account interregional dependencies crossing county boundaries. Focusing on county boundaries can therefore lead to spatial bias, since inventors living in one region could potentially generate their invention in neighboring ones. To capture such interregional interdependencies and to minimize spatial bias, most geographic analyses use functional units (Bettencourt et al. 2007; Youn et al. 2016). In this study, I use 171 *Combined Statistical Areas (CSA)*, which is the largest unit of the Metropolitan Statistical Areas in the United States.

I gathered population data of US counties back to the first documented entries, which were in New York County in 1656. I used Wikipedia as a data source to obtain the information for every US county, then aggregated the population size to the CSA level.[1] The population data are only available for ten-year periods. However, these data allow for constructing a panel covering a long time period.

Methods

Z-scores Approach

Following Uzzi et al. (2013) and Kim et al. (2016), I investigate the combinatorial nature of invention by applying z-score measures at the subclass level of the CPC.[2] Teece et al. (1994) introduced z-scores for estimating the relatedness between industries. Z-scores compare the observed combinations of technology classes to what would be expected under the assumption that combinations are random. More formally, the z-score is expressed as follows

$$z_{i,j} = \frac{o_{i,j} - u_{i,j}}{\sigma_{i,j}} \tag{2}$$

where $o_{i,j}$ is the empirically observed co-occurrence count of technology classes i and j. The expected co-occurrence and standard deviation are $u_{i,j}$ and $\sigma_{i,j}$, respectively. A high value for $o_{i,j}$ can be driven by the combination of i and j or by a high number of patents n for both classes. If n_i and n_j are large, one can expect to observe a fair amount of combinations, even if there is little synergy between them. By contrast, a small n_i and n_j result in a relative small number of combinations. To control for this effect, I compare the observed co-occurrence $o_{i,j}$ to what can be expected given n_i and n_j, if knowledge combinations were random (Teece et al. 1994).

The expected co-occurrence, $u_{i,j}$, represents a hypergeometric distribution and is thus given by the product of the number of patents in both technology classes n_i and n_j divided by the total number of patents N,

[1] I used Wikipedia because it offers data for the entire 174 years of observation. I compared the population size for the most recent years with official data sources, such as www.census.gov, finding no differences.

[2] As a robustness check, I also used the CPC class level (three digits) showing that results are independent of the technological resolution (see Appendix B).

$$u_{i,j} = \frac{n_i n_j}{N} \tag{3}$$

and its standard deviation $\sigma_{i,j}$ is given by

$$\sigma^2_{i,j} = u_{i,j}\left(1 - \frac{n_i}{N}\right)\left(\frac{N - n_j}{N - 1}\right) \tag{4}$$

If i and j were combined more often than expected, equation (2) produces a positive value. A positive z-score indicates a typical class combination and, relatedly, an invention that recombines known elements. Conversely, if the two classes i and j are rarely paired together relative to their expected occurrence, equation (2) produces a negative number. This indicates an atypical knowledge combination and, relatedly, an innovative invention.

I can only consider patents that were assigned to at least two technology classes when discussing knowledge combinations because z-scores measure the typicality of combinations between technology pairs. Single class patents shed no light on the combination process. This provides a total sample of 1,706,499 patents granted to inventors living in US metropolitan areas.

Cumulative Knowledge Combinations

Knowledge accumulates over time, giving rise to the emergence of technological trajectories (Dosi 1982; Nelson and Winter 1982). However, the characteristics of knowledge combinations can vary over time. An atypical combination, for example, can diffuse in the knowledge space, if it is repeated in subsequent inventions. Atypical then becomes typical, under the right circumstances and on a long enough time line. Conversely, a certain combination can lose its typicality over time if it is superseded by newer knowledge combinations. To capture this temporal evolution, I rely on an approach similar to the one applied by Kim et al. (2016). For example, if t is 1950, I consider all patents from the beginning of the observation in 1836 to 1950 to calculate $o_{i,j}$.[3] This approach takes into account the cumulative nature of knowledge production and allows the z-scores to evolve over time.

Scaling Analysis

Urban scaling analyses express the dependency of a certain quantity Y (e.g., air pollution, bike thefts, inventions) on cities' population size N as a power-law relation (Bettencourt et al. 2007)

$$Y = Y_0 N^{\beta} \tag{5}$$

or its linear transformation

$$log(Y) = log(Y_0) + \beta log(N) \tag{6}$$

[3] I also applied a twenty-year rolling window approach in which the history of knowledge combination washes out over time; see Figure A1 in Appendix A. The cumulative and the rolling window approach correlated on average at a high level, with 0.9 < R < 1.

with Y_0 representing a normalization constant. I estimated β by using an ordinary least squares estimation. Thus, β can be interpreted as the exponent of population size N, with β falling into one of three categories: $\beta = 1$ (linear), $\beta < 1$ (sublinear), and $\beta > 1$ (superlinear) (see also "The Geography of Invention"). I use 95 percent confidence intervals to test the significance of the exponents falling into one of the three categories. A superlinear relation, for example, is often associated with increasing returns to urbanization. When N doubles in size, Y increases more than twice as much.

Results

In a first step, I analyzed the scaling relation between technological diversity and the population size of cities. One simple measure of diversity is the number of distinct technologies D in a city. A given technology class belongs to the local portfolio if at least one corresponding patent is filed. The hierarchical nature of the CPC allows the number of distinct technologies at a more granular level to be analyzed. Youn et al. (2016) showed that the resolution by which technologies are considered *distinct* clearly affects the results. I control for this observation by using three different levels of technological resolution as defined by the CPC: subclasses (D_{max} = 654), groups (D_{max} = 10,154), and subgroups (D_{max} = 218,570).

Figure 1 illustrates D as a function of population size at different levels of technological resolution for the whole time span. D is normalized by D_{max} to ensure comparability between resolution levels (Figure 1, panel A). Diversity at the subclass (red dots) and group level (green dots) strongly follows a logarithmic law. The corresponding exponents $\beta_{subclass} = 0.22 \pm 0.02$ and $\beta_{group} = 0.56 \pm 0.03$ imply that diversity relates sublinearly to population size as $\beta < 1$. This finding suggests that larger cities are more diverse but that diversity does not increase disproportionately with city size.

When using the most fine-grained level of distinction, subgroups (blue dots in panels A and D of Figure 1), the exponent changes to $\beta_{subgroup} = 0.95 \pm 0.04$. The corresponding 95 percent confidence interval ranges from 0.86 to 1.03. Hence, the range includes $\beta \approx 1$, which corresponds to a linear relation of diversity and city size. The result is similar to that of Youn et al. (2016), who observed an exponent of $\beta = 0.98 \pm 0.02$ for the relation between diversity of business activities and city size. Accordingly, technological diversity is also strongly related to city size in a linear fashion. This relationship, however, is very sensitive to the level of technological resolution.

Next, I analyzed how the US cities' local diversity relates to knowledge combinations. As was explained in "Theoretical Underpinnings," a proportional increase of diversity shifts knowledge combination opportunities (distinct knowledge combinations). The CPC distinguishes 654 different subclasses (D_{max}), enabling 213,531 distinct class combinations. Using subclasses is sufficient to study knowledge combinations, as cities realize only a small fraction of what is theoretically feasible. The average share of realized combinations across all cities is 3 percent. The most diverse city is New York, with patents in 630 different technologies between 1990 and 2010. New York's knowledge base allows for 198,135 distinct combinations, of which 17,182 were realized (9 percent).

Local diversity can be seen as the endogenous potential for knowledge combinations.

Figure 2 plots the relationship between diversity and distinct class combinations at the city level at four different time periods. In 1850, the relationship was almost linear.

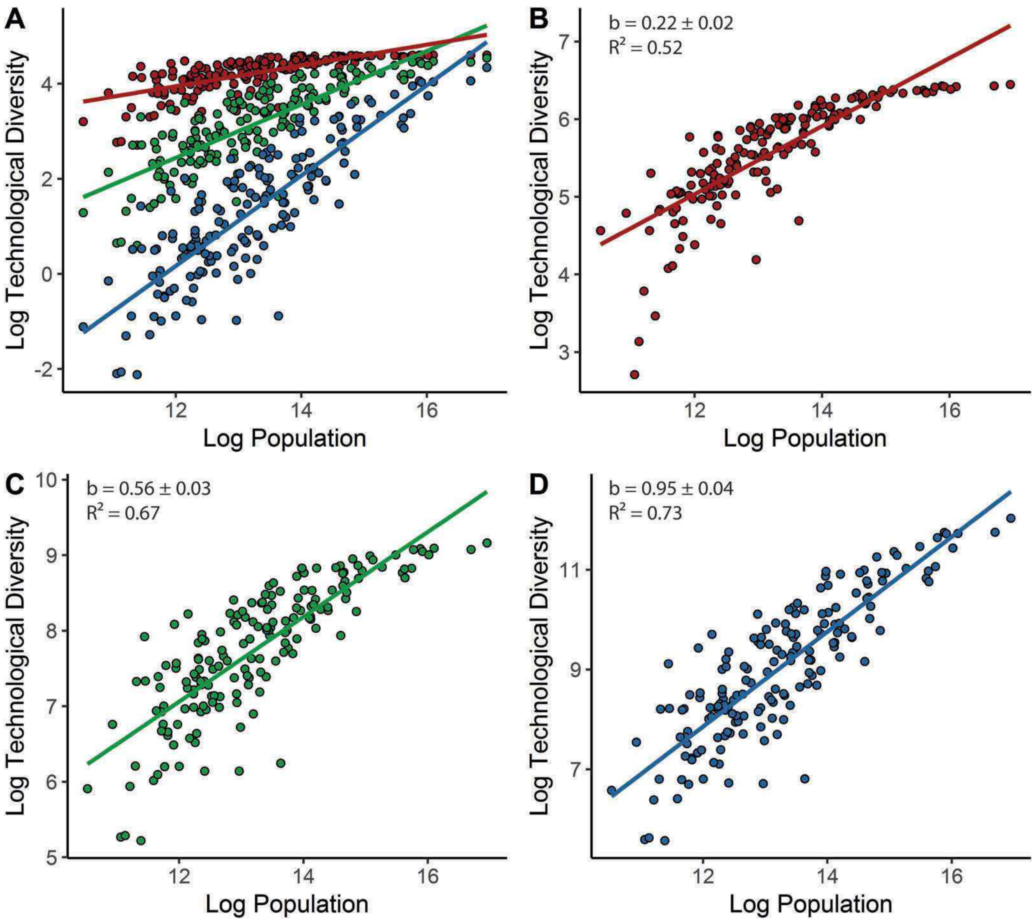

Figure 1. Technological diversity as a function of population size at three different levels of technological resolution. Diversity is normalized in **A** by D_{max} for comparability reasons. Scaling relations between population and technological diversity **B** at the subclass level (D_{max} = 654), **C** at the group level (D_{max} = 10,154), and **D** at the subgroup level (D_{max} = 218,570).

Over the years, the curve became steeper, as cities' technology portfolios grew more diverse.

I investigated the relationship between diversity and knowledge combinations once more by employing the scaling approach. Figure 3 visualizes the development of the scaling exponent, β, over time. The scaling exponent of diversity is larger than one, indicating an overproportionate increase of distinct class combinations with cities' diversity.

In addition, Figure 3 shows that scaling increases over time. I interpret this finding as evidence for growing disparities between the least and the most diverse cities. To understand this finding in greater depth, I divided the sample into two subsamples based on each city's diversity in each year. The most diverse cities belong to the upper quartile, and the least diverse cities to the lower quartile. I compared both groups' sample means and corresponding 95 percent confidence intervals based on the one

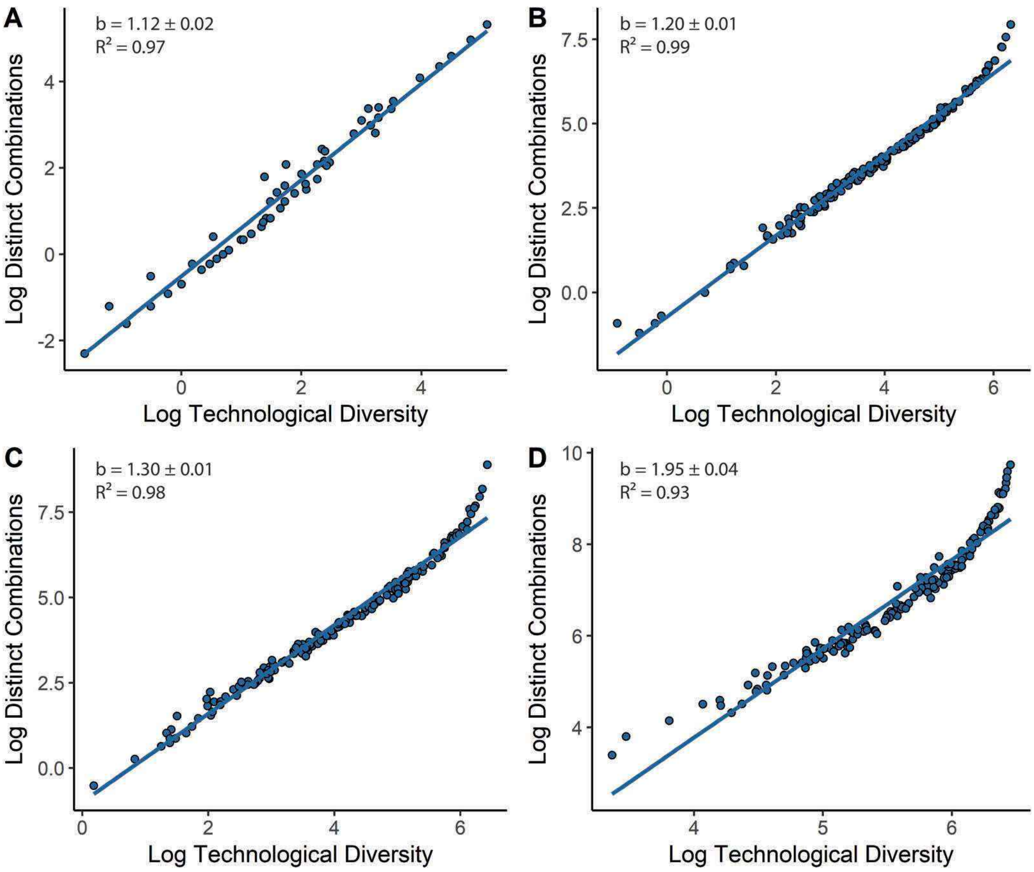

Figure 2. Scaling relationship between technological diversity and the total number of distinct class combinations **A** in 1850, **B** in 1900, **C** in 1950, and **D** in 2010 in US metropolitan areas.

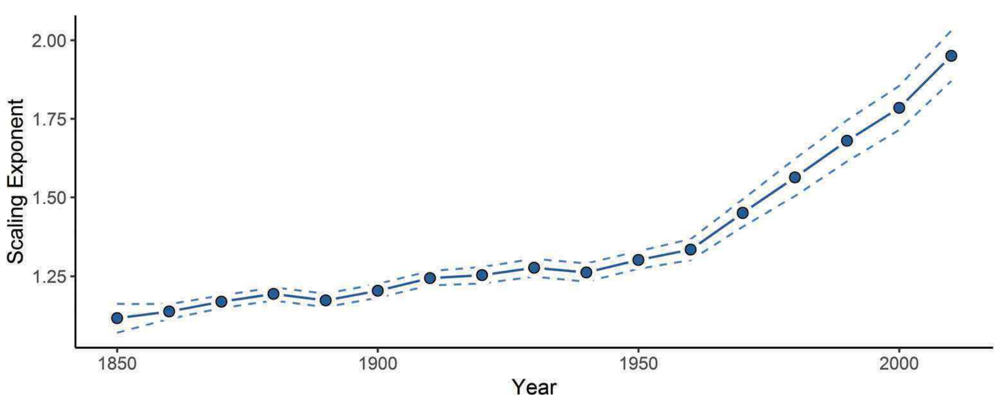

Figure 3. Scaling exponent of diversity with respect to the number of distinct combinations over time.
Note: Dashed lines indicate the 95 percent confidence interval.

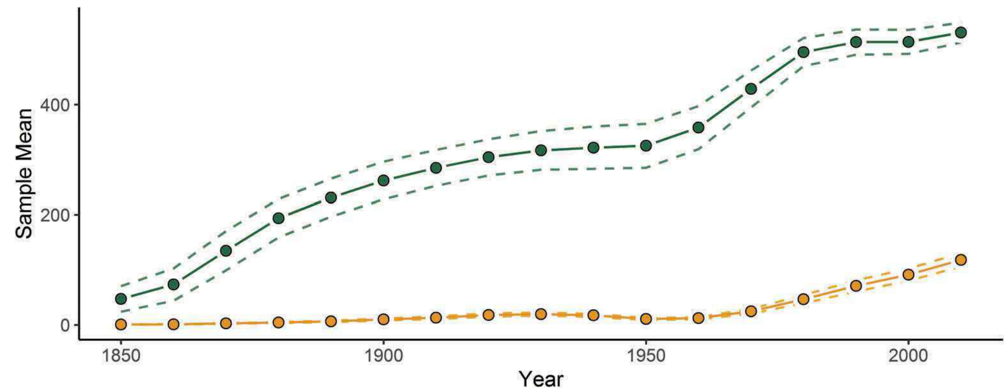

Figure 4. Average number of technologies in the most diversified (green line) and least diversified cities (orange line).
Note: Dashed lines indicate the 95 percent confidence intervals.

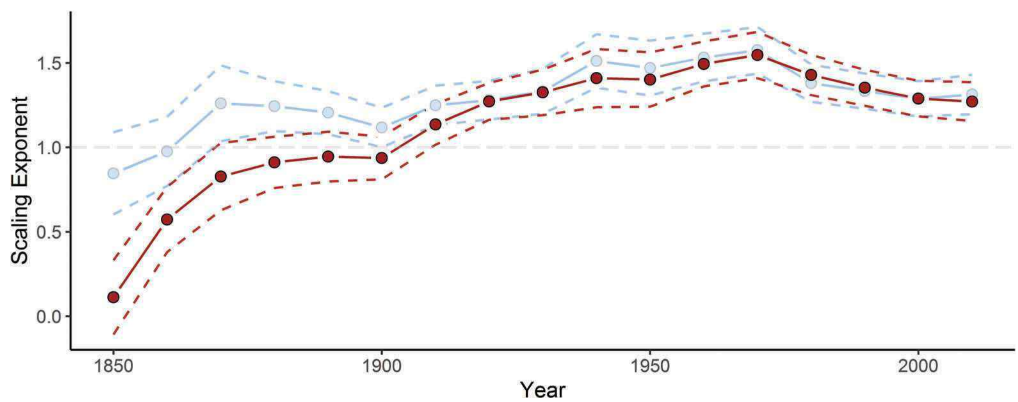

Figure 5. Scaling exponent of population size over time for atypical (red line) and typical combinations (blue line)
Note: Dashed lines indicate the 95 percent confidence interval.

sample *t*-test. Figure 4 visualizes the result. The difference between both sample means is significant and clearly increases over time, emphasizing the increasing disparity between the groups. This disparity is largely driven by the increasing diversity of the most diverse cities, such as New York, Greater Boston, Los Angeles, Chicago, and the Bay Area (in San Francisco).

In a further analysis, I examined the correlation between knowledge combinations typicality and population size. My hypothesis claims that the resources needed for expanding the set of knowledge combinations are especially concentrated in large cities, such that larger cities have more atypical knowledge combinations (see "Geography of Knowledge Combinations").

Figure 5 illustrates β of atypical (red line) and typical (blue line) combinations in relation to population size over time. Typical combinations serve as the *baseline* scaling of knowledge combinations against which atypical combinations are tested and put into relation. The scaling exponent of atypical combinations has increased over the last 174 years. Until 1900, the exponent was smaller than one; this indicates there were no particular benefits of city size at that time. Since then, atypical combinations appear to become an urban phenomenon, with $\beta > 1$ and a maximum of 1.54 ± 0.07 in 1970. Between 1970 and 2010, the scaling exponent, however, has slightly decreased.

Interestingly, urbanization is not just favorable for atypical but also for typical combinations. For most years since 1836, the scaling exponent of typical combinations has been greater than one and larger than the exponent of atypical combinations. That is, cities have been more successful at knowledge exploitation than exploration. In the last decade, both exponents have converged to almost the same value. Based on this finding, I may only partially confirm my hypothesis: both atypical and typical combinations scale superlinearly with city size, but atypical combinations do not scale to a larger extent than typical combinations.

Conclusion

The increasing availability of large and historic data sets opens new possibilities for empirical research. This study is among the first analyzing the geography of invention over almost two centuries. My analysis of American invention history reveals that knowledge exploration clearly concentrates in large cities. That is, atypical combinations scale superlinearly with cities' population size. The scaling exponent significantly increased over the last 174 years, which suggests that large cities drive technological progress not only in quantitative but also in qualitative terms. This finding challenges the prominent death of distance thesis in almost all regards (Friedman 2005).

I attribute the growing importance to the opportunities given in large cities. In particular, knowledge diversity in large cities provides opportunities for knowledge combinations not found in smaller and less diverse towns. Beyond diversity, larger cities also concentrate the skills to exploit the given diversity. Inventors in large cities realize a disproportionate number of distinct knowledge combinations, which also affects the exploration of new combinations. Given the cumulative nature of knowledge, wealth, innovation, and human skill, my results suggest a self-reinforcing process that favors metropolitan centers for knowledge creation. Thus, knowledge creation plays a major role for creating and maintaining spatial inequalities.

Increasing spatial inequalities have profound implications for regional development and policy making. Inequalities unfold in the form of invention activities, as one crucial economic activity that transforms our economy and society. The benefits of knowledge creation in large cities are not shared by all regions and reinforces a widening divergence between large cities—as centers of knowledge exploration—and smaller towns. Given the importance of geography for knowledge generation, it is unlikely that spatial concentration of invention activities will stop. Earlier research, moreover, observes a decreasing productivity of R&D and highlights that more resources and capabilities are necessary to yield useful R&D outcomes (Lanjouw and Schankerman 2004; Wuchty, Jones, and Uzzi 2007; Jones, Wuchty, and Uzzi 2008). Large cities provide the required resources and capabilities in close geographic proximity. Smaller towns lack the requirements to compete, get disconnected, and fall behind. It should be, furthermore, in the interest of policy makers that all places benefit from urban externalities. That is, policy has to consider how to distribute the novelty created in the centers down the urban hierarchy to smaller towns and lagging regions.

However, much research remains to be done. Why did it take longer for atypical combinations to scale that strongly with city size? Has this process stopped, or will it continue? Moreover, atypical knowledge combinations do not automatically imply a high technological impact or economic value. Thus, it remains unclear precisely how (a)typical combinations relate to the economic performance of cities and how they explain local stories of success and failure.

References

Acs, Z. J., Anselin, L., and Varga, A. 2002. Patents and innovation counts as measures of regional production of new knowledge. *Research Policy* 31 (7): 1069–85. doi:10.1016/S0048-7333(01)00184-6.

Ahuja, G., and Lampert., C. M. 2001. Entrepreneurship in the large corporation: A longitudinal study of how established firms create breakthrough inventions. *Strategic Management Journal* 22 (6–7): 521–43. doi:10.1002/smj.176.

Anselin, L., Varga, A., and Acs, Z. 1997. Local geographic spillovers between university research and high technology innovations. *Journal of Urban Economics* 42 (3): 422–48. doi:10.1006/juec.1997.2032.

Arbesman, S., Kleinberg, J. M., and Strogatz, S. H. 2009. Superlinear scaling for innovation in cities. *Physical Review E* 79 (1): 016115. doi:10.1103/PhysRevE.79.016115.

Arcaute, E., Hatna, E., Ferguson, P., Youn, H., Johansson, A., and Batty, M. 2014. Constructing cities, deconstructing scaling laws. *Journal of the Royal Society Interface* 12:102. doi:10.1098/rsif.2014.0745.

Arthur, W. B. 2009. *The nature of technology: What it is and how it evolves*. New York: Free Press.

Arts, S., and Veugelers, R. 2015. Technology familiarity, recombinant novelty, and breakthrough invention. *Industrial and Corporate Change* 24 (6): 1215–46. doi:10.1093/icc/dtu029.

Audretsch, D. B., and Feldman, M. 1996. R&D spillovers and the geography of innovation and production. *American Economic Review* 86 (3): 630–40.

Bacolod, M., Blum, B. S., and Strange, W. C. 2009. Skills in the city. *Journal of Urban Economics* 65 (2): 136–53. doi:10.1016/j.jue.2008.09.003.

Balland, P.-A. 2017. EconGeo: Computing key indicators of the spatial distribution of economic activities. https://github.com/PABalland/EconGeo.

Balland, P.-A., and Rigby, D. 2017. The geography of complex knowledge. *Economic Geography* 93 (1): 1–23. doi:10.1080/00130095.2016.1205947.

Bettencourt, L. M. A., Helbing, D., Kuhnert, C., and West, G. B. 2007. Growth, innovation, scaling, and the pace of life in cities. *Proceedings of the National Academy of Sciences* 104 (17): 7301–06. doi:10.1073/pnas.0610172104.

Bettencourt, L. M. A., Lobo, J., and West, G. B. 2008. Why are large cities faster? Universal scaling and self-similarity in urban organization and dynamics. *European Physical Journal B* 63 (3): 285–93. doi:10.1140/epjb/e2008-00250-6.

Bettencourt, L. M. A., Lobo, J., and Strumsky, D. 2007. Invention in the city: Increasing returns to patenting as a scaling function of metropolitan size. *Research Policy* 36 (1): 107–20. doi:10.1016/j.respol.2006.09.026.

Cairncross, F. 1997. *The death of distance: How the communications revolution will change our lives*. Boston: Harvard Business School Press.

Carlino, G. A., Chatterjee, S., and Hunt, R. M. 2007. Urban density and the rate of invention. *Journal of Urban Economics* 61 (3): 389–419. doi:10.1016/j.jue.2006.08.003.

Castaldi, C., and Los, B. 2017. Geographical patterns in US inventive activity 1977–1998: The 'regional inversion' was underestimated. *Research Policy* 46 (7): 1187–97. doi:10.1016/j.respol.2017.04.005.

Castells, M. 1996. *The rise of the network society. Information age*. Oxford: Blackwell Publishers.

Christaller, W. 1933. *Die zentralen Orte in Süddeutschland: Eine ökonomisch-geographische Untersuchung über die Gesetzmäßigkeit der Verbreitung und Entwicklung der Siedlungen mit städtischen Funktionen* [Central places in Southern Germany: An economic-geographical study on the legality of the distribution and development of settlements with urban functions]. Darmstadt: Wissenschaftliche Buchgesellschaft.

Christopherson, S., Garretsen, H., and Martin, R. 2008. The world is not flat: Putting globalization in its place. *Cambridge Journal of Regions, Economy and Society* 1 (3): 343–49. doi:10.1093/cjres/rsn023.

Co, C. 2002. Evolution of the geography of innovation: Evidence from patent data. *Growth and Change* 33 (4): 393–423. doi:10.1111/1468-2257.00204.

Cohen, W., Nelson, R., and Walsh, J. 2000. Protecting their intellectual assets: Appropriability conditions and why U.S. manufacturing firms patent (or not). Working Paper 7552. Cambridge, MA: National Bureau of Economic Research. doi:10.3386/w7552.

Combes, P.-P., Duranton, G., and Gobillon, L. 2008. Spatial wage disparities: Sorting matters! *Journal of Urban Economics* 63 (2): 723–42. doi:10.1016/j.jue.2007.04.004.

Cowan, R., and Foray, D. 1997. The economics of codification and the diffusion of knowledge. *Industrial and Corporate Change* 6 (3): 595–622. doi:10.1093/icc/6.3.595.

Dahlin, K. B., and Behrens, D. M. 2005. When is an invention really radical? *Research Policy* 34 (5): 717–37. doi:10.1016/j.respol.2005.03.009.

Dosi, G. 1982. Technological paradigms and technological trajectories. *Research Policy* 11 (3): 147–62. doi:10.1016/0048-7333(82)90016-6.

Dumais, G., Ellison, G., and Glaeser, E. L. 2002. Geographic concentration as a dynamic process. *Review of Economics and Statistics* 84 (2): 193–204. doi:10.1162/003465302317411479.

Ellison, G., and Glaeser, E. L. 1999. The geographic concentration of industry: Does natural advantage explain agglomeration? *American Economic Review* 89 (2): 311–16. doi:10.1257/aer.89.2.311.

Ellison, G., Glaeser, E. L., and Kerr, W. R. 2010. What causes industry agglomeration? Evidence from coagglomeration patterns. *American Economic Review* 100 (3): 1195–213. doi:10.1257/aer.100.3.1195.

Feldman, M. P. 1994. *The geography of innovation*. Dordrecht: Kluwer Academic.

Feldman, M. P., and Florida, R. 1994. The geographic sources of innovation: Technological infrastructure and product innovation in the United States. *Annals of the Association of American Geographers* 84 (2): 210–29. doi:10.1111/j.1467-8306.1994.tb01735.x.

Feldman, M. P., and Kogler, D. F. 2010. Stylized facts in the geography of innovation. *Handbook of the economics of innovation*, ed. B. Hall and N. Rosenberg, vol. 1, 381–410. Amsterdam: Elsevier. doi:10.1016/S0169-7218(10)01008-7.

Fleming, L. 2001. Recombinant uncertainty in technological search. *Management Science* 47 (1): 117–32. doi:10.1287/mnsc.47.1.117.10671.

Florida, R. 2002. *The rise of the creative class: And how it's transforming work, leisure, community and everyday life*. New York: Basic Books.

Florida, R., Gulden, T., and Mellander, C. 2008. The rise of the mega-region. *Cambridge Journal of Regions, Economy and Society* 1 (3): 459–76. doi:10.1093/cjres/rsn018.

Frenken, K., van Oort, F., and Verburg, T. 2007. Related variety, unrelated variety and regional economic growth. *Regional Studies* 41 (5): 685–97. doi:10.1080/00343400601120296.

Friedman, T. L. 2005. *The world is flat: A brief history of the twenty-first century.* New York: Farrar, Straus and Giroux.

Gertler, M. S. 2003. Tacit knowledge and the economic geography of context, or the undefinable tacitness of being (there). *Journal of Economic Geography* 3 (1): 75–99. doi:10.1093/jeg/3.1.75.

Glaeser, E. L. 2011. *Triumph of the city: How our greatest invention makes us richer, smarter, greener, healthier, and happier.* New York: Penguin Press.

Glaeser, E. L., and Maré, D. C. 2001. Cities and skills. *Journal of Labor Economics* 19 (2): 316–42. doi:10.1086/319563.

Griliches, Z. 1990. Patent statistics as economic indicators: A survey. Working Paper 3301. Cambridge, MA: National Bureau of Economic Research. doi:10.3386/w3301.

Hall, P., and Markusen, A. R., eds. 1985. *Silicon landscapes*. Boston: Allen and Unwin.

Hargadon, A. 2003. *How breakthroughs happen: The surprising truth about how companies innovate*. Boston: Harvard Business School Press.

Jacobs, J. 1969. *The economy of cities*. New York: Vintage Books.

Jaffe, A. B. 1989. Real effects of academic research. *American Economic Review* 79 (5): 957–70.

Jaffe, A. B., Trajtenberg, M., and Henderson, R. 1993. Geographic localization of knowledge spillovers as evidenced by patent citations. *Quarterly Journal of Economics* 108 (3): 577–98. doi:10.2307/2118401.

Jones, B. F., Wuchty, S., and Uzzi, B. 2008. Multi-university research teams: Shifting impact, geography, and stratification in science. *Science* 322 (5905): 1259–62. doi:10.1126/science.1158357.

Kim, D., Cerigo, D. B., Jeong, H., and Youn, H. 2016. Technological novelty profile and invention's future impact. *EPJ Data Science* 5(1). doi:10.1140/epjds/s13688-016-0069-1.

Kuznets, S. 1960. Population change and aggregate output. *Demographic and economic change in developed countries*, ed. Universities-National Bureau, 324–51. Princeton: Princeton University Press.

Lanjouw, J. O., and Schankerman, M. 2004. Patent quality and research productivity: Measuring innovation with multiple indicators. *Economic Journal* 114 (495): 441–65. doi:10.1111/j.1468-0297.2004.00216.x.

Lee, N., Sissons, P., and Jones, K. 2016. The geography of wage inequality in British cities. *Regional Studies* 50 (10): 1714–27. doi:10.1080/00343404.2015.1053859.

Li, G.-C., Lai, R., D'Amour, A., Doolin, D. M., Sun, Y., Torvik, V. I., Yu, A. Z., and Fleming, L. 2014. Disambiguation and co-authorship networks of the U.S. patent inventor database (1975–2010). *Research Policy* 43 (6): 941–55. doi:10.1016/j.respol.2014.01.012.

Liben-Nowell, D., Novak, J., Kumar, R., Raghavan, P., and Tomkins, A. 2005. Geographic routing in social networks. *Proceedings of the National Academy of Sciences* 102 (33): 11623–28. doi:10.1073/pnas.0503018102.

Louf, R., and Barthelemy, M. 2014. Scaling: Lost in the smog. *Environment and Planning B: Planning and Design* 41 (5): 767–69. doi:10.1068/b4105c.

Lucas, R. E. 1988. On the mechanics of economic development. *Journal of Monetary Economics* 22 (1): 3–42. doi:10.1016/0304-3932(88)90168-7.

March, J. G. 1991. Exploration and exploitation in organizational learning. *Organization Science* 2 (1): 71–87. doi:10.1287/orsc.2.1.71.

Marshall, A. 1890. *Principles of economics*. London: McMillan.

Maskell, P. 1999. Localised learning and industrial competitiveness. *Cambridge Journal of Economics* 23 (2): 167–85. doi:10.1093/cje/23.2.167.

Mori, T., Nishikimi, K., and Smith, T. E. 2008. The number-average size rule: A new empirical relationship between industrial location and city size. *Journal of Regional Science* 48 (1): 165–211. doi:10.1111/j.1467-9787.2008.00550.x.

Nelson, R. R., and Winter, S. G. 1982. *An evolutionary theory of economic change*. Cambridge, MA: Belknap Press of Harvard University Press.

O'Brien, R. 1992. *Global financial integration: The end of geography.* London: Pinter Publishers.

O'hUallichain, B. 1999. Patent places: Size matters. *Journal of Regional Science* 39 (4): 613–36. doi:10.1111/0022-4146.00152.

O'hUallichain, B., and Leslie, T. F. 2005. Spatial convergence and spillovers in American invention. *Annals of the Association of American Geographers* 95 (4): 866–86. doi:10.1111/j.1467-8306.2005.00491.x.

Petralia, S., Balland, P.-A., and Rigby, D. L. 2016. Unveiling the geography of historical patents in the United States from 1836 to 1975. *Scientific Data* 3 (August): 160074. doi:10.1038/sdata.2016.74.

Rodríguez-Pose, A., and Crescenzi, R. 2008. Mountains in a flat world: Why proximity still matters for the location of economic activity. *Cambridge Journal of Regions, Economy and Society* 1 (3): 371–88. doi:10.1093/cjres/rsn011.

Romer, P. M. 1990. Endogenous technological change. *Journal of Political Economy* 98 (5, pt. 2): 71–102. doi:10.1086/261725.

Saxenian, A. 1994. *Regional advantage: Culture and competition in silicon valley and route 128*. Cambridge, MA: Harvard University Press.

Schilling, M. A., and Green, E. 2011. Recombinant search and breakthrough idea generation: An analysis of high impact papers in the social sciences. *Research Policy* 40 (10): 1321–31. doi:10.1016/j.respol.2011.06.009.

Schmidt-Nielsen, K. 1984. *Scaling: Why is animal size so important?* Cambridge: Cambridge University Press.

Schoenmakers, W., and Duysters, G. 2010. The technological origins of radical inventions. *Research Policy* 39 (8): 1051–59. doi:10.1016/j.respol.2010.05.013.

Scott, A. J. 1993. *Technopolis: High-technology industry and regional development in Southern California*. Berkeley: University of California Press.

Sonn, J. W., and Park, I. K. 2011. The increasing importance of agglomeration economies hidden behind convergence: Geography of knowledge production. *Urban Studies* 48 (10): 2180–94. doi:10.1177/0042098010382679.

Sonn, J. W., and Storper, M. 2008. The increasing importance of geographical proximity in knowledge production: An analysis of US patent citations, 1975–1997. *Environment and Planning A* 40 (5): 1020–39. doi:10.1068/a3930.

Storper, M., and Scott, A. J. 2009. Rethinking human capital, creativity and urban growth. *Journal of Economic Geography* 9 (2): 147–67. doi:10.1093/jeg/lbn052.

Teece, D. J., Rumelt, R., Dosi, G., and Winter, S. 1994. Understanding corporate coherence. *Journal of Economic Behavior & Organization* 23 (1): 1–30. doi:10.1016/0167-2681(94)90094-9.

Usher, A. P. 1954. *A history of mechanical inventions*. New York: Dover.

Utterback, J. M. 1996. *Mastering the dynamics of innovation*. Boston: Harvard Business School.

Uzzi, B., Mukherjee, S., Stringer, M., and Jones, B. 2013. Atypical combinations and scientific impact. *Science* 342 (6157): 468–72. doi:10.1126/science.1240474.

Varga, A. 1999. Time-space patterns of US innovation: Stability or change? *Innovation, networks and localities*, ed. M. M. Fischer, L. Suarez-Villa, and M. Steiner, 215–34. Berlin: Springer. doi:10.1007/978-3-642-58524-1_10.

———. 2000. Local academic knowledge transfers and the concentration of economic activity. *Journal of Regional Science* 40 (2): 289–309. doi:10.1111/0022-4146.00175.

Verhoeven, D., Bakker, J., and Veugelers, R. 2016. Measuring technological novelty with patent-based indicators. *Research Policy* 45 (3): 707–23. doi:10.1016/j.respol.2015.11.010.

von Hippel, E. 1994. 'Sticky information' and the locus of problem solving: Implications for innovation. *Management Science* 40 (4): 429–39. doi:10.1287/mnsc.40.4.429.

Weber, A. F. 1899. *The growth of cities in the nineteenth century*. New York: Macmillan.

West, G. B. 1997. A general model for the origin of allometric scaling laws in biology. *Science* 276 (5309): 122–26. doi:10.1126/science.276.5309.122.

Wuchty, S., Jones, B. F., and Uzzi, B. 2007. The increasing dominance of teams in production of knowledge. *Science* 316 (5827): 1036–39. doi:10.1126/science.1136099.

Youn, H., Bettencourt, L. M. A., Lobo, J., Strumsky, D., Samaniego, H., and West, G. B. 2016. Scaling and universality in urban economic diversification. *Journal of the Royal Society Interface* 13 (114). doi:10.1098/rsif.2015.0937.

Zook, M. A. 2000. The web of production: The economic geography of commercial internet content production in the United States. *Environment and Planning A* 32 (3): 411–26. doi:10.1068/a32124.

Appendix

A: Moving Window vs. Cumulative Approach

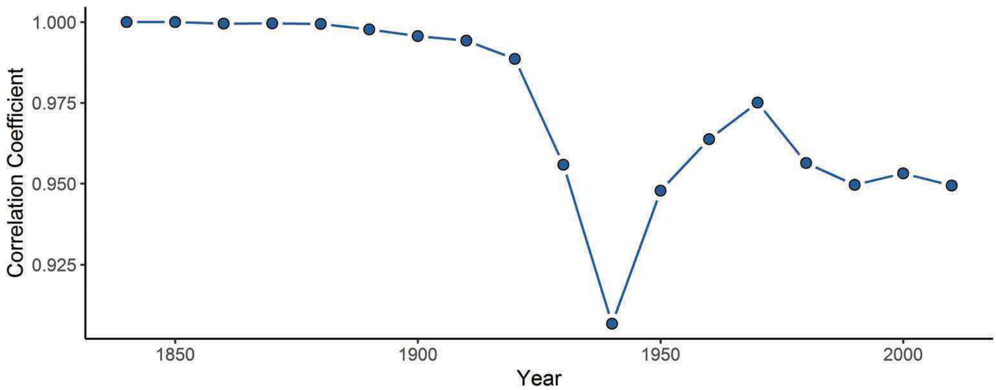

Figure A1. Correlation coefficient between *z*-scores calculated in a rolling window (twenty years) and a cumulative approach (see "Results").

B: Robustness Analysis

To check if the results described in the "Conclusion" are not affected by the choice to use the four-digit CPC level (CPC4), I repeated the analysis by using a different level of technological aggregation, that is, three-digit CPC (CPC3). The CPC3 distinguishes between 127 different technologies. The figures clearly show that my results are relatively robust using the CPC3. As the CPC4 reveals more technological details than CPC3, I decided to use the CPC4 as the main level for my analysis.

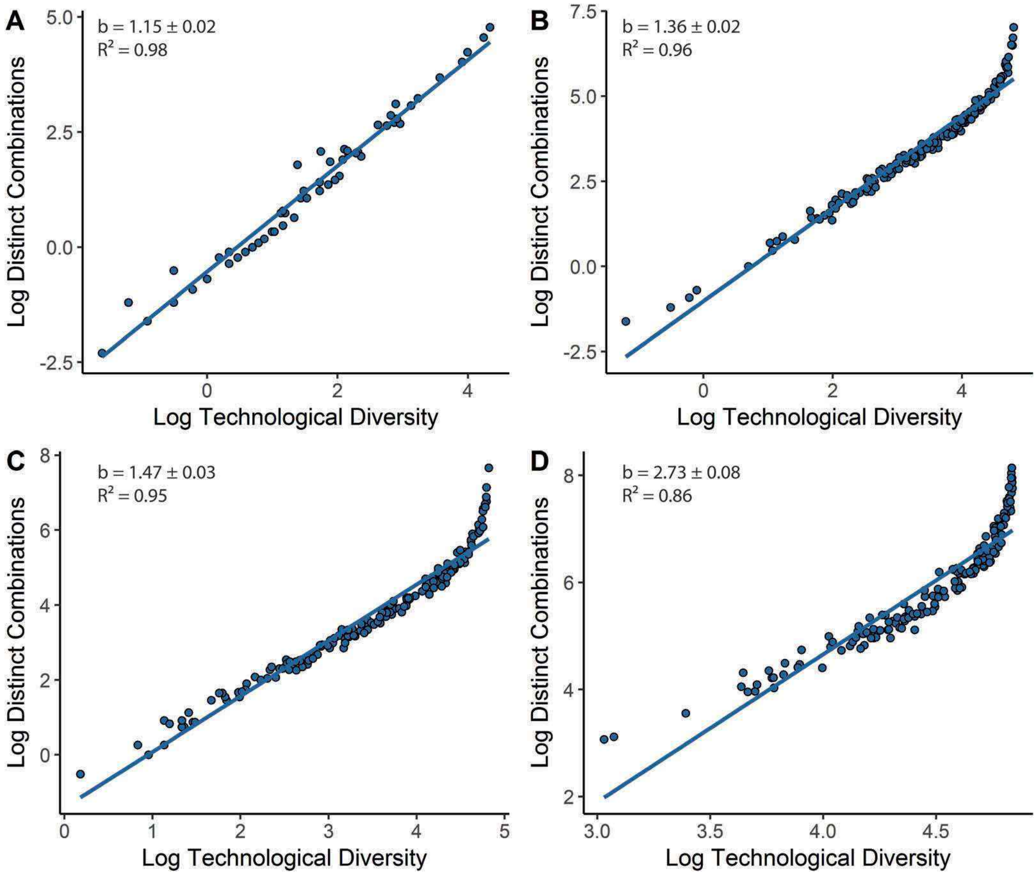

Figure B1. Scaling relationship between technological diversity and the total number of distinct class combinations **A** in 1850, **B** in 1900, **C** in 1950, and **D** in 2010 in US metropolitan areas using the CPC3.

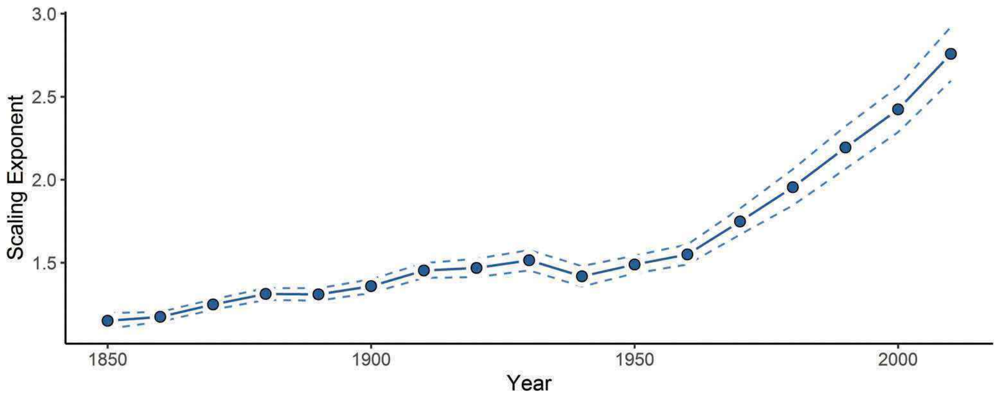

Figure B2. Scaling exponent of diversity with respect to the number of distinct combinations over time using CPC3.
Note: Dashed lines indicate the 95 percent confidence interval.

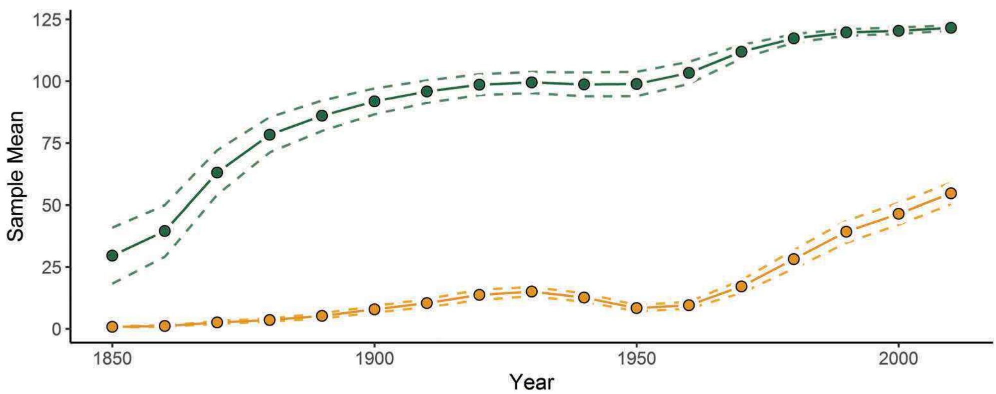

Figure B3. Average number of technologies in the most diversified (green line) and least diversified cities (orange line) using CPC3.
Note: Dashed lines indicate the 95 percent confidence intervals.

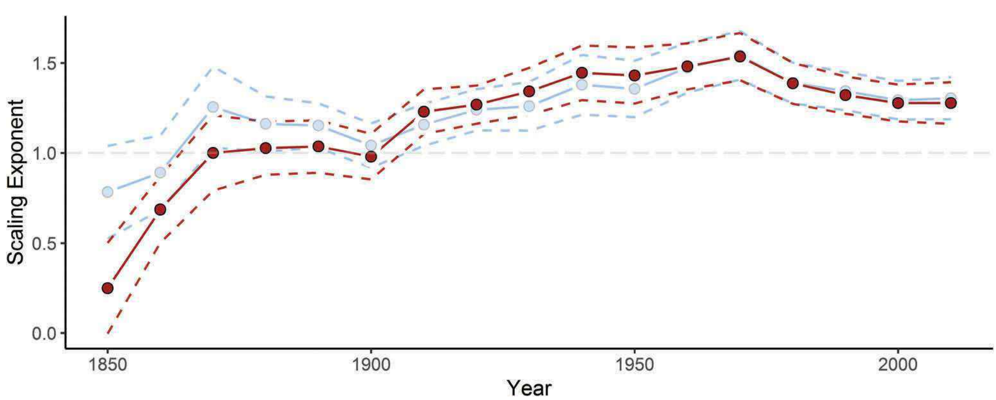

Figure B4. Scaling exponent of population size over time for atypical (red line) and typical combinations (blue line) using CPC3.
Note: Dashed lines indicate the 95 percent confidence interval.