American Economic Journal: Macroeconomics 2022, 14(1): 179–223
https://doi.org/10.1257/mac.20160164

The Rise of the Machines: Automation, Horizontal Innovation, and Income Inequality†

By David Hémous and Morten Olsen*

We build an endogenous growth model with automation (the replacement of low-skill workers with machines) and horizontal innovation (the creation of new products). Over time, the share of automation innovations endogenously increases through an increase in low-skill wages, leading to an increase in the skill premium and a decline in the labor share. We calibrate the model to the US economy and show that it quantitatively replicates the paths of the skill premium, the labor share, and labor productivity. Our model offers a new perspective on recent trends in the income distribution by showing that they can be explained endogenously. (*JEL* D31, E25, J24, J31, O33, O41)

In the past 50 years, the United States has seen dramatic changes in the income distribution. The skill premium increased by 33 percent between 1963 and 2012, and the labor share has declined by 7 percentage points (p.p.) since the 1970s (panels A and B of Figure 1). Meanwhile, several automation technologies (numerically controlled machine tools, automatic conveyor systems, industrial robots…) have been introduced, thereby increasing the range of tasks for which machines can substitute for labor. This is supported by patent data, which suggest that the share of automation innovation has increased over time (panel C of Figure 1 plots the ratio of automation to nonautomation patents in machinery in the United States according to Dechezleprêtre et al. 2019).

Our goal is to assess whether these trends can be explained endogenously as reflecting the transitional dynamics of an economy. To do so, we build a model with high- and low-skill workers that combines horizontal innovation (the creation of

*Hémous: University of Zurich and CEPR (email: david.hemous@econ.uzh.ch); Olsen: University of Copenhagen (email: mgo@econ.ku.dk). Richard Rogerson was coeditor for this article. Morten Olsen gratefully acknowledges the financial sup port of the European Commission under the Marie Curie Research Fellowship program (Grant Agreement PCIG11-GA-2012-321693) and the Spanish Ministry of Economy and Competitiveness (Project ref: ECO2012-38134). We thank three anonymous referees for their suggestions. We thank Daron Acemoglu, Philippe Aghion, Ufuk Akcigit, Pol Antràs, Tobias Broer, Steve Cicala, Per Krusell, Brent Neiman, Jennifer Page, Ofer Setty, Andrei Shleifer, Che-Lin Su, Fabrizio Zilibotti, and Joachim Voth among others for helpful comments. We also thank seminar and conference participants at IIES, University of Copenhagen, University of Warwick, UCSD, UCLA Anderson, USC Marshall, Barcelona GSE Summer Forum, the 6th Joint Macro Workshop at Banque de France, University of Chicago Harris School of Public Policy, the 2014 SED meeting, the NBER Summer Institute, the 2014 EEA meeting, École Polytechnique, the University of Zurich, NUS, London School of Economics, the 2015 World Congress of the Econometric Society, ECARES, Columbia University, EIEF, Brown University, Boston University, Yale University, Collège de France, Washington University, and the CEPR conference on Growth and Inequality. We thank Ria Ivandic and Marton Varga for excellent research assistance.
†Go to https://doi.org/10.1257/mac.20160164 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

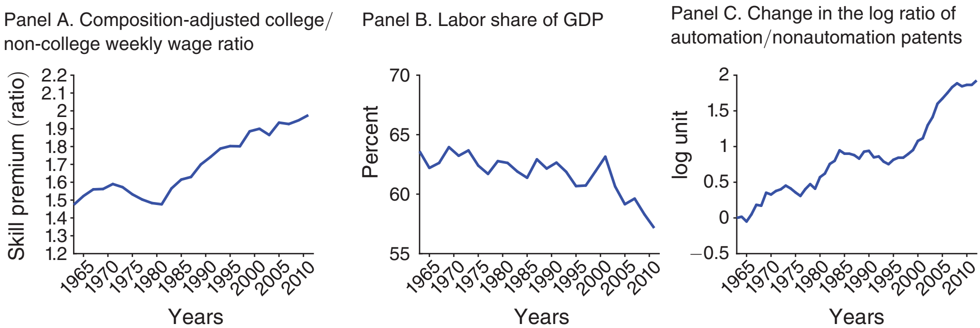

Figure 1. The US Skill premium, Labor Share, and Automation Innovations

Notes: Panel A is taken from Autor (2014). Panel B is from the BLS. Panel C reports the increase in the log ratio of automation to nonautomation innovations in machinery in the United States according to Dechezleprêtre et al. (2019). See further details in Section III.

new products or tasks) and automation. Automation takes place in existing product lines and enables the replacement of low-skill workers with machines. Therefore, our model embodies a task framework where machines can substitute for workers as Autor, Levy, and Murnane (2003), directed technical change as Acemoglu (1998) since innovation endogenously occurs in two different technologies, and capital-skill complementarity as Krusell et al. (2002)—henceforth, KORV. While these papers rely on exogenous shocks (the advent of computers, an increase in the skill supply prompting a change in the direction of innovation, and a drop in the equipment price, respectively) to explain trends in the income distribution, we argue instead that this can be the result of an *endogenous* increase in the share of automation innovations. Moreover, the interplay between automation and horizontal innovation allows us to account for two puzzles in the literature: the stagnation of labor productivity growth despite the rise in automation innovations and the deceleration of the skill premium since the mid-1990s without an apparent decline in skill-biased technical change (SBTC).

We develop our analysis in three steps. First, we present a version of the model in which technical change is exogenous. Horizontal innovation increases both low-skill and high-skill wages. Within a firm, automation increases the demand for high-skill workers but reduces the demand for low-skill workers. At the aggregate level, automation has an ambiguous effect on low-skill wages, and, in line with recent trends, it increases the skill premium and reduces the labor share.

Second, we endogenize innovation, which allows us to rationalize the observed increase in the share of automation innovations. We show that in an economy where low-skill wages are low, there is little automation. As low-skill wages increase with horizontal innovation, the incentive to automate increases and with it the share of automation innovation. As a result, the skill premium rises, the labor share declines, and low-skill wages may temporarily decline. Finally, the economy moves toward an asymptotic steady state where the share of automation innovations stabilizes and low-skill wages grow, though slower than high-skill wages and GDP.

In a third step, to assess how far our "endogenous-transition" approach can go quantitatively, we calibrate an extended version of our model to match the evolution of the skill premium, the labor share, productivity, and the equipment-to-GDP ratio from 1963 to 2012. Our model captures the trends in the data fairly well. In particular, labor productivity growth stagnates as horizontal innovation declines and the skill premium decelerates in the 1990s and 2000s even though innovation is more directed toward automation.[1] Moreover, conditional on our aggregate production function, a model with exogenous technology would not capture trends better.

We model automation as high-skill-biased following a large literature showing that computerization (Autor, Katz, and Krueger 1998; Autor, Levy, and Murnane 2003; and Bartel, Ichniowski, and Shaw 2007) or industrial robots (Acemoglu and Restrepo 2017b and Graetz and Michaels 2018) decrease the relative demand for low-skill labor.[2]

A large macro literature has argued that SBTC can explain the increase in the skill premium since the 1970s. This literature can be divided into three strands. The first emphasizes Nelson and Phelps's (1966) hypothesis that skilled workers adapt better to technological change (Lloyd-Ellis 1999; Caselli 1999; Galor and Moav 2000; Aghion, Howitt, and Violante 2002; Beaudry, Green, and Sand 2016). While such theories explain transitory increases in inequality, our model features widening inequality. Yet we borrow the idea of a shift in production technology spreading through the economy.

A second strand emphasizes the role of capital-skill complementarity: KORV find that the observed increase in the stock of capital equipment can account for most of the variation in the skill premium. Our model also features capital-skill complementarity but differs in several dimensions: it includes low-skill laborsaving innovations; our quantitative exercise is more demanding because we endogenize technology; and we match a decline in the labor share, whereas they have a small increase.

A third branch assumes that technical change is either low- or high-skill labor augmenting and measures the bias of technology (Katz and Murphy 1992, Goldin and Katz 2008, and Katz and Margo 2014). The directed technical change literature (Acemoglu 1998, 2002, 2007) then endogenizes this bias with the skill supply. Such models have no role for labor-replacing technology and cannot generate changes in the labor share (see Acemoglu and Autor 2011). None of these approaches try to explain features of the income distribution through the transitional dynamics of an economy.

[1] Intuitively, this comes from looking at automation as a stock: with a higher share of automated products, there must be more automation innovation to compensate for its depreciation through horizontal innovation. As a result, our model addresses the Card and DiNardo (2002) critique that the slowdown in the skill premium is inconsistent with the SBTC hypothesis.

[2] Autor, Katz, and Kearney (2006, 2008) and Autor and Dorn (2013) relate wage and job polarization to the computer-driven automation of routine tasks often performed by middle-skill workers. We do not distinguish between low- and middle-skill workers, as both have often performed tasks that have later on been automated (a previous version of this paper, Hémous and Olsen 2016, did so). See also Feng and Graetz (2016).

The idea that high wages might incentivize laborsaving technical change dates back to Habakkuk (1962).[3] In Zeira (1998), exogenous increases in TFP raise wages and encourage the adoption of a capital-intensive technology, which further raises wages (while automation can reduce wages in our model). Acemoglu (2010) shows that labor scarcity induces laborsaving innovation. Neither paper analyzes laborsaving innovation in a fully dynamic model nor focuses on income inequality. Peretto and Seater (2013) build a dynamic model of automation where wages are constant. To get a more realistic path for wages, we introduce a second type of innovation, namely the creation of new products or tasks. In work subsequent to our paper, Acemoglu and Restrepo (2017a) also develop a growth model where technical change involves automation and the creation of new tasks. While in our model all tasks are symmetric (except for whether they are automated), in theirs, new tasks are exogenously born with a higher labor productivity. As a result, their model features a balanced growth path, and they focus on the self-correcting elements of the economy after a technological shock, while we focus on accounting for secular trends. Subsequent papers combining automation and horizontal innovations include Rahman (2017), Martinez (2018), and Zeira and Nakamura (2018).[4]

Section I describes the baseline model with exogenous technology. Section II endogenizes the path of technology and rationalizes the increase in the share of automation innovations. Section III calibrates an extended version of the model. Section IV concludes. The main Appendix presents the proofs of the propositions and additional exercises on the quantitative model. The online Appendix presents the proofs of additional results, various extensions, and details of the calibration exercise.

I. A Baseline Model with Exogenous Innovation

This section presents a model with exogenous technology to study the consequences of automation and horizontal innovation on factor prices. Section IC derives comparative statics results and relates them to the evolution of the US income distribution. Section ID analyzes the asymptotic behavior of wages for general paths of technology.

A. *Preferences and Production*

We consider a continuous time infinite-horizon economy populated by H high-skill and L low-skill workers. Both types of workers supply labor inelastically and have identical preferences over a single final good of

$$U_{k,t} = \int_t^{\infty} e^{-\rho(\tau - t)} \frac{C_{k,\tau}^{1-\theta}}{1-\theta} d\tau,$$

[3] A few recent papers provide empirical evidence for the role of wages on technology adoption: Lewis (2011); Hornbeck and Naidu (2014); Manuelli and Seshadri (2014); Clemens, Acemoglu, and Restrepo (2018b); Lewis and Postel (2018); and Dechezleprêtre et al. (2019).

[4] Benzell et al. (2019), following Sachs and Kotlikoff (2012), build a model where a code-capital stock can substitute for labor and show that a technological shock that favors the accumulation of code-capital can lead to lower long-run GDP.

where ρ is the discount rate, $\theta \geq 1$ is the inverse elasticity of intertemporal substitution, and $C_{k,t}$ is consumption of the final good at time t by group $k \in \{H,L\}$. The final good is produced by a competitive industry combining a set of intermediate products, $i \in [0, N_t]$, using a CES aggregator:

$$Y_t = \left(\int_0^{N_t} y_t(i)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}},$$

where $y_t(i)$ is the use of intermediate product i at time t and $\sigma > 1$ is the elasticity of substitution between these products. As in Romer (1990), an increase in N_t represents a source of technological progress.

We normalize the price of Y_t to one at all points in time and drop time subscripts when there is no ambiguity. The demand for each product i is

$$(1) \qquad y(i) = p(i)^{-\sigma} Y,$$

where $p(i)$ is the price of product i and the normalization implies that the ideal price index, $[\int_0^N p(i)^{1-\sigma} di]^{1/(1-\sigma)}$, equals one.

Each product is produced by a monopolist who owns the perpetual rights of production. Production occurs by combining low-skill labor, $l(i)$, high-skill labor, $h(i)$, and, possibly, type-i machines, $x(i)$, according to

$$(2) \qquad y(i) = \left[l(i)^{\frac{\epsilon-1}{\epsilon}} + \alpha(i)\left(\tilde{\varphi}x(i)\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon\beta}{\epsilon-1}} h(i)^{1-\beta},$$

where $\alpha(i) \in \{0,1\}$ is an indicator function for whether or not the monopolist has access to an automation technology, which allows for the use of machines.[5] If the product is not automated ($\alpha(i) = 0$), production takes place using a Cobb-Douglas production function with only low-skill and high-skill labor and a low-skill factor share of β. If it is automated ($\alpha(i) = 1$), machines can be used in the production process as a substitute for low-skill labor with an elasticity $\epsilon > 1$. The parameter $\tilde{\varphi}$ is the relative productivity advantage of machines over low-skill workers, and G denotes the share of automated products. Therefore, automation takes the form of a secondary innovation in existing product lines.[6]

Since each product is produced by a single firm, we identify each product with its firm and refer to a firm that uses an automated production process as an automated firm. We refer to the specific labor inputs provided by high-skill and low-skill workers in the production of different products as "different tasks" performed by these workers so that each product comes with its own tasks. It is because $\alpha(i)$ is not fixed, but can change over time, that our model captures the notion that machines can replace workers in new tasks. A model with a fixed $\alpha(i)$ for each product would

[5] We allow for perfect substitutability ($\epsilon = \infty$) in which case $y(i) = [l(i) + \alpha(i)\tilde{\varphi}x(i)]^{\beta} h(i)^{1-\beta}$.

[6] Secondary innovations in a growth model were introduced by Aghion and Howitt (1996), who study the interplay between applied and fundamental research.

only allow for machines to be used more intensively in production but always for the same tasks.

Although we will refer to x as "machines," our interpretation also includes any form of computer inputs, algorithms, the services of cloud-providers, etc. In Section III, we will identify machines with equipment (excluding transport) and software. In turn, automation innovations refer to innovations that allow machines to accomplish tasks with less need for a human operator. This includes robotics but also computer numerical control machine tools, automatic conveyor belts, computer-aided design, etc.[7]

For now, machines are an intermediate input—this assumption is innocuous, and in Section III machines are a capital input without changing our results qualitatively. Once invented, machines of type i are produced competitively one-for-one with the final good, such that the price of an existing machine is always equal to one and technological progress in machine production follows that in the rest of the economy. Yet our model can capture the notion of a decline in the real cost of equipment, as automation for firm i can equivalently be interpreted as a decline of the price of machine i from infinity to one.

B. *Equilibrium Wages*

In this section, we derive how wages are determined in equilibrium, taking as given the number of products N, the share of automated products G, and the employment of high-skill workers in production $H^P \equiv \int_0^N h(i)\,di$ (we let $H^P \leq H$ to accommodate later sections where high-skill labor is used to innovate).

From equation (2), the unit cost of product i is given by

$$c\big(w_L, w_H, \alpha(i)\big) = \beta^{-\beta}\big(1-\beta\big)^{-(1-\beta)}\left(w_L^{1-\epsilon} + \varphi\alpha(i)\right)^{\frac{\beta}{1-\epsilon}} w_H^{1-\beta}, \tag{3}$$

where $\varphi \equiv \tilde{\varphi}^{\epsilon}$, w_L denotes low-skill wages, and w_H high-skill wages. For all $w_L, w_H > 0$, automation reduces costs ($c(w_L, w_H, 1) < c(w_L, w_H, 0)$). Price is set as a markup over costs: $p(i) = \sigma/(\sigma-1)\cdot c(w_L, w_H, \alpha(i))$. Using Shepard's lemma and equations (1) and (3) delivers the demand for low-skill labor of a single firm:

$$l\big(w_L, w_H, \alpha(i)\big) = \beta\frac{w_L^{-\epsilon}}{w_L^{1-\epsilon} + \varphi\alpha(i)}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma} c\left(w_L, w_H, \alpha(i)\right)^{1-\sigma} Y. \tag{4}$$

The effect of automation on demand for low-skill labor in a firm is generally ambiguous. This is due to the combination of a negative substitution effect (automation allows for substitution between machines and low-skill workers) and a positive scale effect (automation decreases costs, lowers prices, and increases production). As we focus on laborsaving innovation, we impose the condition $\epsilon > 1 + \beta(\sigma - 1)$ throughout the paper, which is necessary and sufficient for

[7] We include IT innovations in our interpretation because our model does not distinguish between low-skill and middle-skill workers.

the substitution effect to dominate at the firm level and ensures $l(w_L, w_H, 1) < l(w_L, w_H, 0)$ for all $w_L, w_H > 0$.

At the aggregate level, since both automated firms and nonautomated firms are symmetric, output can written as

$$Y = N^{\frac{1}{\sigma-1}} \times \left((1-G)^{\frac{1}{\sigma}} \Big(\underbrace{\left(L^{NA}\right)^{\beta} \left(H^{P,NA}\right)^{1-\beta}}_{T_1} \Big)^{\frac{\sigma-1}{\sigma}} + G^{\frac{1}{\sigma}} \Big(\underbrace{\left[\left(L^{A}\right)^{\frac{\epsilon-1}{\epsilon}} + (\tilde{\varphi} X)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon\beta}{\epsilon-1}} \left(H^{P,A}\right)^{1-\beta}}_{T_2} \Big)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

where L^A (respectively L^{NA}) is the total mass of low-skill workers in automated (respectively nonautomated) firms, $H^{P,A}$ (respectively $H^{P,NA}$) is the total mass of high-skill workers hired in production in automated (respectively nonautomated) firms, and $X = \int_0^N x(i)\, di$ is total use of machines. The first term T_1 captures the classic case where production takes place with constant shares between factors (low-skill and high-skill labor). The second term T_2 represents the factors used within automated products and features substitutability between low-skill labor and machines. Note, G is the share parameter of the "automated" products nest, and therefore an increase in G is T_2-biased (as $\sigma > 1$). Finally, $N^{1/(\sigma-1)}$ is a TFP parameter.[8]

With CRS and perfect competition in final good production, the price of the final good is equal to its cost. Using that all intermediate producers charge the same markup $\sigma/(\sigma-1)$ and that final good output obeys equation (5), the price normalization gives

$$\frac{\sigma}{\sigma-1} \frac{N^{\frac{1}{1-\sigma}}}{\beta^{\beta}(1-\beta)^{1-\beta}} \left(G \left(\varphi + w_L^{1-\epsilon} \right)^{\mu} + (1-G)\, w_L^{\beta(1-\sigma)} \right)^{\frac{1}{1-\sigma}} w_H^{1-\beta} = 1, \tag{6}$$

where we define $\mu \equiv \beta(\sigma-1)/(\epsilon-1) < 1$ (by our assumption on ϵ). This relationship defines the *unit isocost curve* in the (w_L, w_H) space in Figure 2. It shows the positive relationship between real wages and the level of technology given by N, the number of products, and G, the share of automated firms.

Applying Shepard's lemma to the cost function defined by the left-hand side of (6), we get the relative labor shares in production:[9]

$$\frac{w_H H^P}{w_L L} = \frac{1-\beta}{\beta} \frac{G + (1-G)\left(1 + \varphi w_L^{\epsilon-1}\right)^{-\mu}}{G\left(1 + \varphi w_L^{\epsilon-1}\right)^{-1} + (1-G)\left(1 + \varphi w_L^{\epsilon-1}\right)^{-\mu}}. \tag{7}$$

[8] For automated firms, this model features an elasticity of substitution between high-skill labor and machines equal to that between high-skill and low-skill labor. This, however, does not hold at the aggregate level, consistent with KORV, who argue that the aggregate elasticity of substitution between high-skill and low-skill labor is greater than that between high-skill labor and machines.

[9] When $\epsilon = \infty$, the skill premium is given by $\frac{w_H}{w_L} = \frac{1-\beta}{\beta}\frac{L}{H^P}$ if $w_L < \tilde{\varphi}^{-1}$ such that no firm uses machines, and $\frac{w_H}{w_L} = \frac{1-\beta}{\beta}\frac{L}{H^P}\frac{G + (1-G)(\tilde{\varphi} w_L)^{-\beta(\sigma-1)}}{(1-G)(\tilde{\varphi} w_L)^{-\beta(\sigma-1)}}$ if $w_L > \tilde{\varphi}^{-1}$.

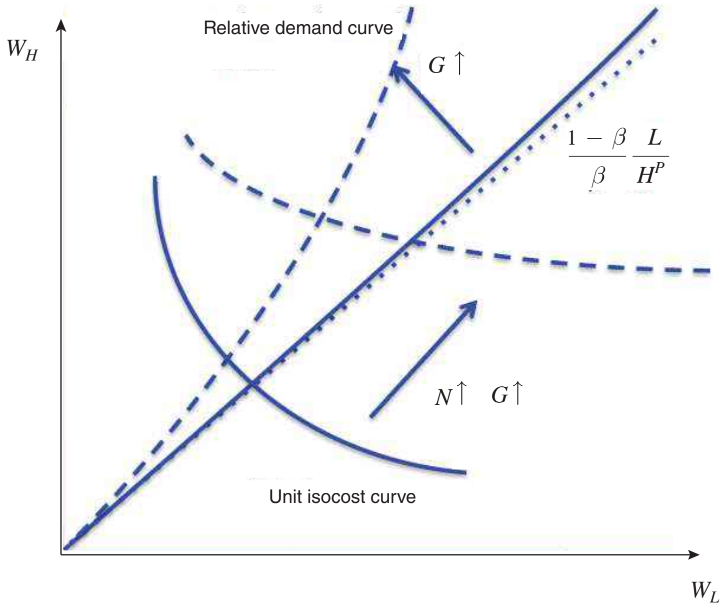

Figure 2. Relative Demand Curve and Isocost Curve for Different Values of N and G

This expression gives the *relative demand curve* for high-skill and low-skill labor drawn in Figure 2. Together (6) and (7) determine real wages uniquely as a function of N, G, and H^P. For $G = 0$, the relative demand curve is a straight line, with slope $(1-\beta)L/(\beta H^P)$, reflecting the constant factor shares in a Cobb-Douglas economy. For $G > 0$, the right-hand side of (7) increases in w_L, so that the relative demand curve is nonhomothetic and bends counterclockwise as w_L grows. Therefore, as long as G tends toward a positive constant, low-skill and high-skill wages cannot grow at the same rate in the long run.

Intuitively, higher low-skill wages increase the ratio of high-skill to low-skill labor share in production for two reasons. First, they induce more substitution toward machines in automated firms as their use relative to low-skill labor obeys $x/l = \varphi w_L^{\epsilon}$, reflected by the term $(1+\varphi w_L^{\epsilon-1})^{-1}$ in (7)—recall that $\epsilon > 1$. Second, higher low-skill wages improve the cost advantage of automated firms and their market share. Using (1) and (3), the relative revenues (and profits) of nonautomated and automated firms are

$$(8)\quad R(w_L, w_H, 0)/R(w_L, w_H, 1) = \pi(w_L, w_H, 0)/\pi(w_L, w_H, 1) = \left(1+\varphi w_L^{\epsilon-1}\right)^{-\mu},$$

which decreases in w_L (reflected by the term $(1+\varphi w_L^{\epsilon-1})^{-\mu}$ in (7)).

The static equilibrium is closed by the final good market clearing condition $Y = C + X$, where $C = C_L + C_H$ is total consumption. Note, *GDP* includes the payment to labor and aggregate profits, which are a share $1/\sigma$ of output. Therefore, GDP and the total labor share *LS* are given by

$$(9)\quad GDP \equiv \frac{1}{\sigma}Y + w_L L + w_H H, \quad LS = 1 - \frac{1}{1+(\sigma-1)(1-\beta)\left(\frac{w_L L}{w_H H^P} + \frac{H}{H^P}\right)},$$

where the second equality uses that payment to high-skill labor in production is a constant share $(1-\beta)(\sigma-1)/\sigma$ of output.

C. *Technical Change and Wages*

We analyze the consequences of technical change on the level of wages using Figure 2. An increase in the number of products, *N*, pushes out the isocost curve and increases both low-skill and high-skill wages. When $G = 0$, both wages grow at the same rate since the relative demand curve is a straight line, but for $G > 0$, the demand curve is nonhomothetic and the skill premium grows. Therefore, an increase in *N* at *constant* $G\ (> 0)$ is high-skill biased.

An increase in the share of automated products *G* has a positive effect on high-skill wages and the skill premium but an ambiguous effect on low-skill wages: Higher automation increases the productive capability of the economy and pushes out the isocost curve (an *aggregate scale* effect), which increases low-skill wages. Yet it also allows for easier substitution away from low-skill labor, which pivots the relative demand curve counterclockwise (an *aggregate substitution* effect), decreasing low-skill wages. Therefore, automation is always high-skill labor biased (w_H/w_L increases), but it is low-skill laborsaving (w_L decreases) if and only if the aggregate substitution effect dominates the aggregate scale effect. Formally, one can show the following result holds (proof in Appendix A1).[10]

PROPOSITION 1: *Consider the equilibrium* (w_L, w_H) *determined by equations* (6) *and* (7). *Assume that* $\epsilon < \infty$. *It holds that*

(*A*) *An increase in the number of products N* (*keeping G and* H^P *constant*) *leads to an increase in both high-skill* (w_H) *and low-skill wages* (w_L). *Provided that* $G > 0$, *an increase in N also increases the skill premium* w_H/w_L *and decreases the labor share.*

(*B*) *An increase in the share of automated products G* (*keeping N and* H^P *constant*) *increases the high-skill wages* w_H, *the skill premium* w_H/w_L, *and decreases the labor share. Its impact on low-skill wages is generally ambiguous, but low-skill wages are decreasing in G if* (*i*) $1 \leq (\sigma-1)(1-\beta)$ *or if* (*ii*) *N and G are high enough.*

(*C*) *An increase in the number of nonautomated products* (*an increase in N keeping GN constant*) *increases both high-skill* (w_H) *and low-skill wages* (w_L). *If N is large enough or* $\epsilon < \sigma$, *it decreases the skill premium.*

[10] In the perfect substitute case, $\epsilon = \infty$, w_H increases in *N* and weakly increases in *G*, w_H/w_L weakly increases in *N* and *G*, and w_L weakly increases in *N* and weakly decreases in *G* provided that $1/(1-\beta) \leq \sigma - 1$ or *G* is large enough. When $\epsilon = \infty$ and $G = 1$, the isocost curve has a horizontal arm and the relative demand curve a vertical one.

Part B gives sufficient conditions under which automation is low-skill laborsaving. The aggregate substitution effect is larger than the scale effect in two cases: (i) The elasticity of substitution σ is large, as newly automated products gain a larger market share; or the cost share of the low-skill labor-machines aggregate β is small as the cost-saving effect of automation is small. (ii) G and N are large: in that case, additional automation hurts low-skill workers more, as there are few nonautomated firms, while most of the aggregate productivity gains are already realized. It is worth comparing the effect of automation with that of an increase in machines' productivity φ (equivalent to a decline in the price of machine). The latter also has an ambiguous effect on low-skill wages resulting from the combination of a substitution effect and a scale effect, but it is less likely to be low-skill laborsaving than automation.[11]

Part C considers an increase in the number of nonautomated products, which corresponds to the "horizontal innovation" to be introduced in Section II. Such technological change pushes out the isocost curve (N increases) but also makes the relative demand curve rotate clockwise (GN stays constant). This increases demand for both types of workers and therefore both wages. Horizontal innovation is low-skill labor biased (it reduces the skill premium) if N is large, in which case w_L is large so that the isocost curve does not move much with horizontal innovation, or if machines and low-skill workers are not too substitute ($\epsilon \leq \sigma$ is a sufficient condition).[12]

Proposition 1 offers important insights into the types of technological change that can simultaneously account for a rising skill premium (panel A of Figure 1) and a decreasing labor share (panel B of Figure 1). Specifically, these trends are consistent with an economy with a growing number of products and a constant or rising share of automated products.[13] Moreover, it suggests that an increase in automation may also give rise to a decrease in wages for low-skill workers. Section II will model innovation and explain why we should expect a rising number of products and a rising share of automated products until it approaches an asymptotic steady-state value.

D. *Asymptotics for General Technological Processes*

We study the asymptotic behavior of the model for given paths of technologies and mass of high-skill workers in production. For any variable a_t (such as N_t), we let $g_t^a \equiv \dot{a}_t / a_t$ denote its growth rate and $g_\infty^a = \lim_{t\to\infty} g_t^a$ if it exists. We focus here on the case where the share of automated products admits an interior limit $G_\infty \in (0,1)$ for which we obtain the following result (proof in Appendix A2; online Appendix B2.1 studies the cases where $G_\infty = 0$ or 1).

[11] Formally, we show that $\partial w / \partial \varphi < 0$ implies that $\partial w / \partial G < 0$ but the reverse is not true—see Appendix A1. Intuitively, this is the case because an increase in automation not only acts as "factor augmenting technical change" for the inputs within automated firms but also as "factor-depleting technical change" for the inputs in nonautomated firms. This point can be seen from equation (5) and is made by Aghion, Jones, and Jones (2019).

[12] In the perfect substitute case, an increase in the number of nonautomated products increases w_H and weakly increases w_L. If $G < 1$ and N is large enough, it decreases the skill premium.

[13] The decreasing relationship between the labor share and the skill premium obtained when machines are an intermediate input generalizes to the case where machines are part of a capital stock with a perfectly elastic supply (see Section III and online Appendix B10).

PROPOSITION 2: *Consider three processes* $[N_t]_{t=0}^{\infty}$, $[G_t]_{t=0}^{\infty}$, *and* $[H_t^P]_{t=0}^{\infty}$, *where* $(N_t, G_t, H_t^P) \in (0,\infty) \times [0,1] \times (0,H]$ *for all t. Assume that* G_t, g_t^N, *and* H_t^P *all admit limits* G_∞, g_∞^N, *and* H_∞^P *with* $G_\infty \in (0,1)$, $g_\infty^N > 0$, *and* $H_\infty^P > 0$. *Then, the asymptotic growth of high-skill wages* w_{Ht} *and output* Y_t *are*

$$g_\infty^{w_H} = g_\infty^Y = g_\infty^N / \big((1-\beta)(\sigma-1)\big), \tag{10}$$

and the asymptotic growth rate of w_{Lt} *is given by*

$$g_\infty^{w_L} = g_\infty^Y / \big(1+\beta(\sigma-1)\big). \tag{11}$$

This proposition first relates the growth rates of output and high-skill wages to the growth rate of the number of products. Note: Y_t is proportional to $N_t^{1/((1-\beta)(\sigma-1))}$ in the long run. This reflects the standard expanding-variety model gains and, in the presence of automation ($G_\infty > 0$), a multiplier effect, which increases in the asymptotic share of machines β, as machines are a reproducible input.

Second, when there is a positive share of nonautomated products asymptotically, $G_\infty < 1$, low-skill workers and machines are imperfect substitutes in the aggregate (even if there are perfect substitutes at the firm level $\epsilon = \infty$).[14] As a result, low-skill wages must grow at a positive rate asymptotically when the number of products grows. Intuitively, a growing stock of machines and a fixed supply of low-skill labor imply that the relative price of a worker (w_{Lt}) to a machine (p_t^x) must grow at a positive rate. Since machines are produced with the same technology as the consumption good, p_t^x equals one and the real wage w_{Lt} must grow at a positive rate.

Third, the proposition shows that if $G_\infty > 0$, low-skill wages cannot grow at the same rate as output. This result follows from Uzawa's theorem: equation (5) shows that an increase in the number of product N_t is not labor augmenting unless $G_\infty = 0$. We get $G_\infty > 0$ as long as the automation intensity is bounded away from zero (see online Appendix B2.2). Further, when $G_\infty < 1$, the demand for low-skill labor increasingly comes from the nonautomated firms (as automation is laborsaving at the firm level), while most of the demand for high-skill labor comes from automated firms. With growing wages, the relative market share of nonautomated firms decreases in proportion to $(1+\varphi w_{Lt}^{\epsilon-1})^{-\mu} \sim \varphi^{-\mu} w_{Lt}^{-\beta(\sigma-1)}$. Then, the growth rate of low-skill wages is a fraction of the growth rate of high-skill wages given by (11). The ratio between the growth rates of high- and low-skill wages increases with a higher importance of low-skill workers (a higher β) or a higher substitutability between automated and nonautomated products (a higher σ) since both imply a faster loss of competitiveness of the nonautomated firms. Yet it is independent of the elasticity of substitution between machines and low-skill workers, ϵ, or of

[14] As long as new nonautomated products are continuously introduced, and the intensity at which nonautomated firms are automated is bounded, the share of nonautomated products is always positive; i.e., $G_\infty < 1$ (see proof in online Appendix B2.2). This ensures that there is no economy-wide perfect substitution between low-skill workers and machines.

the exact asymptotic share of automated products G_∞. In this case, nonautomated products provide employment opportunities for low-skill workers, which limits the relative losses of low-skill workers compared to high-skill workers (their wages grow according to (11) instead of (B5) and $\epsilon > 1 + \beta(\sigma - 1)$). In the model of Section II, the economy endogenously ends up in this case.

Proposition 2 establishes general conditions under which low-skill wages grow asymptotically but slower than high-skill wages. We briefly discuss the robustness of this result. First, one might be concerned that the slower growth in low-skill wages is an artifact of having exogenous supplies of low- and high-skill labor. Online Appendix B3 extends our model to allow for endogenous skill choice. Specifically, we consider a Roy model in which workers have heterogeneous comparative advantage between being low- and high-skill. Low-skill wages still grow slower than high-skill wages asymptotically, and now the share of low-skill workers tends toward zero. Second, the result that imperfect aggregate substitution between machines and low-skill workers leads to positive growth in low-skill wages asymptotically relies on machines and the consumption good sharing the same production technology. Online Appendix B4 relaxes this assumption and allows for negative growth in p_t^x. In that case, low-skill wages may but need not decline asymptotically.

II. Endogenous Innovation

We now model automation and horizontal innovation as the result of investment. In Sections IIA–IIC, we analyze the effect of wages on innovation (the reverse of Proposition 1), study the transitional dynamics of the system, and explain why the economy should experience an increase in the share of automated products as it develops. Section IID explores the interactions between the two innovation processes. Online Appendix B6 provides numerical examples to illustrate this section and shows comparative statics results.

A. *Modeling Innovation*

If a nonautomated firm hires $h_t^A(i)$ high-skill workers to perform automation research, it becomes automated at a Poisson rate $\eta G_t^{\tilde{\kappa}} (N_t h_t^A(i))^\kappa$. Once a firm is automated, it remains so forever. The parameter $\eta > 0$ denotes the productivity of the automation technology; $\kappa \in (0,1)$ measures its concavity; $G_t^{\tilde{\kappa}}$, $\tilde{\kappa} \in [0,\kappa]$, represents possible knowledge spillovers from the share of automated products; and N_t represents knowledge spillovers from the total number of products. The spillovers in N_t ensure that both automation and horizontal innovation may take place in the long run; they exactly compensate for the mechanical reduction in the amount of resources for automation available for each product (namely high-skill workers) when the number of product increases.[15] The presence of spillovers in automation technology

[15] These spillovers can be micro-founded as follows: let there be a fixed mass one of firms indexed by j each producing a continuum N_t of products indexed by i so that production is given by $Y_t = \left(\int_0^1 \int_0^{N_t(j)} y_t(i,j)^{\frac{\sigma-1}{\sigma}} di dj\right)^{\frac{\sigma}{\sigma-1}}$. When a firm hires $\tilde{H}_t^A(j)$ high-skill workers in automation, each of its nonautomated products gets independently automated with a Poisson rate of $\eta G_t^{\tilde{\kappa}}[\tilde{H}_t^A(j)/(1 - G_t(j))]^\kappa$. The aggregate economy would be identical to ours and

($\tilde{\kappa} > 0$) implies a delayed and faster rise in the share of automated products.[16] We assume that $\tilde{\kappa} < 1 - \kappa(1-\beta)$, which ensures that automation always takes off (see Proposition 3).

New products are developed by high-skill workers in a standard manner according to a linear technology with productivity γN_t. With H_t^D high-skill workers pursuing horizontal innovation, the mass of products evolves according to

$$\dot{N}_t = \gamma N_t H_t^D.$$

We assume that firms do not exist before their product is created and therefore cannot invest in automation. As a result, new products are born nonautomated, which means that "horizontal innovation" corresponds to an increase in N_t keeping $G_t N_t$ constant and is low-skill biased under certain conditions (see Proposition 1). This is motivated by the idea that when a task is new and unfamiliar, solving unforeseen problems requires the flexibility and outside experience of human workers. Only as the task becomes routine and potentially codefiable, a machine (or an algorithm) can perform it (Autor 2013).

As nonautomated firms get automated at Poisson rate $\eta G_t^{\tilde{\kappa}} (N_t h_t^A)^{\kappa}$, and new firms are born nonautomated, the share of automated firms obeys

$$\dot{G}_t = \eta G_t^{\tilde{\kappa}} \left(N_t h_t^A\right)^{\kappa} \left(1 - G_t\right) - G_t g_t^N. \tag{12}$$

Therefore, the level of automation in the economy, G_t, can be understood as a "stock" that depreciates through the introduction of new products. As a result, for a given growth rate in the number of products (g_t^N), a higher automation intensity per product ($\eta G_t^{\tilde{\kappa}} (N_t h_t^A)^{\kappa} (1 - G_t)$) is required to increase the share of automated products when this share G_t is already high. This feature plays a role in explaining why the growth rate of the skill premium need not grow the fastest when innovation is the most directed toward automation. It is also one of the main differences between our modeling of automation and a simple reduction in the price of equipment.

Overall, the rate and direction of innovation depends on the equilibrium allocation of high-skill workers between production, automation, and horizontal innovation.[17] We define the total mass of high-skill workers working in automation as $H_t^A \equiv \int_0^{N_t} h_t^A(i)\, di$.[18] High-skill labor market clearing then leads to

$$H_t^A + H_t^D + H_t^P = H. \tag{13}$$

have the same social planner allocation (the decentralized equilibrium would behave similarly, but the externality in the automation technology from the number of products would be internalized).

[16] Whenever $\tilde{\kappa} > 0$, we assume that $G_0 > 0$. Growth models with more than one type of technology often feature similar knowledge spillovers (e.g., Acemoglu 2002). Bloom, Schankerman, and Van Reenen (2013) show empirically that technologies that are closer to each other in the technology space have larger knowledge spillovers.

[17] We focus here on the decentralized equilibrium, but online Appendix B8 studies the social planner's problem. The optimal allocation is qualitatively similar so that our results are not driven by the market structure we impose.

[18] Using that by symmetry the total amount of high-skill workers hired in automation research is $H_t^A = (1 - G_t) N_t h_t^A$, we can rewrite (12) as $\dot{G}_t = \eta G_t^{\tilde{\kappa}} (H_t^A)^{\kappa} (1 - G_t)^{1-\kappa} - G_t g_t^N$.

B. *Innovation Allocation*

We denote by V_t^A the value of an automated firm, by r_t the economy-wide interest rate, and by $\pi_t^A \equiv \pi(w_{Lt}, w_{Ht}, 1)$ the profits at time t of an automated firm. The asset pricing equation for an automated firm is given by

$$r_t V_t^A = \pi_t^A + \dot{V}_t^A. \tag{14}$$

This equation states that the required return on holding an automated firm, V_t^A, must equal the instantaneous profits plus appreciation. An automated firm only maximizes instantaneous profits and has no intertemporal investment decisions to make.

A nonautomated firm invests in automation. Denoting by V_t^N the value of a nonautomated firm and letting $\pi_t^N \equiv \pi(w_{Lt}, w_{Ht}, 0)$, we get the asset pricing equation

$$r_t V_t^N = \pi_t^N + \eta G_t^{\tilde{\kappa}} \left(N_t h_t^A\right)^{\kappa} \left(V_t^A - V_t^N\right) - w_{Ht} h_t^A + \dot{V}_t^N, \tag{15}$$

where h_t^A is the mass of high-skill workers in automation research hired by a single nonautomated firm. This equation is similar to equation (14), but profits are augmented by the instantaneous expected gain from innovation $\eta G_t^{\tilde{\kappa}} (N_t h_t^A)^{\kappa} (V_t^A - V_t^N)$ net of expenditure on automation research, $w_{Ht} h_t^A$. This gives the first-order condition

$$\kappa \eta G_t^{\tilde{\kappa}} N_t^{\kappa} \left(h_t^A\right)^{\kappa - 1} \left(V_t^A - V_t^N\right) = w_{Ht}. \tag{16}$$

Note, h_t^A increases with the difference in value between automated and nonautomated firms and thereby current and future low-skill wages—all else equal.[19]

Free entry in horizontal innovation ensures that the value of creating a new firm equals its opportunity cost when there is strictly positive horizontal innovation ($\dot{N}_t > 0$):

$$\gamma N_t V_t^N = w_{Ht}. \tag{17}$$

The low-skill and high-skill representative households' problems are standard and lead to Euler equations that in combination give[20]

$$\dot{C}_t / C_t = \left(r_t - \rho\right) / \theta, \tag{18}$$

[19] The model predicts that the ratio of high-skill to low-skill labor *in production* is higher for automated than nonautomated firms, though not *overall* since nonautomated firms also hire high-skill workers for the purpose of automating. In particular, new firms do not always have a higher ratio of low- to high-skill workers (and at the time of its birth, a new firm only relies on high-skill workers).

[20] Consumption growth is the same for both households even though low-skill wages grow at a lower rate than high-skill wages in the long run. This is possible because low-skill workers save initially, anticipating a drop in labor income.

with a transversality condition requiring that the present value of all time-t assets in the economy (the aggregate value of all firms) is asymptotically zero:

$$\lim_{t\to\infty}\left(\exp\left(-\int_0^t r_s\,ds\right)N_t\left((1-G_t)V_t^N + G_t V_t^A\right)\right) = 0.$$

C. *Description of the Dynamic Equilibrium*

Appendix A3 shows that the equilibrium can be characterized by a system of four differential equations with two state variables (determining N_t and G_t), two control variables (which give the allocation of high-skill workers in innovation and production), and an auxiliary equation defining low-skill wages. It further establishes the following result.

PROPOSITION 3: *Assume that* $\kappa^{-\kappa}(\gamma(1-\kappa)/\rho)^{\kappa-1}\rho/\eta + \rho/\gamma < \psi H$. *Then:*

(*A*) *The system of differential equations admits an asymptotic steady state with a constant share of automated products* $G_\infty \in (0,1)$ *and positive growth in the number of products* $g_\infty^N > 0$. *The growth rates of output and wages are given by Proposition* 2 *as* (10) *and* (11).

(*B*) *For any* $a > 0$, *and any* $\hat{t} < \infty$, *there exists an* N_0 *sufficiently low such that during the interval* $(0,\hat{t})$, *the automation rate* $\eta G_t^{\tilde{\kappa}}(\hat{h}_t^A)^\kappa < a$, *and the economy behaves arbitrarily close to that of a Romer model where automation is impossible*.

(*C*) *If* g_t^N *admits a positive limit and* G_t *admits a limit*, *then the economy converges to an asymptotic steady state with* $G_\infty \in (0,1)$ *as described in* (*A*).

Proposition 3 establishes three results. First, under the appropriate parameter conditions, there exists an asymptotic steady state where the share of automated products G_∞ is between zero and one.[21,22] Second, for N_0 sufficiently low, the economy behaves close to a Romer model with no automation. Third, if G_t admits a limit and g_t^N a positive limit, the economy must converge toward the asymptotic steady state regardless of the initial values (N_0, G_0). Therefore, the economy must feature a period where the rate of automation innovation increases: it is low for low N_0 but must be positive later on to ensure a positive share of automated products in the

[21] To understand the condition $\kappa^{-\kappa}(\gamma(1-\kappa)/\rho)^{\kappa-1}\rho/\eta + \rho/\gamma < \psi H$, let the efficiency of the automation technology η be arbitrarily large such that the model approaches a Romer model with only automated firms. Then this inequality becomes $\rho/\gamma < \psi H$, which is the condition for positive growth in a Romer model with linear innovation technology. With a smaller η, the present value of a new product is reduced, and the condition is more stringent.

[22] "Asymptotic" because the system of differential equations only admits a fixed point for $N_t = \infty$. Technically, there is a steady state for the transformed system in Appendix A3, where the number of product N_t is replaced by an inversely related variable n_t, which is zero in steady state. Further, multiple steady states with $G_\infty > 0$ are technically possible but unlikely for reasonable parameter values (see online Appendix B5.2). In all our numerical simulations, the steady state was unique and saddle-path stable.

long run. In other words, the path of technological change itself will be unbalanced through the transitional dynamics. To understand this result, we now explain the evolution of the automation incentives, which, we show, are crucially linked to the level of low-skill wages.[23]

Following (16), the mass of high-skill workers in automation ($H_t^A = (1 - G_t) N_t h_t^A$) and therefore the automation intensity rate, given by $\eta G_t^{\tilde{\kappa}} (H_t^A/(1 - G_t))^{\kappa}$, depends on the ratio between the gain in firm value from automation $V_t^A - V_t^N$ and its effective cost, namely the high-skill wage divided by the number of products w_{Ht}/N_t:

$$H_t^A = (1 - G_t) \left(\kappa \eta G_t^{\tilde{\kappa}} \frac{V_t^A - V_t^N}{w_{Ht}/N_t} \right)^{1/(1-\kappa)}. \tag{19}$$

Crucially, as the number of products in the economy increases, the ratio $(V_t^A - V_t^N)/(w_{Ht}/N_t)$ evolves. Combining (14), (15), and (16) gives the difference in value between an automated and a nonautomated firm:

$$V_t^A - V_t^N = \int_t^{\infty} \exp\Big(-\int_t^{\tau} r_u du \Big) \Big(\pi_\tau^A - \pi_\tau^N - \frac{1-\kappa}{\kappa} w_{H\tau} h_\tau^A \Big) d\tau, \tag{20}$$

which is the discounted difference of profit flows adjusted for the cost and probability of automation. Recall that Cobb-Douglas production and isoelastic demand imply that both high-skill wages (for given H_t^P) and aggregate profits are proportional to aggregate output. Therefore, w_{Ht}/N_t is proportional to average profits: $w_{Ht}/N_t = [G_t \pi_t^A + (1 - G_t) \pi_t^N]/[\psi H_t^P]$. As a result, the mass of high-skill workers in automation moves with the discounted flow of profits of automated versus nonautomated firms divided by average firm profits. Intuitively, with a positive discount rate, $V_t^A - V_t^N$ moves like $\pi_t^A - \pi_t^N$ as a first approximation (from equation (20)), so that we get

$$\frac{V_t^A - V_t^N}{w_{Ht}/N_t} \,\tilde{\propto}\, \frac{\pi_t^A - \pi_t^N}{G_t \pi_t^A + (1 - G_t) \pi_t^N} = \frac{1 - \left(1 + \varphi w_{Lt}^{\epsilon - 1}\right)^{-\mu}}{G_t + (1 - G_t)\left(1 + \varphi w_{Lt}^{\epsilon-1}\right)^{-\mu}}, \tag{21}$$

where $\tilde{\propto}$ denotes "approximately proportional" and we used equation (8). This highlights low-skill wages (relative to the inverse productivity of machines $\tilde{\varphi}^{-1}$) as the key determinant of automation innovations. When $w_{Lt} \approx 0$, the incentive for automation innovation is very low, whereas when $w_{Lt} \to \infty$, it approaches a positive constant. This price effect bears similarity to Zeira (1998), where the adoption of a laborsaving technology also depends on the price of labor.[24]

[23] These results extend to the case where the skill supply is endogenous. See online Appendix B9.

[24] Beyond a focus on different empirical phenomena (an increase in inequality versus cross-country productivity differences), there are three important differences between our model and Zeira (1998). He assumes exogenous technical progress and focuses on adoption, while we model two types of endogenous innovation. The innovation cost changes over time in our model, while the cost of adoption is zero in Zeira (1998). As a result, automation is not laborsaving for the aggregate economy in his model.

Low Automation: When the number of products, N_t, is low enough that w_{Lt} is small relative to $\tilde{\varphi}^{-1}$, the difference in profits between automated and nonautomated firms is small relative to average profits. Following (19) and (21), the allocation of high-skill labor to automation, H_t^A, is low, and automation intensity is low. Consequently, as stipulated in Proposition 3B, growth is driven by horizontal innovation, and the behavior of the economy is close to that of a Romer model with a Cobb-Douglas production function with low- and high-skill labor. Both wages approximately grow at a rate $g_t^N/(\sigma - 1)$, and the labor share is approximately constant. Note, G_t depreciates following equation (12).

Rising Automation: As w_{Lt} grows relative to $\tilde{\varphi}^{-1}$, the term $(V_t^A - V_t^N)/(w_{Ht}/N_t)$ increases, which raises the incentive to innovate in automation. Without the externality in the automation technology ($\tilde{\kappa} = 0$), (19) directly implies that H_t^A must rise significantly above zero, and with it the Poisson rate of automation, $\eta(H_t^A/(1 - G_t))^\kappa$ and thereby the share of automated products, G_t. For $\tilde{\kappa} > 0$, the initial depreciation in the share of automated products gradually makes the automation technology less effective, which delays the takeoff of automation. Our initial assumption that knowledge spillovers are not too large ($\tilde{\kappa} < 1 - \kappa(1 - \beta)$) is a sufficient (but not necessary) condition for the takeoff to always happen.[25]

Following Proposition 1, the increases in G_t and N_t lead to an increase in the skill premium and a decline in the labor share.[26] For some parameters, low-skill wages temporarily decline (see numerical examples in online Appendix B6.2).[27] Arguably, this is where our model differs the most from the rest of the literature, notably because a model with fixed G, a capital-skill complementarity model like KORV, or a factor-augmenting technical change model with directed technical change as Acemoglu (1998) does not feature laborsaving innovation and therefore cannot lead to a decline in low-skill wages.

High but Stable Automation: With the share of automated products, G_t, no longer near zero, the gain from automation $V_t^A - V_t^N$ and its effective cost w_{Ht}/N_t grow at the same rate (the right-hand side in (21) is close to $1/G_t$). As a result, the normalized mass of high-skill workers in automation research $(N_t h_t^A)$ stays bounded (see (19)), and so does the Poisson rate of automation, such that G_t converges to a positive constant below one. The economy then converges toward an asymptotic steady state

[25] Alternatively, if we assume that the automation technology obeys $\max\{\eta G_t^{\tilde{\kappa}}, \underline{\eta}\}(N_t h_t^A)^\kappa$ with $\underline{\eta} > 0$, then automation always takes off.

[26] Changes in the mass of high-skill workers in production, H_t^P, also affect the skill premium and the labor share. Increasing G_t requires hiring more high-skill workers in automation innovation (in the same vein as the General Purpose Technology literature, notably Beaudry, Green, and Sand 2016), but at the same time horizontal innovation declines on average during this time period, which increases H_t^P.

[27] Generating a decline in low-skill wages is harder with endogenous than exogenous technical change because a decline in low-skill wages reduces further automation, which is why the decline is temporary. Whether low-skill wages have declined in the data depends on how one accounts for compositional changes in the low-skill population, work benefits, and the deflator (see Section IIIC).

where Proposition 2 applies. High-skill wages grow at the same rate as output, and low-skill wages grow at a positive but lower rate, while the labor share stabilizes again.

With positive growth in low-skill wages, the profits of nonautomated firms become negligible relative to those of automated firms with $g_\infty^{\pi^N} = g_\infty^{\pi^A} - \beta(\sigma - 1)\, g_\infty^{w_L}$. As total profits are proportional to output, the profits and value of automated firms grow at the rate of output minus the growth rate of the number of products: $g_\infty^{V^A} = g_\infty^{\pi^A} = g_\infty^{Y} - g_\infty^{N}$. Asymptotically, the value of a nonautomated firm entirely lies in the possibility of future automation. Therefore, it grows at the same rate as the value of an automated firm: $g_\infty^{V^N} = g_\infty^{V^A}$. In other words, the prospect of future automation eventually guarantees the entry of new products.[28] The model continues then to differ from a generic capital deepening model since the share of automated firms G_t is only constant thanks to the interplay between horizontal innovation and automation while the long-run growth rate depends on automation. Online Appendix B5.6 derives comparative statics in the steady state and shows that the asymptotic growth rates of GDP and low-skill wages increase in the productivity of automation η and horizontal innovation γ.

Overall, our model predicts that we should see an increase in automation as an economy develops, consistent with the increase in automation innovations observed since the 1970s (as documented in panel C of Figure 1). In line with the results of Section I, this increase in automation is associated with an increase in the skill premium and a decline in the labor share, which have also been observed in the United States (panels A and B of Figure 1). This contrasts our paper with most of the growth literature, which relies on exogenous shocks to explain these trends. Section III shows that an extended version of our model can reproduce those trends not only qualitatively but also quantitatively.

D. *Interactions between Automation and Horizontal Innovation*

Before moving to the quantitative exercise, we show how the interactions between automation and horizontal innovation can help account for two puzzles in the literature on inequality and technical change.

Increasing Automation and Decelerating Skill Premium: In recent years, the growth rate of the skill premium has declined (see panel A of Figure 1), while at the same time, innovation has been more directed toward automation (panel C of Figure 1). At first glance, this seems to contradict an explanation of the increase in the skill premium by automation. Yet in our model, there is no one-to-one link between the growth rate of the skill premium and the direction of innovation. First, the share of automated products G_t can be understood as a stock variable that increases with the automation of not-yet automated products but depreciates

[28] The economy would not admit such an asymptotic steady state if automation was entirely undertaken by entrants replacing the incumbents. In that case, the value of creating a new product would only correspond to the discounted flow of profits of a nonautomated firm, which grows slower than the cost of horizontal innovation (high-skill wages normalized by N_t), and horizontal innovation could not be sustained. In contrast, the steady state would still exist if the incumbent also automated with positive probability or captured a share of the surplus created by an automation innovation.

through horizontal innovation. As a result, maintaining a high level for G_t requires a high level of automation innovations. Typically, the skill premium rises the fastest when G_t increases sharply and decelerates when G_t approaches its steady-state value, which may, however, be when innovation is the most intensely directed toward automation (this happens both in Section III below and in the numerical simulation of online Appendix B6.1). Second, since our model does not feature a CES production with factor-augmenting technologies (as Katz and Murphy 1992, Acemoglu 1998, or Goldin and Katz 2008), the elasticities of the skill premium with respect to the two technology variables (G_t and N_t) are not constant.

Automation with No Increase in Growth: A second puzzle is that as the skill premium has increased, *GDP* growth has not accelerated, which casts doubt on whether a technological revolution is happening (see Acemoglu and Autor 2011). Our model offers a potential explanation: horizontal innovation may decline when automation takes off, with an ambiguous net effect on growth. Formally, we find that the rate of horizontal innovation is lower in the steady state than for low N_t (proof in online Appendix B5.5).

COROLLARY 1: *For any G_0, there exists an N_0 sufficiently low that the horizontal innovation rate, g^N, is initially higher than in the asymptotic steady state.*

Intuitively, three effects explain this result: First, once automation sets in, some high-skill workers are hired in automation research, which reduces the amount of high-skill workers in production and therefore reduces horizontal innovation through a classic scale effect. Second, the elasticity of GDP growth with respect to horizontal innovation is larger in the asymptotic steady state, which, from the Euler equation, increases the elasticity of the interest rate with respect to horizontal innovation and reduces horizontal innovation. Third, asymptotically, a new firm makes negligible profits relative to the cost of innovation until it gets automated, which further reduces horizontal innovation.

III. Quantitative Exercise

We conduct a quantitative exercise to compare empirical trends for the United States with the predictions of our model. We proceed in three steps. First, we calibrate our model, then we show that the data call for an increase in the share of automated products G and assess how well our model can do relative to a more flexible setup with exogenous technology. Finally, we analyze the future evolution of the economy.

A. *Extended Model and Calibration*

To match the data quantitatively, we modify the baseline model. First, since the share of high-skill workers has dramatically increased, we let H and L vary over time and use the paths from the data. Second, we assume that producers rent machines from a capital stock. Capital can also be used as structures in both automated and nonautomated firms. Third, we allow for the possibility that low-skill workers are replaced by a composite of machines and high-skill workers. The production function (2) becomes

$$(22) \qquad y(i) = \left[l(i)^{\frac{\epsilon-1}{\epsilon}} + \alpha(i)\left(\tilde{\varphi} h_e(i)^{\beta_4} k_e(i)^{1-\beta_4}\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon\beta_1}{\epsilon-1}} h_s(i)^{\beta_2} k_s(i)^{\beta_3},$$

where $\beta_1 + \beta_2 + \beta_3 = 1$ and $\beta_4 \in [0,1)$. The central difference between equations (2) and (22) is the introduction of $h_e(i)$ as high-skill labor, which—along with machines—performs the newly automated tasks ("e" for equipment). This feature is necessary to capture a relatively low drop in the labor share. Here, $k_s(i)$ is structures and $k_e(i)$ and $k_s(i)$ are both rented from the same capital stock K_t. The value of K_t increases with investment in final goods and depreciates at a fixed rate Δ, so that

$$(23) \qquad \dot{K}_t = Y_t - C_t - \Delta K_t.$$

The cost advantage of automated firms now depends on the ratio between low-skill wage and the price of the high-skill labor capital aggregate, $w_{Lt}/(w_{Ht}^{\beta_4}\tilde{r}_t^{1-\beta_4})$, where $\tilde{r}_t = r_t + \Delta$ is the gross rental rate of capital. The logic of the baseline model directly extends to this case. Proposition 1 still holds, and Proposition 2 holds with $g_\infty^{w_H} = g_\infty^Y = g_\infty^N/((\beta_2 + \beta_1\beta_4)(\sigma - 1))$ and $g_\infty^{w_L} = g_\infty^Y(1 + (\sigma - 1)\beta_1\beta_4)/(1 + \beta_1(\sigma - 1))$. An equivalent to Proposition 3 holds, but the system of differential equations includes three control variables and three state variables. The transitional dynamics are similar to that of the baseline model, but automation innovation now depends on $w_{Lt}/(w_{Ht}^{\beta_4}\tilde{r}_t^{1-\beta_4})$: automation innovation is low when N_t is low, increases later on before stabilizing as the economy approaches its asymptotic steady state. The capital share and the capital output ratio increase when automation increases as equipment replaces low-skill labor in production. Details and proofs are provided in online Appendix B10.

We match our extended model to the data (see online Appendix B11 for details). We identify low-skill workers with non-college-educated workers and high-skill workers with college-educated workers and focus on the years 1963–2012 (workers with "some college" are assigned 50/50 to each category following the methodology of Acemoglu and Autor 2011). We match the skill premium and take the empirical skill ratio as given (and normalize total population to one). We also match the growth rate of real GDP/employment and the labor share. We associate the use of machines with private equipment (excluding transport) and software. As pointed out by Gordon (1990), the NIPA price indices for real equipment are likely to understate quality improvements in equipment and therefore growth in the real stock of equipment. Hence, we use the adjusted price index from Cummins and Violante (2002) for equipment and build (private) equipment and software to GDP ratios from 1963 to 2000.

Our model is not stochastic and cannot be directly estimated. Instead, we take a parsimonious approach and choose parameters to minimize the weighted squared log-difference between observed and predicted paths. We start the simulation 40 years before 1963 to force N_{1963} and G_{1963} to be consistent with the long-run

Table 1—Parameters from Quantitative Exercise

Parameter	σ	ϵ	β_1	γ	$\tilde{\kappa}$	θ	η	κ
Value	5.86	4.28	0.58	0.44	0.55	1	0.40	0.68
Parameter	ρ	β_2	Δ	β_4	$\tilde{\varphi}$	N_{1963}	G_{1963}	
Value	0.033	0.18	0.011	0.73	1.57	10.3	0.02	

behavior of our model.[29] Since the model requires the skill ratio before and after the time period we estimate, we fit a generalized logistical function to the path of the log of the skill ratio and use the predicted values outside 1963–2012 (over that time period, the fit is excellent).

The model features a total of 13 parameters with 2 initial conditions N_{1923} and G_{1923}. We allow all parameters to adjust freely (other than the economically motivated boundaries imposed by the model itself) and then assess whether these parameter estimates fit with other similar estimates. Table 1 gives the resulting parameters and the values of the state variables N_t and G_t in the first matched year, 1963. The elasticity of substitution across products σ is estimated at 5.86, in line with Christiano, Eichenbaum, and Evans (2005), who find that observed markups are consistent with a value of around 6. The elasticity of substitution between machines and workers is estimated at 4.28. The value of $\tilde{\kappa}$ is estimated at 0.55, implying a substantial automation externality, a force that accelerates the increase in the share of automated products. Finally, we find a β_1—the factor share of machines/low-skill workers—of 0.58, which implies sizable room for automation, though a β_4 of 0.73 means that the share of high-skill workers in the composite that replaces low-skill workers is of substantial importance. The preference parameters are within standard estimates with a ρ of 3.3 percent and the implied θ resulting in log-preferences. The only parameter that is estimated outside a common range is the depreciation rate Δ (though it is not precisely identified). Online Appendix B11.3 discusses in detail how the parameters are identified.

Figure 3 further shows the predicted path of the matched data series ("endogenous technology" series—the "exogenous" series will be explained in Section IIIB) along with their empirical counterparts. Panel A demonstrates that the model matches the rise in the skill premium from the early 1980s and the flat skill premium in the period before reasonably well. Though a bit less pronounced than in the data, our model also includes the more recent decline in the growth rate of the skill premium, which, computed over a 5 years moving window, peaks in 1984 at 1.31 percent and drops to 0.52 percent in 2005. The total predicted decline in the labor share (8.7 p.p.) is slightly higher than the actual one (6.6 p.p.).[30] The average growth rate of the economy is matched completely, as shown in panel C. Although the model largely captures the average growth rate of capital equipment over GDP during the

[29] Initial values for G_t and K_t have little impact on the state of the economy several years later. By simulating the economy 40 years prior, we ensure that the simulated moments are nearly independent of the initial values for G_t and K_t. We fix K_{1923} at its steady-state value in a model with no automation.

[30] Our model sees the decline in the labor share as being a gradual phenomenon (instead of a sharp trend break from 2000), in line with the interpretation of Karabarbounis and Neiman (2013) and data on the global labor share.

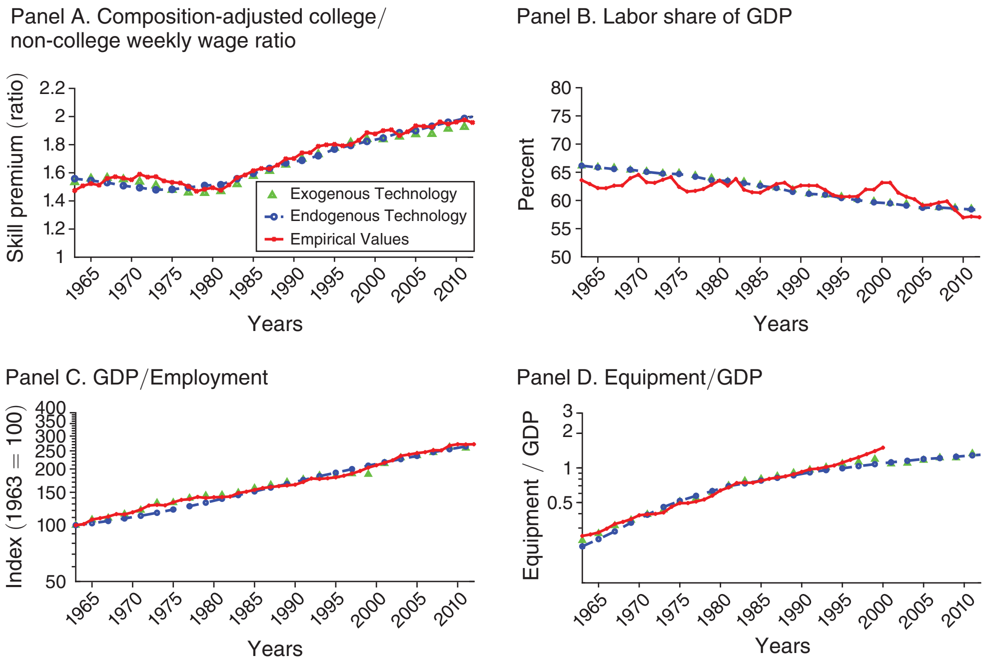

Figure 3. Empirical Paths and Predicted Paths for the Endogenous and Exogenous Technology Models

period, the predicted path differs somewhat from its empirical counterpart, as shown in panel D on log-scale. Whereas the empirical path is close to exponential, the predicted path tapers off somewhat toward the end of the period.[31] Naturally, our model has a harder time capturing higher-frequency movements, such as a temporary increase in the labor share at the end of the 1990s. Further, to assess the predictive power of our model, we reproduce the same exercise but only matching the first 30 years. Appendix A4 reports the results: the parameters are nearly identical, and the calibrated model performs well out of sample.

As shown in Table 1, the share of automated products G_t is low in 1963 at 2.5 percent, far from its steady-state value of 92 percent. Figure 5 below shows that G_t increases sharply through the 1963–2012 time period and reaches 20 percent by 1986 and 64 percent by 2012. Therefore, our calibration strongly suggests that automation has risen in recent decades.

B. *Alternative Technological Processes*

Section IC argued that a rise in the skill premium and a decrease in the labor share are consistent with an increase in the number of products together with either a rising or constant (but positive) share of automated products. It is therefore worth

[31] Recall that the ratio of equipment to GDP in the data is only a proxy for the ratio of machines to GDP in the model since not all equipment is used to replace low-skill workers. Interestingly, more recent data show a slowdown in software investment (see Beaudry, Green, and Sand 2016; Eden and Gaggl 2018).

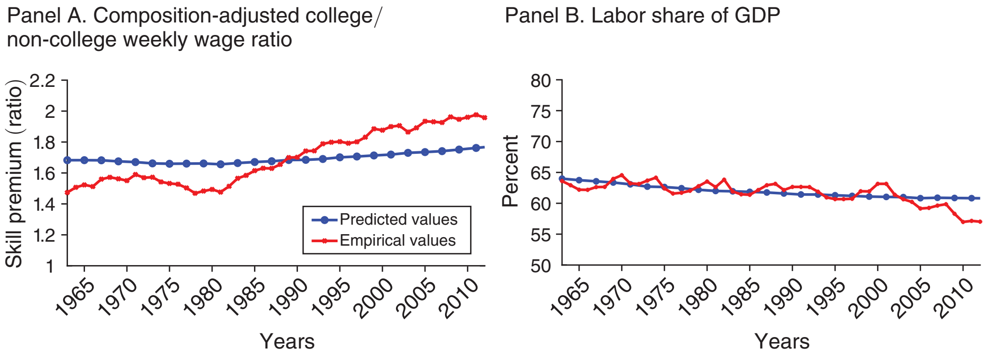

Figure 4. Empirical and Predicted Paths for a Model with Constant G

asking which features of the data require an increase in the share of automated products G_t. To answer this question, we consider an alternative model where G is a constant. We use the same procedure as above to calibrate this alternative economy but let G and the initial capital stock K_{1963} be free parameters. Figure 4 compares the empirical paths of the skill premium and the labor share with the ones predicted by a constant G model and shows that this alternative model does not match the data. Intuitively, a model with constant G has a hard time reconciling a roughly constant skill premium followed by a fast rise without a sharp increase in economic growth. Online Appendix B11.4 shows the two other moments and reports the parameters.

Our paper argues that the evolution of the income distribution in the United States can largely be accounted for with endogenous technical change. This is in contrast with an alternative view that stresses the importance of structural breaks in the data. To evaluate how well our endogenous innovation model performs, we compare it to a model with exogenous technology. We keep the (non-innovation) parameters from Table 1 but let N_t and G_t be free parameters between 1963 and 2012 chosen to match the empirical moments as close as possible.[32] Figure 3 shows the predicted paths for our moments in that case ("exogenous technology" series). This model performs slightly better than the endogenous one, notably when it comes to labor productivity fluctuations, but it does not capture trends better. Figure 5 compares the evolution of N_t and G_t in our endogenous growth model with this exogenous alternative. Panel B shows that the exogenous path for G_t also features a smooth but sharp increase from 1980 but the path of our endogenous growth model is very similar. Panel A shows that the exogenous path for N_t is more volatile than the endogenous path since the exogenous model tries to better capture the short-run fluctuations. The trends, however, are similar. This pattern is robust to including exogenous labor-augmenting technical change (see online Appendix B11.5).

[32] Since K_t still evolves endogenously, this corresponds to a Ramsey model with fully anticipated shocks to N_t and G_t. We let the initial capital stock K_{1963} be a free parameter as well. For the technology paths after 2012, we let N_t grow at a constant rate, which is another free parameter, and assume that G_t stays constant at its 2012 value. To facilitate comparison, we also assume that H_t^P in the exogenous growth model is fixed to be the same as in the endogenous growth model.

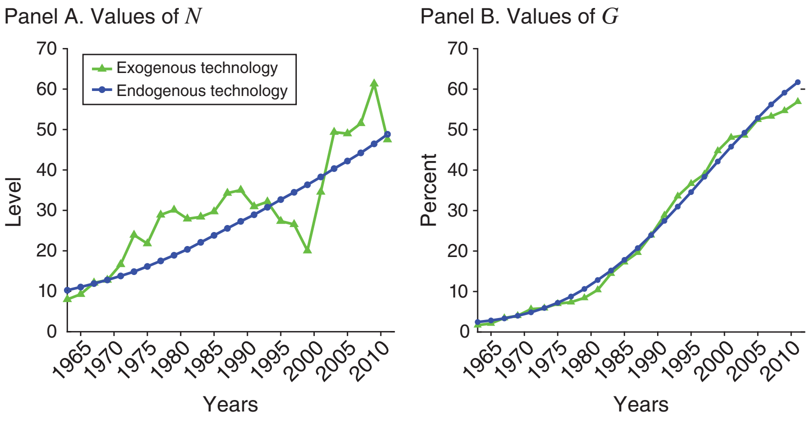

Figure 5. Paths for N_t and G_t in the Endogenous and Exogenous Growth Models

Panel B of Figure 5 shows that the increase in G_t must be sharp to match the data, which is why our estimation procedure found a high value for the automation externality parameter $\tilde{\kappa} = 0.55$. Appendix A4 discusses this further by calibrating the model with $\tilde{\kappa} = 0$ and showing that in this case, the endogenous and exogenous growth models deliver very different paths for G_t.

C. *Evolution of the Calibrated Economy*

Figure 6 further analyzes the behavior of the calibrated economy by plotting the transitional dynamics from 1963 to 2063. Panel A shows that GDP growth slows down past 2012 in line with recent economic trends—as argued in Section IID, our model can account for a slowdown in growth despite a high level of automation innovation thanks to a decline in horizontal innovation. Panels A and B show that the skill premium keeps growing albeit at a slower rate: over the 1980s, the skill premium grew at an average of 1 percent per year according to the model and 0.7 percent in the 2000s, with a predicted growth rate of the skill premium of 0.2 percent in the 2050s as the share of automated products stabilizes. In the context of Proposition 1, for our parameters automation is always low-skill laborsaving and horizontal innovation low-skill biased. In the meantime, the labor share smoothly declines toward its steady-state value of 52.8 percent, and the high-skill labor share increases.[33] Panel C plots the share of automated products G_t, which keeps rising very slowly past 2012 toward its asymptotic value at 92 percent. The share of automation innovations is already above 40 percent in the mid-1970s and increases steadily until the 2000s (panel C). It peaks in 2005, even though the skill premium is decelerating. As argued in Section IID, this occurs in part because the level of automation can be thought of as a stock that depreciates with the entry of new products. This constitutes a response to the critique of the literature on SBTC put forward by

[33] With the decline in horizontal innovation, fewer high-skill workers are allocated to innovation activities overall, which also contributes to the decline in the labor share notably from the mid-1980s.

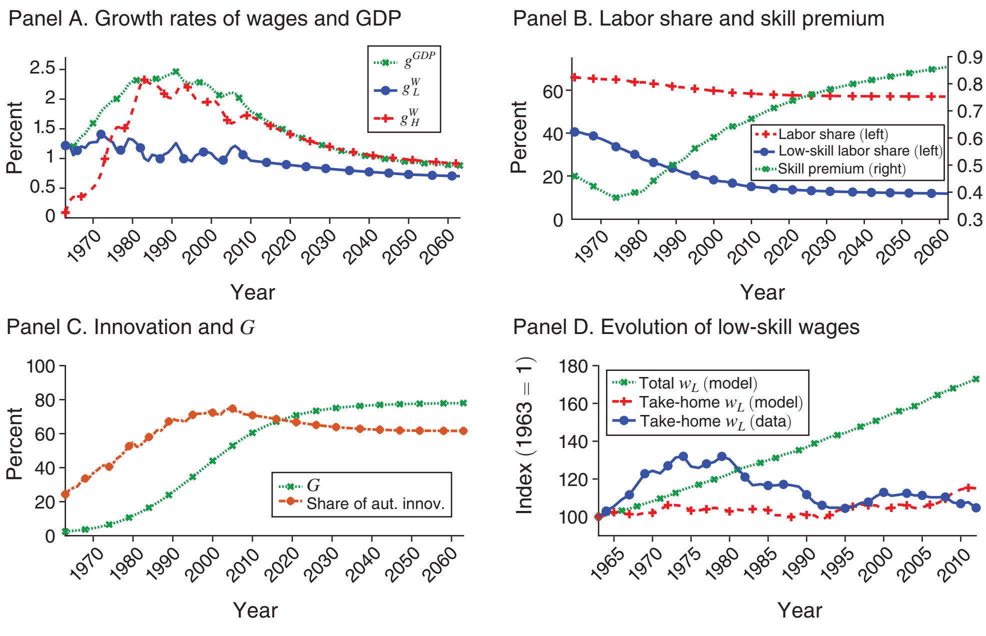

FIGURE 6. TRANSITIONAL DYNAMICS WITH CALIBRATED PARAMETERS

Note: The growth rates are computed over a five-year moving average.

Card and DiNardo (2002), who argue that inequality rising the most in the early to mid-1980s and technological change continuing in the 1990s squares poorly with the predictions of a framework based on SBTC. The model predicts that the share of automation innovation will remain high in the future but at a slightly lower level than in the 2000s.

Acemoglu and Autor (2011) highlight that low-skill wages have declined since the 1980s, and we reproduce their series from 1963 to 2012 in panel D. Our model does not distinguish between wages and other labor costs, so that any mention of wages so far should be understood as labor costs. At the same time, there has been a significant decline in the ratio of wages to total labor costs (from close to 1 to around 2/3 in our data).[34] Panel D plots our predicted low-skill labor costs (w_L) and the take-home low-skill wages (net of work benefits, social contributions, etc.). Our model predicts a very slow growth for low-skill labor costs (70 percent over 50 years), which is consistent with stagnating take-home low-skill wages. Online Appendix B11.6 analyzes the effect of taxes on machines and automation innovations on low-skill wages and output.

Our exercise bears similarities to KORV, who also seek to explain the increase in the skill premium using capital-skill complementarity while matching the labor share. There are four major differences. First, our exercise is more demanding since instead of directly feeding in the empirical path of equipment, we calibrate a general

[34] Obtaining these figures requires combining several indices, which are not perfectly consistent; nevertheless, the trend is clear—see online Appendix B11.2 for details.

equilibrium model that endogenizes both capital and technology.[35] Second, KORV do not attempt to match the evolution of labor productivity: given the large increase in the stock of equipment capital, their model would have to feature a large simultaneous unexplained decrease in the growth rate of TFP. Third, their model does not match a decline in the labor share, but instead shows a slight increase toward the end of their sample period. This is not an artifact of their calibration but a feature of their model. Their production function is a nested CES where low-skill labor is substitute with a CES aggregate of high-skill and equipment, which are complement. Therefore, should the equipment stock keep rising (through investment-specific technological change), the income share of equipment would decline in the long run.[36] Fourth, their model does not feature laborsaving innovation, as an increase in investment-specific technical change increases all wages for perfectly elastic capital (online Appendix B12 elaborates on the last two points).

To summarize, our model makes the case that recent trends in the skill premium, the labor share, labor productivity, and the ratio of equipment to GDP can be quantitatively explained as resulting from an endogenous increase in the share of automation innovations. This provides a different view of these trends as arising from the natural development of an economy instead of exogenous shocks. Furthermore, this viewpoint also explains why the rise in the skill premium was not accompanied by an increase in productivity growth and why the skill premium has decelerated despite a high share of automation innovations. Of course, our model cannot match the data perfectly. In particular, we do not capture high-frequency movements, and we overestimate the decline of the labor share. Our exogenous N, G exercise shows that to better account for the data, one would have to modify the aggregate production function.

D. *Data on Automation Innovations*

We now provide some evidence based on patent data that suggests that the share of automation innovations has increased since the 1970s in line with our model. Classifying patents as automation versus nonautomation is not straightforward, and there are no technological codes in patent data aimed at doing so. Nevertheless, in a recent working paper, Mann and Püttmann (2018) classify US patents as automation versus nonautomation using machine learning techniques (see online Appendix B11.2 for details). Panel A of Figure 7 reports the shares of automation patents according to their analysis and according to our calibrated model. In line with our model, the share of automation patents according to their definition has markedly increased. Relative to their series, our model suggests a higher share of automation innovations (particularly at the beginning of the sample), but the increase is of a similar magnitude.

[35] Others have tried to match similar trends, such as Eden and Gaggl (2018) using a model similar to KORV or Goldin and Katz (2008) using the traditional model of SBTC with factor-augmenting technologies.

[36] If low-skill labor were substitute with a Cobb-Douglas aggregate of high-skill and equipment (as in our automated firms), the capital share would increase in the long run. Yet the growth rate of the skill premium would then increase when investment-specific technological change accelerates, so such a model could not feature a deceleration in the skill premium when innovation seems to be the most biased against low-skill workers, as in our model.

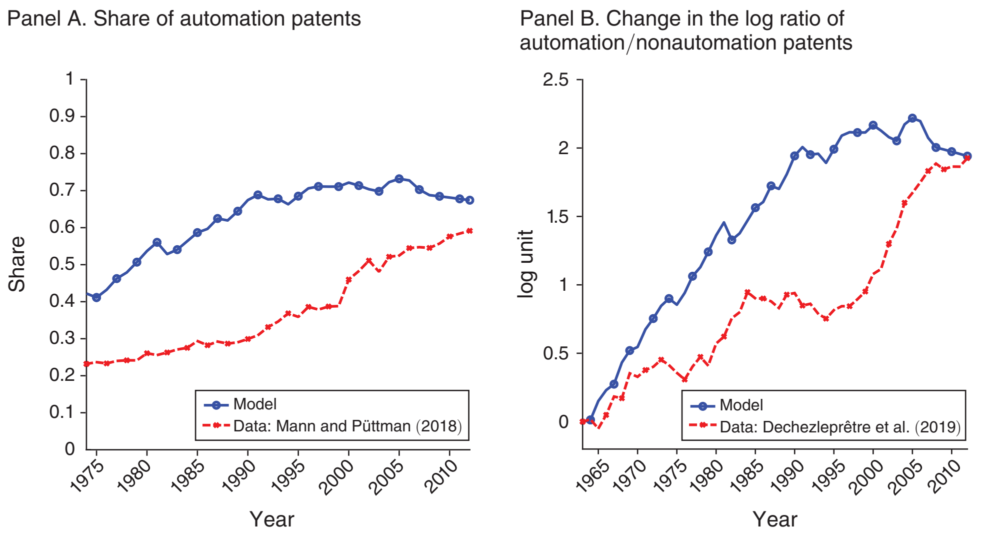

FIGURE 7. TRENDS IN AUTOMATION INNOVATIONS

Dechezleprêtre et al. (2019) offer an alternative classification of automation versus nonautomation patents in machinery, which relies on the technological codes of patents and the presence of certain keywords in the text of patents (see online Appendix B11.2 for details). They show that the share of automation patents in machinery is correlated with a decline in routine tasks across US industries and, using international firm-level data, that higher low-skill wages lead to more automation innovations but not more nonautomation innovations. Their classification only allows for the identification of a subset of automation and nonautomation patents. Yet provided that these subsets are constant shares of both types of innovations, we can use the increase in the log ratio of their automation versus nonautomation patents as a proxy for the increase of automation versus horizontal innovation in our model. We plot the model and data series (indexing the log ratios at zero in 1963) in panel B of Figure 7. Here as well, we find similar trends—except for a small decline between 1984 and 1994 in the data. Any series based on patenting will be a noisy proxy for true underlying automation innovation. Despite this, both of these measures suggest that automation increased since the 1970s and is still very high today, in line with the predictions of our model.

IV. Conclusion

This paper introduces automation in a horizontal innovation growth model. We show that an increase in automation leads to an increase in the skill premium, a decline in the labor share, and possibly a decline in low-skill wages. Moreover, such an increase in the share of automation innovations is the natural outcome of a growing economy since higher low-skill wages incentivize more automation innovations. Quantitatively, our model can replicate the evolution of the US economy since the 1960s: a continuous increase in the skill premium with a more recent slowdown, a decline in the labor share, stagnating labor productivity growth, and an increase in the share of automation innovations. We predict that the skill premium keeps rising in the future albeit at a lower rate and that the labor share stabilizes at a rate below today's.

The increase in the share of automation innovations, which prompts changes in the income distribution, occurs endogenously in our paper. This stands in contrast with most of the literature, which seeks to explain changes in the distribution of income inequality through exogenous changes: an exogenous increase in the stock of equipment as per KORV, a change in the relative supply of skills as per Acemoglu (1998), or the arrival of a general purpose technology as in the related literature. This feature is shared by Buera and Kaboski (2012), who argue that the increase in income inequality is linked to the increase in the demand for high-skill intensive services, which results from nonhomotheticity in consumption.[37]

The present paper is only a step toward a better understanding of the links between automation, growth, and income inequality. Given that automation has targeted either low- or middle-skill workers and that artificial intelligence may now lead to the automation of some high-skill tasks, a natural extension of our framework would include more skill heterogeneity. Another natural next step would be to add firm heterogeneity and embed our framework into a quantitative firm dynamics model. Our framework could also be used to study the recent phenomenon of "reshoring," where US companies that had offshored their low-skill intensive activities to China now start repatriating their production to the US after having further automated their production process.

Appendix A. Main Appendix

A1. *Proof of Proposition 1*

We focus on the imperfect substitute case, and online Appendix B1.1 deals with the perfect substitute case. Rewrite (7) as

$$\text{(A1)} \qquad \frac{w_H}{w_L} = \frac{1-\beta}{\beta}\frac{L}{H^P}\frac{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+1-G}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}+1-G}.$$

Since $0 < \mu < 1$, (A1) establishes w_H as a function of G, H^P, and w_L such that w_H is increasing in w_L and G and decreasing in H^P, with $w_H/w_L > (1-\beta)/\beta \times L/H^P$ for $G > 0$. Equation (6) similarly establishes w_H as a function of N, G, and w_L, which is decreasing in w_L and increasing in N and G. Then, w_H, w_L are jointly uniquely determined by (A1) and (6) for given N, G, and H^P. Both increase in N, and w_H increases in G. In addition, (6) traces a convex isocost curve in the input prices plan.

In addition, (A1) shows that w_H/w_L increases with w_L. Since w_L increases in N, then w_H/w_L increases in N as well. If w_L decreases in G, then since w_H increases in G, w_H/w_L increases in G. If instead w_L increases in G, then the right-hand side

[37] This feature is also shared by Galor and Weil (2000) and Hansen and Prescott (2002), who endogenize the industrial revolution takeoff.

of (A1) increases with G both directly, and because w_L increases, this ensures that w_H/w_L still increases in G. Therefore, both an increase in N and an increase in G are skill-biased and following (9) decrease the labor share. This establishes Parts A and B except for the relationship between w_L and G.

Comparative Statics of w_L with Respect to G: Combine (A1) and (6) to get

$$\text{(A2)} \qquad w_L = \frac{\sigma-1}{\sigma}\beta\left(\frac{H^P}{L}\right)^{(1-\beta)} N^{\frac{1}{\sigma-1}}\left(G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}+(1-G)\right)^{1-\beta}$$
$$\times\left(G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+(1-G)\right)^{\frac{1}{\sigma-1}-(1-\beta)}.$$

Log differentiating with respect to G, one obtains

$$\text{(A3)} \quad \hat{w}_L = \Bigg[\underbrace{\frac{1}{\sigma-1}\frac{\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}-1}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+(1-G)}}_{\text{scale effect}}$$
$$\underbrace{-(1-\beta)\left(\frac{1-\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}+(1-G)}+\frac{\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}-1}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+(1-G)}\right)}_{\text{substitution effect}}\Bigg]\frac{G\hat{G}}{Den},$$

where

$$Den \equiv 1-\frac{\beta\varphi w_L^{\epsilon-1}}{\left(1+\varphi w_L^{\epsilon-1}\right)}\frac{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+(1-G)}$$
$$+\frac{\varphi w_L^{\epsilon-1}(\epsilon-1)(1-\beta)}{\left(1+\varphi w_L^{\epsilon-1}\right)}$$
$$\times\left(\frac{\mu G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+1-G}+\frac{(1-\mu)G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}+1-G}\right).$$

Note, $Den > 0$ as $\epsilon > 1$, $\mu \in (0,1)$, and $\frac{\beta\varphi w_L^{\epsilon-1}}{1+\varphi w_L^{\epsilon-1}}\frac{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+(1-G)} < 1$. In (A3), the scale effect term is positive as $(1+\varphi w_L^{\epsilon-1})^{\mu}-1 > 0$. This term comes from the differentiation of (6) with respect to G at constant w_H (hence, it represents the shift right of the isocost curve). The substitution effect term is negative because $1-(1+\varphi w_L^{\epsilon-1})^{\mu-1} > 0$ since $\mu < 1$, it comes from the differentiation of (7) with respect to G.

First note that if $1/(\sigma-1) \leq 1-\beta$, the scale effect is always dominated by the substitution effect. Hence, w_L is decreasing in G.

If instead $1/(\sigma-1) > 1-\beta$, the scale effect is dominated by the substitution effect provided that $R_W \equiv \frac{1-(1+\varphi w_L^{\epsilon-1})^{\mu-1}}{G(1+\varphi w_L^{\epsilon-1})^{\mu-1}+1-G} / \frac{(1+\varphi w_L^{\epsilon-1})^{\mu}-1}{G(1+\varphi w_L^{\epsilon-1})^{\mu}+1-G}$ is large enough. From (A2), we get

$$w_L = \frac{\sigma-1}{\sigma}\beta N^{\frac{1}{\sigma-1}}\left(\frac{H^P}{L}\frac{1-G\Big(1-\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu-1}\Big)}{G\Big(\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu}-1\Big)+1}\right)^{1-\beta}$$
$$\times\left(G\Big(\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu}-1\Big)+1\right)^{\frac{1}{\sigma-1}}.$$

Using that $G \in [0,1]$, we obtain that

$$w_L\big(1+\varphi w_L^{\epsilon-1}\big)^{1-\beta} > \frac{\sigma-1}{\sigma}\beta\big(H^P/L\big)^{1-\beta}N^{\frac{1}{\sigma-1}},$$

which ensures that $\lim_{N\to\infty} w_L = \infty$ uniformly with respect to G (i.e., for any $\bar{w}_L > 0$, there exist $\bar{N}$ such that for any $N > \bar{N}$ and any G, $w > \bar{w}_L$). Since $\lim_{w_L\to\infty, G\to 1} R_W = \infty$, we get that $\lim_{N\to\infty, G\to 1} R_W = \infty$, so that for N and G large enough the substitution effect dominates. This establishes Part B.

Part C: Log-differentiating (A2) with respect to N gives $\hat{w}_L = \hat{N}/[(\sigma-1)Den]$. We then combine this expression with (A3) and use that for an increase in the number of nonautomated products only, NG is a constant so that $\hat{G} = -\hat{N}$. Denoting $\hat{w}_L^{NT}$ (NT for "new tasks"), the change in w_L, we then get

$$\text{(A4)}\quad \hat{w}_L^{NT} = \left[\frac{(1-\beta)G\Big(\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu}-1\Big)}{G\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu}+(1-G)} + \frac{(1-\beta)G\Big(1-\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu-1}\Big)}{G\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu-1}+(1-G)}\right.$$
$$\left. + \frac{(\sigma-1)^{-1}}{G\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu}+(1-G)}\right]\frac{\hat{N}}{Den}.$$

Hence, low-skill wages always increase with the arrival of nonautomated products. Log-differentiating (7), one gets

$$\hat{w}_H - \hat{w}_L = \left(\frac{\mu G\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu}}{G\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu}+1-G} + \frac{(1-\mu)G\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu-1}}{G\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu-1}+1-G}\right)\frac{(\epsilon-1)\varphi w_L^{\epsilon-1}}{1+\varphi w_L^{\epsilon-1}}\hat{w}_L$$
$$+\left(\frac{G\Big(\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu}-1\Big)}{G\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu}+1-G} + \frac{G\Big(1-\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu-1}\Big)}{G\big(1+\varphi w_L^{\epsilon-1}\big)^{\mu-1}+1-G}\right)\hat{G}.$$

Using (A4) and that $\hat{G} = -\hat{N}$, we get that following an increase in the mass of nonautomated products (keeping NG constant),

$$\hat{w}_H^{NT} - \hat{w}_L^{NT} = \Bigg[\left(\frac{\mu\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+1-G} + \frac{(1-\mu)\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}+1-G}\right)$$

$$\times \frac{(\epsilon-1)\varphi w_L^{\epsilon-1}}{1+\varphi w_L^{\epsilon-1}} \frac{(\sigma-1)^{-1}}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+(1-G)}$$

$$-\left(\frac{\left(\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}-1\right)}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+1-G} + \frac{\left(1-\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}\right)}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}+1-G}\right)$$

$$\times\left(1-\frac{\beta\varphi w_L^{\epsilon-1}}{\left(1+\varphi w_L^{\epsilon-1}\right)} \frac{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+(1-G)}\right)\Bigg]\frac{G\hat{N}}{Den},$$

which simplifies into

$$\text{(A5)}\quad \hat{w}_H^{NT} - \hat{w}_L^{NT} = \frac{\left[\left(\frac{\epsilon-1}{\sigma-1}-\beta\right)\frac{1}{1+\varphi w_L^{\epsilon-1}}-(1-\beta)\right]\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}\varphi w_L^{\epsilon-1}}{\left(G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+1-G\right)\left(G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}+1-G\right)}\frac{G\hat{N}}{Den}.$$

Therefore, an increase in the mass of nonautomated products reduces the skill premium (and increases the labor share) if and only if $1-\beta > ((\epsilon-1)/(\sigma-1)-\beta)/(1+\varphi w_L^{\epsilon-1})$. This in turn is true for w_L sufficiently large (that is N large enough) or for $\epsilon < \sigma$.

Combining (A5) with (A4), we further get

$$\hat{w}_H^{NT} = \frac{(\sigma-1)^{-1}}{G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu}+(1-G)}$$

$$\times\left(1+\frac{G(1-\mu)\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}}{\left(G\left(1+\varphi w_L^{\epsilon-1}\right)^{\mu-1}+1-G\right)}\frac{(\epsilon-1)\varphi w_L^{\epsilon-1}}{1+\varphi w_L^{\epsilon-1}}\right)\frac{\hat{N}}{Den}.$$

Therefore, an increase in the mass of nonautomated products leads to higher high-skill wages. This establishes Part C. ∎

A2. *Proof of Proposition 2*

To see that w_{Lt} is bounded from below, assume that $\liminf w_{Lt} = 0$. Then as H_t^P and G_t admit positive limits, (7) implies that $\liminf w_{Ht} = 0$. Plugging this in (6) gives $\liminf N_t = 0$, which is impossible since g_t^N admits a positive limit.

Therefore, w_{Lt} must be bounded below, so that (6) gives $g_\infty^{w_H} = \psi g_\infty^N$, where $\psi \equiv ((1-\beta)(\sigma-1))^{-1}$. Further, using that H_t^P admits a limit and that

$$w_{Ht} H_t^P = (1-\beta)\frac{\sigma-1}{\sigma} Y_t \tag{A6}$$

gives the growth rate of Y_t. To derive the asymptotic growth rate of w_{Lt}, we consider in turn the cases $\epsilon < \infty$ and $\epsilon = \infty$.

Subcase with $\epsilon < \infty$.—We use equation (A2), which gives w_{Lt} as a function of N_t, G_t, and H_t^P. Assuming that $\liminf w_{Lt}$ is finite leads to a contradiction, so that w_{Lt} tends to ∞. Then (A2) implies that for $G_\infty < 1$,

$$w_{Lt} \sim \left(\left(\frac{\sigma-1}{\sigma}\beta\right)^{\frac{1}{1-\beta}}(1-G_\infty)\frac{H_\infty^P}{L}(G_\infty \varphi^\mu)^{\psi-1}\right)^{\frac{1}{1+\beta(\sigma-1)}} N_t^{\frac{\psi}{1+\beta(\sigma-1)}},$$

where $x_t \sim y_t$ signifies $x_t/y_t \to 1$. This delivers (11).

Subcase with $\epsilon = \infty$.—Low-skill wages are now defined as described in online Appendix B1.1, and equation (11) immediately follows from $G_\infty < 1$. ∎

A3. *Analytical Appendix to Section 2*

In this Appendix, we derive the system of normalized equation that characterizes the equilibrium and prove Proposition 3. Online Appendix B5 contains the proofs of intermediate results, of Corollary 1, and additional results mentioned in the text.

A3.1 Normalized System of Differential Equations.—Following Proposition 2, high-skill wages, output, and consumption grow asymptotically proportionately to N_t^ψ with $\psi = ((1-\beta)(\sigma-1))^{-1}$ when $g_\infty^N > 0$ and $G_\infty > 0$. Therefore, to study the behavior of the system, we introduce the normalized variables $\hat{v}_t \equiv w_{Ht} N_t^{-\psi}$ and $\hat{c}_t \equiv c_t N_t^{-\psi}$. As h_t^A mechanically tends to zero as the mass of nonautomated firms grows, we introduce $\hat{h}_t^A \equiv N_t h_t^A$. We define $\chi_t \equiv \hat{c}_t^{\,\theta}/\hat{v}_t$, which allows to simplify the system (χ_t is related to the mass of high-skill workers in production and therefore, given $\hat{h}_t^A$, to H_t^D and g_t^N). Since the economy does not feature a nonasymptotic steady state, we need to keep track of the level of N_t by introducing $n_t \equiv N_t^{-\beta/[(1-\beta)(1+\beta(\sigma-1))]}$, which tends toward zero as N_t tends toward infinity. Finally, we define $\omega_t \equiv (w_{Lt} N_t^{-\psi/(1+\beta(\sigma-1))})^{\beta(1-\sigma)}$, which asymptotes a finite positive number.

We now derive the system of differential equations satisfied by the normalized variables (n_t, G_t, h_t, χ_t). By definition, we get

$$\dot{n}_t = -\frac{\beta}{(1-\beta)(1+\beta(\sigma-1))} g_t^N n_t. \tag{A7}$$

Rewriting (12) with $\hat{h}_t^A$ gives

$$\dot{G}_t = \eta G_t^{\tilde{\kappa}} \left(\hat{h}_t^A\right)^{\kappa} (1 - G_t) - G_t g_t^N. \tag{A8}$$

Defining normalized profits $\hat{\pi}_t^A \equiv N_t^{1-\psi} \pi_t^A$ and $\hat{\pi}_t^N \equiv N_t^{1-\psi} \pi_t^N$ and the normalized values of firms $\hat{V}_t^A \equiv N_t^{1-\psi} V_t^A$ and $\hat{V}_t^N \equiv N_t^{1-\psi} V_t^N$, we can rewrite (14) and (15) as

$$\left(r_t - (\psi - 1)\, g_t^N\right) \hat{V}_t^A = \hat{\pi}_t^A + \dot{\hat{V}}_t^A, \tag{A9}$$

$$\left(r_t - (\psi - 1)\, g_t^N\right) \hat{V}_t^N = \hat{\pi}_t^N + \eta G_t^{\tilde{\kappa}} \left(\hat{h}_t^A\right)^{\kappa} \left(\hat{V}_t^A - \hat{V}_t^N\right) - \hat{v}_t \hat{h}_t^A + \dot{\hat{V}}_t^N. \tag{A10}$$

Equation (16) can similarly be rewritten as

$$\kappa \eta G_t^{\tilde{\kappa}} \left(\hat{h}_t^A\right)^{\kappa - 1} \left(\hat{V}_t^A - \hat{V}_t^N\right) = \hat{v}_t. \tag{A11}$$

Equation (17) implies that $\hat{V}_t^N = \hat{v}_t / \gamma$, therefore using (A11) into (A10), we get

$$\left(r_t - (\psi - 1)\, g_t^N\right) \hat{v}_t = \gamma \hat{\pi}_t^N + \gamma \frac{1 - \kappa}{\kappa} \hat{v}_t \hat{h}_t^A + \dot{\hat{v}}_t. \tag{A12}$$

Since $w_{Lt}^{\beta(1-\sigma)} = \omega_t n_t$, we have using (8),

$$\hat{\pi}_t^N = \omega_t n_t \left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^{-\mu} \hat{\pi}_t^A. \tag{A13}$$

Combining (A8)–(A13), we derive in online Appendix B5.1

$$\begin{aligned} \dot{\hat{h}}_t^A = {} & \frac{\gamma \hat{h}_t^A \omega_t n_t \left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^{-\mu} \hat{\pi}_t^A}{(1 - \kappa)\, \hat{v}_t} \\ & + \frac{\gamma \left(\hat{h}_t^A\right)^2}{\kappa} - \frac{\kappa \eta G_t^{\tilde{\kappa}} \left(\hat{h}_t^A\right)^{\kappa} \left(1 - \omega_t n_t \left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^{-\mu}\right) \hat{\pi}_t^A}{(1 - \kappa)\, \hat{v}_t} \\ & + \eta G_t^{\tilde{\kappa}} \left(\hat{h}_t^A\right)^{\kappa + 1} + \frac{\tilde{\kappa} \hat{h}_t^A}{1 - \kappa} \left(\eta G_t^{\tilde{\kappa} - 1} \left(\hat{h}_t^A\right)^{\kappa} (1 - G_t) - g_t^N\right). \end{aligned} \tag{A14}$$

Rewriting (18) leads to $r_t = \rho + \theta \dot{\hat{c}}_t / \hat{c}_t + \theta \psi g_t^N$. Using this with (A12) and (A13), we get

$$\begin{aligned} \dot{\chi}_t = \chi_t \Bigg(& \gamma \omega_t n_t \left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^{-\mu} \frac{\hat{\pi}_t^A}{\hat{v}_t} \\ & + \gamma \frac{1 - \kappa}{\kappa} \hat{h}_t^A - \rho - (\theta \psi - \psi + 1)\, g_t^N \Bigg). \end{aligned} \tag{A15}$$

Together equations (A7), (A8), (A14), and (A15) form a system of differential equations that depends on ω_t, $\hat{\pi}_t^A/\hat{v}_t$, and g_t^N. To determine $\hat{\pi}_t^A/\hat{v}_t$, note that profits are given by

$$\text{(A16)} \qquad \pi\left(w_L, w_H, \alpha(i)\right) = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} c\left(w_L, w_H, \alpha(i)\right)^{1-\sigma} Y.$$

Using (3) and the definition of ω_t, one gets

$$\text{(A17)} \quad \pi_t^A = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma}\left(\beta^\beta(1-\beta)^{1-\beta}\right)^{\sigma-1}\left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^\mu w_{Ht}^{-\psi^{-1}} Y_t.$$

Rearranging terms in (6) gives

$$\text{(A18)} \quad \hat{v}_t = \left(\frac{\sigma-1}{\sigma}\right)^{\frac{1}{1-\beta}} \beta^{\frac{\beta}{1-\beta}}(1-\beta)\left(G\left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^\mu + (1-G)\,\omega_t n_t\right)^\psi.$$

Using the relationship between the high-skill wage bill and output in (A6), we get

$$\text{(A19)} \qquad Y_t = \sigma\psi\hat{v}_t H_t^P N_t^\psi.$$

Therefore, rewriting (A17) with (A18) and (A19), one gets

$$\text{(A20)} \qquad \frac{\hat{\pi}_t^A}{\hat{v}_t} = \frac{\psi\left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^\mu H_t^P}{G\left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^\mu + (1-G)\,\omega_t n_t},$$

which still requires finding H_t^P. Using (3), (4), $x/l = \varphi w_L^\epsilon$, and aggregating over all automated firms, one obtains the total demand of machines:

$$X_t = \beta G_t N_t \varphi\left(\frac{\sigma-1}{\sigma}\right)^\sigma\left(\beta^\beta(1-\beta)^{(1-\beta)}\right)^{\sigma-1}\left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^{\mu-1} w_{Ht}^{-\psi^{-1}} Y_t.$$

Using (A18), this expression can be rewritten as

$$\text{(A21)} \qquad X_t = \frac{\sigma-1}{\sigma}\frac{\beta G_t\varphi\left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^{\mu-1}}{G\left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^\mu + (1-G)\,\omega_t n_t} Y_t.$$

This together with (A19) implies that $\hat{c}_t$ obeys

$$\hat{c}_t = \left(1 - \frac{\sigma-1}{\sigma}\frac{\beta G_t\varphi\left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^{\mu-1}}{G\left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^\mu + (1-G)\,\omega_t n_t}\right)\sigma\psi\hat{v}_t H_t^P.$$

Combining this equation with (A18) leads to

$$\text{(A22)} \quad H_t^P = \frac{\left(\beta^\beta\frac{\sigma-1}{\sigma}\right)^{\frac{\theta^{-1}-\beta}{1-\beta}}(1-\beta)^{\frac{1}{\theta}}\chi_t^{\frac{1}{\theta}}\left(G_t\left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^\mu + (1-G_t)\,\omega_t n_t\right)^{\psi\left(\frac{1}{\theta}-1\right)+1}}{G_t\left(\left(1-\beta\frac{\sigma-1}{\sigma}\right)\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)\left(\varphi + (\omega_t n_t)^{\frac{1}{\mu}}\right)^{\mu-1} + (1-G_t)\,\omega_t}.$$

Using the definition of H_t^D, one can rewrite (13) for high-skill workers as

$$g_t^N = \gamma\Big(H - H_t^P - (1 - G_t)\,\hat{h}_t^A\Big). \tag{A23}$$

Together (A20), (A22), and (A23) determine $\hat{\pi}_t^A/\hat{v}_t$ and g_t^N as a function of the original variables $n_t, G_t, \hat{h}_t^A, \chi_t$ and of ω_t, which still needs to be determined. To do so, combine (7) and (6) to obtain an implicit definition of ω_t:

$$\omega_t = \Bigg[\Big(\frac{\sigma-1}{\sigma}\beta\Big)^{\frac{1}{1-\beta}}\frac{H_t^P}{L}\Big(G_t\Big(\varphi+(\omega_t n_t)^{\frac{1}{\mu}}\Big)^{\mu-1}(\omega_t n_t)^{\frac{1-\mu}{\mu}}+(1-G_t)\Big)$$
$$\times\Big(G_t\Big(\varphi+(\omega_t n_t)^{\frac{1}{\mu}}\Big)^{\mu}+(1-G_t)\omega_t n_t\Big)^{\psi-1}\Bigg]^{\frac{\beta(1-\sigma)}{1+\beta(\sigma-1)}}. \tag{A24}$$

Therefore, the system of differential equations satisfied by $n_t, G_t, \hat{h}_t^A, \chi_t$ is defined by (A7), (A8), (A14), and (A15), with $\hat{\pi}_t^A/\hat{v}_t$, H_t^P, g_t^N, and ω_t given by (A20), (A22), (A23), and (A24). The state variables are n_t and G_t and the control variables $\hat{h}_t^A$ and χ_t.

A3.2 Proof of Proposition 3A.—To prove Proposition 3A, we show that the system described in Section A3.1 admits a steady state.

LEMMA A.1: *The system of differential equations admits a steady state* $(n^*, G^*, \hat{h}^{A*}, \chi^*)$ *with* $n^* = 0, 0 < G^* < 1$, *and positive growth* $(g^N)^* > 0$ *if*

$$\kappa^{-\kappa}\big(\gamma(1-\kappa)/\rho\big)^{\kappa-1}\rho/\eta + \rho/\gamma < \psi H. \tag{A25}$$

PROOF:

We look for a steady state with positive long-run growth ($(g^N)^* > 0$) for the system defined by (A7), (A18), (A14), and (A15) and denote such a (potential) steady state $n^*, G^*, \hat{h}^{A*}, \chi^*$ (we denote all variables at steady state with a *). Following (A7), we immediately get $n^* = 0$. Using (A8), G^* obeys

$$G^* = \frac{\eta\,(G^*)^{\tilde{\kappa}}\,\big(\hat{h}^{A*}\big)^{\kappa}}{\eta\,(G^*)^{\tilde{\kappa}}\,\big(\hat{h}^{A*}\big)^{\kappa} + g^{N*}}. \tag{A26}$$

We focus on a solution with $G^* > 0$ (when $\tilde{\kappa} > 0$, $G^* = 0$ is also a solution); (A26) implies that with $(g^N)^* > 0, G^* < 1$. Then, with $\mu \in (0,1)$, (A24) implies that

$$\omega^* = \Bigg[\Big(\frac{\sigma-1}{\sigma}\beta\Big)^{\frac{1}{1-\beta}}\frac{H^{P*}}{L}\,(1-G^*)\,(G^*\varphi^{\mu})^{\psi-1}\Bigg]^{\frac{\beta(1-\sigma)}{1+\beta(\sigma-1)}}.$$

Using (A20), (A15) implies that in steady state,

$$\text{(A27)} \qquad \hat{h}^{A*} = \frac{\kappa}{\gamma(1-\kappa)}\Big(\rho + \big((\theta-1)\psi + 1\big)\, g^{N*}\Big),$$

which uniquely defines $\hat{h}^{A*}$ as an increasing function of g^{N*} (recall that $\theta \geq 1$) with $\hat{h}^{A*} > 0$ if $g^{N*} > 0$. Then, for $G^* > 0$, (A26) combined with (A27) defines G^* uniquely as an increasing function of g^{N*}. Equation (A23) also uniquely defines H^{P*} as a function of g^{N*}:

$$\text{(A28)} \qquad H^{P*} = H - \frac{g^{N*}}{\gamma} - \left(1 - G^*\right)\hat{h}^{A*}.$$

Equations (A20) and (A26) allow us to rewrite (A14) in steady state as

$$\text{(A29)} \qquad \frac{\eta\kappa\left(G^*\right)^{\tilde{\kappa}-1}\left(\hat{h}^{A*}\right)^{\kappa}}{1-\kappa}\,\psi H^{P*} = \frac{\gamma}{\kappa}\left(\hat{h}^{A*}\right)^{2} + \eta G_t^{\tilde{\kappa}}\left(\hat{h}^{A*}\right)^{\kappa+1}.$$

Since G^*, $\hat{h}^{A*}$, and H^{P*} are functions of g^{N*}, one can rewrite (A29) as an equation determining g^{N*}. A steady state with positive growth-rate is a solution to

$$\text{(A30)} \qquad f\left(g^{N*}\right) \equiv \frac{1-\kappa}{\kappa}\frac{\gamma G^* \hat{h}^{A*}}{\psi H^{P*}}\left(\frac{1}{\kappa\eta\left(G^*\right)^{\tilde{\kappa}}}\left(\hat{h}^{A*}\right)^{1-\kappa} + \frac{1}{\gamma}\right) = 1,$$

with $g^{N*} > 0$. Indeed, (A22) simply determines χ^* as

$$\text{(A31)} \qquad \chi^* = \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{1-\beta}(1-\theta\beta)} \frac{\left(1 - \beta\frac{\sigma-1}{\sigma}\right)^{\theta}\left(H^{P*}\right)^{\theta}}{\left(1-\beta\right)\beta^{\frac{\beta}{1-\beta}(1-\theta)}\left(G^*\varphi^{\mu}\right)^{\psi(1-\theta)}},$$

which achieves the characterization of a steady state for the system of differential equations defined by (A7), (A8), (A14), and (A15).

To establish the sufficiency of equation (A25) for positive growth, note that as $g^{N*} \to 0$, then equations (A27), (A26), and (A28) imply that

$$f(0) = \frac{\rho}{\psi H}\left(\frac{1}{\eta\kappa^{\kappa}\left(1-\kappa\right)^{1-\kappa}}\left(\frac{\rho}{\gamma}\right)^{1-\kappa} + \frac{1}{\gamma}\right).$$

In addition, $(g^{N*}/\gamma) + (1-G^*)\,\hat{h}^{A*}$ is always greater than (g^{N*}/γ), therefore for a sufficiently large g^{N*} (smaller than γH), H^{P*} is arbitrarily small, while for the same value G^* and $\hat{h}^{A*}$ are bounded below and above. This establishes that for g^{N*} large enough, $f(g^{N*}) > 1$. Therefore, a sufficient condition for the existence of at least one steady state with positive growth and positive G^* is that $f(0) < 1$ (such that $f(g^{N*}) = 1$ has a solution), which is equivalent to condition (A25). The assumption that $\theta \geq 1$ further ensures that the transversality condition always holds. ∎

The steady state $(n^*, G^*, \hat{h}^{A*}, \chi^*)$ corresponds to an asymptotic steady state for our original system of differential equations (as $N_t \to \infty$ when $n_t \to 0$). Since $G_\infty = G^* \in (0,1)$, $g^N_\infty = g^{N*} > 0$, and $H^P_\infty = H^{P*} > 0$, Proposition 2 applies (which is also in line with the normalized variables being constant in steady state). Online Appendix Section B5.4 provides further details on the behavior of the economy close to the steady state.

A3.3 Proof of Proposition 3B.—Here we prove Proposition 3B. Combining (20) and (19), we can write

$$N_t h_t^A = \left(\kappa\eta G_t^{\tilde{\kappa}}\left(\int_t^{\infty}\exp\left(-\int_t^{\tau} r_u du\right)\right.\right.$$
$$\left.\left.\times\left(\frac{N_t}{w_{Ht}}\left(\pi_\tau^A - \pi_\tau^N\right) - \frac{1-\kappa}{\kappa}\frac{N_t}{N_\tau}\frac{w_{H\tau}}{w_{Ht}}\left(N_\tau h_\tau^A\right)\right)d\tau\right)\right)^{\frac{1}{1-\kappa}}.$$

Using (A6) and that aggregate profits $\Pi_t = N_t(G_t\pi_t^A + (1-G_t)\pi_t^N)$ are a share $1/\sigma$ of output, we can rewrite this equation as

$$\text{(A32)}\quad \hat{h}_t^A = \left(\kappa\eta G_t^{\tilde{\kappa}}\left(\int_t^{\infty}\exp\left(-\int_t^{\tau} r_u du\right)\right.\right.$$
$$\left.\left.\times\left(\psi H_t^P\frac{\pi_\tau^A - \pi_\tau^N}{G_t\pi_t^A + (1-G_t)\pi_t^N} - \frac{1-\kappa}{\kappa}\frac{N_t}{N_\tau}\frac{\hat{v}_\tau}{\hat{v}_t}\hat{h}_\tau^A\right)d\tau\right)\right)^{\frac{1}{1-\kappa}}.$$

Recalling (8), we can write

$$\hat{h}_t^A = \left(\kappa\eta G_t^{\tilde{\kappa}}\left(\int_t^{\infty}\left(\psi H_t^P\frac{\left(1+\varphi w_{L\tau}^{\epsilon-1}\right)^{\mu}-1}{G_t\left(1+\varphi w_{Lt}^{\epsilon-1}\right)^{\mu}+1}\exp\left(\int_t^{\tau}\left(g_u^{\pi^N} - r_u\right)du\right)\right.\right.\right.$$
$$\left.\left.\left.-\frac{1-\kappa}{\kappa}\exp\left(\int_t^{\tau}\left(g_u^{v} - g_u^{N} - r_u\right)du\right)\hat{h}_\tau^A\right)d\tau\right)\right)^{\frac{1}{1-\kappa}}.$$

Consider a fixed $\hat{t} > 0$. For an arbitrarily large T, if w_{L0} is sufficiently small relative to $\tilde{\varphi}^{-1}$, w_{Lt} remains small relative to $\tilde{\varphi}^{-1}$ over $(0, \hat{t}+T)$. For any $\tau \in (0, \hat{t}+T)$, we have that $\frac{(1+\varphi w_{L\tau}^{\epsilon-1})^{\mu}-1}{G_t(1+\varphi w_{Lt}^{\epsilon-1})^{\mu}+1} = \mu\varphi w_{L\tau}^{\epsilon-1} + o(\varphi w_{L\tau}^{\epsilon-1})$.[38] Then for any $t \in (0,\hat{t})$,

$$\left(\hat{h}_t^A\right)^{1-\kappa} \le \kappa\eta G_t^{\tilde{\kappa}}\left(\int_t^{\hat{t}+T}\psi H_t^P\left(\mu\varphi w_{L\tau}^{\epsilon-1} + o\left(\varphi w_{L\tau}^{\epsilon-1}\right)\right)\exp\left(\int_t^{\tau}\left(g_u^{\pi^N} - r_u\right)du\right)d\tau\right.$$
$$\left.+\int_{\hat{t}+T}^{\infty}\psi H_t^P\frac{\left(1+\varphi w_{L\tau}^{\epsilon-1}\right)^{\mu}-1}{G_t\left(1+\varphi w_{Lt}^{\epsilon-1}\right)^{\mu}+1}\exp\left(\int_t^{\tau}\left(g_u^{\pi^N} - r_u\right)du\right)d\tau\right).$$

[38] The notation $o(z)$ denotes negligible relative to z (that is $f(z) = o(z)$, if $\lim_{z\to 0} f(z)/z = 0$), and $O(z)$ will denote of the same order or negligible in front of z ($f(z) = O(z)$ if $\limsup_{z\to 0}|f(z)/z| < \infty$).

Further, $r_u = \rho + \theta g_u^C$ with $\theta \geq 1$. In addition, $C_u = Y_u - X_u$, with X_u the aggregate spending on machines (initially negligible and later on a share of output bounded away from 1), and π_u^N initially grows like Y_u/N_u (and from then on will grow slower), therefore, $r_u - g_u^{\pi^N} > \rho$. Hence, one can write

$$\left(\hat{h}_t^A\right)^{1-\kappa} \leq \kappa\eta G_t^{\tilde{\kappa}}\left(\int_t^{\hat{t}+T}\mu\psi H_t^P \varphi w_{L\tau}^{\epsilon-1} e^{\int_t^\tau\left(g_u^{\pi^N}-r_u\right)du} d\tau + o\left(\varphi w_{L\hat{t}+T}^{\epsilon-1}\right) + o\left(e^{-\rho\left(T+\hat{t}-t\right)}\right)\right).$$

Since $r_u - g_u^{\pi^N} > \rho$, then $\int_t^{\hat{t}+T} e^{\int_t^\tau (g_u^{\pi^N}-r_u)du} d\tau \leq (1/\rho)(1-e^{-\rho(\hat{t}+T-t)})$ and we have

$$\left(\hat{h}_t^A\right)^{1-\kappa} \leq \kappa\eta G_t^{\tilde{\kappa}}\left(\frac{\mu\psi H_t^P\varphi}{\rho}\max_{\tau\in\left(t,\hat{t}+T\right)}\left(w_{L\tau}^{\epsilon-1}\right) + o\left(\varphi w_{L\hat{t}+T}^{\epsilon-1}\right) + o\left(e^{-\rho T}\right)\right).$$

Therefore, since T is large and $\varphi w_{L\tau}^{\epsilon-1}$ is small, then $\hat{h}_t^A$ must be small too. In fact, we get that $\hat{h}_t^A = O((\varphi w_{L\hat{t}+T}^{\epsilon-1})^{\frac{1}{1-\kappa}}) + o(e^{-\rho T})$.

For any $t \in (0,\hat{t})$, we can then rewrite (A15) as

$$\text{(A33)} \qquad \frac{\dot{\chi}_t}{\chi_t} = \gamma\psi H^P - \rho - \left(\theta\psi - \psi + 1\right) g_t^N + O\left(\left(\varphi w_{L\hat{t}+T}^{\epsilon-1}\right)^{\frac{1}{1-\kappa}}\right) + o\left(e^{-\rho T}\right).$$

Next, (4) and the corresponding equation for high-skill labor demand in production imply

$$\frac{L^{NA}}{L^A} = \frac{\left(1-G_t\right)\left(1+\varphi w_{Lt}^{\epsilon-1}\right)^{-\mu-1}}{G_t} \quad \text{and} \quad \frac{H^{P,NA}}{H^{P,A}} = \frac{\left(1-G_t\right)\left(1+\varphi w_{Lt}^{\epsilon-1}\right)^{-\mu}}{G_t}.$$

Using these expressions in (5), and knowing that $w_{Lt} = O(Y_t/L)$ so that $\varphi^{\frac{1}{\epsilon}} Y_t/L = O(\varphi^{\frac{1}{\epsilon}} w_{Lt})$, we get

$$\text{(A34)} \qquad Y_t = \left(1 + O\left(\varphi w_{Lt}^{\epsilon-1}\right)\right) N^{\frac{1}{\sigma-1}} L^\beta \left(H_t^P\right)^{1-\beta}.$$

One then gets that wages obey

$$\text{(A35)} \qquad w_{Ht} = \left(1 + O\left(\varphi w_{Lt}^{\epsilon-1}\right)\right)\frac{\sigma-1}{\sigma}\left(1-\beta\right) N_t^{\frac{1}{\sigma-1}} L^\beta \left(H_t^P\right)^{-\beta},$$

$$\text{(A36)} \qquad w_{Lt} = \left(1 + O\left(\varphi w_{Lt}^{\epsilon-1}\right)\right)\frac{\sigma-1}{\sigma}\beta N_t^{\frac{1}{\sigma-1}} L^{\beta-1} \left(H_t^P\right)^{1-\beta}.$$

Using (A21), we obtain $C_t = Y_t - X_t = (1 + O(G_t\varphi w_{Lt}^{\epsilon-1}))Y_t$, then using the definition of χ_t, (A34) and (A35), we get

$$\chi_t = \left(1 + O\left(\varphi w_{Lt}^{\epsilon-1}\right)\right)\sigma\psi L^{\beta(\theta-1)} \left(H_t^P\right)^{(1-\beta)\theta+\beta} N_t^{\frac{(1-\theta)\beta}{(\sigma-1)(1-\beta)}}.$$

Differentiating and plugging into (A33) and using (A23), we get (recalling (A36) so that $d\ln(1 + O(\varphi w_{Lt}^{\epsilon-1}))/dt$ will be of order $O(\varphi w_{Lt}^{\epsilon-1})$ as well)

$$\left((1-\beta)\theta + \beta\right)\frac{\dot{H}_t^P}{H_t^P} = \gamma\psi H_t^P - \rho - \left(\frac{\theta-1}{\sigma-1}+1\right)\gamma\left(H - H_t^P\right)$$

$$+ O\left(\left(\varphi w_{L\hat{t}+T}^{\epsilon-1}\right)^{\frac{1}{1-\kappa}}\right) + o\left(e^{-\rho T}\right);$$

we dropped terms in $\varphi w_{Lt}^{\epsilon-1}$ since these are negligible in front of $(\varphi w_{L\hat{t}+T}^{\epsilon-1})^{\frac{1}{1-\kappa}}$. The exact counterpart of this system admits a BGP with H_t^P constant without transitional dynamics as in Romer (1990). Therefore, we have over the interval $(0,\hat{t})$

$$H_t^P = \frac{\left(\frac{\theta-1}{\sigma-1}+1\right)H + \frac{\rho}{\gamma}}{\psi + \frac{\theta-1}{\sigma-1}+1} + O\left(\left(\varphi w_{L\hat{t}+T}^{\epsilon-1}\right)^{\frac{1}{1-\kappa}}\right) + o\left(e^{-\rho T}\right)$$

and

$$g_t^N = \frac{\gamma H\psi - \rho}{\psi + \frac{\theta-1}{\sigma-1}+1} + O\left(\left(\varphi w_{L\hat{t}+T}^{\epsilon-1}\right)^{\frac{1}{1-\kappa}}\right) + o\left(e^{-\rho T}\right), \tag{A37}$$

which is positive under assumption (A25). With a low $\hat{h}_t^A$, (12) can be solved as $G_t = G_0\exp\left(-\frac{\gamma H\psi - \rho}{\psi + \frac{\theta-1}{\sigma-1}+1}t\right) + O((\varphi w_{L\hat{t}+T}^{\epsilon-1})^{\frac{1}{1-\kappa}}) + o(e^{-\rho T})$. This characterizes the solution over the interval $(0,\hat{t})$ for w_{Lt} sufficiently small relative to $\tilde{\varphi}^{-1}$. A small w_{Lt} in return can be generated by a sufficiently small N_0, so that for any $a > 0$, there is an N_0 low enough that the automation rate is lower than a. This establishes Proposition 3B. ■

A3.4 Proof of Proposition 3C.—We now prove Part C. The proof relies on two lemmas proved in online Appendix B5.3.

LEMMA A.2: *If $\kappa(1-\beta) + \tilde{\kappa} < 1$ and g_t^N has a positive limit, then G_t cannot converge toward zero.*

This lemma shows that automation must take off at some point; the second lemma is more technical.

LEMMA A.3: *If G_t is bounded above zero, then $\hat{h}_t^A$ is bounded.*

Under the assumptions of the Proposition, G_t has a limit G_∞ and g_t^N a positive limit g_∞^N. Then Lemma A.2 implies that $G_\infty > 0$ and Lemma A.3 together with

(A8) that $G_\infty < 1$. Following Proposition 2, we then get that $w_{Lt} = O\left(N_t^{\frac{\psi}{1+\beta(\sigma-1)}}\right)$ or $\omega_t = O(1)$. Therefore, we can rewrite the system as (A8),

$$\dot{\hat{h}}_t^A = \frac{\gamma\left(\hat{h}_t^A\right)^2}{\kappa} + \eta G_t^{\tilde{\kappa}}\left(\hat{h}_t^A\right)^\kappa\left(\hat{h}_t^A - \frac{\kappa\,\hat{\pi}_t^A}{(1-\kappa)\,\hat{v}_t}\right)$$

$$+ \frac{\tilde{\kappa}\left(\eta G_t^{\tilde{\kappa}-1}\left(\hat{h}_t^A\right)^{\kappa+1}(1-G_t) - g_t^N\hat{h}_t^A\right)}{1-\kappa} + O(n_t),$$

$$\dot{\chi}_t = \chi_t\left(\gamma\frac{1-\kappa}{\kappa}\hat{h}_t^A - \rho - (\theta\psi - \psi + 1)\,g_t^N\right) + O(n_t);$$

knowing that

$$\text{(A38)}\quad H_t^P = \left(\frac{\sigma-1}{\sigma}\right)^{\frac{1}{1-\beta}\left(\frac{1}{\theta}-\beta\right)}\frac{(1-\beta)^{\frac{1}{\theta}}\beta^{\frac{\beta}{1-\beta}\left(\frac{1}{\theta}-1\right)}\chi_t^{\frac{1}{\theta}}\left(G_t\varphi^\mu\right)^{\psi\left(\frac{1}{\theta}-1\right)}}{G_t\varphi^\mu\left(1-\beta\frac{\sigma-1}{\sigma}\right)} + O\left(n_t^{\frac{1}{\mu}}\right),$$

$$\text{(A39)}\quad \frac{\hat{\pi}_t^A}{\hat{v}_t} = \frac{\psi H_t^P}{G_t} + O\left(n_t^{\frac{1}{\mu}}\right),$$

and (A23). Using that g_t^N and G_t have limits in (A8) implies that $\hat{h}_t^A$ must also have a limit. Using (A23), this implies that H_t^P must also have a limit and therefore using (A38) that χ_t must have a limit. In other words, the equilibrium path tends toward the steady state $(\hat{h}^{A*}, G^*, \chi^*)$ with $n_t \rightarrow 0$ defined in Lemma A.1. ∎

A4. *Appendix to the Quantitative Exercise*

This Appendix presents two exercises that complement Section III: a calibration on the first 30 years of data and an analysis of the role played by the automation externality. Online Appendix B11 contains additional details on the quantitative exercise.

A4.1 Out-of-Sample Prediction.—We reproduce our calibration exercise but only matching the first 30 years of data. Figure A1 reports the results, and Table A1 gives the new parameters. The model behaves very well out of sample. The predicted path and parameters are close to those of the baseline case. The calibrated model over the first 30 years slightly underestimates the pace of the rise of the skill premium. The biggest parameter difference is a lower elasticity of substitution between low-skill workers and machines at 4.3 instead of 5.8. This decreases the incentive to automate, slows the growth in the skill premium, and lowers the growth rate of equipment (panel D of Figure A1).

A4.2 The Role of the Automation Externality.—To analyze the role played by the automation externality, we recalibrate our model without it (i.e., we impose that $\tilde{\kappa} = 0$). Table A2 gives the resulting parameters, and Figure A3 reports the results.

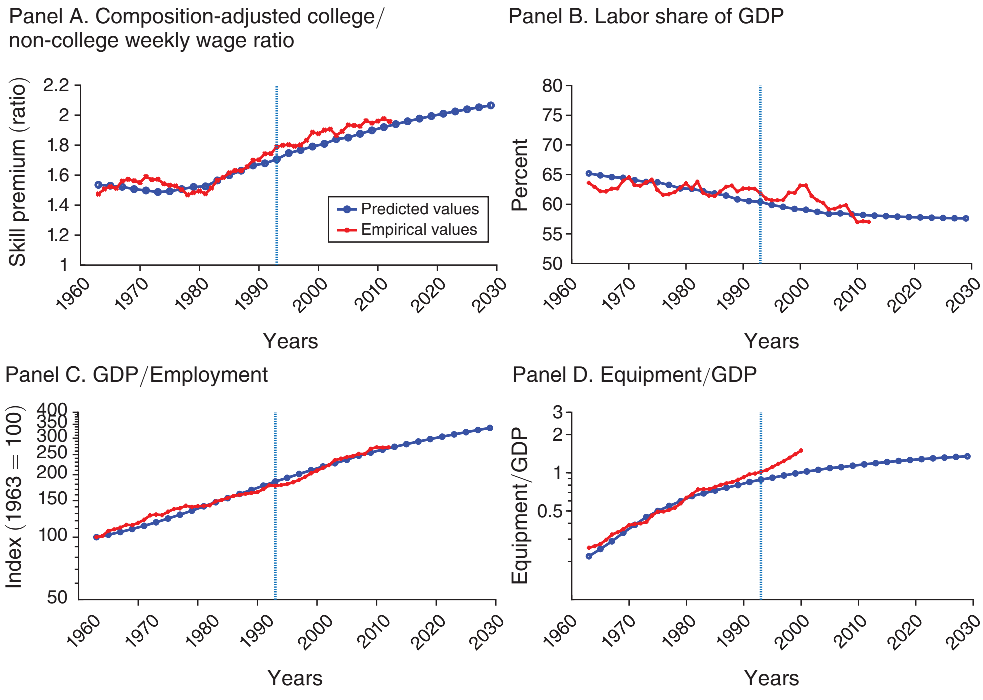

Figure A1. Predicted and Empirical Time Paths Only Matching the First 30 Years (*until 1992*)

Table A1—Parameters (*only matching the first 30 years*)

Parameter	σ	ϵ	β_1	γ	$\tilde{\kappa}$	θ	η	κ
Value	5.80	4.34	0.60	0.43	0.55	1	0.40	0.66
Parameter	ρ	β_2	Δ	β_4	$\tilde{\varphi}$	N_{1963}	G_{1963}	
Value	0.034	0.16	0.011	0.76	1.56	14.8	0.03	

Table A2—Parameters (*when* $\tilde{\kappa} = 0$)

Parameter	σ	ϵ	β_1	γ	$\tilde{\kappa}$	θ	η	κ
Value	4.95	4.95	0.66	0.51	0	1.15	0.30	0.87
Parameter	ρ	β_2	Δ	β_4	$\tilde{\varphi}$	N_{1963}	G_{1963}	
Value	0.039	0.14	0.011	0.76	1.47	18.4	0.12	

The model still reproduces the paths for the labor share, GDP/employment, and equipment/GDP. Yet it does not capture the evolution of the skill premium. Indeed, the fast rise in the skill premium in the 1980s and 1990s requires a fast increase in automation, which, given the moderate decline in the labor share and the stable economic growth, can only be brought about by a positive automation externality. The data clearly favor a positive automation externality (even though the exact value of $\tilde{\kappa}$ is not precisely estimated; see online Appendix Table B4).

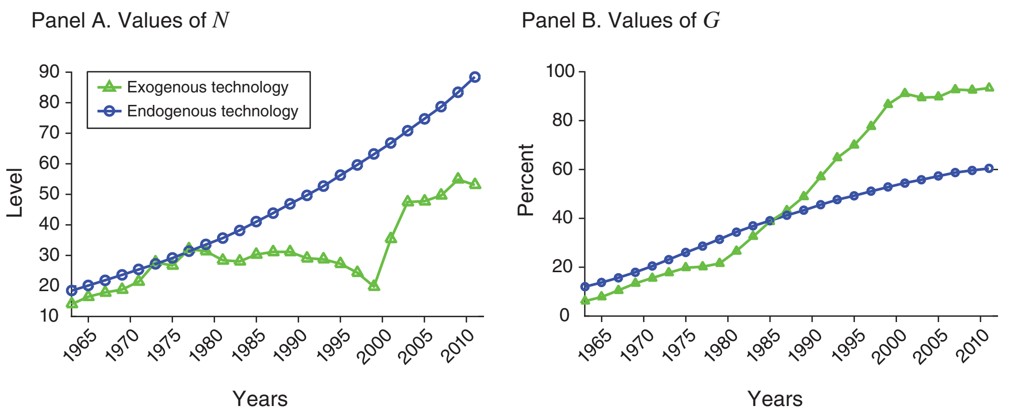

Figure A2. Paths for N_t and G_t in the Endogenous and Exogenous Growth Models without Automation Externality

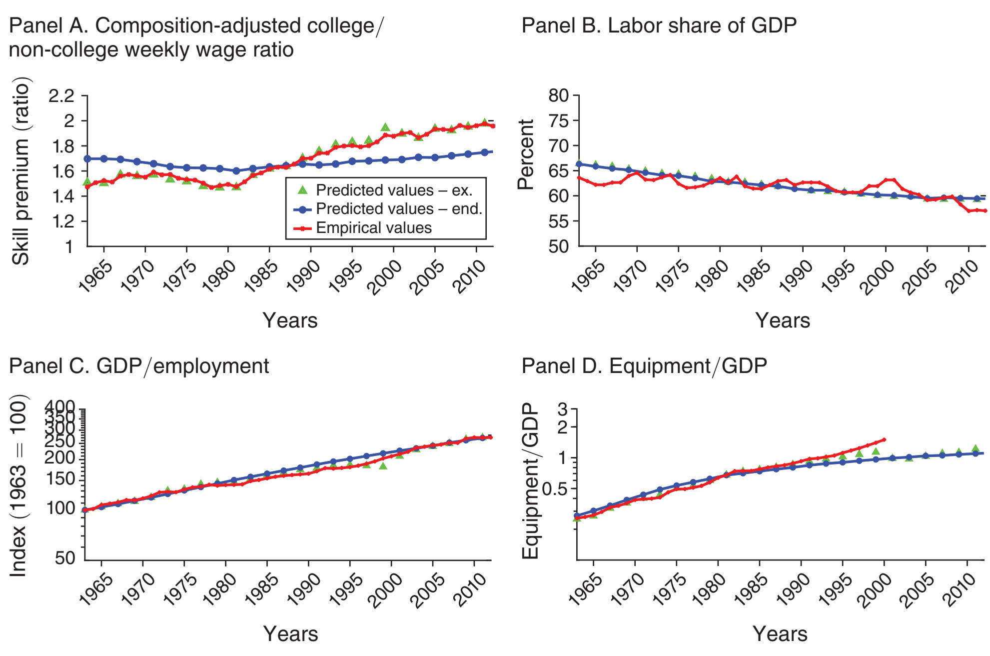

Figure A3. Empirical Paths and Predicted Paths for the Endogenous and Exogenous Technology Models for $\tilde{\kappa} = 0$

Furthermore, we reproduce the exercise of Section IIIB without an automation externality. We fix the parameters to their values in the recalibrated model with $\tilde{\kappa} = 0$ and look for the (exogenous) paths of N_t and G_t that minimize the distance between the model and the data. Figure A2 shows that with this set of parameters, an exogenous technological model would indeed require a faster increase in G_t to better match the data than in the endogenous growth model: the path looks closer to that obtained in the endogenous model with $\tilde{\kappa} \neq 0$ than with $\tilde{\kappa} = 0$. If anything, it exhibits an even sharper increase in the 1980s and 1990s than in panel B of Figure 5. Figure A3 shows that this exogenous growth model still matches the data well.

REFERENCES

Acemoglu, Daron. 1998. "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality." *Quarterly Journal of Economics* 113 (4): 1055–89.

Acemoglu, Daron. 2002. "Directed Technical Change." *Review of Economic Studies* 69 (4): 781–809.

Acemoglu, Daron. 2007. "Equilibrium Bias of Technology." *Econometrica* 75 (5): 1371–1409.

Acemoglu, Daron. 2010. "When Does Labor Scarcity Encourage Innovation?" *Journal of Political Economy* 118 (6): 1037–78.

Acemoglu, Daron, and David Autor. 2011. "Chapter 12—Skills, Tasks and Technologies: Implications for Employment and Earnings." In *Handbook of Labor Economics*, Vol. 4B, edited by David Card and Orley Ashenfelter, 1043–1171. Amsterdam: North-Holland.

Acemoglu, Daron, and Pascual Restrepo. 2018a. "The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment." *American Economic Review* 108 (6): 1488–1542.

Acemoglu, Daron, and Pascual Restrepo. 2018b. "Demographics and Automation." NBER Working Paper 24421.

Acemoglu, Daron, and Pascual Restrepo. 2020. "Robots and Jobs: Evidence from US Labor Markets." *Journal of Political Economy* 128 (6): 2188–2244.

Aghion, Philippe, and Peter Howitt. 1996. "Research and Development in the Growth Process." *Journal of Economic Growth* 1 (1): 49–73.

Aghion, Philippe, Peter Howitt, and Giovanni L. Violante. 2002. "General Purpose Technology and Wage Inequality." *Journal of Economic Growth* 7 (4): 315–45.

Aghion, Philippe, Benjamin F. Jones, and Charles I. Jones. 2019. "Artificial Intelligence and Economic Growth." In *The Economics of Artificial Intelligence: An Agenda,* edited by A. Agrawal, J. Gans, A. Goldfarb, 237–90. Chicago: University of Chicago Press.

Autor, David H. 2013. "The 'Task Approach' to Labor Markets: An Overview." *Journal for Labour Market Research* 46: 185–99.

Autor, David. H. 2014. "Skills, Education, and the Rise of Earnings Inequality among the 'Other 99 Percent.'" *Science* 344 (6186): 843–51.

Autor, David H., Lawrence F. Katz, and Melissa S. Kearney. 2006. "The Polarization of the U.S. Labor Market." *American Economic Review* 96 (2): 189–94.

Autor, David H., Lawrence F. Katz, and Melissa S. Kearney. 2008. "Trends in U.S. Wage Inequality: Revising the Revisionists." *Review of Economics and Statistics* 90 (2): 300–323.

Autor, David H., Lawrence F. Katz, and Alan B. Krueger. 1998. "Computing Inequality: Have Computers Changed the Labor Market?" *Quarterly Journal of Economics* 113 (4): 1169–1213.

Autor, David H., Frank Levy, and Richard J. Murnane. 2003. "The Skill Content of Recent Technological Change: An Empirical Exploration." *Quarterly Journal of Economics* 118 (4): 1279–1333.

Bartel, Ann, Casey Ichniowski, and Kathryn Shaw. 2007. "How Does Information Technology Affect Productivity? Plant-Level Comparisons of Product Innovation, Process Improvement and Worker Skills." *Quarterly Journal of Economics* 122 (4): 1721–58.

Beaudry, Paul, David A. Green, and Benjamin M. Sand. 2016. "The Great Reversal in the Demand for Skill and Cognitive Tasks." *Journal of Labor Economics* 34 (S1): S199–S247.

Benzell, Seth G., Laurence J. Kotlikoff, Guillermo LaGarda, and Jeffrey D. Sachs. 2019. "Robots Are Us: Some Economics of Human Replacement." Boston University Department of Economics Working Papers Series WP2020-003.

Bloom, Nicholas, Mark Schankerman, and John Van Reenen. 2013. "Identifying Technology Spillovers and Product Market Rivalry." *Econometrica* 81 (4): 1347–93.

Buera, Francisco J., and Joseph P. Kaboski. 2012. "The Rise of the Service Economy." *American Economic Review* 102 (6): 2540–69.

Card, David, and John E. DiNardo. 2002. "Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles." *Journal of Labor Economics* 20 (4): 733–83.

Caselli, Francesco. 1999. "Technological Revolutions." *American Economic Review* 89 (1): 78–102.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113 (1): 1–45.

Clemens, Michael A., Ethan G. Lewis, and Hannah M. Postel. 2018. "Immigration Restrictions as Active Labor Market Policy: Evidence from the Mexican Bracero Exclusion." *American Economic Review* 108 (6): 1468–87.

Cummins, Jason G., and Giovanni L. Violante. 2002. "Investment-Specific Technical Change in the United States (1947–2000): Measurement and Macroeconomic Consequences." *Review of Economic Dynamics* 5 (2): 243–84.

Dechezleprêtre, Antoine, David Hémous, Morten Olsen, and Carlo Zanella. 2019. "Automating Labor: Evidence from Firm-Level Patent Data." CEPR Discussion Paper DP14249.

Eden, Maya, and Paul Gaggl. 2018. "On the Welfare Implications of Automation." *Review of Economic Dynamics* 29: 15–43.

Feng, Andy, and Georg Graetz. 2016. "Rise of the Machines: The Effects of Laborsaving Innovations on Jobs and Wages." IZA Discussion Paper 8836.

Galor, Oded, and Omar Moav. 2000. "Ability-Biased Technological Transition, Wage Inequality, and Economic Growth." *Quarterly Journal of Economics* 115 (2): 469–97.

Galor, Oded, and David N. Weil. 2000. "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond." *American Economic Review* 90 (4): 806–28.

Goldin, Claudia Dale, and Lawrence F. Katz. 2008. *The Race between Education and Technology*. Cambridge, MA: Harvard University Press.

Gordon, Robert J. 1990. *The Measurement of Durable Goods Prices*. Chicago: University of Chicago Press.

Graetz, Goerg, and Guy Michaels. 2018. "Robots at Work." *Review of Economics and Statistics* 100 (5): 753–68.

Habakkuk, H.J. 1962. *American and British Technology in the Nineteenth Century*. Cambridge, UK: Cambridge University Press.

Hansen, Gary D., and Edward C. Prescott. 2002. "Malthus to Solow." *American Economic Review* 92 (4): 1205–17.

Hémous, David, and Morten Olsen. 2016. "The Rise of the Machines: Automation, Horizontal Innovation and Income Inequality." CEPR Discussion Paper DP 10244.

Hémous, David, and Morten Olsen. 2022. "Replication data for: The Rise of the Machines: Automation, Horizontal Innovation, and Income Inequality." American Economic Association [publisher], Inter-university Consortium for Political and Social Research [distributor]. https://doi.org/10.3886/E120390V1.

Hornbeck, Richard, and Suresh Naidu. 2014. "When the Levee Breaks: Black Migration and Economic Development in the American South." *American Economic Review* 104 (3): 963–90.

Karabarbounis, Loukas, and Brent Neiman. 2014. "The Global Decline of the Labor Share." *Quarterly Journal of Economics* 129 (1): 61–103.

Katz, Lawrence F., and Robert A. Margo. 2014. "Technical Change and the Relative Demand for Skilled Labor: The United States in Historical Perspective." In *Human Capital in History: The American Record*, edited by Leah Platt Boustan, Carola Frydman, and Robert A. Margo, 15–57. Chicago: University of Chicago.

Katz, Lawrence F., and Kevin M. Murphy. 1992. "Changes in Relative Wages, 1963–1987: Supply and Demand Factors." *Quarterly Journal of Economics* 107 (1): 35–78.

Krusell, Per, Lee E. Ohanian, José-Víctor Ríos-Rull, and Giovanni L. Violante. 2000. "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis." *Econometrica* 68 (5): 1029–53.

Lewis, Ethan. 2011. "Immigration, Skill Mix, and Capital Skill Complementarity." *Quarterly Journal of Economics* 126 (2): 1029–69.

Lloyd-Ellis, Huw. 1999. "Endogenous Technological Change and Wage Inequality." *American Economic Review* 89 (1): 47–77.

Mann, Katja, and Lukas Püttmann. 2018. "Benign Effects of Automation: New Evidence from Patent Texts." https://papers.ssrn.com/sol3/Delivery.cfm/SSRN_ID3233471_code2678418.pdf?abstractid=2959584&mirid=1&type=2.

Manuelli, Rodolfo E., and Ananth Seshadri. 2014. "Frictionless Technology Diffusion: The Case of Tractors." *American Economic Review* 104 (4): 1368–91.

Martinez, Joseba. 2018. "Automation, Growth and Factor Shares." Society for Economic Dynamics 2018 Meeting Papers 736.

Nakamura, Hideki, and Joseph Zeira. 2018. "Automation and Unemployment: Help Is on the Way." CEPR Discussion Paper DP12974.

Nelson, Richard R., and Edmund S. Phelps. 1966. "Investment in Humans, Technological Diffusion, and Economic Growth." *American Economic Review* 56 (1/2): 69–75.

Peretto, Pietro F., and John J. Seater. 2013. "Factor-Eliminating Technical Change." *Journal of Monetary Economics* 60 (4): 459–73.

Rahman, Ahmed S. 2017. "Rise of the Machines Redux: Education, Technological Transition and Long-Run Growth." US Naval Academy Department of Economics Departmental Working Paper 61.

Romer, Paul M. 1990. "Endogenous Technological Change." *Journal of Political Economy* 98 (5): S71–S102.

Sachs, Jeffrey D., and Laurence J. Kotlikoff. 2012. "Smart Machines and Long-Term Misery." NBER Working Paper 18629.

Trimborn, Timo, Karl-Josef Koch, and Thomas Steger. 2008. "Multidimensional Transitional Dynamics: A Simple Numerical Procedure." *Macroeconomic Dynamics* 12 (3): 301–19.

Zeira, Joseph. 1998. "Workers, Machines, and Economic Growth." *Quarterly Journal of Economics* 113 (4): 1091–1117.