NOTES AND LETTERS

• *ABSTRACT*

This paper represents a preliminary attempt to bring the field of mathematics within the scope of the sociology of science. An analysis is performed to determine whether citations are a rough measure of quality in mathematics. Citations to work published by mathematicians who have been elected to the US National Academy of Sciences are compared with those to the work of a random sample of university-based American mathematicians. The study examines productivity and citation counts, to determine if there is a relationship between a mathematician's age and achievement, and to test the claim that younger mathematicians are more apt to do important work. Finally, this paper explores the difficulties peculiar to the sociological study of mathematics, which account for the reluctance of most sociologists of science to study this field.

Age and Achievement in Mathematics: A Case-Study in the Sociology of Science

Nancy Stern

In his classic work, *Little Science, Big Science,* Derek Price considered the apparent doubling of the scientific population every 10 to 15 years, and predicted that '[s]cientific doomsday is . . . less than a century distant'.[1] He anticipated a future growth pattern significantly different from the previous exponential expansion. In light of recent reductions in the recruitment of scientists, and of resources devoted to R & D, scholars are beginning to speculate as to what that future pattern might be, and to study possible consequences of reduced support. One concern is whether a substantial decline in scientific growth will have deleterious effects on the quality of scientific work. If so, what should policy makers, administrators, or others concerned with the future of science do about it? Stephen Cole has investigated one aspect of this problem by looking at the relationship between scientists' age and achievement — where achievement is measured both by productivity and its quality, as characterized by scientific citations.[2] Cole's study deals with whether the predicted decline in the number of *young* people who enter scientific professions will have serious ramifications for the quality of science.

Many famous scientists are responsible for promoting the view that young scientists are most productive and most likely to produce work of major significance: 'Einstein once said "A person who has not made his great contribution to science before the age of thirty will never do so." '.[3] Einstein's hypothesis was substantiated in work by H.C. Lehman,[4] While Lehman's methodology has since been criticized as inadequate,[5] his hypothesis has nonetheless received widespread acceptance. If such statements are correct, then a decline in the number of young people who enter scientific professions may well mean a disproportionate decline

Social Studies of Science (SAGE, London and Beverly Hills), Vol.8 (1978), 127-40

in the overall quality of future scientific work. If, on the other hand, it could be demonstrated that age is only peripherally related to scientific productivity and to the quality of scientific work (if, indeed, it is related at all), then the projected decrease in the number of young scientists would not necessarily produce any dramatic negative consequences.

To be more specific, Cole's study could have demonstrated one of three possible relationships. First, scientific performance might decrease with increasing age, as Einstein suggested. The cause of such a relationship might be either physiological or sociological: that is, if younger scientists actually produce more and better work, it might be because their mental faculties are sharper, or because of their particular position in the scientific community. Second, productivity and quality of output might increase with increasing age, suggesting that older scientists have an 'edge'. This could be the result of accumulated advantage gained from experience, or of the increasing ability of senior scientists to attract graduate students, funds and equipment. Finally, there might be, in fact, no relationship whatever between age and performance, and middle-aged scientists may be neither more nor less likely to produce important work than their younger or older colleagues.

In Cole's study, random samples of scientists at PhD-granting institutions in the United States were selected from the five disciplines of chemistry, geology, physics, psychology and sociology. Data were collected on the quantity and quality of current scientific output, where 'current output' was defined as papers published by the sample scientists between 1965 and 1969. Cole's results indicated the following about age and current productivity:

> Productivity rates rise, peaking either in the [age range of the] late 30s or 40s and then drop off. The same curvilinear relationship was observed in all five fields with the point of inflection varying slightly. In general, we can conclude that age explains little variance on productivity.[6]

Similarly, Cole used citations as a rough measure of the quality of work to demonstrate the relationship between age and quality. The results were standardized for the five fields:

> There are basically no differences in the quality of work published by scientists between the ages of 30 and 50. Scientists over the age of 50 are slightly less likely to publish high quality research.[7]

Cole concluded that a scientist's age is not significantly correlated with productivity or with quality of work published: 'We can reject the commonly held belief that the creativity of scientists declines after the age of 35.'[8] Hence, there would appear to be little reason to be overly concerned about the future of science. However, although the disciplines studied by Cole were meant to represent a cross-section of scientific fields, there is a major omission: mathematics. Indeed, sociologists of science have, in general, tended to ignore mathematics,[9] presumably because it appears to be different from most other sciences. This paper is a preliminary attempt to make good this deficiency. My purpose is to argue that Cole's conclusions also apply to mathematics.

The reluctance of most sociologists of science to study mathematics is not difficult to understand. Mathematicians have traditionally been characterized as unique.

However, historians of mathematics (and some mathematicians) have recently set out to destroy the myth that mathematicians are a breed without parallel:

> Those who have never known a professional mathematician may be rather surprised on meeting some. Only by seeing in detail what manner of men some of the great mathematicians were and what kind of lives they lived, can we recognize the ludicrous untruth of the traditional portrait of the mathematician. . . . There have been eccentrics in mathematics, of course; but the percentage is no higher than in commerce or the professions.[10]

A study of age and achievement in mathematics can serve two important purposes. First, it can test Cole's findings that age and scientific achievement are not significantly correlated. The suggestion that it is the young who are the most prolific and most qualified is made more often and more confidently about mathematics than almost any other field. Second, and perhaps more importantly, it can demonstrate that mathematics, whatever its differences from the natural sciences, does share many sociological characteristics with them. This paper uses the same methodology as that employed by Cole.

THE USE OF CITATIONS AS A MEASURE OF QUALITY IN MATHEMATICS

The use of citations is not without inherent difficulties. To begin with, the value of citations as a measure of the quality of scientists' work is itself a controversial issue.[11] Citations can be interpreted as a measure of either quality or recognition, and although the two are not unrelated, they cannot simply be equated; in addition,citation studies which focus on age cohorts do not, in general, adequately consider the possible effects of selective mortality. In such studies the older scientists may be those who have, in their earlier years, received some reinforcement in terms of rewards and recognition, which motivated them to continue their academic careers. Many younger people who fail to receive such reinforcement may, in time, become 'drop-outs' from academic life. Hence any study which compares citation data for different age cohorts may be giving an 'edge' to the older cohorts.[12]

Despite the limitations of citation studies, they do appear to provide an approximate indicator of quality of work in the sciences, however crude and biased that measure might be.[13] But it is by no means clear that citations have any validity in a study of mathematics. It may be that the most prestigious mathematicians are not, in general, the most cited ones. If this proves to be the case, then the use of citations as a measure of quality would be suspect, and the belief that mathematicians can be sociologically studied in much the same way as other scientists would be seriously undermined. Thus, it must first be demonstrated that citations can be used as a crude measure of quality in mathematics. To this end, citations to work by mathematicians who are currently members of the US National Academy of Sciences (NAS) were studied to determine if they are in general greater than citations to the work of average mathematicians. Since there is no major award (such as the Nobel Prize) given to a wide range of mathematicians,[14] which might clearly indicate the mathematical élite, honorary election to the NAS will be taken as serving the same purpose. However, use of this sample of distinguished mathematicians has an inherent difficulty: it may consist of men and women who have 'passed their prime' and are no longer very productive. Since membership in the Academy is reserved for the few, highly prestigious mathematicians, one might expect that current members have already made their most significant contributions.

The productivity and citation counts for the 60 currently active mathematicians who are members of the NAS were compared to similar figures for a random sample of mathematicians on the faculty of PhD-granting institutions. The universities were selected from the 1969 American Council of Education study of departmental prestige.[15] Beginning with the university ranked highest in the field of mathematics, every other university from the first 40 on the list was selected — a sample of 20 schools. Similarly, every other member of the mathematics faculty of these schools, as obtained from university catalogues, was included in the sample — a total population of 435 mathematicians. These 435 people were compared to the 60 NAS mathematicians with respect to current output and the number of citations received (both to current work and to all past work). 'Current output' was defined as the total number of single-authored and coauthored works published in the period 1970-74. This information was obtained from the 1975 issue of the *Science Citation Index.*[16] For each mathematician, the number of citations to all work published in 1970-74, to work published in 1965-69, and to work published before 1965 was obtained by counting the citations listed in the 1975 *SCI.* Results for both NAS mathematicians and the sample of university mathematicians were then averaged (see Table 1).

Table 1.
A Comparison of NAS Mathematicians with a Random Sample of Mathematicians at PhD-Granting Institutions

Quantity	**NAS**	**University Sample**
Mean number single authored papers published 1970-74	2.80	3.12
Mean number coauthored papers published 1970-74	1.93	2.49
Total: Mean number papers published 1970-74	4.73	5.60
Quality		
Mean number citations to 1970-74 work (single & coauthored works incl.)	11.80	5.07
Mean number citations to 1965-69 work	15.17	7.24
Mean number citations to all pre-1965 work	41.92	6.48
Total: Mean number citations to all work published	68.88	18.79

It might appear that the NAS mathematicians are less prolific than the university sample. In 1970-74, the NAS mathematicians published less single-authored and coauthored papers than the sample population, suggesting that the quantity of work bears either no relationship or an inverse relationship to distinction in the field. Such a simple comparison is, however, misleading since the average age of the mathematicians in the university sample is 47, while that of the NAS sample is 60. Controlling for age, mathematicians aged 60 and over in the university sample produced 3.43 single-authored papers in the years 1970-74, as compared to 2.80 for the NAS mathematicians, and 3.11 for the entire sample of university mathematicians.

Despite the obvious differences noted in Table 1, these results should *not* be used to argue that NAS mathematicians are less productive than their university counterparts, for one important reason. Many mathematicians in the NAS are professors who are approaching retirement, or are in semi-retirement, while all mathematicians in the university sample are fully active. Thus it is not at all surprising that the NAS members produced somewhat less work than the university sample: it is perhaps more surprising that they produced as much as they did. In any case, the broad similarity in productivity between the élite and the average in mathematics is itself striking, and is not characteristic of other sciences.

When we compare the number of citations to work published by NAS mathematicians with that to work by the university sample, we find a statistically significant difference. Despite the fact that NAS mathematicians produced somewhat less than their university counterparts, their work was cited, for each category indicated, more than twice as often. This suggests that citations are directly related to publicly-acknowledged distinction in the field. There was an even wider divergence between the mean number of citations to pre-1965 work, but this is somewhat misleading. Since NAS members are on the average 13 years older than the average mathematician, they have published longer. But even allowing for the edge that NAS members have because of their professional age, it is clear that their work is cited more often than the average. This suggests that citations are a valid measure of the quality of mathematical work.

POTENTIAL DIFFICULTIES PECULIAR TO THE SOCIOLOGICAL STUDY OF MATHEMATICS

This section of the paper will indicate why, despite the statistical results presented above, some scholars will still be reticent to use citations as a measure of quality in mathematics.

While the most cited mathematicians prove to be, in general, those elected to the NAS, many distinguished mathematicians are less frequently cited, if cited at all. For example, while the mean number of citations received by NAS mathematicians was 68.88, 40 of the 60 NAS members received less than that number, suggesting that a few very important mathematicians receive a disproportionately high number of citations. In short, citations may be one suitable measure of quality but, at least in mathematics, it may not be the only one — or even a necessary or sufficient one.

Sceptical assessments of the validity of citations as a measure of quality tend to occur more often in mathematics than in other sciences. There are several reasons for this. First, it is claimed that some important works in mathematics are considered really significant because they solve some heretofore unresolved problems —

thus eliminating controversy (and even discussion), and ultimately reducing the number of mathematicians working in that area. Some really significant work may thus be rarely cited because it becomes a kind of 'terminus'. Charles Fisher has studied this tendency and claims that it has historical validity:

> In 1888 David Hilbert, age 26, surprised the mathematical world by producing a proof of a general finiteness theorem. His result was spectacular, because using techniques from outside Invariant Theory he proved the theorem in the space of four quarto pages . . . As mentioned before, the number of research contributions which were classified as invariant-theoretic slowly declined . . . Many outside of Invariant Theory looked back on the work of Hilbert as having killed the subject by solving all of its problems.[17]

If Fisher's conclusions concerning Invariant Theory apply to mathematics in general, then citations may be an inadequate measure of the quality of mathematical work. It may also be that mathematical specialties are so highly differentiated, and contain so few mathematicians, that even if every paper published in a particular specialty in a given year were to cite a specific work, it would still receive a relatively small number of citations. In the absence of data on the distribution of papers by specialty, statements about the overall merits of citation analysis are not necessarily definitive. Moreover, an important work may be distinguished more by its style than by its substantive content. Mathematicians often evaluate each others' work on the basis of its aesthetic quality or 'elegance'. Summarizing the results of interviews with many mathematicians, Warren Hagstrom claims: 'In mathematics, the style of a proof, its "elegance", is often considered as important to its merit as the truth of the theorem proved'.[18] Similarly, in discussing the evolution of mathematical concepts, Raymond Wilder suggests that mathematicians are often primarily motivated by aesthetic considerations.[19] If Hagstrom and Wilder are correct, then a mathematical work which provides a simpler, more elegant proof for an already established theory may be judged to be as 'important' in the field as one which adds to or alters the theory. We can safely asume that the latter would lead to a high citation count: but since elegance does not specifically alter the store of information, the former might or might not be highly cited. If style and form are aspects of 'quality' in mathematics that do not earn high citations, then citation counts would not reliably reflect mathematical achievement.

Finally, it may be that mathematics is more highly individualistic, or 'anomic', than other sciences, and that this contributes to mathematical development in ways which defy traditional sociological interpretation. Hagstrom claims that mathematicians suffer from 'anomie' because there are no established barriers separating specialties:

> Mathematics has come to the paradoxical situation of intense specialization without having clearly defined specialties. Many mathematicians find it difficult to identify their colleagues or themselves. The audience to which they address themselves is unknown or almost nonexistent.[20]

Anomie is defined by Hagstrom as the absence of opportunities to receive recognition.[21] His interviews indicate that mathematicians do not generally view citations as an adequate measure of recognition. Diana Crane, in an effort to test

Hagstrom's claim, provides statistical evidence to show that communication lines in mathematics are not as effective as they are in other fields:

> While this area [mathematics] does not exhibit the acute social isolation that Hagstrom described, it is clear that some members were less involved in the communication network than others.[22]

Hagstrom and Crane base their view of mathematics as somewhat anomic on mathematicians' opinions of themselves. Although the internalized norm of isolation and independence may, as Crane and Hagstrom suggest, be characteristic of mathematicians, this norm may bear little if any resemblance to the *actual* relationship between mathematicians. Despite the way in which mathematicians view their own discipline, citations (as demonstrated in this study) are correlated with recognition; this suggests that anomie may not be a major factor. Hence, mathematicians may think of their field as anomic because they see themselves as independent isolates: but the statistical evidence appears to belie their claim. Similarly, historians of mathematics (and mathematicians themselves) promote the view that this field is closer to the arts than to the sciences, and hence represents a unique discipline, in which productivity and quality of work cannot be measured in any scientific sense.

> It is not surprising that many professional mathematicians consider mathematics to be an art, for certainly creative work in mathematics does share many common features with such artistic pursuits as music and painting. Moreover, the inspiration for many advances in mathematics has come from the artistic impulses of their creators.[23]

All these claims emphasize the isolation and individualism of mathematicians, and their tendency to resist norms. If it is true that mathematicians avoid aligning themselves within specific theoretical constructs, or resist being categorized into intellectual 'camps', then the use of citations may prove to be an inadequate method of evaluating mathematical work.

In summary, citations may be inadequate as a measure of mathematical quality, and should be treated with some caution, pending further careful research on their validity. One can only say that, at best, citations may prove to be as good a measure of quality in mathematics as in any other scientific field; or, at worst, that citations may prove to be a less reliable measure of quality, because one or more of the factors discussed above plays a critical role.

AGE AND ACHIEVEMENT IN MATHEMATICS

I have already presented evidence that citation counts can be taken as a crude indicator of quality of work in mathematics (and hence that mathematics shares at least some of the sociological characteristics of the other sciences). I will now follow Cole's procedures, and use productivity and citation counts in the random sample of 435 mathematicians at PhD-granting institutions to determine if there is a relationship between a mathematician's age and achievement.

The ages of the members of the sample had first to be determined. The dates of birth of 387 of the sample were listed in the twelfth edition of *American Men and*

Women of Science, but the remaining 48 were not listed there, because they had only recently received their PhD degrees. For these, the date of PhD was obtained from the *Dissertation Abstracts* (1972-75), and an approximate date of birth was derived by subtracting 27 — 27 being the average age when PhDs were granted for all the others.

Table 2. Age and Mathematical Productivity 1970-74. Mean Number of Papers Published in 1970-74 by Mathematicians of Different Ages

Ages	Mean Number Single Authored Papers[1]	Mean Number Co-Authored Papers[1]	Mean Number Total[1]	N
Under 35	3.27	1.73	5.12	101
35-39	3.97	3.36	7.33	96
40-44	3.24	2.94	6.24	67
45-49	2.37	1.13	3.49	63
50-59	2.16	3.03	5.22	73
60+	3.43	2.69	6.11	35
TOTAL	3.11	2.49	5.64	435

[1]These figures are roughly equivalent to those in Table 1. Any slight variations are a result of the fact that this table was computed to two decimal positions, whereas the previous one was computed to three decimal posisitions.

The mean numbers of single-authored papers and of coauthored papers were then determined for each age group. The results appear in Table 2. Note that while there are variations among age groups, there is no apparent overall relationship between age and mathematical productivity. A mathematician who is less than 35 years old, for example, has published, on the average, 5.12 papers in 1970-74, while a mathematician who is aged 60 or more has published, on the average, 6.11 papers in the same period. While there are fluctuations between these age groups, the notion that younger mathematicians are, as it were, 'physiologically' more able to produce papers would appear to be in error. In general, we can state categorically that age explains very little, if anything, about productivity.

However, there may still be a relationship between age and quality of work: younger scientists may tend to produce *more important* papers. To test this, Table 3 presents information on the relationship between age and the number of citations received to work published in 1970-74. Here again, there is no apparent relationship between age and the quality of work. If anything, it seems that older mathematicians, on the average, may produce better quality work than their younger colleagues. This is not however, the claim I wish to make since it is more probable (as Cole suggests about scientists in general) that older mathematicians have a greater degree of recognition, and hence are more likely to be cited. In short, no clear-cut relationship exists between age and productivity, or between age and quality of work. The claim that younger mathematicians (whether for physiological or sociological reasons) are more apt to create important work is, then, unsubstantiated.[24]

Table 3. Age and Citations to Work Published 1970-74

Age	Mean Number Citations to Single-Authored and First-Authored 1970-74 Work
Under 35	2.73 (101)
35-59	3.80 (96)
40-44	5.79 (67)
45-49	3.44 (63)
50-59	5.63 (73)
60+	5.09 (35)
TOTAL	4.22 (435)

Citation analyses which use the mean as a basis for comparison sometimes mask skewed or disproportionate distributions. To determine if a disproportionately small number of mathematicians were responsible for inflating the mean by producing a disproportionately large number of papers, I compiled listings, by age, of the percentage of mathematicians who published more than the mean number of papers in 1970-74, and of the percentage cited more than the mean number of times for work published during this period. A skewed distribution would be reflected in a very small percentage of those publishing more than the mean. If a few, highly distinguished mathematicians have inflated the mean number of citations by producing far more than most others, this, too, would be reflected in a small percentage cited more than the mean number of times.

Table 4 provides a breakdown of these percentages. The results follow the same general pattern as that presented in Tables 2 and 3: 24.8% of mathematicians under 35 years old produced more than the mean number of papers, whereas 25.7% of mathematicians 60 years old and older produced more than the mean number of papers (a percentage difference of 0.9%, which is statistically insignificant). However, the citation results are less consistent. Only 10.9% of mathematicians under age 35 were cited more than the mean number of times, whereas 28.6% of mathematicians 60 and older were cited more than the mean: that is, 11 of 101 mathematicians under 35 were responsible for approximately half the citations of that age group, while 10 of 35 mathematicians 60 and over were responsible for half their citations. This implies that if an effort had been made in this study to eliminate or normalize disproportionately great numbers of citations to a few works (that is, citation counts that appear to be anomalous), then the citation rates for younger scientists would be somewhat less than the numbers actually obtained. Younger mathematicians, as a whole, are probably cited less frequently than the numbers in Tables 2 and 3 suggest. Thus, the belief that younger mathematicians have an 'edge', a notion already deemed erroneous, is even more discredited.

Table 4.
Percent Publishing More than the Mean Number of Papers in 1970-74 by Age and Percent Cited More Than the Mean Number of Times for Work Published in 1970-74 by Age

Age	Percent Publishing More than the Mean	Percent Cited More than mean number of Times for Work Published 1970-74	N
Under 35	24.8 (25)	10.9 (11)	101
35-39	51.0 (49)	25.0 (24)	96
40-44	40.3 (27)	13.4 (9)	67
45-49	17.5 (11)	17.5 (11)	63
50-59	28.8 (21)	23.3 (17)	73
60+	25.7 (9)	28.6 (10)	35

ADDITIONAL FINDINGS

I have argued that, if citation counts are an adequate measure of quality, then the sociological characteristics which have been found to apply to the natural sciences in general may also apply to mathematics. I have found no clear relationship between age and achievement in mathematics — just as Stephen Cole did in five other sciences. If younger mathematicians were more able to produce significant work, they would be cited more often; they are not.

There are, however, some anomalous features which require further explanation. A closer look at Tables 2 and 3 indicates either a relatively stable, nearly linear relationship between age and achievement, or a curvilinear relationship where achievement appears to increase with age. This latter, apparently direct relationship between age and achievement has been attributed to social factors: increasing age often results in accrued benefits such as increased recognition, funding, equipment, released time and number of graduate students — all of which give older scientists an edge in terms of productivity and citation counts. While the benefits of increased resources in general probably play a less decisive role in mathematics than in other sciences (where equipment is a more vital aspect of the research effort), the significance of other aspects of accumulated advantage may still be important.

One anomaly which has not so far been considered is that mathematicians in the 45-49 age range seem, on the average, to produce less work, and less important work, than both their younger and older colleagues. This may be a result of one of several factors. First, it may be that the sample used in this study includes unusual or extraordinary cases which tend to skew or perhaps even distort the outcome. A second possibility is that the ages between 45 and 49 may be the most likely period in a mathematician's career when an administrative or 'gatekeeping' role is assumed. Zuckerman and Merton define the gatekeeping role by stating that . . .

> Although it is often (and loosely) included under 'administration', a fourth role of the scientist needs to be distinguished from the others since it is basic to the systems of evaluation and the allocation of roles and resources in science. This is the gatekeeping role. Variously distributed within the organization and institutions of science, it involves continuing or intermittent assessment of the performance of scientists at every stage of their career, from the phase of youthful novice to that of ancient veteran, and providing or denying access to opportunities.[25]

They claim that gatekeepers are among the élite in science:

> . . . the gatekeeping function seems to involve a mixture of Turner's types of mobility . . .. in which *élites or their agents* help recruit their successors fairly early.[26]

They also demonstrate that administrative responsibilities increase with increasing age, but their study does not test the extent of such responsibilities for scientists beyond the age of 50.[27] If it happens that such administrators and gatekeepers are among the élite in mathematics, and are most frequently 45-49 years old, then the decreased productivity of this age group would need no further explanation.

To test this hypothesis, career patterns of NAS mathematicians were examined, using the twelfth edition of *American Men and Women of Science,* and *Who's Who in Science*, for biographical data. The age at which these people assumed their first major administrative role was recorded. A 'major administrative role' was defined as a position with a scientific agency, an executive position in a mathematical society, editorship of a mathematical journal or chairmanship of a department. Clearly these roles are not inclusive and vary in responsibility depending upon the society or institution, and the nature of tasks to be performed. It was assumed, however, that if mathematicians included such positions in their entries for *American Men and Women of Science* or *Who's Who in Science,* and specified the corresponding dates of tenure, such roles must have been important to them, either in terms of status or of achievement. In either case, the position probably required a fair share of the mathematician's time.

In the NAS sample, 58% listed a major administrative position with corresponding dates of service.[28] The average age at which the mathematicians assumed these roles was 47. For the university sample, the percent of mathematicians who held such positions was somewhat less (42%), but the average age was 48, strikingly similar to the NAS group. Since the average age at which a mathematician assumes a major administrative position falls within the 45-49 year old period in which productivity and achievement decline, it is quite possible that the two are related.

To determine precisely why mathematicians in the 45-49 age range produce less significant work, it would be necessary to select a cross-section of mathematicians and follow their careers back through time. This approach has many potential benefits. In particular, by focussing attention on the details of individual biographies, it could provide insight into factors that might tend to be neglected or obscured in a purely statistical analysis. Allison and Stewart, in their study on accumulated advantage in science, discuss this technique:

> Our central hypothesis is that the distribution of productivity among scientists becomes increasingly dispersed with the passage of time. Since persons continually enter and leave the population of scientists, the ideal method would be to measure the variation in productivity for one or more cohorts at several time points during their career history.[29]

Allison and Stewart suggest that a purely statistical analysis of citation counts might tend to distort individual patterns.

In short, statistical data can be used to support the hypothesis that age and achievement are not significantly correlated, but statistical techniques of this kind are not without important limitations.

• NOTES

I wish to express my appreciation to Professor Stephen Cole and to David Edge for their assistance in the preparation of this paper. I am also grateful for the support and encouragement of Professor Ruth Schwartz Cowan. The data collection for this study was supported by NSF grant SOC 72-05324 to Columbia University in the Sociology of Science.

1. Derek J. de Solla Price, *Little Science, Big Science* (New York: Columbia University Press, 1963), 19.
2. Stephen Cole, 'Age and Scientific Performance' (unpublished paper, SUNY at Stony Brook, 1976).
3. S. Brodestsky, *Nature*, Vol. 150 (1942), 699, as quoted in C.W. Adams, 'The Age at which Scientists do Their Best Work', *Isis*, Vol. 36 (1946), 166-69.
4. Harvey C. Lehman, *Age and Achievement* (Princeton, NJ: Princeton University Press, 1953).
5. Wayne Dennis, 'Age and Productivity Among Scientists', *Science*, Vol. 123 (1956), 724.
6. Cole, op. cit. note 2, 5.
7. Ibid., 6.
8. Ibid., 7.
9. Diana Crane and Warren Hagstrom are exceptions, since their works do include studies in the field of mathematics.
10. E.T. Bell, *Men of Mathematics* (New York: Simon and Schuster, 1965), 8.

11. See *Social Studies of Science,* Vol. 7, No. 2 (May 1977), and the *4S Newsletter,* Vol. 2, No. 3 (Summer 1977), for a current assessment of this controversy.

12. To adequately address the issue of selective mortality, one could analyze the career patterns of industrial and government scientists to determine if, in fact, they are 'dropouts' from academia. One could also analyze the career patterns of a wide range of scientists over an extended period. Stephen Cole is currently using the latter technique to study the degree to which selective mortality affects citation analysis.

13. Harriet Zuckermann, in her recent book *Scientific Elite* (New York: Free Press, 1977), 37, lends credence to this perspective:

> With all their limitations, citation counts have been found to be a useful though crude indicator of the impact of research on subsequent scientific development.

14. There are, of course, prestigious awards given in mathematics, such as the Fields Medal, Chauvenet Prize and Bochner Prize. Such awards, however, are given to only a small number of American mathematicians who, in almost all cases, are also members of the NAS. Since mathematicians in the NAS are apt to be those whose contributions to their field are widely recognized, such a sample has been considered most appropriate.

15. Kenneth D. Roose and Charles J. Andersen, *A Rating of Graduate Programs* (Washington, DC: American Council of Education, 1970).

16. The *Source Index* of *SCI* was found to be a far more comprehensive source than any of the abstracting journals in mathematics. Mathematicians who coauthored works are listed in the *Source Index* regardless of the order in which their names appear in the publication. Thus coauthored works for mathematicians in this study include those in which they were named first, as well as those in which they were not named first.

17. Charles S. Fisher, 'The Last Invariant Theorists: A Sociological Study of the Collective Biographies of Mathematical Specialists', *European Journal of Sociology,* Vol. VIII (1967), 226-27.

18. Warren O. Hagstrom, *The Scientific Community* (New York and London: Basic Books, 1965, reprinted Carbondale, Ill.: Southern Illinois University Press, 1975), 17.

19. Raymond L. Wilder, *Evolution of Mathematical Concepts* (New York: John Wiley and Sons, Inc., 1968), 13.

20. Hagstrom, op. cit. note 18, 227-28.

21. Ibid., 228.

22. Diana Crane, *Invisible Colleges* (Chicago: The University of Chicago Press, 1972), 63.

23. Wilder, op. cit. note 19, 13.

24. The claim made about youth really has a dual significance. It can mean that the most significant work is produced by younger mathematicians, or that mathematicians produce their best work when they are younger. In either case, the evidence invalidates the claim.

25. Harriet Zuckerman and Robert K. Merton, 'Age, Aging and Age Structure in Science', in R.K. Merton (ed.), *The Sociology of Science* (Chicago: The University of Chicago Press, 1973), 521.

26. Ibid., 522 (emphasis added).

27. Ibid., 525.

28. Actually 71% of the NAS sample listed a major administrative position, but

13% of them did not indicate dates. Hence the 13% could not be used in any age analysis.

29. Paul D. Allison and John A. Stewart, 'Productivity Differences Among Scientists: Evidence for Accumulative Advantage', *American Sociological Review*, Vol. 39 (August 1974), 598.

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