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THE PHASE TRANSITION IN HUMAN COGNITION

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This article attempts to build a bridge between cognitive psychology and computational neuroscience, perhaps allowing each group to understand the other's theoretical insights and sympathize with the other's methodological challenges. In briefly discussing a collection of conceptual demonstrations, neural network and dynamical system simulations, and human experimental results, we highlight the importance of the concept of phase transition to understand cognitive function. Our goal is to show that viewing cognition as a self-organizing process (involving phase transitions, criticality, and autocatalysis) affords a more natural explanation of these data over traditional approaches inspired by a sequence of linear filters (involving detection, recognition, and then response selection).

Keywords: Cognition; dynamical systems; emergence; phase transition.

1. The Phase Transition Itself

In a variety of sciences, the identification of a phase transition is often exploited as evidence for two separate regimes of behavior in the same substrate, each of which then requires its own independent form of explanatory account.[1,2] Overemphasizing this divide-and-conquer perspective on the scientific function of phase transitions could be missing out on an important opportunity to potentially discover mechanisms that drive those transitions. By also focusing our measurements on the phase transition itself, to understand the behavior of the system in question during that interregnum between one stable phase and the other stable phase, a deeper understanding of the mechanisms may be revealed.

A phase transition is an impressively sudden shift in a system's behavior. Take, for example, water transitioning into vapor. There is a wide range of temperature increases (from 33° to about 211° Fahrenheit, depending on the air pressure) where a body of water's behavior, i.e. standing still, does not change at all. Then, suddenly, increasing the temperature by a couple more degrees above 211 causes the water to take on a life of its own, bubbling and roiling. In the other direction, a few degrees below 33 causes a rapid restructuring of the body of water, and intricate lattices of molecules form a solid surface on which hockey can suddenly be played. These are all one substrate, but vastly different behaviors appear quite suddenly under different conditions.

Like other signatures of complex dynamical systems that show up in human data, such as $1/f$ scaling[3] and sensitivity to stochastic resonance,[4] exhibiting a phase transition may also be a prominent signature in human cognition — where initial increases in some parameter have little or no effect on the system behavior but at some point a threshold is crossed, and tiny increases in that same parameter suddenly induce massive changes in system behavior. This article describes experimental laboratory data on phase transitions in human action, perception, language, and cognition, with an eye toward drawing some parallels with similar swift jumps in the behavior of complex dynamical systems and simulations.

There indeed appear to be a range of psychological phenomena that exhibit qualitative shifts from one stable state to another stable state, almost as if one symbolic representation of mental contents is being turned off and another turned on.[5] At relatively coarse time scales of measurement, this account may be embraced from a perspective of explanatory pluralism.[6] Nonetheless, such discrete-like behavior admits of a finer-grain time scale of measurement that exposes details *in the transition itself*, uncovering the lower-lever mechanisms that enable the emergence of symbol-like behavior. If a dynamical system actually allowed extreme parameter changes from one instant in time to the next (as would be required by genuine symbolic processing) it would run the risk of oscillating *around*, rather than settling at, the regime it is moving toward during a phase transition.[7] Therefore, most dynamical systems that appear to undergo succinct and sudden phase transitions from one stable regime to another may actually be functioning on a finer time scale than is being measured. That is, the mechanisms that are carrying out the transition itself are doing so at an iteration-by-iteration scale that has the system spending a number of timesteps in a region of parameter space that is in between the two identified phases.

The pioneering work of Walter Freeman has served to lead the field toward this important path of understanding neural dynamics *using* dynamics. Among his numerous influential contributions, his work on the phase dynamics of the rabbit's olfactory bulb provides perhaps one of the earliest indications of the importance of this dynamical process (reviewed in Refs. 8 and 9). In these studies, it is found that the olfactory bulb has a baseline level of neural activity that is chaotic in nature. Upon receiving afferent sensory input to the bulb, a phase transition can occur, where neural firing patterns engage in synchronized waves in the gamma frequency range, and thus underlie the recognition of some particular known scent. Following repeated stimulation — such as in conditioning a new scent — a broader change in the system's chaotic dynamics can occur, in which the overall baseline activity is transformed substantially (though still gamma, and chaotic). Still, the animal can recognize both the previous and new scents. The precise identity of the neural firing does not manifest the recognition of the odor — it is instead these chaotic dynamics giving way to context-dependent phase transitions.

The phase transition is thus a potentially crucial characteristic of an organism's perceptual dynamics. Following on this fundamental insight, this paper will briefly touch on several areas relevant to neural and cognitive processes where the concept of a phase transition can be especially illuminating, including rhythmic movement patterns, visual perception, language processing, and problem solving. The latter portion will focus on some recent experimental results where a phase transition appears to occur between incoherent and coherent comprehension of sentences. The key lessons for cognitive science in this exploration are: (i) sharp transitions in behavior need not be attributed to formally discrete logical processes, but instead can emerge from non-linear dynamics in the continuous interactions within an aggregate system, and (ii) finding the appropriate temporal scale of analysis is crucial for identifying those non-linear (but numerically continuous) dynamics.

2. Phase Transitions Between Rhythmic Movement Patterns

The phase transition can be demonstrated using surprisingly simple systems. One dynamical system that demonstrates sharp changes in state is extraordinarily simple and often used as an illustration of "bifurcation" — points at which stable states double in number, thus creating a period-doubling regimen under sharp transitions. This system, known as the logistic map, is an iterated dynamical system given by the equation: $x_t = rx_{t-1}\ (1 - x_{t-1})$. The value r is a control parameter that can dictate the stable values that the system's state x can achieve. The system's behavior changes dramatically at critical values of r, and the number of stable x values doubles, or bifurcates. This is shown in Fig. 1. For a range of low r values, only one stable state occurs after multiple iterations. At a critical juncture in this control variable, the number of states doubles. Now the system oscillates between two values. These are visited in alternating order, so the system's evolution has a period of two. This period doubles again at a juncture of about $r = 3.4$. Four stable states now occur, and are visited in a particular order by the system (thus having a period of four).

These transitions are often referred to as bifurcations, but they serve as another demonstration of phase transition behavior, albeit simple. In an animal's motor system, a similar transitioning can occur in systematic ways. This is particularly true of the "modes" of rhythmic behavior that organisms exhibit. Probably the

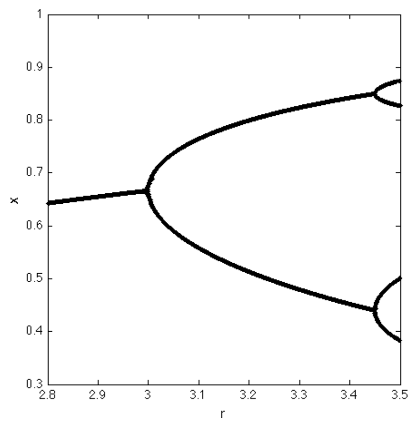

Fig. 1. Representation of stable states (x) achieved by logistic map iterations as the control parameter (r) increases. At about $r = 3$, the system bifurcates, and two stable, oscillating values occur. At approximately 3.4, this phase transition occurs again, and the system occupies 4 states.

most frequent rhythmic movements exhibited by organisms are those involved in locomotion. Consider the horse's gait. There are three prominent modes of locomotion that this animal exhibits: walk, trot, and gallop. These terms connote certain rates of movement, the next being at a faster rate of locomotion than the previous. The actual locomotory patterns themselves are manifested by differences in the relative phase of the horse's limbs. For example, during gallop, the front limbs bound forward and backward in relative synchrony, while in a trot one forelimb coordinates with its opposing hind limb. Controlled experiments demonstrate that these gaits will transition from one to another predictably by the rate at which a treadmill is moving.[10] These rhythmic movements thus exhibit a phase transition, with distinct changes in limb phase patterns under boundaries of the control parameter of movement rate. The same can be shown even in the American cockroach. At a low rate of movement, the gait of this insect involves all six limbs, moving in an alternating tripod manner.[11] As the rate of movement is increased, the cockroach will move into a quadrupedal phase, then even a bipedal locomotory phase at sufficiently high speeds. This phase transition between modes of locomotion is predictable by the rate at which the movement is occurring, much as one can predict bifurcation of the logistic map under changes in the values of r. Interestingly, the phase transition described in locomotory modes above are also shown by humans when arms and legs move together, with phase transitions at critical rate points as well.[12]

A prominent model for this kind of phase transition in rhythmic movement originated with the work of Haken, Kelso, and Bunz (HKB) who initially modeled the biphasic manual coordination present in finger movements.[13,14] This classic "finger twiddling" experiment is quite simple, and demonstrates the exact sort of phase transition described in non-human animals above. In this experiment, participants are instructed, with hands held out together, to move their index fingers rhythmically (side to side) in step with a metronome. At a slow rate, movement has two stable phases, either antiphase (symmetric muscle groups operating in opposite phase, i.e. fingers moving left or right together) or in phase (muscle groups acting in the same patterns, i.e. fingers moving outward and inward together). As the rate of movement increases, however, a phase transition occurs if the current stable mode is the antiphase pattern: Participants involuntarily switch to the inphase mode of movement. Shortly before this motoric change in phase takes place, instabilities in the motor movement itself (and in the accompanying neural synchrony) are detectable, presaging the dissolving of the old phase and assembling of the new.[15]

The HKB model characterizes this relative phase of the movements as a differential equation that describes the potential wells into which the finger movement may settle (or remain). This function changes according to parameters that essentially reflect the frequency of the required movements. Figure 2 shows two potential functions, reflecting opposing endpoints in gradual change of this parameter. The phase shift from antiphase to inphase can thus occur as a relatively rapid shift under this rate parameter. This has been substantiated thoroughly in a variety of cognitive and rhythmic motor tasks (see Ref. 24 for review).

These phase transitions in modes of rhythmic movements show what may be termed an "intrinsic dynamics"[16] that an organism's motor system can naturally

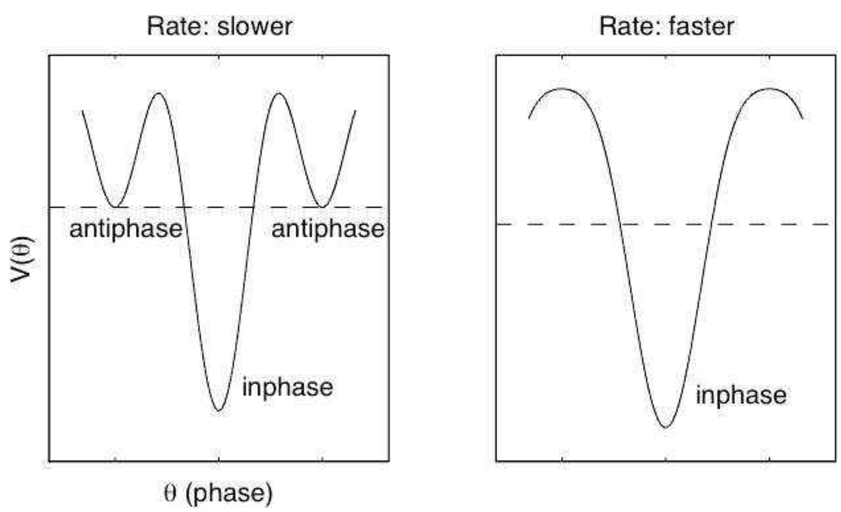

Fig. 2. The Haken-Kelso-Bunz (HKB) model of coordination dynamics. The x-axis shows relative phases of two-finger movement. The y-axis displays the potential, with wells representing possible stable phases of finger movement. During slower movement of the fingers (left panel) two basic stable phases are possible (antiphase or inphase). At higher rates of movement (right panel), there is only one stable pattern of inphase. As finger-movement rate increases, participants performing antiphase movement will transition into inphase.

exhibit. It is possible, however, that these intrinsic dynamics serve as an important characteristic of a motor system that is coupling to oscillatory perceptual information in the environment. A classic experiment by Schmidt, Carello, and Turvey[17] has shown that when finger twiddling becomes leg swaying, these phase transitions between rhythmic modes can even occur *between people*. They had pairs of participants each sway a single leg while seated beside each other. When they were able to perceive each other's leg movements, the same phase tendencies as in Fig. 2 were exhibited by this two-person system. Moreover, when subtle modulation of metronome rate occurs (even unbeknownst to subjects), rapid compensation can occur in the relative asynchrony of tapping shown by participants.[18,19] For example, while participants tap to a metronome pulse, inter-pulse intervals that change by ± 10 ms can be rapidly compensated for within just 2 or 3 taps. This may be interpreted as rapid and "subliminal" phase transitions in sensorimotor coordination. In fact, the same sorts of phase transition seen in these intrinsic motor dynamics can also be shown in the dynamics of visual perception.

3. Phase Transitions in Visual Processing

Transitioning from one phasic pattern to another phasic pattern is exactly what the Lorenz attractor is known for. Edward Lorenz discovered this three-dimensional strange attractor in 1963 while exploring simplified equations for convection rolls in the atmosphere. Often described as looking like a butterfly, the Lorenz system is the result of three simple equations whose products feed into each other. The typical parameters that produce its famous pattern are as follows:

$$dx/dt = 10(y - x)$$
$$dy/dt = x(28 - z) - y$$
$$dz/dt = xy - (8/3)z$$

The Lorenz attractor is deterministic yet unpredictable, and thus is referred to as chaotic. It also exhibits sensitivity to initial conditions. In a time series where each iteration increments the values by 0.01, the trajectory of the system alternates erratically between two expanding orbits. Figure 3 displays the Lorenz attractor, starting with random x, y, and z values, and unerringly settling into its butterfly pattern. We have highlighted in solid lines the transitional portions of the trajectory as it crosses over from the outer edge of one wing to the inner edge of the other, 20 time-steps before and after the midpoint ($x = 0$). The transitional sections are some of the fastest moving sections of the Lorenz trajectory, but they are not instantaneous.

The x dimension by itself displays quasi-stable activity hovering around $x = 8$ that suddenly transitions to quasi-stable activity that hovers around $x = -8$ and then returns to the $x = 8$ region, etc. Looking simply at the sign of these x values produces a series of alternating transitions (Fig. 4) that are similar to the perceptual

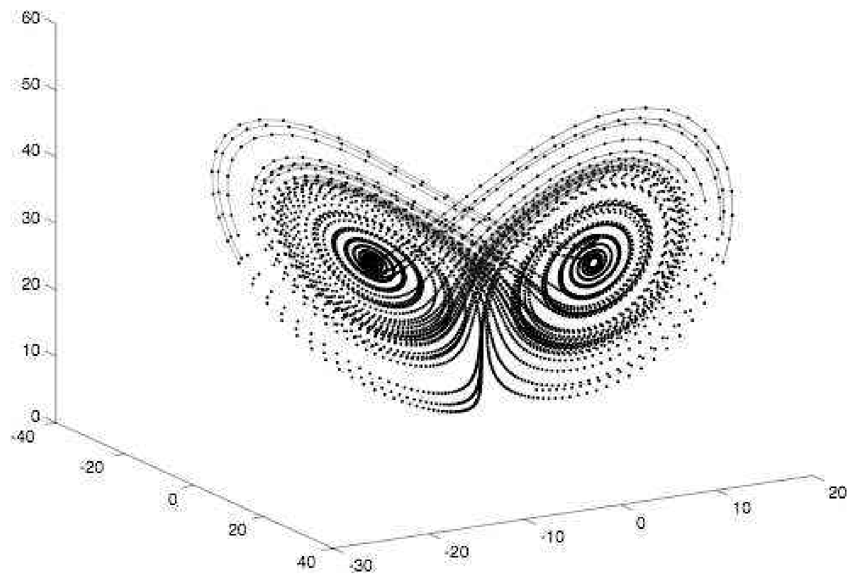

Fig. 3. The Lorenz attractor with transitional sections highlighted as solid lines.

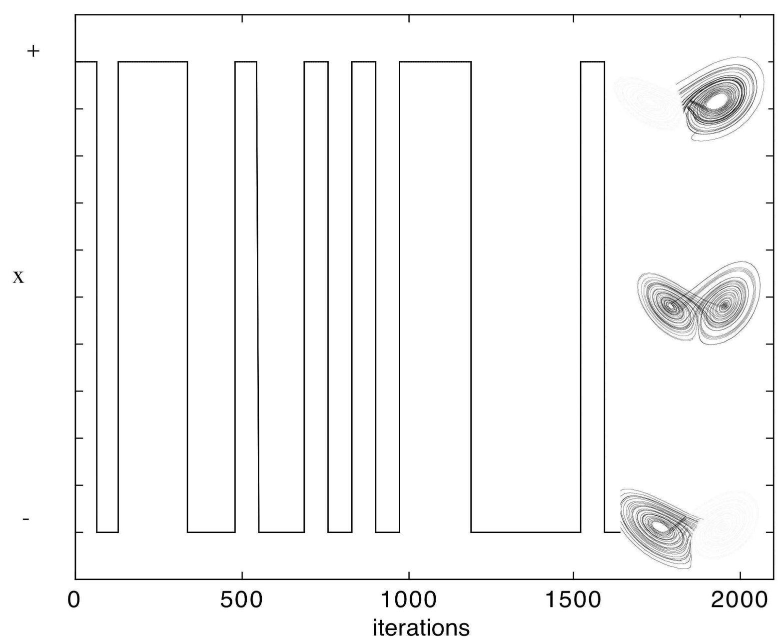

Fig. 4. Sign of x-values over time in the Lorenz attractor.

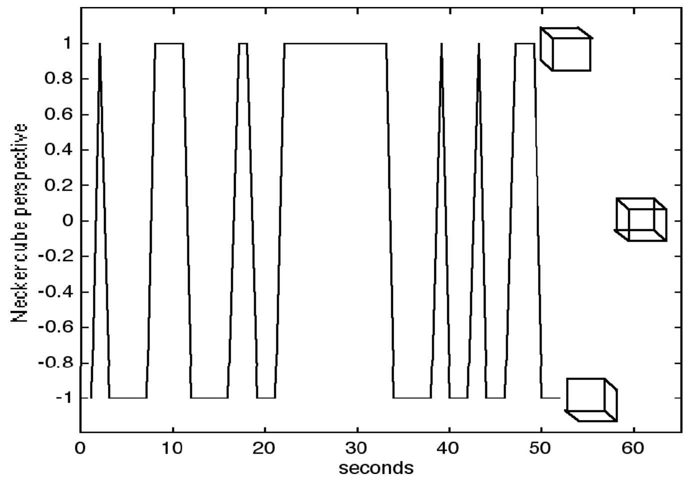

Fig. 5. Alternating perspectives over time while viewing the Necker cube.

reversals that people experience when gazing at an ambiguous figure such as the famous Necker cube (Fig. 5).

The vast majority of research on the Necker cube focuses on what kinds of contextual perturbations influence which perspective becomes more dominant on average.[20,21] However, some researchers have conducted statistical analyses of the time series of dwell times, i.e. how long a perspective holds sway each time before giving way to the alternative, and often find evidence for fractal structure, or $1/f$ scaling, in the data[22] — suggesting that the human brain is a self-organized system poised at criticality between stability and chaos.[23] In fact, by infusing their neural network with a self-organized criticality parameter, Aks and Sprott[24] were able to simulate a sequence of dwell times that shared the same statistical structure as human dwell times (see also Ref. 25 for early work on this).

Still, what researchers have yet to focus on with the Necker cube is the time it takes to carry out the shift from one stable percept to the other, i.e. the transition itself. Just like with the Lorenz attractor, this transition is rapid, but not instantaneous. Informal inquiry with dozens of observers suggests that the perceived transition time may average around one-third of a second (see Ref. 26, Chapter 1; also Ref. 27 for a corresponding numerically derived prediction).

In the field of visual perception, one-third of a second is a substantial amount of time to be *in-between* the two possible perceptual states afforded by a stimulus. Interestingly, analyses of multi-cell recordings in the superior temporal sulcus of the monkey (an object/face recognition area) show a gradual increase, over the course of about 1/3 to 1/2 of a second, in information content in the firing rates for discriminating a face or object.[28] Much like the generic phase transition pattern, this information content curve rises quickly at first (over the first couple hundred milliseconds; see also Ref. 29) and then the slope shallows as it approaches asymptote (over the next couple hundred milliseconds).

Binocular rivalry, where differing images presented to each eye generate object representations that compete with one another, also exhibits these alternating transitions.[30] Importantly, a different phase-locked pattern of activity in visual cortex appears to underlie each of the two percepts, and thus the transition between the two percepts is quite literally a transition between two phasic patterns[31] (see also related work in Ref. 32, showing coordinated phase transitions across various sensory brain areas). Like the Lorenz system and the Necker cube, these transitions are not instantaneous, but take at least a couple hundred milliseconds. What this reveals is that on the way toward achieving a stable percept, the brain spends a significant amount of its time in regions of phase space that do not neatly correspond to any of the labeled categories that language, or the experimenter, or society itself, has laid out before it.[26] Inside these interregna, we may find the secrets to understanding the real-time dynamics of cognition.

4. On Finding the Optimal Averaging of Measurements

Importantly, plumbing the insights into the mind that hide inside the phase transition often requires finding the optimal window of temporal resolution. For example, quasiperiodic behavior such as that in logistic map or in the Lorenz attractor can appear very different to an observer depending on how many time steps the observer's measurement is averaging over. This is why one must endeavor to get one's epistemic measurement process as close as possible to the raw ontic data stream itself.[33,34] Take, for example, what happens when the Lorenz attractor's time series is averaged over a sliding temporal window that is the inverse of the iterative incrementing amount, in this case 100 times steps. Figure 6 shows what we call the Mardi Gras Mask version of the Lorenz system with this averaging window. The figure-eight in the middle of the mask results from portions of the original trajectory that follow only one orbit around one of the lobes before transitioning back to the other lobe, whereas the tight loops on the left and right sides are portions of the original trajectory that remained in orbit around one lobe for longer periods of time.

As further evidence for the need to get closer to the raw ontic stream, not only can it be misleading to average over chunks of time that are too large, but it can also be misleading to summarily average over experimental participants or stimulus items (as is too often done in cognitive psychology experiments). Individual stimulus items or individual participants in an experiment can often exhibit very sharp transitions as a function of some continuous manipulation. Note the sharp transitions from 0 to 1 (dashed lines) in Fig. 7 that occur at a wide variety of locations along the x-axis (with an even distribution). Each event, in terms of how it actually occurs, is clearly a sudden phase transition, but when they are averaged (solid line with asterisks) they produce a steady linear function that can lead the

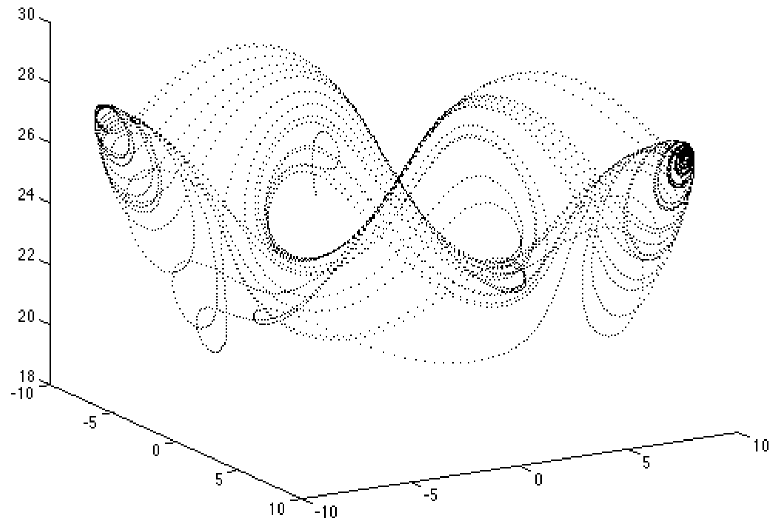

Fig. 6. The Lorenz attractor plotted with a sliding averaged-window of 100 time steps.

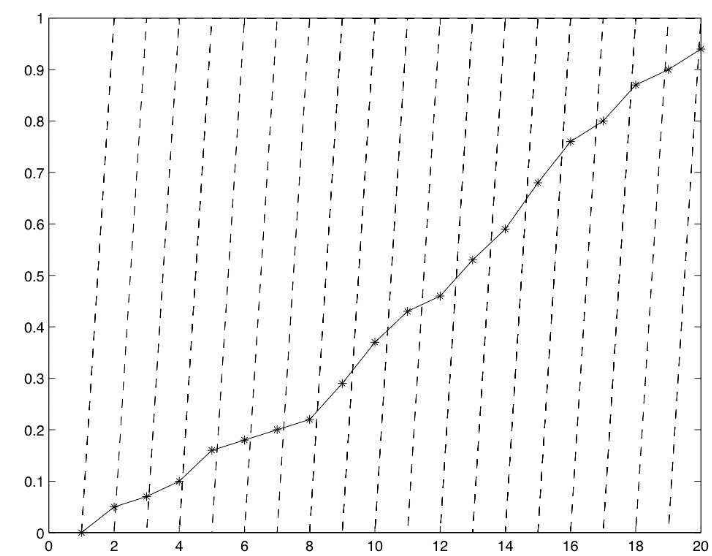

Fig. 7. When individual step-functions are averaged (multiple dashed lines are level at 0 until they jump to 1), this average can produce a linear function that will mislead inquiry into the underlying mechanisms.

observer to assume that the process underlying the transition from 0 to 1 is a linear process. By examining the phase transitions that take place in individual participants or in individual stimuli, misleading interpretations such as this can be avoided.

5. Phase Transitions in Language Processing

Speech perception is one field where this concern about averaging over experimental participants has come to the fore of major theoretical debates.[35,36] As an idealized example, if one participant shows a sharp transition between two ways of categorizing a linguistic stimulus when the phonetic parameter is set around 4 in Fig. 7, another exhibits his sharp transition around 10 on the x-axis, and a third participant produces her sharp transition around 16 on that x-axis, then averaging across those three participants would cause one to miss the fact that the categorical transition, when it actually happens, is quite sharp.

For half a century, it has been known that a continuous linear change in certain phonetic parameters, such as Voice Onset Time (VOT, the latency between when the air is first released and when the vocal chords begin to vibrate), can produce a remarkably discontinuous non-linear change in perception.[37] When the VOT for the phoneme /b/ is synthetically altered from, say, 0 ms to 50 ms in 5 ms steps, what results is a continuum of voicing for that speech sound that steadily spans from a typical-sounding /b/ to a typical-sounding /p/. However, a listener's perception of this continuum is anything but steady. Native English speakers usually perceive the first five tokens of that voicing continuum as equally acceptable instances of the phoneme /b/, even the one with 25 ms of VOT! Likewise, the last five tokens of the voicing continuum are usually perceived as equally acceptable instances of the phoneme /p/. Thus, as one gradually increases VOT in this synthesized speech sound, at around 30 ms of VOT a phase transition occurs where the speech sound rather suddenly shifts from being confidently perceived as a /b/ to being confidently perceived as a /p/ (Fig. 8).

Traditional approaches to cognitive psychology tended to interpret this finding as evidence for speech perception involving a specialized domain-specific neural mechanism that immediately slotted noisy imperfect acoustic signals into neat and tidy linguistic categories, such as "voiced" and "unvoiced." However, the process itself is not quite immediate. Even when the intermediate speech sounds (with VOTs of 20 or of 30 ms) are systematically perceived as belonging to the same category trial after trial, the response times on those trials are reliably longer than on trials with more extreme VOTs.[38] That is, despite the fact that a 20 ms VOT stimulus is almost always categorized as a /b/, participants take about 100 ms longer to settle on that categorization. This, in fact, fits perfectly with the predictions of dynamical neural network simulations in which both category representations are partially active and compete against one another for the privilege to drive motor output.[39,40]

Figure 9 shows activation curves from six simulations of categorical speech perception with a localist attractor network (for details, see Ref. 26, Chapter 6). Each simulation produces a symmetric bifurcation where one category representation (corresponding to a particular pattern of neural activity) rises in activation over time (solid lines) and its competing category representation declines over time (dotted

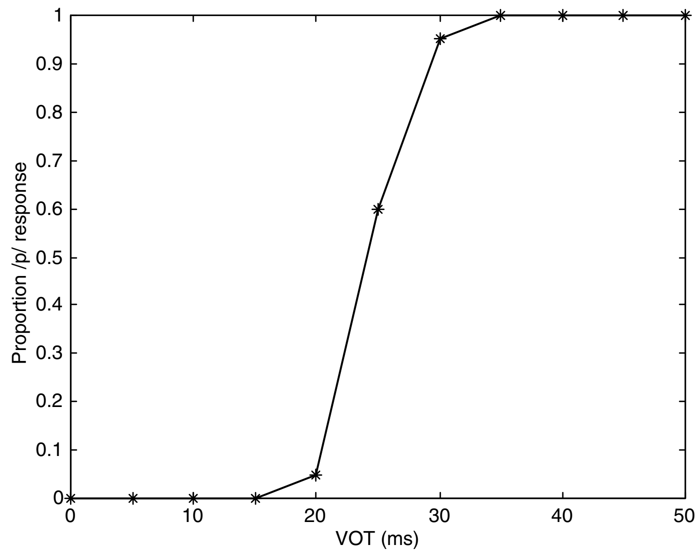

Fig. 8. Example categorization function for a phonetic continuum, describable as a phase transition occurring near a particular phonetic parameter value.

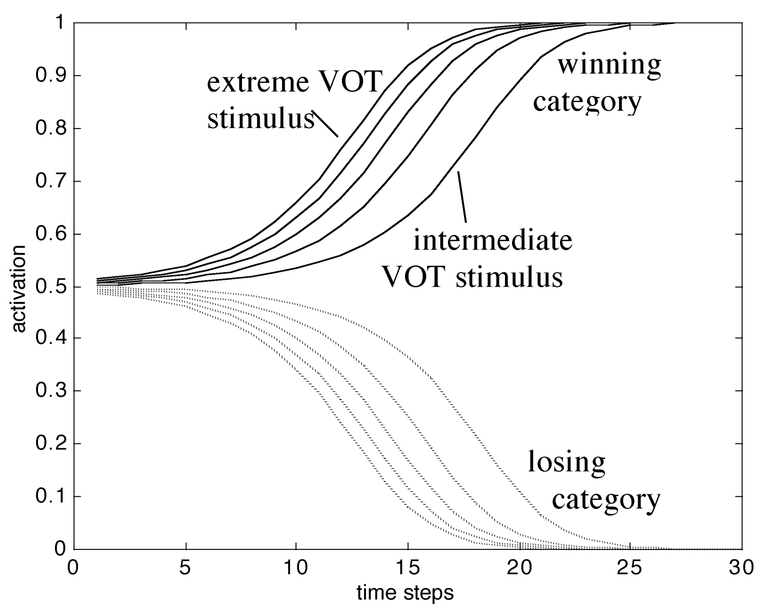

Fig. 9. Activation curves of two categories competing over time. (Each pair of diverging curves comes from a different simulation.)

lines). With stimuli that are more ambiguous or intermediate in their classification, the winning activation pattern takes longer to reach asymptote, hence longer reaction times despite the same categorical result. Importantly, this behavior arises from a relatively generic interactive dynamical system that spends a notable portion of its time in regions of state space that are consistent with both possible categories being treated as partially acceptable simultaneously — not a domain-specific system that immediately slots its inputs into linguistic categories without being affected by the idiosyncracies of the original acoustic signal. Eye-movement data show corroborative evidence for brief simultaneous consideration of both competing alternatives when the speech stimulus has an intermediate VOT.[41]

Another area of language research where a sudden shift in cognitive processing seems to occur is word learning. However, instead of calling it a "phase transition," developmental psychologists have taken to calling it a "vocabulary spurt."[42] Around their second birthday, many children begin to exhibit a dramatic increase in the rate of new words being learned, more than 10 new words a day, and thus thousands of new words over the next year of life. In the field of cognitive psychology, this observation is often attributed in part to hypothesized domain-specific learning mechanisms that are active during the accelerated learning phase[43] or that precede and follow it.[44] However, the natural statistics of a dynamic word-accrual process that is continuous and parallel can very easily lead to a sudden ramp up in word accumulation rate all by itself.[45] In such a situation, a vocabulary spurt emerges rather simply from the aggregate behavior of the entire neural system doing the learning, rather than being caused by some specialized accelerated learning modules that get turned on and then turned off.

For example, take the simplifying assumption that the majority of words that a child has to learn are of medium difficulty, and a smaller proportion of words are very easy to learn or very hard to learn. Accordingly, imagine a normal distribution for the frequency histogram of "how long it takes" a child to learn her first 10,000 words, where hundreds of words can be learned in several months, thousands of words can be learned in a dozens of months, and hundreds of words require a few years or more to learn (Fig. 10(a)). All that we have to do is relinquish the implicit assumption from traditional word learning theories that a word is either known or not known, and instead accept the idea that a word can be partially-known. That is, its attractor basin in the dynamics of the child's brain can be *partly* formed. If all of these words are being learned in parallel, in that the neural activation patterns that will eventually become their "representations" are all simultaneously finding their places on the same attractor landscape, then it becomes trivial to see how a vocabulary spurt would arise.

During the first couple of years, the easy words are being acquired, such that a few of those attractors per day become sufficiently well-formed enough to allow the child to correctly produce those words, and the parent then writes them down as a "learned words." During the next couple of years, the medium-difficulty words

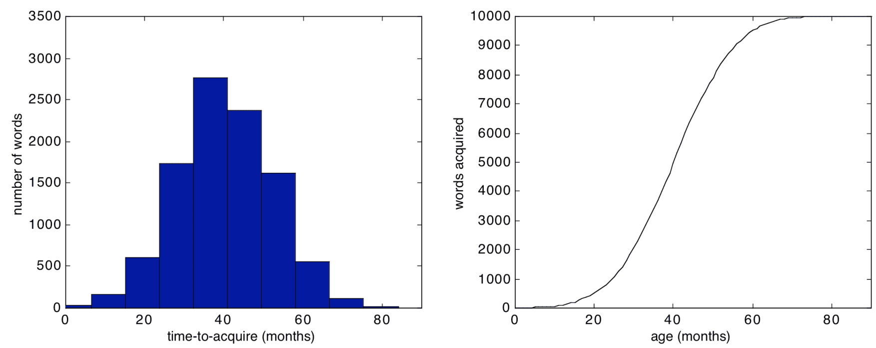

Fig. 10. (a) A hypothetical normal distribution of learning-times for 10,000 words. (b) The resulting logistic curve where the slope indicates the rate of accrual.

begin to accrue, and since there are many more of them, the rate of accrual cannot help but increase sharply at around 24 months and continue until at least 48 months (Fig. 10(b)). For the next couple of years after that, there are relatively few difficult words to be had, among these first 10,000, so the accumulation rate would slow down again (but, of course, words beyond those first 10,000 are also being learned at that time). For an in-depth treatment of this general argument, see Ref. 45.

As can be seen, despite its not always using the terminology, language research is no stranger to the phase transition. In wider support of the view of language processing as a complex dynamical system in the human brain (rather than a sequence of encapsulated modules), it exhibits a variety of recognized "signatures" of self-organized criticality, in addition to phase transitions. For example, the pattern of data over the course of many trials in a categorical speech perception task clearly show hysteresis effects.[46] Moreover, when the same spoken word is uttered several hundred times, the statistical variation across frequency bands in the spectral patterns exhibit unmistakable $1/f$ scaling properties.[47]

6. Phase Transitions in Problem Solving

Compared to language research, the field of problem solving is even further from the rising tide of dynamical system frameworks and neuroscientific evidence. However, this does not protect the data from *exhibiting* dynamical phenomena — they just rarely get analyzed as such. Take, for example, what cognitive psychologists call "insight problem solving." While standard linear problems, such as arithmetic or certain puzzles, imbue the solver with a sense of steady gradual effortful approach to the solution, insight problems usually cause the solver to go through a period of impasse, where (after some failed proposed solutions) no viable option seems forthcoming, and then suddenly out of nowhere (with an "Aha!") some portion of solvers experience the correct solution popping into their minds all by itself.

Knoblich, Ohlsson, and Raney describe this as a "restructuring of the problem representation."[48]

The actual process of this restructuring, however, does not quite arise "out of nowhere." There are intriguing laboratory hints of the gradual build up to the insight itself. For example, to test for evidence of partial activation of an "insight," researchers presented participants with a pair of remote-associate word triplets.[49] One of them would have a coherent solution and the other would not. For example, "What one word makes a compound with the words *still*, *pages*, and *music*?" and "What one word makes a compound with the words *playing*, *credit*, and *report*?" Participants were asked to find the solution to the coherent triplet that actually has a solution, and barring that, at least guess which word triplet has a solution at all. In trials where participants could not find the solution to the coherent word triplet, they could still nonetheless identify, more often than not, which triplet *had* a solution. Thus, some form of implicit knowledge was present in their brains, a subtle suspicion that *playing/credit/report* somehow was more likely to be the triplet that had a coherent solution — even when that coherent solution itself was not forthcoming. In fact, using the same kind of remote-associates task, Bowden and Jung-Beeman[50] have recorded lexical decision times to reveal significant priming for the *undiscovered* correct answers to remote-associate problems, such as "What one word makes a compound with the words *back*, *step*, and *screen*?"

As a further example of hints that precede a phase transition in reasoning, shortly before achieving the correct insight in a diagram-based version of Duncker's notoriously difficult tumor-and-lasers radiation problem, participants showed an increase in eye movements to a particular portion of the diagram[51] — and when the display lured their eyes to that region, solution rates doubled. In fact, even when a secondary task happened to force participants to move their eyes in that pattern, solution rates increased.[52]

Even outside of the realm of "insight problem solving," standard problem solving can sometimes show sudden realizations of how to streamline a solution. When participants are looking at displays of sequentially connected gears, and trying to determine what direction the last one goes, they often start out mentally animating each gear, meticulously reversing the rotation each time, and often assisting their mental animation with finger and eye movements.[53] After several such trials, participants suddenly realize that an even number of gears always reverses the initial gear's direction and an odd number of gears always maintains it. Importantly, shortly before they achieve this realization, while they are still using their mental animation strategy, the data reveal detectable instability in that strategy, as if it is beginning to dissemble to make way for the impending new strategy.[54] During the five trials that precede this cognitive phase transition in strategies, the records of their hand and eye movements show a reliable increase in entropy. Thus, even though they are still using this inefficient strategy on those trials, the neural pattern that generates this strategy is clearly becoming unstable.

Clearly, the data in the problem solving literature are in fact replete with phase transitions, even including some of the dynamical properties that go along with them, such as preparatory instabilities.[15,54] As the brain begins to shift from one task strategy to another, the phasic neural patterns associated with the to-be-discarded strategy begin to decohere, and different phasic patterns for the new strategy begin to take form.[55]

7. Phase Transitions Between Incoherence and Coherence

In this section, we introduce new data suggesting a phase transition between coherent comprehension and incoherent failure to comprehend. Let us start with an idealized simulation of a phase transition that Erdös and Renyi introduced 50 years ago.[56] In their random graph theory, one starts with a set of nodes and then one adds edges that connect those nodes (a little bit like a neural connection between a pair of units). If you start with 100 nodes, then the maximum number of edges possible (if you exclude edges that loop onto the node itself) is 9,900. However, it takes far fewer than that number of edges to have all 100 nodes form a single connected whole, such that every node is (at least) indirectly connected to every other node. Such a "giant component" in the network can, in principle, arise with as few as 99 edges. But if you added edges randomly from the start, how many would it take for that giant component to emerge?

As it turns out, that function is the phase transition that Erdös and Renyi discovered. The smooth curve in Fig. 11 shows the average of 10 runs of this simulation, where the largest connected component of nodes resulting from randomly placing

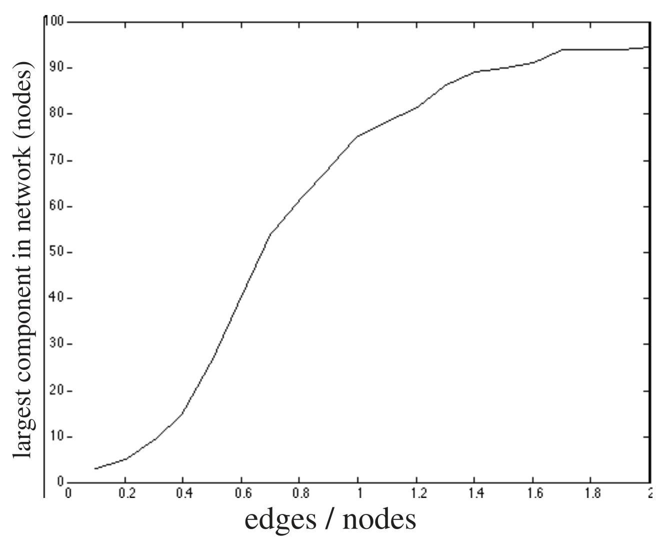

Fig. 11. Erdös and Renyi's phase transition showing the sudden emergence of a giant connected component among nodes being connected by randomly placed edges.

40 edges among those 100 nodes was a mere 15 nodes in size, but adding 20 more edges expands that component to about 40 nodes, and adding another 20 edges expands it to about 60 nodes. This elegant demonstration of structure arising quite abruptly out of a random serial accumulation process is often used as a keystone example of the concept of emergence.[57,58]

A strikingly similar emergence of *meaning comprehension* appears to take place with sentences that have had their characters and spaces randomly scrambled. By drawing an analogy between the accrual of randomly placed edges in a graph and the accrual of correctly placed characters in a sentence, we can predict an abrupt transition from scrambled gibberish to interpretable sentence (containing perhaps a handful of only mildly distracting typos remaining). Take, for example, the following four different sentences, with 21, 15, 9, and 3 characters randomly relocated, respectively.

(i) Noahr would iit e possble in many csaei aflor t hem io vebann eaaltt or y efectivenes on hhst thir wouncftersparts broad aere pid.
(ii) He flew abut tghew lace tmaksigou thtese andjents and t was obvious hat hiat hie was doin was the fr uitmof lpnocg experene.
(iii) H cuouoraeosly defendeed the rights of smal dnaions, ang he stood his grunda-galinst tt hesavage attacks of the Communist bloc.
(iv) t weighs n the tons, so the proximity of factory and exhibitiotn area makeIs it possible foir an outstanding exhibi each year.

Forty-seven undergraduates from the University of Memphis and Cornell University participated in the experiment for payment or extra course credit. Forty sentences were randomly selected from the online Brown Corpus[59] of written language use, with the following constraints: all had between 120 and 140 characters and spaces, and contained no more than one proper noun, no apostrophes, and no numerical digits. These sentences were then run through a Matlab script that randomly relocated between 3 and 21 (in increments of two) of its characters, creating ten scrambled versions of each of the original sentences. Ten lists were created, such that each participant saw all 40 sentences, but only one scrambled version of each. Participants were told that they would be reading sentences full of varying degrees of typos, and that their job was to try to figure out what each sentence was intended to convey before the typos were introduced. For each sentence, they were asked to answer two yes/no questions. First, they were asked whether they could tell what the basic topic of the sentence was. Second, they were asked if they thought they could correctly answer a comprehension question about the content of the sentence. For the present report, we focus on the result from this second more stringent question. The experiment took between 5 and 10 minutes.

Since each participant was only able to see one level of coherence for any given sentence, and participants have varying degrees of self-confidence, there is unavoidably a substantial amount of noise in the data. This resulted in a few instances of non-monotonic functions, such that slightly less coherent versions of a sentence showed slightly higher comprehension rates. It is expected that further data collection with additional participants will smooth out those non-monotonicities.

When all the data are averaged, the result is a linear function of steady increase in coherence as more and more of the letters are left intact — looking quite similar to that in Fig. 7. However, much like the lesson embedded in the discussion of that figure, the proper analysis of these data requires examining individual stimulus items. The reason the overall averaged data produce a linear function of increasing coherence is because some sentences begin their abrupt rise in coherence with many characters out of place (Fig. 12, upper middle panel) and others do not begin their abrupt rise in coherence until rather few characters are out of place (Fig. 12, bottom right panel).

When stimuli are analyzed individually, the data reveal clear phase transitions for the vast majority of sentences. Figure 12 shows several examples of these individual-stimulus results, where each one exhibits a rather abrupt phase transition much like that seen in Erdös and Renyi's random graph analysis (Fig. 11). Thus, for any given sentence, as more characters and spaces are correctly positioned, in increments of 2, there tends to be a point at which meaning suddenly begins to be

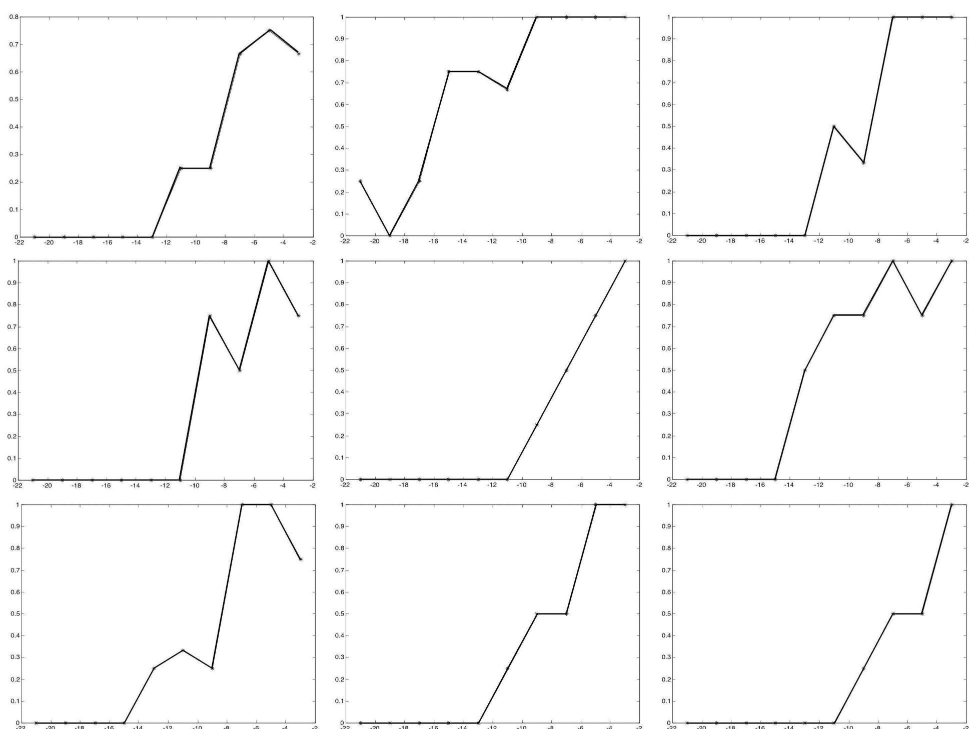

Fig. 12. Results from nine representative stimuli where the sentence is more and more coherent left to right, from 21 characters randomly relocated to 3 characters randomly relocated, in steps of 2. Along the y-axis is shown the proportion of participants who reported successful comprehension.

extractable, and coherent comprehension rises very quickly with just a few additional characters and spaces put in their proper locations. By noting the rapidity of this shift, we can see how sentence meaning exhibits the complex dynamical property of emergence. At the same time, by looking closely at the phase transition itself, we can see the gradations in this emergence, where the arrival of meaning clearly has an analog (non-binary) underlying nature.

8. Conclusion

For decades, human cognitive phenomena that exhibit sharp sudden transitions (nearly step-functions) from one state to another have captured the attention of cognitive scientists and lured many to proclaim that human mental functioning is purely instantiated in discrete and logic-based operations.[60–63] Indeed, some continue today with new versions of such proclamations: "Computational nativism is clearly the best theory of the cognitive mind that anyone has thought of so far" (Ref. 64, p. 3). Unfortunately for these proclaimers, it is quite easy to examine neural systems and conclude that no such capability for genuinely discrete logic exists. One is still left with the question of how neural systems are in fact responsible for the nearly-discrete properties of behavior that exhibit phase transitions. By examining the non-linear dynamics of these systems, a number of cognitive scientists[26,65–68] with Walter Freeman leading the tip of the spear, have begun to provide insight into how populations of neurons (acting as dynamic resonators) can give rise to cognitive processes that provide very close approximations to discrete categorical shifts in processing modes.

From a broader perspective, this phase transition behavior may permit a "rapprochement" between these two schools of thought.[69] The epistemological value of discrete computational descriptions in many ways continues in high-level cognitive domains, though our understanding of the low-level dynamics of neural systems is gradually scaling up. When such accounts meet in the middle, dynamical descriptions may show how discrete-like modes emerge, while discrete computational descriptions may finally be provided a solid anchor in neural dynamic processes — yet retaining their own unique epistemological value at certain temporal and spatial scales.[70–73] In this sense, the phase transition may be a fundamental aspect of an integrative approach to this puzzle of cognitive science. The extent to which such accounts could "co-exist" in explanations of cognitive phenomena of vast spatial and temporal scales is still under debate in the field.[6,74]

The foregoing simulations and empirical review demonstrate the range of application of this fundamental concept. Nevertheless, it is not the first time anyone has noted the commonality between findings like these. In fact, in the 1950s, the famous Gestalt psychologist Wertheimer was already drawing connections between sudden insight during problem solving and the sudden shifts in perspective during viewing of bistable figures.[75] Only recently has cognitive science, by looking to dynamical formalisms in other disciplines, been able to characterize and quantify the underlying mechanisms of the phase transition and other surprisingly general dynamical phenomena that cognition exhibits. Such developments have also accompanied the growth of computer-based methods for applying dynamical systems to analyzing data[76] as well as laboratory equipment that allows a more continuous-time data stream such as recording EEG or eye movements or reaching movements.[26] There has also been a recent move in cognitive science to integrate neural networks and dynamical systems as closely related frameworks for uncovering mechanisms driving cognition.[77]

The new century therefore holds much promise to expand and integrate dynamical systems with our understanding of the mind and brain. We anticipate that Walter Freeman's pioneering approach to the mind[8,9] by way of dynamics of the brain, having already changed the field, still has much to say in this respect. This explicit focus on neural dynamics as a means to understanding complex human cognition is therefore a "frontier" issue. We hope this review encourages those with interest in this domain to contribute to the exploration of these new territories.

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