THE LEGEND OF JOHN VON NEUMANN

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John von Neumann was a brilliant mathematician who made important contributions to quantum physics, to logic, to meteorology, to war, to the theory and applications of high-speed computing machines, and, via the mathematical theory of games of strategy, to economics.

Youth. He was born December 28, 1903, in Budapest, Hungary. He was the eldest of three sons in a well-to-do Jewish family. His father was a banker who received a minor title of nobility from the Emperor Franz Josef; since the title was hereditary, von Neumann's full Hungarian name was Margittai Neumann János. (Hungarians put the family name first. Literally, but in reverse order, the name means John Neumann of Margitta. The "of", indicated by the final "i", is where the "von" comes from; the place name was dropped in the German translation. In ordinary social intercourse such titles were never used, and by the end of the first world war their use had gone out of fashion altogether. In Hungary von Neumann is and always was known as Neumann János and his works are alphabetized under N. Incidentally, his two brothers, when they settled in the U.S., solved the name problem differently. One of them reserves the title of nobility for ceremonial occasions only, but, in daily life, calls himself Neumann; the other makes it less conspicuous by amalgamating it with the family name and signs himself Vonneuman.)

Even in the city and in the time that produced Szilárd (1898), Wigner (1902), and Teller (1908), von Neumann's brilliance stood out, and the legends about him started accumulating in his childhood. Many of the legends tell about his memory. His love of history began early, and, since he remembered what he learned, he ultimately became an expert on Byzantine history, the details of the trial of Joan of Arc, and minute features of the battles of the American Civil War.

Paul Halmos claims that he took up mathematics because he flunked his master's orals in philosophy.

He received his Univ. of Illinois Ph.D. under J.L. Doob. Then he was von Neumann's assistant, followed by positions at Illinois, Syracuse, M. I. T. 's Radiation Lab, Chicago, Michigan, Hawaii, and now is Distinguished Professor at Indiana Univ. He spent leaves at the Univ. of Uruguay, Montevideo, Univ. of Miami, Univ. of California, Berkeley, Tulane, and Univ. of Washington. He held a Guggenheim Fellowship and was awarded the MAA Chauvenet Prize.

Professor Halmos' research is mainly measure theory, probability, ergodic theory, topological groups, Boolean algebra, algebraic logic, and operator theory in Hilbert space. He has served on the Council of the AMS for many years and was Editor of the Proceedings of the AMS and Mathematical Reviews. His eight books, all widely used, include *Finite-Dimensional Vector Spaces* (Van Nostrand, 1958), *Measure Theory* (Van Nostrand, 1950), *Naive Set Theory* (Van Nostrand, 1960), and *Hilbert Space Problem Book* (Van Nostrand, 1967).

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He could, it is said, memorize the names, addresses, and telephone numbers in a column of the telephone book on sight. Some of the later legends tell about his wit and his fondness for humor, including puns and off-color limericks. Speaking of the Manhattan telephone book he said once that he knew all the numbers in it — the only other thing he needed, to be able to dispense with the book altogether, was to know the names that the numbers belonged to.

Most of the legends, from childhood on, tell about his phenomenal speed in absorbing ideas and solving problems. At the age of 6 he could divide two eight-digit numbers in his head; by 8 he had mastered the calculus; by 12 he had read and understood Borel's *Théorie des Fonctions*.

These are some of the von Neumann stories in circulation. I'll report others, but I feel sure that I haven't heard them all. Many are undocumented and unverifiable, but I'll not insert a separate caveat for each one: let this do for them all. Even the purely fictional ones say something about him; the stories that men make up about a folk hero are, at the very least, a strong hint to what he was like.)

In his early teens he had the guidance of an intelligent and dedicated high-school teacher, L. Rátz, and, not much later, he became a pupil of the young M. Fekete and the great L. Fejér, "the spiritual father of many Hungarian mathematicians". ("Fekete" means "Black", and "Fejér" is an archaic spelling, analogous to "Whyte".)

According to von Kármán, von Neumann's father asked him, when John von Neumann was 17, to dissuade the boy from becoming a mathematician, for financial reasons. As a compromise between father and son, the solution von Kármán proposed was chemistry. The compromise was adopted, and von Neumann studied chemistry in Berlin (1921–1923) and in Zürich (1923–1925). In 1926 he got both a Zürich diploma in chemical engineering and a Budapest Ph.D. in mathematics.

Early work. His definition of ordinal numbers (published when he was 20) is the one that is now universally adopted. His Ph.D. dissertation was about set theory too; his axiomatization has left a permanent mark on the subject. He kept up his interest in set theory and logic most of his life, even though he was shaken by K. Gödel's proof of the impossibility of proving that mathematics is consistent.

He admired Gödel and praised him in strong terms: "Kurt Gödel's achievement in modern logic is singular and monumental — indeed it is more than a monument, it is a landmark which will remain visible far in space and time. ... The subject of logic has certainly completely changed its nature and possibilities with Gödel's achievement." In a talk entitled "The Mathematician", speaking, among other things, of Gödel's work, he said: "This happened in our lifetime, and I know myself how humiliatingly easily my own values regarding the absolute mathematical truth changed during this episode, and how they changed three times in succession!"

He was Privatdozent at Berlin (1926–1929) and at Hamburg (1929–1930). During this time he worked mainly on two subjects, far from set theory but near to one another: quantum physics and operator theory. It is almost not fair to call them two

subjects: due in great part to von Neumann's own work, they can be viewed as two aspects of the same subject. He started the process of making precise mathematics out of quantum theory, and (it comes to the same thing really) he was inspired by the new physical concepts to make broader and deeper the purely mathematical study of infinite-dimensional spaces and operators on them. The basic insight was that the geometry of the vectors in a Hilbert space has the same formal properties as the structure of the states of a quantum-mechanical system. Once that is accepted, the difference between a quantum physicist and a mathematical operator-theorist becomes one of language and emphasis only. Von Neumann's book on quantum mechanics appeared (in German) in 1932. It has been translated into French (1947), Spanish (1949), and English (1955), and it is still one of the standard and one of the most inspiring treatments of the subject. Speaking of von Neumann's contributions to quantum mechanics, E. Wigner, a Nobel laureate, said that they alone "would have secured him a distinguished position in present day theoretical physics".

Princeton. In 1930 von Neumann went to Princeton University for one term as visiting lecturer, and the following year he became professor there. In 1933, when the Institute for Advanced Study was founded, he was one of the original six professors of its School of Mathematics, and he kept that position for the rest of his life. (It is easy to get confused about the Institute and its formal relation with Princeton University, even though there is none. They are completely distinct institutions. The Institute was founded for scholarship and research only, not teaching. The first six professors in the School of Mathematics were J. W. Alexander, A. Einstein, M. Morse, O. Veblen, J. von Neumann, and H. Weyl. When the Institute began it had no building, and it accepted the hospitality of Princeton University. Its members and visitors have, over the years, maintained close professional and personal relations with their colleagues at the University. These facts kept contributing to the confusion, which was partly clarified in 1940, when the Institute acquired a building of its own, about a mile from the Princeton campus.)

In 1930 von Neumann married Marietta Kövesi; in 1935 their daughter Marina was born. (In 1956 Marina von Neumann graduated from Radcliffe *summa cum laude*, with the highest scholastic record in her class. In 1972 Marina von Neumann Whitman was appointed by President Nixon to the Council of Economic Advisers.) In the 1930's the stature of von Neumann, the mathematician, grew at the rate that his meteoric early rise had promised, and the legends about Johnny, the human being, grew along with it. He enjoyed life in America and lived it in an informal manner, very differently from the style of the conventional German professor. He was not a refugee and he didn't feel like one. He was a cosmopolite in attitude and a U.S. citizen by choice.

The parties at the von Neumanns' house were frequent, and famous, and long. Johnny was not a heavy drinker, but he was far from a teetotaller. In a roadside

restaurant he once ordered a brandy with a hamburger chaser. The outing was in honor of his birthday and he was feeling fine that evening. One of his gifts was a toy, a short prepared tape attached to a cardboard box that acted as sounding board; when the tape was pulled briskly past a thumbnail, it would squawk "Happy birthday!" Johnny squawked it often. Another time, at a party at his house, there was one of those thermodynamic birds that dips his beak in a glass of water, straightens up, teeter-totters for a while, and then repeats the cycle. A temporary but firm house rule was quickly passed: everyone had to take a drink each time that the bird did.

He liked to drive, but he didn't do it well. There was a "von Neumann's corner" in Princeton, where, the story goes, his cars repeatedly had trouble. One often quoted explanation that he allegedly offered for one particular crack-up goes like this: "I was proceeding down the road. The trees on the right were passing me in orderly fashion at 60 miles an hour. Suddenly one of them stepped in my path. Boom!"

He once had a dog named "Inverse". He played poker, but only rarely, and he usually lost.

In 1937 the von Neumanns were divorced; in 1938 he married Klára Dán. She learned mathematics from him and became an expert programmer. Many years later, in an interview, she spoke about him. "He has a very weak idea of the geography of the house. ...Once, in Princeton, I sent him to get me a glass of water; he came back after a while wanting to know where the glasses were. We had been in the house only seventeen years. ...He has never touched a hammer or a screwdriver; he does nothing around the house. Except for fixing zippers. He can fix a broken zipper with a touch."

Von Neumann was definitely not the caricatured college professor. He was a round, pudgy man, always neatly, formally dressed. There are, to be sure, one or two stories of his absentmindedness. Klári told one about the time when he left their Princeton house one morning to drive to a New York appointment, and then phoned her when he reached New Brunswick to ask: "Why am I going to New York?" It may not be strictly relevant, but I am reminded of the time I drove him to his house one afternoon. Since there was to be a party there later that night, and since I didn't trust myself to remember exactly how I got there, I asked how I'd be able to know his house when I came again. "That's easy," he said; "it's the one with that pigeon sitting by the curb."

Normally he was alert, good at rapid repartee. He could be blunt, but never stuffy, never pompous. Once the telephone interrupted us when we were working in his office. His end of the conversation was very short; all he said between "Hello" and "Goodbye" was "Fekete pestis!", which means "Black plague!" Remembering, after he hung up, that I understood Hungarian, he turned to me, half apologetic and half exasperated, and explained that he wasn't speaking of one of the horsemen of the Apocalypse, but merely of some unexpected and unwanted dinner guests that his wife just told him about.

On a train once, hungry, he asked the conductor to send the man with the sandwich

tray to his seat. The busy and impatient conductor said "I will if I see him". Johnny's reply: "This train is linear, isn't it?"

Speed. The speed with which von Neumann could think was awe-inspiring. G. Pólya admitted that "Johnny was the only student I was ever afraid of. If in the course of a lecture I stated an unsolved problem, the chances were he'd come to me as soon as the lecture was over, with the complete solution in a few scribbles on a slip of paper." Abstract proofs or numerical calculations — he was equally quick with both, but he was especially pleased with and proud of his facility with numbers. When his electronic computer was ready for its first preliminary test, someone suggested a relatively simple problem involving powers of 2. (It was something of this kind: what is the smallest power of 2 with the property that its decimal digit fourth from the right is 7? This is a completely trivial problem for a present-day computer: it takes only a fraction of a second of machine time.) The machine and Johnny started at the same time, and Johnny finished first.

One famous story concerns a complicated expression that a young scientist at the Aberdeen Proving Ground needed to evaluate. He spent ten minutes on the first special case; the second computation took an hour of paper and pencil work; for the third he had to resort to a desk calculator, and even so took half a day. When Johnny came to town, the young man showed him the formula and asked him what to do. Johnny was glad to tackle it. "Let's see what happens for the first few cases. If we put $n = 1$, we get..." — and he looked into space and mumbled for a minute. Knowing the answer, the young questioner put in "2.31?" Johnny gave him a funny look and said "Now if $n = 2$,...",and once again voiced some of his thoughts as he worked. The young man, prepared, could of course follow what Johnny was doing, and, a few seconds before Johnny finished, he interrupted again, in a hesitant tone of voice: "7.49?" This time Johnny frowned, and hurried on: "If $n = 3$, then...". The same thing happened as before — Johnny muttered for several minutes, the young man eavesdropped, and, just before Johnny finished, the young man exclaimed: "11.06!" That was too much for Johnny. It couldn't be! No unknown beginner could outdo him! He was upset and he sulked till the practical joker confessed.

Then there is the famous fly puzzle. Two bicyclists start twenty miles apart and head toward each other, each going at a steady rate of 10 m.p.h. At the same time a fly that travels at a steady 15 m.p.h. starts from the front wheel of the southbound bicycle and flies to the front wheel of the northbound one, then turns around and flies to the front wheel of the southbound one again, and continues in this manner till he is crushed between the two front wheels. Question: what total distance did the fly cover? The slow way to find the answer is to calculate what distance the fly covers on the first, northbound, leg of the trip, then on the second, southbound, leg, then on the third, etc., etc., and, finally, to sum the infinite series so obtained. The quick way is to observe that the bicycles meet exactly one hour after their start, so that the fly had just an hour for his travels; the answer must therefore be 15 miles. When the

question was put to von Neumann, he solved it in an instant, and thereby disappointed the questioner: "Oh, you must have heard the trick before!" "What trick?" asked von Neumann; "all I did was sum the infinite series."

I remember one lecture in which von Neumann was talking about rings of operators. At an appropriate point he mentioned that they can be classified two ways: finite versus infinite, and discrete versus continuous. He went on to say: "This leads to a total of four possibilities, and, indeed, all four of them can occur. Or — let's see — can they?" Many of us in the audience had been learning this subject from him for some time, and it was no trouble to stop and mentally check off all four possibilities. No trouble — it took something like two seconds for each, and, allowing for some fumbling and shifting of gears, it took us perhaps 10 seconds in all. But after two seconds von Neumann had already said "Yes, they can," and he was two sentences into the next paragraph before, dazed, we could scramble aboard again.

Speech. Since Hungarian is not exactly a *lingua franca*, all educated Hungarians must acquire one or more languages with a popular appeal greater than that of their mother tongue. At home the von Neumanns spoke Hungarian, but he was perfectly at ease in German, and in French, and, of course, in English. His English was fast and grammatically defensible, but in both pronunciation and sentence construction it was reminiscent of German. His "Sprachgefühl" was not perfect, and his sentences ten ded to become involved. His choice of words was usually exactly right; the occasional oddities (like "a self-obvious theorem") disappeared in later years. His spelling was sometimes more consistent than commonplace: if "commit", then "ommit". S. Ulam tells about von Neumann's trip to Mexico, where "he tried to make himself understood by using 'neo-Castilian', a creation of his own — English words with an 'el' prefix and appropriate Spanish endings".

He prepared for lectures, but rarely used notes. Once, five minutes before a nonmathematical lecture to a general audience, I saw him as he was preparing. He sat in the lounge of the Institute and scribbled on a small card a few phrases such as these: "Motivation, 5 min.; historical background, 15 min.; connection with economics, 10 min.;..."

As a mathematical lecturer he was dazzling. He spoke rapidly but clearly; he spoke precisely, and he covered the ground completely. If, for instance, a subject has four possible axiomatic approaches, most teachers content themselves with developing one, or at most two, and merely mentioning the others. Von Neumann was fond of presenting the "complete graph" of the situation. He would, that is, describe the shortest path that leads from the first to the second, from the first to the third, and so on through all twelve possibilities.

His one irritating lecturing habit was the way he wielded an eraser. He would write on the board the crucial formula under discussion. When one of the symbols in it had been proved to be replaceable by something else, he made the replacement not by rewriting the whole formula, suitably modified, but by erasing the replaceable

symbol and substituting the new one for it. This had the tendency of inducing symptoms of acute discouragement among note-takers, especially since, to maintain the flow of the argument, he would keep talking at the same time.

His style was so persuasive that one didn't have to be an expert to enjoy his lectures; everything seemed easy and natural. Afterward, however, the Chinese-dinner phenomenon was likely to occur. A couple of hours later the average memory could no longer support the delicate balance of mutually interlocking implications, and, puzzled, would feel hungry for more explanation.

Style. As a writer of mathematics von Neumann was clear, but not clean; he was powerful but not elegant. He seemed to love fussy detail, needless repetition, and notation so explicit as to be confusing. To maintain a logically valid but perfectly transparent and unimportant distinction, in one paper he introduced an extension of the usual functional notation: along with the standard $\phi(x)$ he dealt also with something denoted by $\phi((x))$. The hair that was split to get there had to be split again a little later, and there was $\phi(((x)))$, and, ultimately, $\phi((((x))))$. Equations such as

$$(\psi(((((a)))))) ^2 = \phi((((a))))$$

have to be peeled before they can be digested; some irreverent students referred to this paper as von Neumann's onion.

Perhaps one reason for von Neumann's attention to detail was that he found it quicker to hack through the underbrush himself than to trace references and see what others had done. The result was that sometimes he appeared ignorant of the standard literature. If he needed facts, well-known facts, from Lebesgue integration theory, he waded in, defined the basic notions, and developed the theory to the point where he could use it. If, in a later paper, he needed integration theory again, he would go back to the beginning and do the same thing again.

He saw nothing wrong with long strings of suffixes, and subscripts on subscripts; his papers abound in avoidable algebraic computations. The reason, probably, is that he saw the large picture; the trees did not conceal the forest from him. He saw and he relished all parts of the mathematics he was thinking about. He never wrote "down" to an audience; he told it as he saw it. The practice caused no harm; the main result was that, quite a few times, it gave lesser men an opportunity to publish "improvements" of von Neumann.

Since he had no formal connections with educational institutions after he was 30, von Neumann does not have a long list of students; he supervised only one Ph.D. thesis. Through his lectures and informal conversations he acquired, however, quite a few disciples who followed in one or another of his footsteps. A few among them are J. W. Calkin, J. Charney, H. H. Goldstine, P. R. Halmos, I. Halperin, O. Morgenstern, F. J. Murray, R. Schatten, I. E. Segal, A. H. Taub, and S. Ulam.

Work habits. Von Neumann was not satisfied with seeing things quickly and clearly; he also worked very hard. His wife said "he had always done his writing at home during the night or at dawn. His capacity for work was practically unlimited." In addition to his work at home, he worked hard at his office. He arrived early, he stayed late, and he never wasted any time. He was systematic in both large things and small; he was, for instance, a meticulous proofreader. He would correct a manuscript, record on the first page the page numbers where he found errors, and, by appropriate tallies, record the number of errors that he had marked on each of those pages. Another example: when requested to prepare an abstract of not more than 200 words, he would not be satisfied with a statistical check — there are roughly 20 lines with about 10 words each — but he would count every word.

When I was his assistant we wrote one paper jointly. After the thinking and the talking were finished, it became my job to do the writing. I did it, and I submitted to him a typescript of about 12 pages. He read it, criticized it mercilessly, crossed out half, and rewrote the rest; the result was about 18 pages. I removed some of the Germanisms, changed a few spellings, and compressed it into 16 pages. He was far from satisfied, and made basic changes again; the result was 20 pages. The almost divergent process continued (four innings on each side as I now recall it); the final outcome was about 30 typescript pages (which came to 19 in print).

Another notable and enviable trait of von Neumann's was his mathematical courage. If, in the middle of a search for a counterexample, an infinite series came up, with a lot of exponentials that had quadratic exponents, many mathematicians would start with a clean sheet of paper and look for another counterexample. Not Johnny! When that happened to him, he cheerfully said: "Oh, yes, a theta function...", and plowed ahead with the mountainous computations. He wasn't afraid of anything.

He knew a lot of mathematics, but there were also gaps in his knowledge, most notably number theory and algebraic toplogy. Once when he saw some of us at a blackboard staring at a rectangle that had arrows marked on each of its sides, he wanted to know that what was. "Oh just the torus, you know — the usual identification convention." No, he didn't know. The subject is elementary, but some of it just never crossed his path, and even though most graduate students knew about it, he didn't.

Brains, speed, and hard work produced results. In von Neumann's *Collected Works* there is a list of over 150 papers. About 60 of them are on pure mathematics (set theory, logic, topolog cal groups, measure theory, ergodic theory, operator theory, and continuous geometry), about 20 on physics, about 60 on applied mathematics (including statistics, game theory, and computer theory), and a small handful on some special mathematical subjects and general non-mathematical ones. A special number of the *Bulletin of the American Mathematical Society* was devoted to a discussion of his life and work (in May 1958).

Pure mathematics. Von Neumann's reputation as a mathematician was firmly

established by the 1930's, based mainly on his work on set theory, quantum theory, and operator theory, but enough more for about three ordinary careers, in pure mathematics alone, was still to come. The first of these was the proof of the ergodic theorem. Various more or less precise statements had been formulated earlier in statistical mechanics and called the ergodic hypothesis. In 1931 B. O. Koopman published a penetrating remark whose main substance was that one of the contexts in which a precise statement of the ergodic hypothesis could be formulated is the theory of operators on Hilbert space — the very subject that von Neumann used earlier to make quantum mechanics precise and on which he had written several epoch-making papers. It is tempting to speculate on von Neumann's reaction to Koopman's paper. It could have been something like this: "By Koopman's remark the ergodic hypothesis becomes a theorem about Hilbert spaces — and if that's what it is I ought to be able to prove it. Let's see now....." Soon after the appearance of Koopman's paper, von Neumann formulated and proved the statement that is now known as the mean ergodic theorem for unitary operators. There was some temporary confusion, caused by publication dates, about who did what before whom, but by now it is universally recognized that von Neumann's theorem preceded and inspired G. D. Birkhoff's point ergodic theorem. In the course of the next few years von Neumann published several more first-rate papers on ergodic theory, and he made use of the techniques and results of that theory later, in his studies of rings of operators.

In 1900 D. Hilbert presented a famous list of 23 problems that summarized the state of mathematical knowledge at the time and showed where further work was needed. In 1933 A. Haar proved the existence of a suitable measure (which has come to be called Haar measure) in topological groups; his proof appears in the *Annals of Mathematics*. Von Neumann had access to Haar's result before it was published, and he quickly saw that that was exactly what was needed to solve an important special case (compact groups) of one of Hilbert's problems (the 5th). His solution appears in the same issue of the same journal, immediately after Haar's paper.

In the second half of the 1930's the main part of von Neumann's publications was a sequence of papers, partly in collaboration with F. J. Murray, on what he called rings of operators. (They are now called von Neumann algebras.) It is possible that this is the work for which von Neumann will be remembered the longest. It is a technically brilliant development of operator theory that makes contact with von Neumann's earlier work, generalizes many familiar facts about finite-dimensional algebra, and is currently one of the most powerful tools in the study of quantum physics.

A surprising outgrowth of the theory of rings of operators is what von Neumann called continuous geometry. Ordinary geometry deals with spaces of dimension 1, 2, 3, etc. In his work on rings of operators von Neumann saw that what really determines the dimension structure of a space is the group of rotations that it admits. The group of rotations associated with the ring of *all* operators yields the familiar dimensions. Other groups, associated with different rings, assign to spaces dimensions

whose values can vary continuously; in that context it makes sense to speak of a space of dimension 3/4, say. Abstracting from the "concrete" case of rings of operators, von Neumann formulated the axioms that make these continuous-dimensional spaces possible. For several years he thought, wrote, and lectured about continuous geometries. In 1937 he was the Colloquium Lecturer of the American Mathematical Society and chose that subject for his topic.

Applied mathematics. The year 1940 was just about the half-way point of von Neumann's scientific life, and his publications show a discontinuous break then. Till then he was a topflight pure mathematician who understood physics; after that he was an applied mathematician who remembered his pure work. He became interested in partial differential equations, the principal classical tool of the applications of mathematics to the physical world. Whether the war made him into an applied mathematician or his interest in applied mathematics made him invaluable to the war effort, in either case he was much in demand as a consultant and advisor to the armed forces and to the civilian agencies concerned with the problems of war. His papers from this point on are mainly on statistics, shock waves, flow problems, hydrodynamics, aerodynamics, ballistics, problems of detonation, meteorology, and, last but not least, two non-classical, new aspects of the applicability of mathematics to the real world: games and computers.

Von Neumann's contributions to war were manifold. Most often mentioned is his proposal of the implosion method for bringing nuclear fuel to explosion (during World War II) and his espousal of the development of the hydrogen bomb (after the war). The citation that accompanied his honorary D.Sc. from Princeton in 1947 mentions (in one word) that he was a mathematician, but praises him for being a physicist, an engineer, an armorer, and a patriot.

Politics. His political and administrative decisions were rarely on the side that is described nowadays by the catchall term "liberal". He appeared at times to advocate preventive war with Russia. As early as 1946 atomic bomb tests were already receiving adverse criticism, but von Neumann thought that they were necessary and (in, for instance, a letter to the *New York Times*) defended them vigorously. He disagreed with J. R. Oppenheimer on the H-bomb crash program, and urged that the U.S. proceed with it before Russia could. He was, however, a "pro-Oppenheimer" witness at the Oppenheimer security hearings. He said that Oppenheimer opposed the program "in good faith" and was "very constructive" once the decision to go ahead with the super bomb was made. He insisted that Oppenheimer was loyal and was not a security risk.

As a member of the Atomic Energy Commission (appointed by President Eisenhower, he was sworn in on March 15, 1955), having to "think about the unthinkable", he urged a United Nations study of world-wide radiation effects. "We willingly pay 30,000–40,000 fatalities per year (2% of the total death rate)," he wrote, "for the advantages of individual transportation by automobile." He mentioned a

fall-out accident in an early Pacific bomb test that resulted in one fatality and danger to 200 people, and he compared it with a Japanese ferry accident that "killed about 1,000 people, including 20 Americans — yet the...fall-out was what attracted almost world-wide attention." He asked: "Is the price in international popularity worth paying?" And he answered: "Yes: we have to accept it as part payment for our more advanced industrial position."

Game theory. At about the same time that he began to apply his analytic talents to the problems of war, von Neumann found time and energy to apply his combinatorial insight to what he called the theory of games, whose major application was to economics. The mathematical cornerstone of the theory is one statement, the so-called minimax theorem, that von Neumann proved early (1928) in a short article (25 pages); its elaboration and applications are in the book he wrote jointly with O. Morgenstern in 1944. The minimax theorem says about a large class of two-person games that there is no point in playing them. If either player considers, for each possible strategy of play, the *maximum* loss that he can expect to sustain with that strategy, and then chooses the "optimal" strategy that *minimizes* the maximum loss, then he can be statistically sure of not losing more than that minimax value. Since (and this is the whole point of the theorem) that value is the negative of the one, similarly defined, that his opponent can guarantee for himself, the long-run outcome is completely determined by the rules.

Mathematical economics before von Neumann tried to achieve success by imitating the technique of classical mathematical physics. The mathematical tools used were those of analysis (specifically the calculus of variations), and the procedure relied on a not completely reliable analogy between economics and mechanics. The secret of the success of the von Neumann approach was the abandonment of the mechanical analogy and its replacement by a fresh point of view (games of strategy) and new tools (the ideas of combinatorics and convexity).

The role that game theory will play in the future of mathematics and economics is not easy to predict. As far as mathematics is concerned, it is tenable that the only thing that makes the Morgenstern-von Neumann book 600 pages longer than the original von Neumann paper is the development needed to apply the abstruse deductions of one subject to the concrete details of another. On the other hand, enthusiastic proponents of game theory can be found who go so far as to say that it may be "one of the major scientific contributions of the first half of the 20th century".

Machines. The last subject that contributed to von Neumann's fame was the theory of electronic computers and automata. He was interested in them from every point of view: he wanted to understand them, design them, build them, and use them. What are the logical components of the processes that a computer will be asked to perform? What is the best way of obtaining practically reliable answers from a machine with unreliable components? What does a machine need to "remember", and what is the best way to equip it with a "memory"? Can a machine be built that can not

only save us the labor of computing but save us also the trouble of building a new machine — is it possible, in other words, to produce a self-reproducing automaton? (Answer: in principle, yes. A sufficiently complicated machine, embedded in a thick chowder of randomly distributed spare parts, its "food", would pick up one part after another till it found a usable one, put it in place, and continue to search and construct till its descendant was complete and operational.) Can a machine successfully imitate "randomness", so that when no formulae are available to solve a concrete physical problem (such as that of finding an optimal bombing pattern), the machine can perform a large number of probability experiments and yield an answer that is statistically accurate? (The last question belongs to the concept that is sometimes described as the Monte Carlo method.) These are some of the problems that von Neumann studied and to whose solutions he made basic contributions.

He had close contact with several computers — among them the MANIAC (Mathematical Analyzer, Numerical Integrator, Automatic Calculator), and the affectionately named JONIAC. He advocated their use for everything from the accumulation of heuristic data for the clarification of our intuition about partial differential equations to the accurate long-range prediction and, ultimately, control of the weather. One of the most striking ideas whose study he suggested was to dye the polar icecaps so as to decrease the amount of energy they would reflect — the result could warm the earth enough to make the climate of Iceland approximate that of Hawaii.

The last academic assignment that von Neumann accepted was to deliver and prepare for publication the Silliman lectures at Yale. He worked on that job in the hospital where he died, but he couldn't finish it. His notes for it were published, and even they make illuminating reading. They contain tantalizing capsule statements of insights, and throughout them there shines an attitude of faith in and dedication to knowledge. While physicists, engineers, meteorologists, statisticians, logicians, and computers all proudly claim von Neumann as one of theirs, the Silliman lectures prove, indirectly by their approach and explicitly in the author's words, that von Neumann was first, foremost, and always a mathematician.

Death. Von Neumann was an outstanding man in tune with his times, and it is not surprising that he received many awards and honors. There is no point in listing them all here, but a few may be mentioned. He received several honorary doctorates, including ones from Princeton (1947), Harvard (1950), and Istanbul (1952). He served a term as president of the American Mathematical Society (1951–1953), and he was a member of several national scientific academies (including, of course, that of the U. S.). Somewhat to his embarrassment, he was elected to the East German Academy of Science, but the election didn't seem to take — in later years no mention is made of it in the standard biographical reference works. He received the Enrico Fermi award in 1956, when he already knew that he was incurably ill.

Von Neumann became ill in 1955. There was an operation, and the result was a diagnosis of cancer. He kept on working, and even travelling, as the disease progressed. Later he was confined to a wheelchair, but still thought, wrote, and attended meetings. In April 1956 he entered Walter Reed Hospital, and never left it. Of his last days his good friend Eugene Wigner wrote: "When von Neumann realized he was incurably ill, his logic forced him to realize that he would cease to exist, and hence cease to have thoughts. ...It was heartbreaking to watch the frustration of his mind, when all hope was gone, in its struggle with the fate which appeared to him unavoidable but unacceptable."

Von Neumann was baptized a Roman Catholic (in the U. S.), but, after his divorce, he was not a practicing member of the church. In the hospital he asked to see a priest — "one that will be intellectually compatible". Arrangements were made, he was given special instruction, and, in due course, he again received the sacraments. He died February 8, 1957.

The heroes of humanity are of two kinds: the ones who are just like all of us, but very much more so, and the ones who, apparently, have an extra-human spark. We can all run, and some of us can run the mile in less than 4 minutes; but there is nothing that most of us can do that compares with the creation of the Great G-minor Fugue. Von Neumann's greatness was the human kind. We can all think clearly, more or less, some of the time, but von Neumann's clarity of thought was orders of magnitude greater than that of most of us, all the time. Both Norbert Wiener and John von Neumann were great men, and their names will live after them, but for different reasons. Wiener saw things deeply but intuitively; von Neumann saw things clearly and logically.

What made von Neumann great? Was it the extraordinary rapidity with which he could understand and think and the unusual memory that retained everything he had once thought through? No. These qualities, however impressive they might have been, are ephemeral; they will have no more effect on the mathematics and the mathematicians of the future than the prowess of an athlete of a hundred years ago has on the sport of today.

The "axiomatic method" is sometimes mentioned as the secret of von Neumann's success. In his hands it was not pedantry but perception; he got to the root of the matter by concentrating on the basic properties (axioms) from which all else follows. The method, at the same time, revealed to him the steps to follow to get from the foundations to the applications. He knew his own strengths and he admired, perhaps envied, people who had the complementary qualities, the flashes of irrational intuition that sometimes change the direction of scientific progress. For von Neumann it seemed to be impossible to be unclear in thought or in expression. His insights were illuminating and his statements were precise.