The Despicable Doctor Fischer's (Bayesian) Bomb Party

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How greedy are the rich? In Graham Greene's novel "Doctor Fischer of Geneva or The Bomb Party"[1] this is a question that fascinates the despicable Doctor Fischer. He invites wealthy guests to a series of parties where he derives huge amusement from observing that they appear consistently willing to suffer all manner of indignities in order to receive expensive presents that they, being so rich, could easily afford to buy for themselves. For his final party he has contrived a macabre experimental test of his theory—extreme even by his disgusting standards.

When they are all gathered he shows his six guests a barrel of bran in a corner of the garden in which are six Christmas crackers. Five of the crackers, he explains, each contain a cheque for two million Swiss francs. The sixth contains enough explosive so primed as to end the life of the person who pulls the cracker. One guest, Monsieur Belmont, shocked by what he has been told, says that if anyone were to be killed then it would be murder. But Doctor Fischer refutes this suggestion by explaining that it would not be murder, or even suicide, but more like Russian roulette. He adds that anyone who does not wish to play must leave at once. At this point another guest, Mr. Kips, announces he will not play and gets up to leave. In spite of Doctor Fischer telling him that there are five chances to one in his favour Mr. Kips departs saying he considers gambling for money highly immoral.

Then one of the five remaining guests stands and, while pausing to gather courage to approach the barrel, is beaten to it by another, Mrs. Montgomery, who runs to the barrel first.

The story continues: "...Perhaps she had calculated that the odds would never be as favourable again.

Belmont had probably been thinking along the same lines, for he protested, "We should have drawn for turns" (p. 124).

When she pulls her cracker there is a small pop and a cheque for two million francs falls out. Belmont then pulls his cracker and also wins a cheque.

The story continues: "What about you, Jones?" Doctor Fischer said, "The odds are narrowing."

"I prefer to watch your damned experiment to the end. Greed is winning, isn't it?"

"If you watch you must eventually play—or leave like Mr. Kips."

"Oh I'll play, I promise you that. I'll bet on the last cracker. That gives better odds to the Divisionaire" (p. 125). (Jones has been contemplating suicide for several days and pities the Divisionaire, who, despite his high military rank, has never heard a shot fired in anger and has been taunted by Doctor Fischer for having no record of any act of bravery.)

Then the third cracker, which also turns out to contain two million francs is pulled by another guest. This leaves three crackers and two guests, Jones and the Divisionaire, who is realising that he is too afraid—too cowardly—to take the risk:

"I haven't the courage. I should have gone to the tub first, when the odds were better..." (p. 126).

The comments of the characters at the bomb party imply that the longer you wait your turn to pull a cracker the more dangerous the risk becomes. But is this correct reasoning? Is there anything you can do in this situation to maximise the possibility of obtaining two million francs while at the same time minimising the possibility of being blown to pieces?

Perhaps the first thing to notice is that while Doctor Fischer is correct when he says the odds on winning are narrowing after the first two crackers are pulled, this is conditional on neither of them containing the bomb; if the bomb had been detonated by the first guest to take the risk, the others could have (and no doubt gleefully would have) pulled the remaining crackers secure in the knowledge that they would benefit financially rather than be exploded. So, the longer you wait your turn the odds on survival shorten *providing the bomb cracker is not pulled*; but a direct corollary is that the longer you wait the more likely it is that someone else will pull the bomb before you.

Perhaps if Doctor Fischer and his guests had taken this factor into account their reasoning might have been somewhat different. It is tempting to suppose that an intermediate position in the order, by taking some advantage of both tendencies, might provide the optimal solution. However, actual computation of the probabilities shows that the chances of being blown up are equal in each serial position; there is therefore no advantage (or disadvantage) to be gained by risking to be first to the barrel or waiting till the end or by choosing any other turn. The calculation is quite straightforward. The first guest obviously has a 1 in 6 chance of pulling the lethal cracker and 5 chances in 6 of surviving. The second guest has a 1 in 5 chance *if* the first guest *does not* blow up; the probability of the second guest blowing up is therefore $\frac{1}{5} \times \frac{5}{6}$ which is $\frac{1}{6}$. Similarly the third guest blows up with a probability of $\frac{1}{6}$ $\left(\frac{1}{4} \times \frac{4}{5} \times \frac{5}{6}\right)$, and so on. In fact, for any number of guests drawing any number of crackers in which any number of bombs have been concealed, the position of a guest in the queue does not affect his probability of being blown up.

There is, however, one complication that we have not yet considered. In the story Mrs. Montgomery, before pulling the first cracker, accuses Doctor Fischer of joking about the presence of the bomb. Doctor Fischer responds by teasing his guests about the possibility of him bluffing them. Although he gives them his word that the cheques are there, of the possibility that he is only joking about the presence of the bomb he says: "Perhaps I am. You will know by the end of the party whether I am or not. Isn't the gamble worth while?..." (p. 121). Now, the fact of the bomb's existence has been made uncertain. Under these circumstances perhaps it pays a guest at a bomb party to think again about when to pull a cracker. This is because if we entertain any degree of doubt concerning the presence of a bomb in *any* of the crackers then that doubt will be fuelled the more crackers that are pulled without a bomb exploding.

It is possible to represent degrees of doubt or belief as probabilities—subjective probabilities—which can then be incorporated in the calculations we can perform to work out the likelihood of surviving or blowing up at the bomb party. For instance, suppose that we suspect Doctor Fischer is bluffing about the presence of a bomb but, knowing him as we do as a man whose sense of humour is rather abominably cruel, we are not really very sure. If our belief in the presence of a bomb was rather uncertain we might say the odds were even, or $\frac{1}{2}$. This means that we think the chances of being blown up are half what they would be if we were absolutely certain that one of the crackers contained a bomb. So the chances of the first guest blowing up are $\frac{1}{2} \times \frac{1}{6}$ or $\frac{1}{12}$. But what about the chances of the second and subsequent guests blowing up?

Imagine you are an observer, at a whole series of bomb parties of which half turn out to be hoaxes. At any one party, as time goes by and guests pull crackers without exploding, you would start to get more confident that you are witnessing a hoax. If we are not at a hoax bomb party we would expect the bomb to explode before the last cracker is pulled $\frac{5}{6}$ of the time $\left(\text{only } \frac{1}{6} \text{ of the time will the last cracker contain the bomb}\right)$. But at hoax parties there is never a bomb so while there are fewer real bomb parties that keep you in suspense till the end, all the hoax parties do. This means that the relative proportion of occasions at which we witness a bomb party and ultimately discover it to be a hoax will increase as a function of the number of innocuous crackers that have been pulled.

So how do we calculate the probabilities for each guest being blown up? Implausible as it may sound a posthumously published theorem derived by the Reverend Thomas Bayes, a pastor who preached at Tunbridge Wells in the eighteenth century, provides the wherewithall to complete the task. Bayes' theorem allows us to compute probabilities for hypotheses (such as the likelihood that we are at a real or a hoax bomb party) providing we can state an initial belief in these hypotheses and the likelihood under each hypothesis of events occurring (such as the pulling of explosive or non-explosive crackers). Using Bayes' theorem at the bomb party would tell us that our degree of belief in the hypothesis that we are at a real, rather than a hoax, bomb party after one cracker is pulled and fails to explode is reduced from $\frac{1}{2}$ to $\frac{5}{11}$ (see Appendix for calculations). After two fail to explode Bayes tells us that the probability goes down to $\frac{4}{10}$. After three it is $\frac{3}{9}$, after four $\frac{2}{8}$ and after five $\frac{1}{7}$.

The probability of the second guest blowing up can now be calculated. If there really is a bomb in one of the crackers and *if* the first guest does not blow up then the probability is $\frac{1}{5}$ (under these two conditions there would be five crackers left and one will contain a bomb). So to take these uncertainties into account we must multiply that $\frac{1}{5}$ by the probability that there really is a bomb in one of the crackers and by the probability that the *first* guest *does not* blow up. Obviously if the first guest blows up all the others know they are safe. Remember the Reverend Bayes' theorem tells us that if the first guest fails to explode, this should be considered sufficient evidence to reduce belief in the likelihood of the existence of a bomb from $\frac{1}{2}$ to $\frac{5}{11}$. $\frac{1}{5}\times\frac{5}{11}=\frac{1}{11}$ so the second guest should feel that there is a $\frac{1}{11}$ chance of being exploded—if the first guest does not explode. Remember we said the chances of the first guest blowing up were $\frac{1}{12}$, so the chances of the first guest surviving must be $\frac{11}{12}$. If we multiply $\frac{1}{11}$ by $\frac{11}{12}$ we get the true likelihood of the second guest blowing up. $\frac{1}{11}\times\frac{11}{12}=\frac{1}{12}$, the same probability as the first guest!

By now it may not be so astonishing to discover that all the guests have the same chance of being destroyed. Using the same reasoning as before, and combining the probabilities for the relevant conditions (that of the crackers left you will pull one containing the bomb, that the whole party is not a hoax and that an earlier guest has not already detonated the bomb) the chances always work out the same So guest number three's chances are $\frac{1}{4}\times\frac{10}{12}\times\frac{4}{10}=\frac{1}{12}$. Guest four's chances are $\frac{1}{3}\times\frac{9}{12}\times\frac{3}{9}=\frac{1}{12}$. Guest five's are $\frac{1}{2}\times\frac{8}{12}\times\frac{2}{8}=\frac{1}{12}$. Guest six's are $1\times\frac{7}{12}\times\frac{1}{7}=\frac{1}{12}$. On the basis of probability then there is no rationale for a preference for going in any particular position in the order.

Perhaps it is not altogether surprising that the characters in the story misunderstand the probabilities at the bomb party. After all, it is not immediately obvious that the probabilities are as they turn out on closer consideration to be. However, it is intriguing to note that the author, Graham Greene, has experience of something rather akin to the situation in which Doctor Fischer placed his guests. In his autobiographical book "A Sort of Life"[2] Greene records that in the autumn of 1923 he had been reading a book which described how the White Russian officers, condemned to inaction in southern Russia at the tail-end of the counter-revolutionary war, invented the game now known as Russian roulette with which to escape boredom. One man would place a bullet in one of the six chambers of a revolver and turn the chambers at random before passing the gun to a companion who would put it to his head and pull the trigger. Greene notes that the chances here are 5 to 1 in favour of life (just as Doctor Fischer had reminded Mr. Kips before he left the bomb party).

Greene describes how, having read about Russian roulette, he found a revolver together with some bullets that belonged to his elder brother. Without hesitation he took it and walked across the local common to a remote spot where having loaded it with one bullet he put the muzzle into his right ear and pulled the trigger. He describes the intense jubilation that was provoked by the tiny click that signalled his survival of this experience—looking at the chamber he saw that the bullet had now moved into the firing position, "I was out by one" (p. 94). He played Russian roulette on several other occasions but the thrill gradually diminished:

"It was back in Berkhamsted during the Christmas of 1923 that I paid a permanent farewell to the drug. As I inserted my fifth dose, which corresponded in my mind to the odds against death, it occurred to me that I was not even excited: I was beginning to pull the trigger as casually as I might take an aspirin tablet. I decided to give the revolver—since it was six-chambered—a sixth and last chance. I twirled the chambers round and put the muzzle to my ear for a second time, then heard the familiar empty click as the chambers shifted." (p. 95)

Of course, on each separate game of Russian roulette the chances of surviving are $\frac{5}{6}$. However, the chances of surviving six games (providing the chamber is spun randomly after each game—otherwise there is no chance) are $\left(\frac{5}{6}\right)^6$ which is $\frac{15\,625}{46\,656}$ which is just a little over $\frac{1}{3}$.

Russian roulette, as played by Graham Greene, is a little different from Doctor Fischer's bomb party even though, initially at least, the odds "are 5 to 1 in favour of life" in both cases. The bomb party is a case of sampling without replacement; the guests either blow up or keep their prize; but they do not replace the crackers they remove. This means that as the party progresses, *providing no-one blows up* (and we have seen how significant this qualification is), the chances of survival do indeed narrow. Russian roulette on the other hand is a case of sampling *with* replacement; here the chamber is spun afresh each time the game is played—at least in the version played by Green it was—and if the gun did go off another bullet would have to be placed in the gun before anyone could play again. So if a group of people sat down to play Russian roulette in this fashion we would imagine there would be no particular preference for going first—although, as we have shown, there is no rational basis for going first at a bomb party.

There are other examples of literary mention of probability concepts being introduced by writers with first-hand experience to advise them. Fyodor Dostoyevsky was a Russian with a passion for roulette. His novel "The Gambler"[3] describes the absurd and desperate straits that befall compulsive gamblers. The inspiration for this work undoubtedly came from his own rather traumatic episodes at various casinos throughout Europe. The first time he played he managed to come away from the tables with some profit. However, after that, the usual pattern was that he would lose every penny, pawn anything of value and, being unable to pay his hotel bill or buy meals, write to his wife to ask for money to rescue him from destitution. When she sent it he would, after settling his bills, stop off at the roulette tables on his way to the train and again lose everything so that he would not even be able to pay for the ticket home.

In spite of his experiences, however, (or maybe this explains them) he, and the characters he describes in "The Gambler" entertain some misconceptions about the nature of chance. In 1863, after his first, winning acquaintance with roulette he wrote to his sister-in-law saying: "Please don't think I am so pleased with myself for not losing that I am showing off when I say that I do know the secret of how not to lose but win. I really do know the secret; it is terribly silly and simple and consists of keeping one's head the whole time, whatever the state of the game, and not getting excited. That is all, and it makes losing simply impossible and winning a certainty" (*Ibid* p. 11). These comments seem rather more like symptoms of the psychopathology of gambling than helpful prescriptive advice to roulette players.

In the novel the narrator, Alexis Ivanovich, attempts to advise "Grandmamma" how to stake at roulette: "Grandmamma, *zero* has only just turned up," I said, "so now it won't turn up again for a long time. You'll lose a lot of stakes; wait a little while." (p. 90). Of course, there is nothing to be gained by waiting; roulette wheels have no memory so the chances of any outcome occurring are unaffected by the recency with which it last occurred. As it turns out *zero* turns up on two of the next three plays and Grandmamma wins a fortune and, tragically, is convinced that she can win again.

Although examples of fallacious reasoning are evident throughout the book we can give Dostoyevsky some credit for spotting fallacious reasoning himself: "Two days earlier I had been told that the previous week red had won twenty-two times running; nobody could remember such a thing happening in roulette, and people were talking about it with amazement. Of course, in such a case everybody immediately stops staking on red, and after it has come up say ten successive times, hardly anybody at all risks a stake on it. But at such times no experienced player will stake on the opposite colour, black, either. Experienced players know the meaning of such "freakish chances." One might suppose, for example, that after red has come up sixteen times, on the seventeenth it will inevitably be black that does so. Novices rush to this conclusion in crowds, double and treble their stakes, and lose heavily" (p. 132).

Although somewhat loosely defined this is an account of what is now widely known as the "gambler's fallacy"—that is the erroneous belief that long runs of one particular outcome must be balanced out by a consequently stronger tendency for the alternative outcome(s) to occur.

Reasoning with chance is notoriously perilous and there are many cases of eminent and respected intellects having been conned by their own vulnerable intuitions into making mistakes. Novelists, who have often been credited with having greater insight than psychologists into the motives that govern people's choices in a largely uncertain world, can provide a useful source of convincing and vivid descriptive accounts of human reasoning. But, when it comes to deciding what you *ought* to think, rather than what people do seem to think, do not trust them either.

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References

1. Greene, G., "Doctor Fischer of Geneva or The Bomb Party," London, The Bodley Head, 1980.
2. Greene, G., "A Sort of Life," London, The Bodley Head, 1971.
3. Dostoyevsky, F. M., "The Gambler," translation by Coulson, J., Harmondsworth, Penguin, 1966.

Appendix

From Bayes theorem we know that:

$$\frac{p(H_1/D)}{p(H_2/D)} = \frac{p(H_1)}{p(H_2)} \times \frac{p(D/H_1)}{p(D/H_2)}$$

If the first cracker is pulled and does not explode then, substituting the likelihoods and prior beliefs $\left(\frac{1}{2}\right)$ gives

$$\frac{p(\text{real bomb party/ first cracker fails to explode})}{p(\text{hoax bomb party/ first cracker fails to explode})} = \frac{0.5}{0.5} \times \frac{\frac{5}{6}}{1.0}$$

As we are either at a real or a hoax party then the numerator and the denominator on the left-hand side of the equation must add to one. Thus the probability that we are at a real bomb party given that the first cracker does not explode is $\frac{5}{11}$.

Similarly, the probabilities that we are at a real bomb party if the later crackers do not explode are:

$$\frac{4}{10}, \frac{3}{9}, \frac{2}{8}, \quad \text{and} \quad \frac{1}{7}.$$

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