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The Productivity J-Curve: How Intangibles Complement General Purpose Technologies†

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General purpose technologies (GPTs) like AI enable and require significant complementary investments. These investments are often intangible and poorly measured in national accounts. We develop a model that shows how this can lead to underestimation of productivity growth in a new GPTs early years and, later, when the benefits of intangible investments are harvested, productivity growth overestimation. We call this phenomenon the ***Productivity J-curve****. We apply our method to US data and find that adjusting for intangibles related to computer hardware and software yields a TFP level that is 15.9 percent higher than official measures by the end of 2017.* (*JEL* E22, E23, G31, L63, L86)

> *After I left academe in 2014, I joined the technical organization at iRobot. I quickly learned how challenging it is to build deliberative robotic systems exposed to millions of individual homes. In contrast, the research results presented in papers (including mine) were mostly linked to a handful of environments that served as a proof of concept.*
>
> Alexander Kleiner[1]

Robert Solow (1987) pointed out that "a technological revolution, a drastic change in our productive lives" had curiously been accompanied by "a slowing-down of productivity growth, not by a step up." His famous productivity paradox, that one "can see the computer age everywhere but in the productivity statistics," named a challenge for economists seeking to reconcile the emergence of exciting technological breakthroughs with tepid productivity growth (Brynjolfsson 1993).

Solow's Paradox is not unique. In this paper, we argue it is one example of a more general phenomenon resulting from the need for intangible investments in early stages of new general purpose technologies. General purpose technologies (GPTs) are "engines for growth." Specifically, they are pervasive, improve over time, and

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† Go to https://doi.org/10.1257/mac.20180386 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

[1] Kleiner, Alexander. 2018. "The Low-Cost Evolution of AI in Domestic Floor Cleaning Robots" *AI Magazine*, 39(2): 89.

lead to complementary innovation (Bresnahan and Trajtenberg 1995). However, along with installing more easily measured items like new types of physical equipment and structures, we emphasize that realizing their potential also requires large intangible investments and a fundamental rethinking of the organization of production itself. Firms must create new business processes, develop managerial experience, train workers, patch software, and build other intangibles. This raises productivity measurement issues because intangible investments are not readily tallied on a balance sheet or in the national accounts.

The presence and timing of this sort of intangible investment is one reason why Solow's Paradox could occur. When a new GPT emerges, there will be a period, possibly of considerable length, during which measurable resources are committed and measurable output forgone, of building new, unmeasured inputs that complement the GPT.[2] For example, the technologies driving the British industrial revolution led to "Engels' Pause," a half-century-long period of capital accumulation, industrial innovation, and wage stagnation (Allen 2009, Acemoglu and Robinson 2013). In the later GPT case of electrification, it took a generation for the nature of factory layouts to be reinvented in order to fully harness the new technology's benefits (David 1990). Solow highlighted a similar phenomenon roughly two decades into the IT era.

I. The Productivity J-Curve

We call the measurement aspect of this phenomenon the *Productivity J-curve*. As firms adopt a new GPT, total factor productivity growth will initially be underestimated because capital and labor are used to accumulate unmeasured intangible capital stocks. Later, measured productivity growth overestimates true productivity growth because the capital service flows from those hidden intangible stocks generate measurable output. The error in measured total factor productivity growth therefore follows a J-curve shape, initially dipping while the investment rate in unmeasured capital is larger than the investment rate in other types of capital, then rising as growing intangible stocks begin to contribute to measured production. In the long run, as intangible investments and capital stocks reach their steady-state growth rates, the return-adjusted value of the unmeasured intangible capital stock service flows (in expectation) approaches the value of the initial unmeasured investment. This means that some of the mismeasurement effects on productivity growth can persist even in the long run.

In our more general discussion of the current productivity paradox in the context of Artificial Intelligence (AI) in Brynjolfsson, Rock, and Syverson (2019), we explained the basic idea of the Productivity J-curve, building on earlier work by Yang and Brynjolfsson (2001). This paper formalizes and expands on this concept and provides a set of quantitative methods designed to measure the productivity effects of intangible investments. Namely, we augment a traditional growth accounting framework to include intangible capital, deriving an expression for productivity mismeasurement as a function of the growth rates, size, and shadow

[2] In Hornstein and Krusell (1996) and Greenwood and Yorukoglu (1997), the key intangibles are the skills and knowledge required to put new GPT-related capital to use, and these investments are made over a "learning" period.

values of intangible capital. Next, following Hall (2001 and 2004), Yang and Brynjolfsson (2001), and Hall (2006), we use a set of measures derived from stock market valuations to obtain measures of these intangible values. The basic idea of this approach is that market valuations reflect the value of intangibles even if they are otherwise hidden on firms' balance sheets. We then use these measures of implied intangibles to compute productivity growth mismeasurement associated with a four technologies: generic R&D investments, computer hardware, software, and more speculatively, AI.

II. Technology, Investment Theory, and Productivity Growth

Economic historians have emphasized the transformative effects of GPTs. We mentioned the work of David (1990), Allen (2009), and Acemoglu and Robinson (2013) above. Rosenberg and Trajtenberg (2004) identify the Corliss steam engine as an "icon of the Industrial Revolution," shifting population centers from rural to urban areas as water power was abandoned in favor of steam. Crafts (2004) explores the contribution of steam power to growth for the British economy during the Industrial Revolution. Lipsey, Carlaw, and Bekar (2006) offer a list of possible GPTs (including electrification, mass production, and the factory system) while relating those inventions to the presence of a productivity paradox. Bresnahan (2010) conducts a wide review of the GPT concept, making the point that the information and communication technologies (ICTs) of the modern era broadly constitute a GPT. Particularly relevant to our analysis is Helpman and Trajtenberg (1994), which notes how GPTs can generate alternating periods of investment and harvesting. Likewise, Jacobs and Nahuis (2002) suggest that GPTs can cause an initial productivity slowdown as high-skilled workers invest in knowledge instead of production.

An important motivation for our analysis is the most recent potential GPT: artificial intelligence (AI). AI, and in particular the subfield of AI called machine learning, is pervasive, improves over time, and spawns complementary innovation, thereby meeting Bresnahan and Trajtenberg's (1995) three canonical GPT criteria. Accordingly, after an implementation lag period, AI might significantly impact economic growth as other GPTs have (Aghion, Jones, and Jones 2017; Agrawal, McHale, and Oettl 2018; Cockburn, Henderson, and Stern 2018; Trajtenberg 2018; Brynjolfsson, Rock, and Syverson 2019). That said, the formal arguments presented here are applicable to other technologies and intangible capital accumulation more generally. Indeed, we empirically apply the approach to investments in R&D, computer hardware, and software in recent decades, generating a set of updated productivity series.

Our approach applies Tobin's q-theory of investment to infer productivity mismeasurement attributable to unmeasured intangible capital. We estimate the quantity of intangible investment using market value regressions where part of q is interpreted as reflecting intangible capital. Intuitively, when we observe a firm's market value rise by an amount greater than observed investment, we infer the difference as reflecting the value of intangible capital investments that were correlated with the tangible investment. We call these *intangible correlates*. Our framework also handles the case in which intangible capital is used to produce more intangible capital.

This approach has antecedents in the literature. Yang and Brynjolfsson (2001) note that combining q-theory of investment (Hayashi 1982, Wildasin 1984, Hayashi and Inoue 1991) and neoclassical growth accounting (Solow 1956; Solow 1957; Barro 1999; Corrado, Hulten, and Sichel 2009; Oliner and Sichel 2000; Oliner, Sichel, and Stiroh 2008) can deal simultaneously with the magnitudes of the intangible component of GPT-related investment and implementation lags. The recognition that q could reflect intangibles has led to numerous proposed updates to standard growth accounting frameworks and an emphasis in recent productivity studies on IT's role in productivity dynamics (Jorgenson and Stiroh 2000; Marrano, Haskel, and Wallis 2009; Corrado, Hulten, and Sichel 2009; McGrattan and Prescott 2010; Byrne, Fernald, and Reinsdorf 2016), and specifically in the ICT-as-GPT case discussed in Basu, Fernald, Oulton, and Srinivasan (2003).[3]

The broader notion that intangibles might have a substantial role and growing role in productivity and growth is not new. See, for example, Brynjolfsson and Hitt (2000); Hall (2000, 2001); Brynjolfsson, Hitt, and Yang (2002); McGrattan and Prescott (2010); Tambe, Hitt, and Brynjolfsson (2012); Saunders and Brynjolfsson (2016); and McGrattan (2017). Haskel and Westlake (2017) summarize and evaluate many of the relevant arguments.

The Productivity J-curve that we describe in this paper is related to, but distinct from, the trade balance J-curve of Magee (1973) and Rose and Yellen (1989).[4] Their J-curve describes how trade balances react over time to changes in real exchange rates.[5] The similarity between the two J-curves is that there is a change in the sign of derivatives of focal quantities with respect to time as time passes (trade balances in the earlier case, productivity in this one), reflecting the adjustment of production processes in response to an external shock. In Rose and Yellen, the shock comes from a large change in exchange rates. In our paper, the shock comes from a large technological innovation.

III. Growth Accounting in the Presence of Intangibles

Our setup builds on the approach of Yang and Brynjolfsson (2001). Assume the aggregate (economy- or industry-wide) production function is the product of Hicks-neutral total factor productivity A and a function $F(\cdot)$ that is weakly increasing and has constant returns to scale in inputs K and L (each potentially vectors). Further assume that markets are perfectly competitive. Then

$$Y = AF(K,L) \tag{1}$$

[3] We also note that the existence of significant intangible assets can explain the relatively poor historical performance of Tobin's Q (the ratio of a firm's market-to-book value) in explaining capital investment (Crouzet and Eberly 2018). Accounting for organizational investments, human capital, and business processes can strengthen the link between observed investment and asset prices (Hall 2000; McGrattan and Prescott 2001; Eisfeldt and Papanikolaou 2013, 2014; Peters and Taylor 2017; Kogan et al. 2017; Andrei, Mann, and Moyen 2018).

[4] We thank Larry Summers for suggesting how the dynamics we model are similar to the trade J-curve.

[5] Assuming export prices between countries are sticky, depreciation of a country's currency initially makes sticky-priced imports more expensive, which leads to more spending on imports. Later, consumption habits and production systems adjust so foreign import demand decreases.

where Y is output (and the numeraire), which can be either consumed or invested as capital.[6] With flexible capital and factor prices r and w equaling the aggregate marginal product values of capital and labor, we have the following (g denotes a growth rate):

$$(2)\quad g_Y \equiv \frac{dY}{Y} = \frac{AF_K dK + AF_L dL + F(K,L)dA}{Y} = \left(\frac{rK}{Y}\right)g_K + \left(\frac{wL}{Y}\right)g_L + g_A;$$

$$g_K \equiv \frac{dK}{K}; \quad g_L \equiv \frac{dL}{L}; \quad g_A \equiv \frac{dA}{A}.$$

Productivity growth is then

$$(3)\qquad g_A = g_Y - \left(\frac{rK}{Y}\right)g_K - \left(\frac{wL}{Y}\right)g_L.$$

This is the familiar Solow Residual. It is the growth in output not accounted for by the growth in capital or labor inputs, with each input weighted by its payment share of output. Under the assumptions of the model, the Solow Residual represents an improvement in productive efficiency, or more modestly a kind of "measure of our ignorance" about how producers convert inputs to outputs.[7] Equation (3) is the basis for traditional growth accounting.

Now suppose there are unmeasured intangible capital investments and capital service flows emanating from the resulting accumulating intangible stocks. Despite being unmeasured, these intangibles are true outputs when created as investment goods and, when put into place, inputs into the aggregate production function. In this case, the aggregate production becomes a combination of factor neutral productivity A^* (to denote its difference from A in the standard production function above) and a constant returns to scale function of tangible capital K, labor L, and intangible capital U.[8] Output now consists of both tangible output Y and intangible investment I_U, which has a price of ϕ relative to the numeraire, again with perfect competition in all markets. Therefore, using F^* to denote the production function that includes unmeasured intangible capital stocks, we have

$$(4)\qquad Y + \phi I_U = A^* F^*(K,U,L).$$

We can write growth in total factor productivity in this intangible-inclusive economy as

$$(5)\quad g_{A^*} = \left(\frac{Y}{Y+\phi I_U}\right)\left(g_Y - \left(\frac{rK}{Y}\right)g_K - \left(\frac{r_U U}{Y}\right)g_U - \left(\frac{wL}{Y}\right)g_L\right) + \left(\frac{\phi I_U}{Y+\phi I_U}\right)g_I$$

[6] $F(K,L)$ might, for example, take the Cobb-Douglas form $Y = AK^{\alpha}L^{1-\alpha}$, in which case $\alpha = \frac{rK}{Y} = 1 - \frac{wL}{Y}$.
[7] Abramovitz (1956).
[8] This factor-neutral productivity is once again in terms of measured output for equation (5) to hold, not the total output including unmeasured intangibles. The Appendix has detailed derivations.

where the factor price of intangible capital is r_U.[9] We have kept the factor prices for both types of capital and labor constant between F^* and F by assumption. In practice, these prices are often taken from empirical sources to calculate capital service flows.

Incorporating intangibles leads to two adjustments to the standard model. First, capital services from the stock of intangibles U are an input into production.[10] Their influence on measured productivity growth is readily apparent in the $(r_U U/Y) g_U$ term on the right-hand side of equation (5). The second difference is in the final term. Because output now includes intangible capital investment I_U, initial production of these intangibles shows up positively in productivity to the extent that they represent part of total output. Thus intangibles influence both the input and output parts of the growth accounting framework. Note that just like tangible investment goods, intangible investment output is produced by a combination of all three inputs: tangible capital, labor input, and intangible capital.[11]

Equation (5) may represent a more accurate picture of productivity growth, but its key additions of intangible capital inputs and outputs are difficult to measure in practice.

The difference between the Solow Residuals implied by these two measurement contexts elucidates the sources of productivity growth mismeasurement when intangibles exist but standard measurement techniques are applied. Let Δ denote (3) minus (5), the difference between the standard Solow Residual and true productivity growth in the presence of intangibles. Rearranging terms yields an expression that offers an intuitive decomposition of how intangibles lead to a difference between measured and actual productivity growth:

$$(6) \quad \Delta = g_A - g_{A^*}$$
$$= \left(\frac{\phi I_U}{Y + \phi I_U}\right)\left(g_Y - \left(\frac{rK}{Y}\right)g_K - \left(\frac{wL}{Y}\right)g_L - g_{I_U}\right) + \left(\frac{Y}{Y + \phi I_U}\right)\left(\frac{r_U U}{Y}\right)g_U.$$

The first term on the right-hand side of (6) is productivity mismeasurement because of the fact that the standard productivity growth measure does not count intangible investment goods as output when they are produced. That causes measured productivity growth to understate true productivity growth (i.e., makes Δ negative). The second term reflects an overstatement of true productivity due to the fact that the standard Solow Residual attributes outputs made by intangible inputs to productivity rather than those inputs. This term is weighted by measured output's share of total output. Whether Δ is positive or negative depends on the relative size of these two terms. Note that, as reflected in the first term, influences that would raise the standard Solow Residual will increase the overstatement of measured relative

[9] In this case, the Cobb-Douglas version of the production function is $Y + \phi I_U = A^* K^\alpha L^\beta U^{1-\alpha-\beta}$ and $\alpha = (rK/(Y + \phi I_U))$, $\beta = (wL/(Y + \phi I_U))$.

[10] We discuss intangibles U as a type of capital, but unmeasured labor service flows would be treated analogously.

[11] There is a theoretical case when only intangible capital is used to produce only intangible capital and no other outputs. In that situation, measured output and productivity reflect only the measurable component of the economy, but within that domain there is no mismeasurement.

to true productivity growth, a discrepancy that increases in unmeasured intangible investment's share of true output.

A related way to think about productivity growth mismeasurement in the context of intangibles is to recognize that there are four possible ways input service flows can yield output in the presence of unmeasured intangibles:

(i) measured inputs produce measured output

(ii) measured inputs produce unmeasured output

(iii) unmeasured inputs produce measured output

(iv) unmeasured inputs produce unmeasured output

The standard Solow Residual in equation (3) handles the first and the fourth cases (the fourth by ignoring it entirely). Equation (5) includes all four. Equation (6) explicitly describes the source of these differences.

We can express the difference between measured and true productivity growth as a function of the four possible ways inputs can yield output of both types. Define $Y_t^* = Y_t + \phi_t I_{U_t}$ and the ratio η_t to represent the proportion of Y_t^* represented by unmeasured intangible investment (subscript t denotes the time period).[12] Then,

$$(7) \qquad Y_t = (1-\eta_t)Y_t^*,$$

$$\eta_t = \frac{\phi_t I_{U_t}}{Y_t + \phi_t I_{U_t}} = \frac{Y_t^* - Y_t}{Y_t^*}.$$

In any given period t, the difference between the Solow Residuals implied by these different measurement contexts reveals another expression for the drivers of productivity growth mismeasurement:

$$(8) \qquad \Delta = \left(\frac{dY}{Y} - \frac{dY^*}{Y^*}\right) + \left(\left(\frac{rK}{Y^*}\right)\left(\frac{dK}{K}\right) - \left(\frac{rK}{(1-\eta_t)Y^*}\right)\left(\frac{dK}{K}\right)\right)$$
$$+ \left(\left(\frac{wL}{Y^*}\right)\left(\frac{dL}{L}\right) - \left(\frac{wL}{(1-\eta_t)Y^*}\right)\left(\frac{dL}{L}\right)\right) + \left(\frac{r_U U}{Y^*}\right)\left(\frac{dU}{U}\right).$$

Rearranging terms and combining with equation (7), we get:

$$(9) \quad \Delta = \underbrace{-\eta_t\left(\left(\frac{rK}{Y}\right)g_K + \left(\frac{wL}{Y}\right)g_L\right)}_{\substack{\text{Intangible Capital}\\ \text{Investment Produced}\\ \text{by Tangible Capital Inputs}\\ \text{and Labor}}} + \underbrace{(1-\eta_t)\left(\frac{r_U U}{Y}\right)g_U}_{\substack{\text{Contribution to}\\ \text{Measured Output}\\ \text{of Intangible Capital Stock}}} + \underbrace{g_{(1-\eta_t)}}_{\substack{\text{Growth in the}\\ \text{Measured Share of Output}}}.$$

[12] We thank Richard Rogerson for suggesting this formulation.

Similarly to the terms in (6), the first two components of (9) have opposite effects on the difference between measured and actual productivity growth. The first term is the unmeasured intangible capital investment output produced by tangible capital stock service flows and labor inputs. This causes measured productivity to understate true productivity. The second term is the measured final goods output produced by the unmeasured intangible capital stock service flows. It causes measured productivity to overstate true productivity. The last term is zero in the case that the share of unmeasured intangible output is constant. Should unmeasured output grow as a share of total true output, this third term will be negative. This implies that measured productivity growth will be otherwise less than true productivity growth, holding the first two difference terms fixed.[13]

Equation (9) captures measured inputs producing unmeasured output in its first term and unmeasured inputs producing measured output in its second term.[14] The main difference between Δ in (6) and Δ in (9) is whether unmeasured intangible output is first subtracted from the input side (as in equation (6)) or if it maintained as output (as in equation (9)) in calculating differences between adjusted and unadjusted Solow Residuals. These two formulations are equivalent.[15]

This framework does not mathematically imply a generalized pattern for the relative size of the two measurement effects over time, but the economics of the situation make certain dynamics likely. When a new intangible enters the economy (when a new GPT is invented, for example), it must first be produced. This intangible investment output is the first term in (9). At the same time, because the intangible is new, its stock and share of payments in the second term of (9) are likely to be small. Thus initially the first term is likely to be larger than the second, and measured productivity growth will understate true productivity growth. Later, however, as the stock of intangible capital continues to build, the second term grows and, eventually, begins to dominate. At this point, productivity is overstated because intangible inputs are not being "credited" with their contribution to output.[16] This pattern of productivity, initially understated and then rising to eventually being overstated, creates the *Productivity J-curve*. In the long run, the capital service flows from unmeasured intangible capital stocks will in expectation be equal to the return-adjusted present value of other inputs used to create the capital stock in the first place. Thus, in the very long run, productivity levels will be mismeasured less and less even if intangibles remain significant. Over an infinite time horizon, the productivity-level contribution of unmeasured service flows will be equivalent to

[13] The quantitative effect of a changing unmeasured output share on productivity mismeasurement could be substantial, perhaps in the tenths of a percent per year, under certain circumstances. See the Appendix for details.

[14] There is a similar description in equation (5) of Brynjolfsson, Rock, and Syverson (2017), where the mismeasured components of investment and capital stock work against each other to generate the difference between measured and actual productivity growth.

[15] See Appendix.

[16] The path of mismeasurement in growth is contingent on the relative size of these effects. In very early stages of a new technology, it is possible that the growth rate of unmeasured inputs in the extreme might exceed the growth rate of unmeasured investment. The relative effects of unmeasured investment goods output and unmeasured input are neither dominated by unmeasured investment output nor unmeasured capital input as a rule. Clearly, however, the absolute size of the effects are not likely to be meaningful when unmeasured components are a very small component of output.

the unmeasured investment output. Figure 1 shows a stylized version of the resulting J-curve.[17]

However, another measurement problem arises in typical income accounting practice. The analysis above assumes inputs' income shares are separately observable. However, measuring payments to capital (even observable capital) is difficult. As a result, national accountants and researchers often leverage the constant returns to scale assumption to measure capital's share as the residual of labor's share (i.e., $rK/Y = 1 - (wL/Y)$) when constructing the standard Solow Residual. This is incorrect if intangibles are present. Instead, still assuming constant returns to scale, $rK/Y = 1 - \left(wL/Y\right) - \left(r_U U/Y\right) + \left(\phi I_U/Y\right)$. Plugging these respective expressions into (3) and (5) and recomputing Δ as in (6) gives

$$\begin{aligned} (10) \qquad \Delta &= \left(\frac{Y}{Y+\phi I_U}\right)\left(\left(\frac{r_U U}{Y}\right)(g_U - g_K) - \left(\frac{\phi I_U}{Y}\right)(g_{I_U} - g_K)\right) \\ &\quad + \left(\frac{\phi I_U}{Y+\phi I_U}\right)\left(g_Y - \left(\frac{wL}{Y}\right)g_L - \left(1 - \frac{wL}{Y}\right)g_K\right) \\ &= (1-\eta)\left(\frac{r_U U}{Y}\right)(g_U - g_K) + \eta\left(g_A - g_{I_U}\right), \end{aligned}$$

and the total growth in productivity equation is

$$\begin{aligned} (11) \qquad g_{A^*} = \left(\frac{Y}{Y+\phi I_U}\right)\Bigg[g_Y - \left(\frac{wL}{Y}\right)g_L - \left(1 - \frac{wL}{Y}\right)g_K - \left(\frac{r_U U}{Y}\right)(g_U - g_K) \\ + \left(\frac{\phi I_U}{Y}\right)(g_{I_U} - g_K)\Bigg]. \end{aligned}$$

Equations (10) and (11) describe discrepancies using Y as *measured* output and g_Y as *measured* output growth. With this setup it is easier to plug in estimated values for measured quantities of outputs and inputs to retrieve g_{A^*}, the true productivity growth under the stated assumptions. Now, because of the error introduced by substituting for capital's share of income, productivity mismeasurement still depends on the growth rates of intangible investments and stocks, but instead of their absolute values, it depends on their growth rates *relative* to the growth of observable capital.

The intuition for relative growth rates mattering in (10) is as follows. Because using the residual of labor's share mismeasures observable capital's share, capital's contribution to output growth will be misstated. Early on, when intangible investment outputs are likely larger than the contribution of intangible inputs, the residual share will be smaller than the true share. This results in an understatement of tangible capital's growth effect on output growth, with the gap instead attributed to productivity growth. This productivity overstatement effect will counteract the direct productivity understatement effect of missing intangible investment discussed

[17] The logic presented here applies principally to productivity growth rates, but we present in the Appendix a version that applies to productivity levels as well. We have assumed "smooth" investment and capital growth in early periods for illustrative purposes. Toy economy J-curve parameters and spreadsheet file available at http://drock.mit.edu/research.

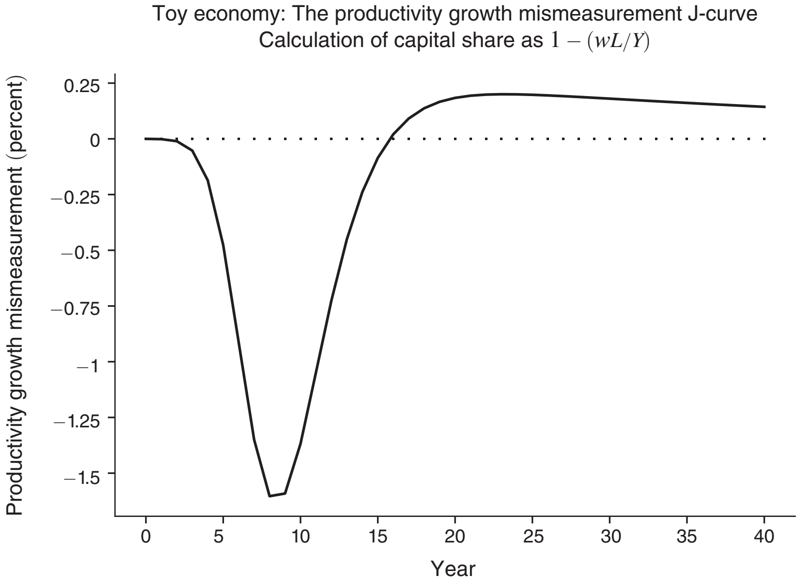

Figure 1. Productivity Growth Mismeasurement

above. The net unmeasured investment effect depends on the relative growth rates of intangible investment output and observable capital. The larger the latter, then the larger is the countervailing "share-misattribution" effect, which will, depending on its size, either mitigate productivity understatement or actually cause net overstatement of productivity growth. Part of the overall mismeasurement of productivity growth will come from the components making up the ordinary Solow Residual as that productivity is allocated to unmeasured output. This is one of the cases described earlier: measured growth being allocated to unmeasured output.

Later, when intangibles become more important as inputs and this effect starts to dominate missing intangible outputs, observable capital's share will be overstated because some of the output used to pay intangible inputs will be mistakenly attributed to observable capital. This overstates capital's contribution to output growth, which in isolation is a productivity growth understatement. Analogously to the intangible output effect described in the prior paragraph, this counteracts the direct productivity overstatement effect of intangible inputs. Net measurement depends on the relative growth rate of the intangible and observable capital stocks. In the special case that the growth rate of the unmeasured intangible stock is equal to both the growth rate of the measured capital stock and the growth rate of unmeasured intangible investment, then the standard growth accounting method produces the correct productivity growth estimate. Figure 2 shows the path of net measured investment shares of measured output for research and development, computer hardware, and computer software investments. These three types of capital will be central to our empirical investigations below.[18]

[18] We calculate these as the measured net investment from the BEA net fixed asset investment series in each type of capital, divided by measured contemporaneous GDP.

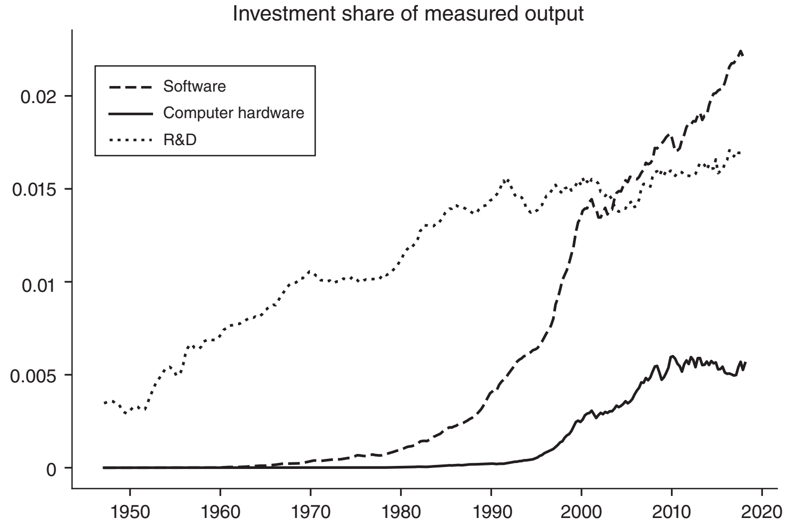

Figure 2. Measured Investment Shares of Measured Output (GDP) for Selected Capital Varieties

In (10) and (11) the differences in output growth (and productivity growth measurement) depend on the growth rates of the intangible input stock and intangible investment output. The first component is the unmeasured intangible output share multiplied by the difference in growth rates between intangible and tangible capital (third term in the brackets of the right side of equation (11)). If the intangible stock grows faster than the tangible stock, actual productivity will be lower than measured productivity. Typically, however, we do not know the intangible output share nor do we have an independent measure of intangible capital. However, if tangible and intangible investments must be made together, the growth rate of tangible capital stocks and correlated intangible capital stocks will be similar (McGrattan 2017 offers an argument for this). This would allow adjustment of productivity growth estimates with only an estimate of the relative quantities of the two capital types. We therefore make the simplifying assumption that $g_U = g_K$.[19] The new aggregate capital share (combining U and K) is defined as

$$(12) \qquad \frac{r_{total} K_{total}}{Y} = 1 - \frac{wL}{Y} + \left(\frac{\phi I_U}{Y}\right).$$

With factor neutral productivity, growth is then properly decomposed as follows:

$$(13) \qquad g_Y = \left(\frac{wL}{Y}\right) g_L + \left(1 - \frac{wL}{Y}\right) g_K - \left(\frac{\phi I_U}{Y}\right)(g_I - g_K) + \left(\frac{Y + \phi I_U}{Y}\right) g_{A^*}.$$

[19] Some might argue that intangibles have become an increasingly important component of capital investment. Historically, however, there has always been substantial unmeasured intangible capital in the economy. For example, the widespread increase of literacy rates accompanied a proliferation of intangible know-how. Likewise, the business process redesign associated with the dynamo, as described in David (1990), is another example of intangible capital investment going unmeasured. Writ large, this type of capital has always been an important production input.

Given the aforementioned assumptions, this leads to a convenient formulation for the adjusted productivity growth value as

$$g_{A^*} = (1 - \eta)(g_A) + \eta(g_{I_U} - g_K). \tag{14}$$

The intangible-adjusted productivity growth rate is a convex combination of the standard productivity growth rate and the difference between the growth rates of unmeasured investment and overall capital, with the unmeasured investment share of true output as the weight on the growth rate differential. If this growth rate differential is zero or not too positive, adjusted productivity growth will be less than standard measured productivity growth. If capital inputs grow fast enough relative to intangible investment, however, adjusted productivity growth will be larger than the standard measured value. We refer to the unmeasured investment share of output-weighted investment growth and capital stock growth components respectively as the "investment" and "capital stock" effects.

Again, equations (12) and (13) describe discrepancies using Y as measured output and g_Y as measured output growth. The ideal approach would be to measure each input quantity and its associated factor cost with high fidelity rather than assume that observable factor shares sum to one. But absent sufficient data to follow that route, the difference between the investment growth rate and the capital stock growth rate informs the extent to which there is a drag (or acceleration) of productivity growth. The third term of the right side of equation (13) is typically unmeasured. If it is negative (positive), productivity growth will have to be larger (smaller) than under the standard growth accounting conventions to compensate for the negative (positive) addition. For example, if investment is positive, unmeasured investment unit prices are larger than measured investment prices, and investment is growing faster than capital stocks, then true productivity growth will be larger than measured productivity growth.

The intuition behind needing to subtract the overall growth rate of capital is that part of this effect would otherwise be captured by overestimating capital's share of output. The third term of (13) serves two functions. First, it purges capital's share of output of the hidden investment share introduced in the standard approach by treating capital's share as the residual to labor's share. This subtracted capital growth rate need not correspond to the specific type of capital that has gone unmeasured. Faster growth of intangible capital relative to tangible capital nevertheless introduces an overestimation of productivity growth into standard growth accounting methods, going against the direction of the investment growth effects.

The type of capital growth differenced from the investment growth rate must correspond to the type that is incorrectly attributed factor payments from unmeasured capital service flows. Most growth accounting approaches adjust the capital share term to make factor shares add up to one. Whenever a factor share is estimated as the leftover share of output, it will partially reflect payments to intangibles. The third term of equation (13) differences this out. Mechanically this also means that an increasing output share of capital (both measured and unmeasured) will be taken out of the labor share of output.[20] If measured capital growth and labor growth are constant, but the unmeasured intangible share of output is increasing, labor's share will decline. For measurement purposes, capital services growth need not be differenced out if all output shares are measured directly by dividing service flow values by output (see, for example, Barkai and Benzell 2018).

The third term of equation (13) also separates the production of unmeasured capital output from productivity growth. Equation (13) now accounts for the unmeasured intangible capital stock via the increased capital share of output and unmeasured intangible capital investment through the third term on the right side. Productivity growth g_{A^*} is therefore more accurately measured.

In the short run, the use of different types of resources to produce measured and unmeasured outputs can influence the extent of productivity growth mismeasurement. Unmeasured intangible capital stocks might produce even more unmeasured intangible assets, in which case the hidden output and hidden input effects can offset. If intangible capital production accelerates and uses increasingly large quantities of measured capital and labor services, the J-curve effects will be more pronounced.[21] This will also occur if the quantity of intangible investments correlated with each unit of tangible investment grows over time.

IV. A Method to Measure Intangibles

To adjust measured productivity growth for intangibles in practice, we of course need to estimate intangible investments. In terms of the equation (13), we need a measure of the intangible investment value ϕI_U.

If z is defined as the price of measured investment output, the investment share of output zI/Y and the growth rate of investment g_{I_U} can be taken from observable investment and output series. One way of estimating intangible investments is to assume that each unit of measured investment is the observable component of a combined investment unit that also includes intangibles. In other words, assume that there are ϕ units of unmeasured intangible investment per z units of measured investment (the "intangible correlates" mentioned above).

We apply a method to estimate these intangible correlates. The Appendix details its microfoundations, but the basic logic is to use the q-theory of investment to recover the amount of intangibles correlated with tangible capital from the relationship between firms' market valuations and their stocks of measured capital. $(\phi + z)/z = \lambda/z$ represents the ratio of total asset value to book asset value; it is a measure of Tobin's q for the aggregate economy (Tobin 1969, Hayashi 1982, Hayashi and Inoue 1991). Following the setup in the Appendix, the market value at

[20] McGrattan (2017) investigates in detail the ties between intangible capital output and labor input measurement.

[21] As mentioned earlier, there is also a degenerate scenario in which firms shift toward focusing on intangible output production using intangible assets. In this case, the standard productivity measurement apparatus loses its value.

$t = 0$ for firms where each unit of observable capital variety K_j coincides with ϕ_j more units of unmeasured intangible capital is

$$V(0) = \sum_{j=1}^{J} \lambda_j(0) K_j(0). \tag{15}$$

That is, the value of the firm $V(0)$ is equal to the sum of the value of the asset varieties, each priced at their shadow values inclusive of unmeasured intangible correlate assets λ_j. If the market prices firms correctly, the equilibrium valuations will reflect all varieties of productive capital independent of measurement. Equation (15) implies that the shadow prices can be recovered by taking advantage of omitted variable bias in a regression of market value on observable asset types.

If, for example, measured R&D is correlated with \$1 of additional intangible investment, a regression of market value on the dollar value of R&D capital across firms will return a coefficient of 2. That is, $\lambda/z = q = 2$. (This is roughly what we find below.) This q in excess of 1 captures the unmeasured intangible correlate assets at the firm. In practice it might also reflect any capitalized investment adjustment costs associated with observable investments. These too can also be considered a type of intangible asset, as replicating \$1 of the (installed) observable R&D capital stock would also require adjustment costs to be capitalized. Either the intangible correlate or adjustment cost story is consistent with the idea that applying R&D requires complementary intangible investments to reorganize production.

Now consider how this logic relates to the influence of GPTs on intangible investments and productivity measurement. Suppose firms adopting a new GPT must invest proportionately in two assets: computer equipment and firm-specific GPT specialist training (e.g., training AI engineers). Further assume that asset and securities markets efficiently price firms inclusive of both measured and unmeasured assets. For a firm with a measurable quantity of tangible computer equipment, the market price for the computer equipment investment will exceed the replacement cost of computer equipment by the value of the complementary training. The training is not capitalized on the firm's formal balance sheet, yet the financial market must also value the future service flows from training (and any other correlated business process innovations) if no arbitrage conditions are to hold. The market value premium over book value implies a Tobin's q above unity; the firm's value is higher than the simple replacement cost of its observed assets. If we can measure q, we can infer the amount of intangibles and adjust measured productivity growth using the framework in the prior section.[22]

Note that we are not investigating the question of whether unmeasured intangibles cause market value here. Instead, we are looking for descriptive price coefficients of the market value and intangible investment equilibrium as a starting point

[22] There are a couple of caveats to our approach. First, our method requires intangibles have a nonzero correlation with measured capital. If any intangible investments are completely uncorrelated with measurable assets, we will not be able to measure them. Second, using firm-level data to infer the aggregate ratio of intangible to tangible capital requires an assumption about aggregation that may be problematic in some circumstances (Houthakker 1955, Basu and Fernald 1997). Those issues noted, we conduct our exercises below using multiple values of this ratio to allow readers to gauge for themselves how the particular value influences implied productivity mismeasurement.

for valuing the intangible investment in the economy. This brings in another set of concerns about whether $\lambda/z = (\phi + z)/z$ (that is, the q value) for a particular capital variety should represent an estimated *average* valuation or an estimated *marginal* valuation over all vintages of unmeasured intangible capital. The answer lies in matching the timing of the growth accounting pricing to the timing of the observed pricing of investment quantities and capital service flows. If the presence of measured tangible assets indicates the presence of unmeasured intangible assets, and both are valued by investors, then firms' valuations net of their tangible assets can help account for intangible capital (Hall 2001). We aim to measure on an annual basis the total value of the new unmeasured intangible investment output and pair it with the value of the unmeasured intangible capital service flow through the year. For this purpose, coefficients should reflect the average price across firms of unmeasured intangible assets by year when possible. A firm-level regression of market value on measurable capital types that are expected to have strong correlation with hidden intangible assets can quantify that intangible shadow value.

V. Deploying the Framework: R&D, Software, and Computer Hardware Investment

We begin by exploring whether technology-related intangible investments in recent decades have created J-curve dynamics.

Specifically, we estimate the per-unit magnitudes of intangible capital investment that coincide with observable investments in R&D, computer hardware, and software capital. We then use those values to adjust total factor productivity estimates using our framework above and explore the adjusted series to see if substantial J-curve effects exist for those capital types. To estimate the magnitude of intangible investments, we use the approach described above for obtaining intangible capital shadow values by comparing firms' observable investments to their market capitalization. We use these to build up time series estimates of the distinct intangible stocks correlated with R&D, software, and hardware investments over 1961–2017. We embed these intangible shadow values for all three capital varieties in the adjustment method described by equations (13) and (15) since this most directly applies to the data available to researchers.

We obtain our productivity baselines, net capital stocks for measured capital varieties including computer hardware and software, and investments by these capital varieties from Fernald (2014) quarterly series on total factor productivity (TFP), extended through 2017 (Bureau of Economic Analysis 2018).[23] We take estimates of the total stock of R&D capital and the total stock of capitalized selling, general, and administrative (SG&A) expense from Peters and Taylor (2017). We extend these measures through 2017 as well, using Wharton Research Data Services data and following the guidelines in their paper (the Peters and Taylor Total Q series) (WRDS 2019). This data is merged into Compustat-CapitalIQ firm-level data to construct a panel from 1961–2017 of market values, book values, R&D capital, "organizational" capital (the capitalized SG&A expenditure), and other identifiers

[23] Capital stock estimates for these series are also available from the US Bureau of Economic Analysis (BEA).

of all publicly held companies in the United States (S&P 2019).[24] We define industry by four-digit NAICS code.

R&D capital provides a useful context for understanding Productivity J-curve dynamics for a few reasons. Corporate research leads to the development of new technologies that diffuse over time, and there has been a steady flow of investment into R&D for decades. Further, the link between R&D investment and market value is well established (Hall 1993 and 2006). Because investment in R&D has persisted over the long term, we are more likely to find investment in R&D at nearly steady-state levels. This implies that the intangible-related challenges for productivity estimation coming from R&D are likely to be minimal at present. In fact, that is exactly what we find below. (Recall that as the growth rates of intangible investment and stocks converge, productivity mismeasurement falls to zero even when intangibles are present.)

In contrast, large investments in software and computer hardware are a more recent phenomenon in which firm behavior might not yet have entirely matured, so J-curve dynamics may affect productivity estimates. We find evidence of this as well.

The first step in estimating the productivity mismeasurement effect of intangible correlates is estimating $\lambda_j(t)$ as described above. We begin with R&D and capitalized SG&A stock measures from Peters and Taylor (2017), which follow a perpetual inventory approach to build stocks of these assets out of the expense measures. We then use these stocks in a market value regression of the style in Hall (1993) and Brynjolfsson, Hitt, and Yang (2002).

Specifically, the market value of firm i in industry j at time t is

$$MarketValue_{ijt} = \beta_0 + \beta_1 TotalAssets_{it} + \beta_2 R\&D_{it} + \eta_{jt} + \epsilon_{it}.$$

The coefficient on R&D picks up the ratio of dollars of market value created per unit of the firm's R&D stock in a given year. This, which we refer to as the *intangible multiplier*, is the ratio λ/z from our analysis above. We estimate specifications both including and excluding capitalized SG&A and industry-year fixed effects. The results are in Table 1.

The coefficients on total assets are very close to 1. In other words, each \$1 of property, plant, and equipment is valued by the market at \$1, as would be expected in an efficient financial market. On the other hand, estimated coefficients for R&D are significantly higher than 1. Even after including firm and year fixed effects, the point estimate remains above 2. Including capitalized SG&A does decrease the R&D coefficients, though in all cases they remain significantly larger than 1. Furthermore, capitalized SG&A itself has point estimates greater than 1 (though not always significantly so). Thus, these models suggest that on average each \$1 of capitalized R&D is correlated with intangible capital valued at around \$1 or, depending on if one interprets capitalized SG&A as being a similar type of observable capital to capitalized R&D, perhaps as much as \$2.50 $(= 1.753 + 1.755 - 1)$

[24] See Wharton Research Data Services for more information: http://www.whartonwrds.com/wp-content/uploads/2016/07/Peters-and-Taylor-Total-Q.pdf.

Table 1—Market Value Regressions on R&D and SG&A Stocks

Market value regressions (1962–2017)	Basic R&D (1)	Basic R&D and SG&A (2)	Industry-year FE: R&D (3)	Industry-year FE: R&D and SG&A (4)	Firm and year FE: R&D (5)	Firm and year FE: R&D and SG&A (6)
Total assets	1.016	0.998	1.015	0.999	1.013	0.997
	(0.002)	(0.002)	(0.009)	(0.011)	(0.007)	(0.011)
R&D stock	2.730	1.753	2.841	1.953	2.161	1.509
	(0.105)	(0.097)	(0.479)	(0.399)	(0.297)	(0.278)
SG&A stock		1.755		1.657		1.453
		(0.102)		(0.399)		(0.374)
Constant	656.8	458.7				
	(14.32)	(18.06)				
Firm-year observations	268,687	268,687	266,795	266,795	267,683	267,683
R^2	0.987	0.988	0.989	0.989	0.993	0.993
Industry-year fixed effects	No	No	Yes	Yes	No	No
Firm and year fixed effects	No	No	No	No	Yes	Yes

Notes: Total assets are the total assets on the firm's balance sheet, Industry is the four-digit NAICS code. Market value is the sum of the book value of debt, preferred stock, and the end-of-year equity share price multiplied by common shares outstanding. Specifications 5 and 6 include firm and year fixed effects, but not firm-year fixed effects. Standard errors are in parentheses (robust for 1 and 2, clustered by industry in 3 and 4, clustered by firm in 5 and 6).

of intangibles. These intangible correlates show up both in the cross section and within the firm over time.

To examine the stability of the relationships between observables and intangible correlates, we also estimate our market value regression while allowing the coefficients on observable capital types to vary by year. (We include industry fixed effects in this specification.) Figure 3 plots the time series of R&D coefficient estimates for that specification.[25] The year-by-year regressions reveal substantial variation in the shadow value of R&D-related intangible assets. To allow such dynamics to potentially influence productivity measurement, we use this set of estimates to compute the implied productivity growth adjustments for our model. Figure 4 shows the same coefficient estimates for total assets, which are considerably lower in comparison (note the vertical scale is an order of magnitude smaller than Figure 3).

Using the yearly estimates of the amount of intangible correlates per unit of R&D investment, we adjust standard productivity growth Solow Residuals to include the missing intangible outputs and inputs. Figure 5 shows the time series of total factor productivity (TFP) growth (five-year moving averages), both as measured in Fernald's data and adjusting for unmeasured R&D-related intangibles. Figure 6 shows the effects in level terms, obtained by integrating the growth rates.

The unadjusted series differs very little from the net adjusted series. The reason is that, as mentioned above, R&D capital investment rates have been relatively steady over the observation period, roughly canceling out the countervailing influences of

[25] The full table of coefficients is available in the online Appendix. Available: http://drock.mit.edu/Research.

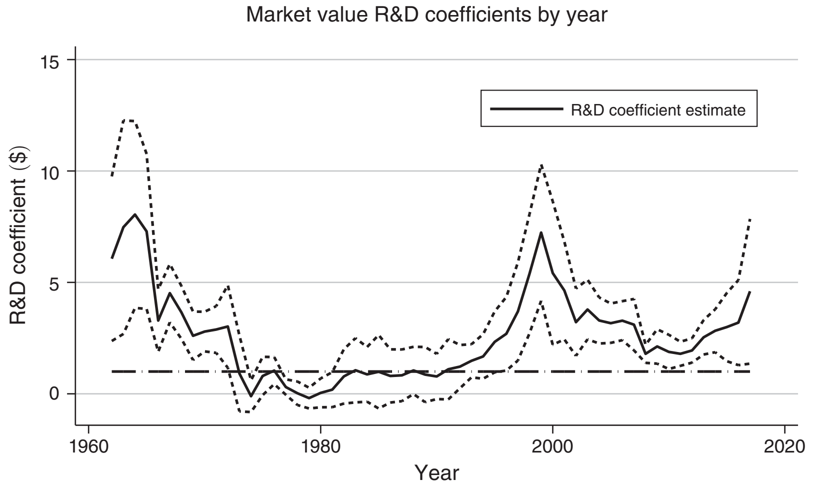

Figure 3. R&D Market Value Year-by-Year Regression Coefficients 1962–2017

Notes: Coefficients estimates come from regressions within year of market value on total assets, capitalized R&D stocks, capitalized SG&A stocks, and 4-digit NAICS fixed effects. Dotted lines represent the 95 percent confidence interval with standard errors clustered by industry.

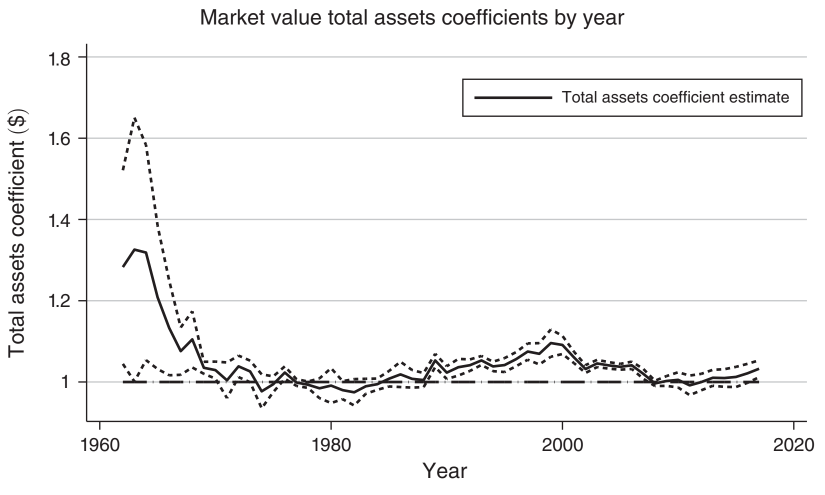

Figure 4. Total Asset Market Value Regression Coefficients 1962–2017

Notes: Coefficients estimates come from regressions within year of market value on total assets, capitalized R&D stocks, capitalized SG&A stocks, and 4-digit NAICS fixed effects. Dotted lines represent the 95 percent confidence interval with standard errors clustered by industry.

intangible outputs and intangible inputs. This is made clear by the dotted and lines in Figure 6, which isolate the influence of the twin effects of intangible investment growth and misattribution of intangible capital input growth. The higher dotted line reflects the upward adjustment to productivity due to uncounted outputs tied to

Figure 5. R&D-Related Intangible Capital-Adjusted Annual Total Factor Productivity Growth

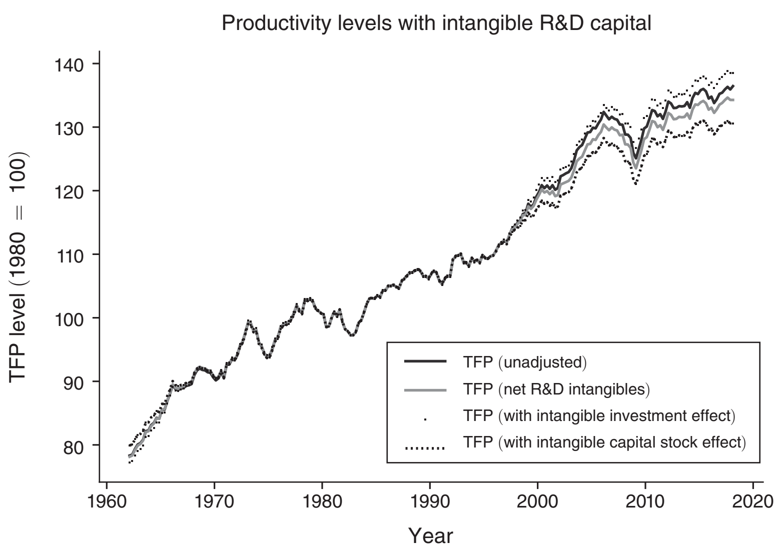

Figure 6. R&D-Related Intangible Capital-Adjusted Total Factor Productivity Levels

intangible investment. The lower dotted line shows the nearly equal-sized downward adjustment to measured productivity due to the failure to measure intangible capital input service flows. The net adjusted productivity level is 1.6 percent lower than the measured series as of the end of 2017.

Although the net measurement effects of R&D-related intangibles are negligible, the same is not true for software and computer investments. We do not have similar firm-level data on IT capital stocks and investment to run the market value regression above for IT, so we conduct the analysis using a range of plausible values for the intangible multiplier λ/z. Here, we are guided by the Brynjolfsson, Hitt, and Yang (2002), who estimate that each unit of observable software and computer hardware is associated with roughly \$12 (standard error \$4) of firm market value, and Saunders and Brynjolfsson (2016), who found similar values. Hence we will employ a value of λ/z of 10, but we will also explore how our estimates change for values of 5, 3, and 2.

The calculations imply that, in contrast to the adjustment for R&D-related intangibles, the Productivity J-curves for both computer hardware and software capital (we separately analyze each) have yet to reach positive territory in terms of levels.

Figures 7 and 8, analogously to Figures 5 and 6 for R&D-related intangibles, show the analysis for computer hardware-related intangibles. They plot adjusted and measured TFP growth and levels assuming $\lambda/z = 10$ for each dollar of hardware investment. The divergence between measured and corrected TFP becomes most noticeable in the 1990s. Figure 8 breaks out the implied TFP level as measured (black), adjusted for intangibles (grey), isolating only the missing intangible inputs effect (lower dotted line), and isolating only the missing intangible outputs effect (higher dotted line). To show the influence of our assumption about the quantity of intangibles correlated with each \$1 of observable hardware capital, Figure 9 compares the adjusted series for an intangibles value of \$10, \$5, \$3, \$2, and \$1 (unadjusted). These effect differences are weighted by the unmeasured intangible share of total output following the difference derivations in the previous section and the Appendix.

We see that the accumulated mismeasurement due to hardware-correlated intangibles is noticeable but modest. Adjusted TFP (assuming $\lambda/z = 10$) at the end of 2016 is 3.2 percent higher than the measured series. Figures 10 and 11 show the magnitudes of the deviations in TFP growth and levels between the measured and corrected series (i.e., the vertical distances between the adjusted and measured series in Figures 7 and 8) for different levels of the intangible multiplier. Interestingly, the recent slowdown in the rate of hardware investment has actually caused a small *overstatement* of productivity growth for parts of the past two decades. As a result, adjusted TFP has moved back toward measured TFP in levels. The reversal started following the dot-com bust, then reverted as computer hardware investment rebounded in the following years, and then reversed again at the start of the Great Recession. The growth overestimate was about 0.3 percent at the end of our sample. In level terms, productivity understatement has stabilized.

We move on to software, which has the least mature J-curve of the three capital varieties we investigate in this section. Software investment has been growing faster than overall capital investment, and its level is sufficiently large to suggest that it might have a substantial mismeasurement effect. Figures 12 and 13 plot the five-year moving average of annualized quarterly growth rates and levels of measured TFP and software-intangible-adjusted TFP. The differences between measured and corrected estimates are starkly larger than those arising from R&D.

The J-curve dynamics of software investment began in the 1990s and have not waned since. If we assume an intangible multiplier of 10—somewhat lower than

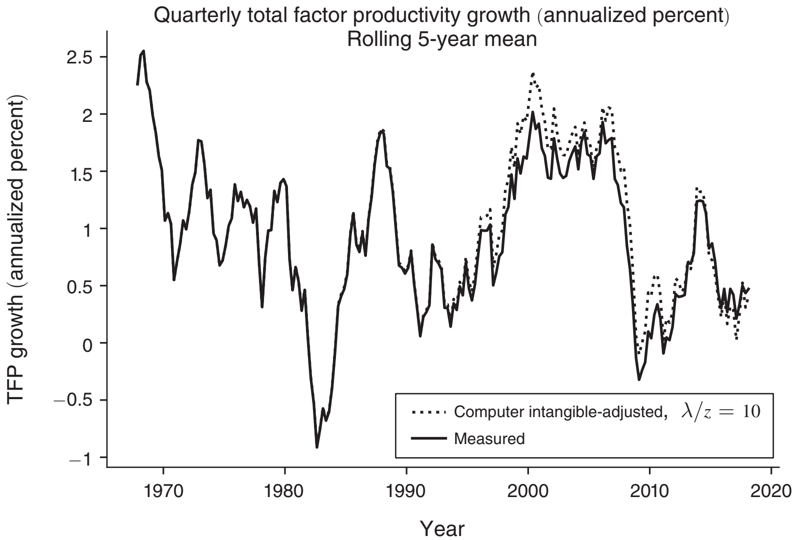

FIGURE 7. COMPUTER HARDWARE-RELATED INTANGIBLE CAPITAL-ADJUSTED ANNUAL TFP GROWTH RATES

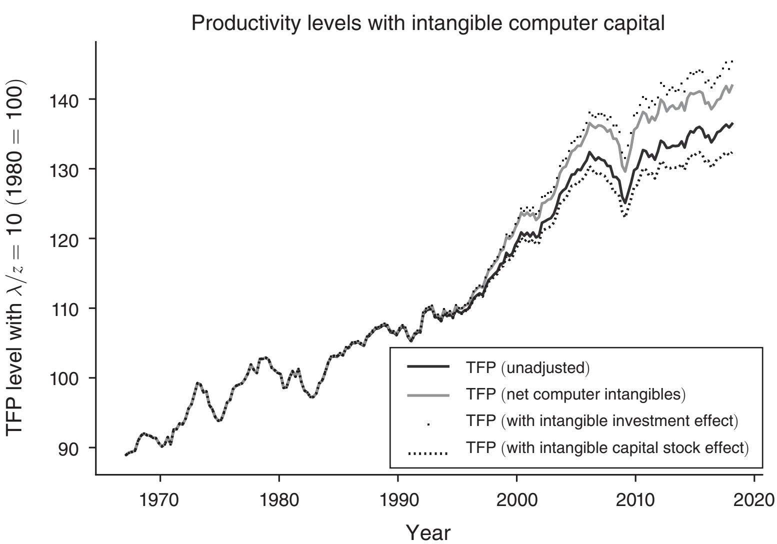

FIGURE 8. COMPUTER HARDWARE-RELATED INTANGIBLE CAPITAL-ADJUSTED TFP LEVELS

the levels estimated in Brynjolfsson, Hitt, and Yang (2002) and Rock (2019)—then the net adjusted TFP level is over 12.4 percent higher than measured TFP at the beginning of 2017. Figure 14 shows the productivity-level adjustments for more conservative intangible multipliers. Even for lower levels of the multiplier, the productivity-level differences are notable and growing.

The growing understatement of productivity due to software-related intangibles reflects the still-growing rate of software investment. Aside from brief periods

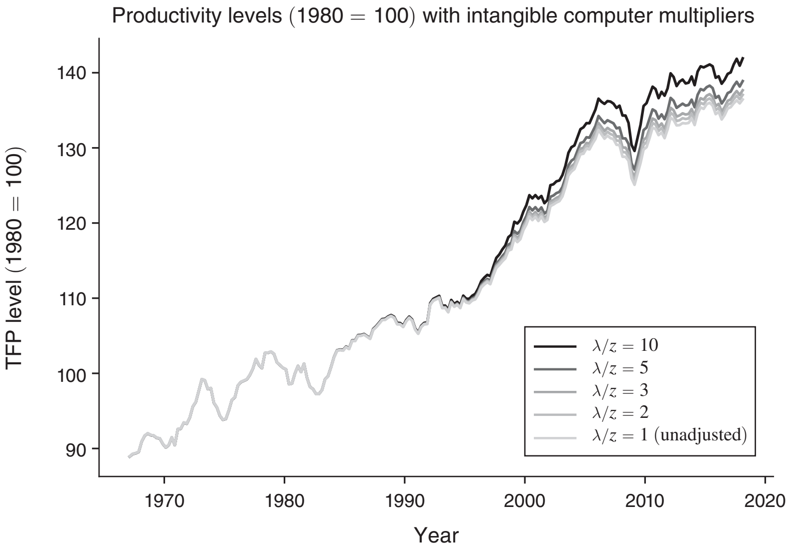

Figure 9. TFP Levels Adjusted Assuming Different Hardware Intangible Multipliers

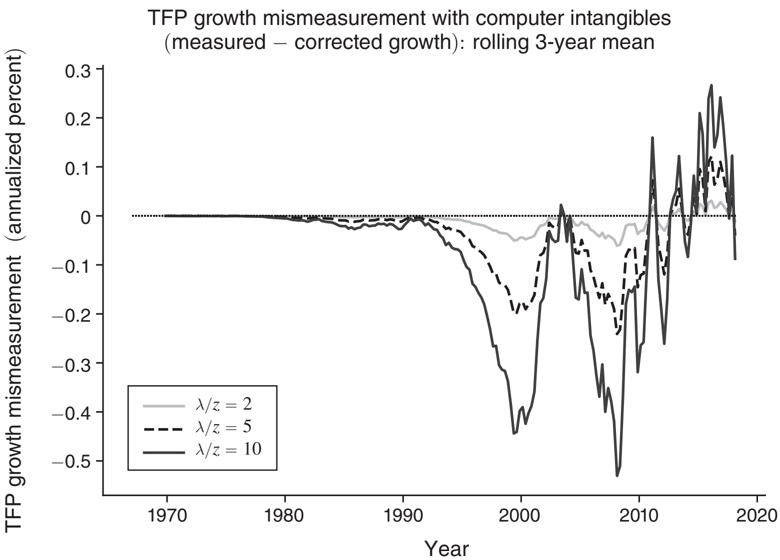

Figure 10. Hardware-Related Annual TFP Growth Mismeasurement

following the dot-com bust and the financial crisis, investment in software has typically grown significantly faster than other investments. As a result, software-related intangible investment rates are far from steady state. Our J-curve model shows that when the investment growth rate dominates the growth of the stock of intangible inputs, productivity growth is understated. Since 2010, when the productivity growth mismeasurement effect was near zero, average annualized quarterly productivity growth underestimation increased to 0.30 percent by the end of 2016. The

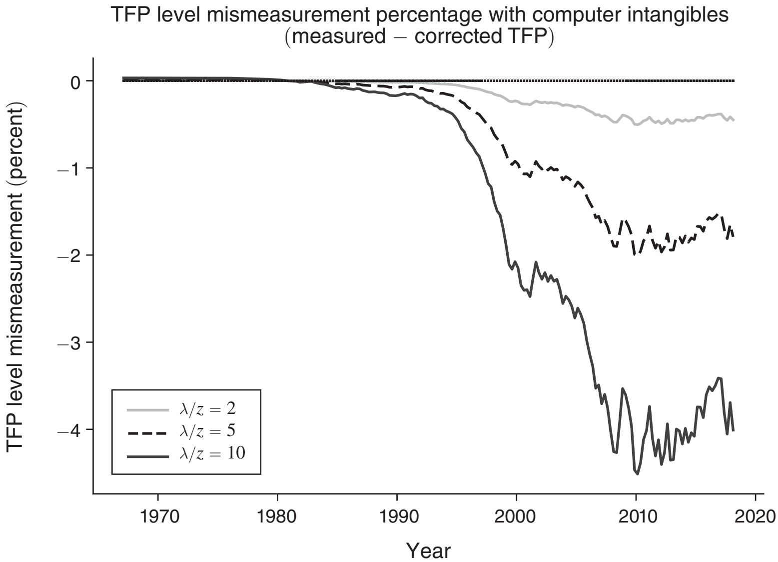

Figure 11. Hardware-Related TFP Level Mismeasurement

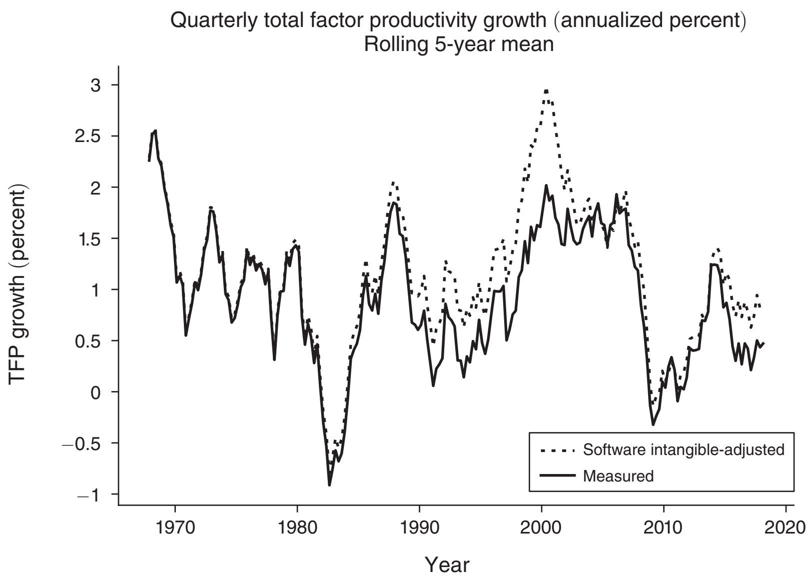

Figure 12. Software-Related Intangible Capital-Adjusted Annual TFP Growth

implied understatement was even larger at the end of the 1990s, where the three-year rolling mean of the understatement of measured productivity growth relative to software-adjusted productivity growth was as much as approximately 1.25 percent. Figures 15 and 16 show the respective mismeasurements of TFP growth and levels for software-related intangible capital outputs for intangible multipliers of 2, 5, and

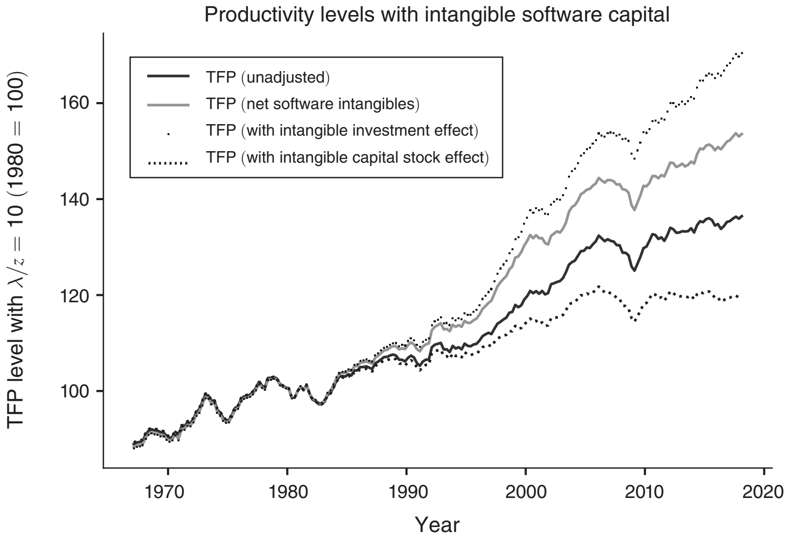

Figure 13. Software-Related Intangible Capital-Adjusted TFP Levels

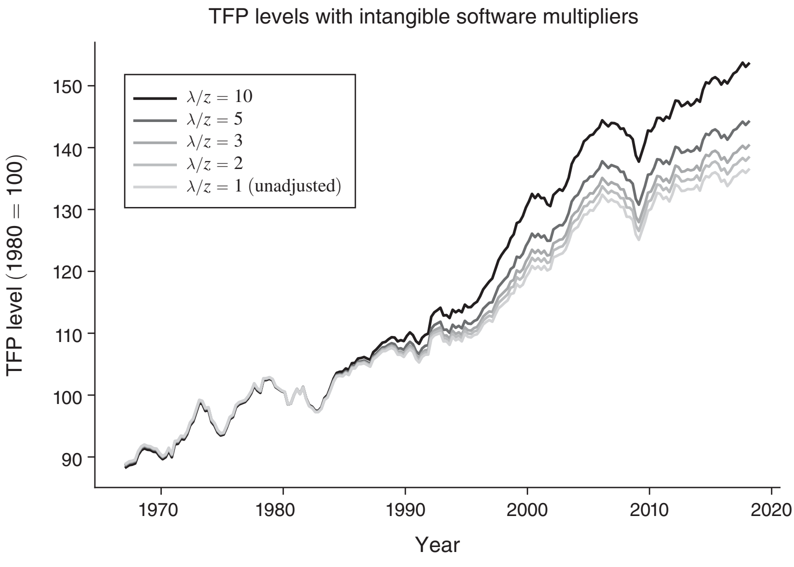

Figure 14. TFP Levels Corrected for Different Software Intangible Multipliers

10. At least in level terms, we are still in the capital accumulation phase of a deep Productivity J-curve.[26] This is reflected in the relative changes of the measured output shares of investment for these three capital varieties. The output share of

[26]Tables in the online Appendix show the productivity growth adjustments for R&D, computer software, and computer hardware from 1967 to 2017. Available at http://drock.mit.edu/Research.

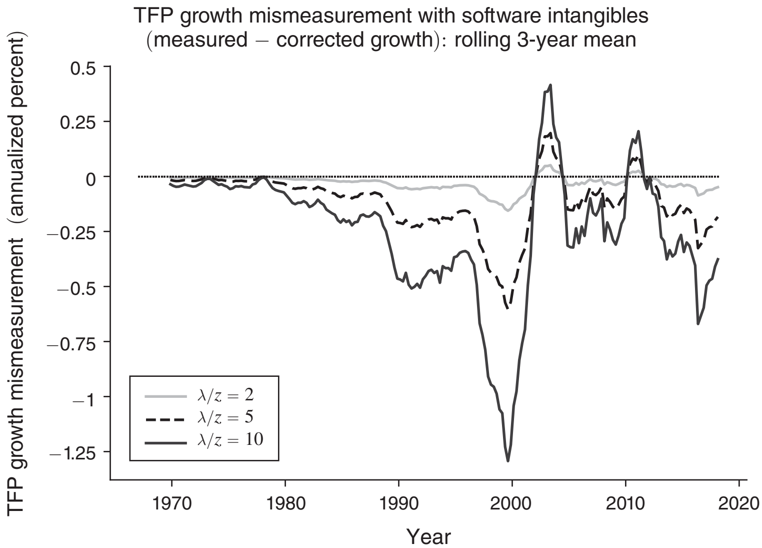

FIGURE 15. COMPUTER SOFTWARE-RELATED TFP GROWTH MISMEASUREMENT

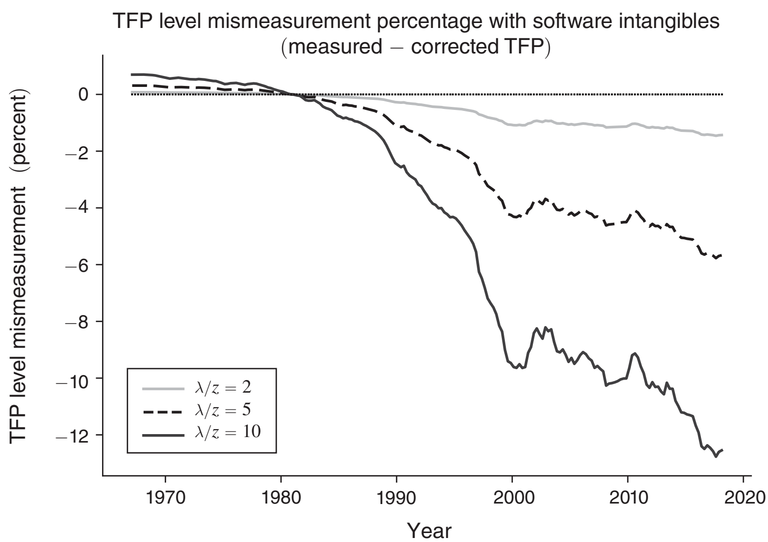

FIGURE 16. COMPUTER SOFTWARE-RELATED TFP LEVEL MISMEASUREMENT

software investment is growing rapidly over time, whereas the growth of computer hardware investment's output share has slowed.

VI. Can Intangible Capital Outputs Explain the Productivity Slowdown?

We now take the above estimates of the TFP adjustments due to intangible capital related to R&D, computer hardware, and software to ask if the measured productivity slowdown after 2004 (see, e.g., Gordon 2015, Summers 2015, Syverson

2017) can be accounted for by such intangibles. Some role seems plausible; while our calculations above imply intangibles related to software and (to a lesser extent) hardware started having a noticeable influence on true TFP in the 1990s, they also contributed in more recent periods. If these recent effects are larger than their earlier influence, they would in part explain the measured productivity slowdown.

The slowdown in measured annual TFP growth from 1995–2004 to 2005–2017 was approximately 1.23 percent per year.[27] Had measured TFP grown since 2005 at the same rate it did from 1995–2004, and holding labor and tangible capital inputs fixed at their observed levels, US GDP at the end of 2017 would have been about \$3.5 trillion higher than it was.[28]

To see if intangible capital accumulation tied to R&D, computer hardware, and software investments can account for any of this shortfall, we use our calculated TFP growth adjustments above to construct an intangible-adjusted TFP series. As discussed above, the adjusted productivity levels are substantially higher than the measured values in the post-slowdown period. Adjusted annual TFP growth over 2005–2017 was 0.71 percent, as opposed to the measured value of 0.40 percent.

However, the adjusted series was also larger *before* the productivity slowdown, averaging 2.20 percent growth per year from 1995–2004, higher than the measured value of 1.63 percent. The productivity slowdown therefore also exists in the adjusted series. Indeed, at 1.49 percent per year it is *larger* than the measured slowdown of 1.23 percent.[29]

Of course, this analysis assumes that the multiplier for intangibles—the amount of intangibles associated with each dollar of tangible investments—is constant throughout the period. If it were higher in recent periods, mismeasurement would be greater in recent periods. The opposite would hold if it were lower more recently.

Intangibles, at least in the simplest formulation with a constant multiplier, do not explain the productivity slowdown, and actually somewhat deepen it. This fact does not imply that intangibles' influence on productivity levels or total GDP is small. Adjusted TFP (again holding observed labor and tangible capital constant) is 11.3 percent higher than observed at the end of 2004, and 15.9 percent higher than observed at the end of 2017. In other words, in addition to all the measured assets, including housing, property plant and equipment, and so on that the US economy produced over the past several decades, there are also trillions of dollars' worth of unmeasured intangible capital. The long-lived nature itself of intangibles' effects causes these upward adjustments to be differenced out when seeking to explain the slowdown.

[27] We calculate this as the difference between the average quarterly TFP growth values for 1995–2004 and 2005–2017, respectively. We then annualize this average difference.

[28] At the end of 2017, counterfactual TFP would be 1.235 $(= 1.00407^{52})$ times its level at the end of 2004, where 0.407 percent was average quarterly TFP growth over 1995–2004. Measured TFP was instead 1.052 times larger. Assuming observed labor and capital inputs remain as observed, counterfactual GDP at the end of 2017 would thus be 1.174 $(= 1.235/1.052)$ times larger than the observed value of \$19.83 trillion. The difference, \$3.46 trillion, is 17.4 percent of \$19.83 trillion.

[29] For the adjusted series, counterfactual TFP is 1.33 $(= 1.0055^{52})$ times its end of 2017 level at the end of 2004, where 0.55 percent is the average quarterly adjusted TFP growth over 1995–2004. Measured TFP was 1.095 times larger in 2017 than in 2004 in adjusted terms.

VII. Is Hidden AI Capital Investment Already Causing a Productivity Shortfall?

Gross domestic product in the United States in 2017 was \$19.5 trillion and in real terms grew at an average annual rate of 2.17 percent over 2010 to 2017, down from 2.72 percent per year from 2000 to 2007 (the eight years prior to the Great Recession). This implies that unmeasured intangible capital investment over 2010 to 2017 would need to average \$107 billion per year ($= 19.5$ trillion $\times$ $[2.72\% - 2.17\%]$) in 2017 dollars to explain the entire slowdown in GDP growth. How much of this slowdown could be explained by a Productivity J-curve for investment in AI and related intangibles?

The economy is very early in the AI adoption cycle, but recent growth has been impressive. There has been a rapid increase in the use of AI and robotics technology over the past decade (Furman and Seamans 2018). Startup funding for AI has increased from \$500 million in 2010 to \$4.2 billion by 2016 (growing by 40 percent between 2013 and 2016) (Himel and Seamans 2017). Though concentrated heavily in the IT sector, overall measurable corporate investment in AI in 2016 was \$26–39 billion, marking 300 percent growth since 2013 (Bughin et al. 2017). Similarly, international industrial robot shipments since 2004 have nearly doubled overall and almost quadrupled in the consumer electronics industry (Furman and Seamans 2018). Nonetheless, relatively little AI has translated into change in business processes or new products. Despite large investments, we are yet not aware of any autonomous taxis in regular service replacing human drivers or machine learning systems reading images in lieu of human radiologists.

For AI to account for the 0.55 percent of "lost" output growth in 2017 GDP, the quantity of correlated intangible investments per unit of tangible investment must be between roughly 2.7 and 4.1 times the observable investment values (using the Bughin et al. 2017 estimate).[30] This is not implausible. Brynjolfsson, Hitt, and Yang (2002) find that the total market value of measured computer capital investments is as much as \$11.8 per \$1 in measured expenditure, with a standard error of \$4.025. Similarly, Rock (2019) finds an estimated \$11.9 of market value per \$1 of engineering worker wage bill expenditure, with a standard error of \$4.93. None of the intangible "shadow" output value will show up in the productivity statistics. Because the foregone output cannot be explained by growth in labor or observable capital inputs alone, the output shortfall will be attributed to slower productivity growth. Further, this investment (discounted and risk-adjusted) will later generate a capital service flow that produces measurable output and will be partially attributed to capital growth.

Of course, these numbers are just for 2017, when measured AI investment was several multiples of what it was only a few years prior. Thus analogous pre-2017 values would be notably smaller, and it is unlikely that much of the GDP slowdown gaps in those earlier years would be attributable to AI-related intangibles.

[30] The required forgone output in 2017 was \$107 billion ($=$ \$19.5 trillion $\times$ 0.55%). Dividing by the low observed investment figure of \$26 billion implies a required intangible investment that was $107/26 = 4.1$ times the observed investment. Using the larger \$39 billion figure implies intangibles that were $107/39 = 2.7$ times observed investment.

However, AI investments are likely to continue growing quickly. Further, existing AI capital has a high market valuation and as such suggests a considerable shadow value for intangible correlates, which indicates that we may be entering the period in which AI-as-GPT will have noticeable impacts on estimates of productivity growth.

VIII. Conclusion

Our approach has shown how accounting for intangible investments that are correlated with observable investment in new technology can meaningfully change estimates of productivity growth and dynamics. Intangibles are both capital inputs and capital outputs. Productivity is underestimated when the contribution of intangibles to outputs exceeds their contribution as inputs, and it is overestimated when the opposite holds. The output effect tends to dominate early in the capital accumulation cycle, when firms and organizations devote resources to building unmeasured intangible capital. The input effect dominates later, when these unmeasured assets generate capital services that increase measured output. Finally, when the capital accumulation reaches steady state, there is no longer any mismeasurement. These dynamics generate what we call the Productivity J-curve.

Because technological improvement often leads to the creation of new capital varieties and necessitates investment in intangible complements, the introduction of a new GPT is especially likely to cause such a J-curve to occur. In fact, the more transformative the new technology, the more likely its productivity effects will initially be underestimated. We analyze a series of recent, overlapping J-curves and show how productivity has been mismeasured for IT-related capital in recent decades. Our calculations suggest that trillions of dollars of intangibles output has been produced but not counted in the national income accounts, resulting in a 15.9 percent underestimate of TFP levels 2017. There is also some evidence that the phenomenon appears to have begun again, very recently, for AI-related intangible investments.

The mere presence of intangible correlate investment is not a guarantee of the existence of the Productivity J-curve. Although R&D investments are large and are associated with large intangibles, we find that mismeasurement related to R&D investments currently has a negligible effect on the estimation of productivity growth. On the other hand, computer hardware and to a greater extent software have had noticeable effects. The difference reflects the interaction of three quantities: the investment share of output of the asset type, the intangible correlate quantity and adjustment costs per unit of observable investment, and the difference between the growth rate of investment in the asset and the growth rate of capital services. In the case of R&D, the investment share is large. But, as a mature asset type, the difference between the growth rate of R&D investment and the growth rate of capital is not significant. Software, in contrast, has a smaller but still meaningfully large investment share of output, while the investment growth rate in software has substantially exceeded the growth of capital services overall.

By integrating aspects of the q-theory of investment and traditional growth accounting methods, we offer a means of adjusting the productivity statistics such that new, seemingly omnipresent GPTs might show up in the productivity statistics.

Assuming that capital markets price corporate securities efficiently in expectation, then market value regressions provide a way to estimate the value of intangible correlates and adjustment costs per unit of observable capital. The forward-looking nature of market valuation means that lags in capital services would rationally be considered correctly on average. Of course, these multipliers reflect a risk-adjusted discounted expected value of the accumulated asset stock which might to come to bear. The mismeasurement issues might accordingly be sensitive to differences in the timing of expected returns. Lower interest rates, for example, could encourage longer duration investments and therefore prolong the effects of the J-curve. This investment timing component of productivity mismeasurement is left to future research.

The J-curve method also suggests an indicator of whether or not a new technology is indeed a GPT. If measures of the investment in a given new technology fail to generate economically significant intangibles, that particular technology at that moment in time would not qualify as a GPT. This framework also might inform whether intangible capital accounts for the wide differences between frontier and median productivity firms (Andrews, Criscuolo, and Gal 2015).

The Productivity J-curve helps explain why productivity paradoxes can be both a recurrent and expected phenomenon when important new technologies are introduced. Adjusting productive processes to take advantage of new types of capital requires intangible investments that official statistics miss. To paraphrase Solow: In the future, after making appropriate adjustments accounting for the Productivity J-curve, we will see new technologies everywhere *including* the productivity statistics.

Appendix

A. *Derivations of Differences between Standard Productivity Growth and Unmeasured Intangible Capital Output-Adjusted Productivity Growth*

We begin with the standard growth accounting framework as in (1):

$$Y = AF(K,L) \tag{A1}$$

leading to

$$g_Y = \frac{dY}{Y} = \frac{AF_K dK + AF_L dL + F(K,L)dA}{Y}, \tag{A2}$$

$$g_Y = \frac{rK}{Y} g_K + \frac{wL}{Y} g_L + g_A, \tag{A3}$$

noting that the marginal products of capital and labor are $AF_K = r$ and $AF_L = w$. This yields

$$g_A = g_Y - \frac{rK}{Y} g_K - \frac{wL}{Y} g_L \tag{A4}$$

as in equation (3).

Now for the case that total output includes measured output Y and unmeasured investment output ϕI_U, we have

$$Y^* = Y + \phi I_U = A^* F^*(K, U, L), \tag{A5}$$

where ϕ is the price of unmeasured output, U is the stock of unmeasured capital, and other inputs are as before. This is equation (4). Differencing the unmeasured intangible investment from the output side,

$$Y = A^* F^*(K, U, L) - \phi I_U. \tag{A6}$$

Importantly, Y still corresponds to measured output. Following similar steps as above,

$$dY = F^*(K, U, L) dA^* + A^* F_K^* dK + A^* F_L^* dL + A^* F_U^* dU - \phi dI_U \tag{A7}$$

with the same observed marginal factor prices (plus a different cost of capital for unmeasured intangible capital), $A^* F_K^* = r$, $A^* F_U^* = r_U$, and $A^* F_L^* = w$. Then,

$$\frac{dY}{Y} = \frac{F^*(K, U, L)}{Y} dA^* + \frac{A^* F_K^* K}{Y} \frac{dK}{K} + \frac{A^* F_L^* L}{Y} \frac{dL}{L} + \frac{A^* F_U^* U}{Y} \frac{dU}{U} - \frac{\phi I_U}{Y} \frac{dI_U}{I_U}, \tag{A8}$$

or in growth terms,

$$g_Y = \frac{Y + \phi I_U}{Y} g_{A^*} + \frac{rK}{Y} g_K + \frac{wL}{Y} g_L + \frac{r_U U}{Y} g_U - \frac{\phi I_U}{Y} g_{I_U}. \tag{A9}$$

This allows for direct comparison between the growth rates of productivity in (3) and (5). Productivity growth in (5) is then:

$$g_{A^*} = \left(\frac{Y}{Y + \phi I_U}\right)\left(g_Y - \frac{rK}{Y} g_K - \frac{wL}{Y} g_L - \frac{r_U U}{Y} g_U + \frac{\phi I_U}{Y} g_{I_U}\right). \tag{A10}$$

This is equivalent to equation (5). Notably we assume that the marginal products of F and F^* are the same to get the same prices r and w for capital and labor, respectively. Since these capital and labor service flows and attendant output shares are in this framework measured directly, we can compare productivity growth rates.

Let $\eta = I_U / Y^*$ and therefore $Y^*(1 - \eta) = Y$. We will hold this constant for now but relax that assumption later. Then,

$$\begin{aligned} g_A - g_{A^*} = {} & g_y - (1 - \eta) g_y - \frac{rK}{Y} g_K + (1 - \eta) \frac{rK}{Y} g_K - \frac{wL}{Y} g_L \\ & + (1 - \eta) \frac{wL}{Y} g_L + (1 - \eta)\left(\frac{r_U U}{Y} g_U - \frac{\phi I_U}{Y} g_{I_U}\right) \end{aligned} \tag{A11}$$

and $\phi I_U/Y = \eta/(1-\eta)$, therefore

$$\text{(A12)} \qquad g_A - g_{A^*} = g_y - (1-\eta)g_y - \frac{rK}{Y}g_K + (1-\eta)\frac{rK}{Y}g_K - \frac{wL}{Y}g_L$$
$$+ (1-\eta)\frac{wL}{Y}g_L + (1-\eta)\frac{r_U U}{Y}g_U - \eta g_{I_U}$$
$$= \eta g_y - \eta g_{I_U} - \eta\Big(\frac{rK}{Y}g_K + \frac{wL}{Y}g_L\Big) + (1-\eta)\frac{r_U U}{Y}g_U$$
$$= \eta g_y - \eta\frac{rK}{Y}g_K - \eta\frac{wL}{Y}g_L + (1-\eta)\frac{r_U U}{Y}g_U - \eta g_{I_U}$$
$$= \eta(g_A - g_{I_U}) + (1-\eta)\frac{r_U U}{Y}g_U$$
$$= \left(\frac{\phi I_U}{Y+\phi I_U}\right)(g_A - g_{I_U}) + \left(\frac{Y}{Y+\phi I_U}\right)\left(\frac{r_U U}{Y}\right)g_U$$
$$= \Delta.$$

This is the derivation of equation (6). We call this the "right-hand-side method." This is a useful framing for adjusting empirical estimates of productivity growth. Another approach leaves unmeasured investment on the output side of the equations, such that output includes measured output and unmeasured capital investment. This is the "left-hand-side approach" and leads to equation (9) as well.

As before,

$$\text{(A13)} \qquad Y^* = Y + \phi I_U = A^* F^*(K, U, L)$$

and then

$$\text{(A14)} \qquad \frac{dY^*}{Y^*} = \frac{F^*(K,U,L)}{Y^*}dA^* + \frac{A^* F_K^* K}{Y^*}\frac{dK}{K} + \frac{A^* F_L^* L}{Y^*}\frac{dL}{L} + \frac{A^* F_U^* U}{Y^*}\frac{dU}{U}.$$

Now productivity refers to the aggregate economy's productivity in producing all types of output. As such, the growth of output reflects all output as well. Then,

$$\text{(A15)} \qquad g_{Y^*} = g_{A^*} + \frac{rK}{Y^*}g_K + \frac{wL}{Y^*}g_L + \frac{r_U U}{Y^*}g_U.$$

Rearranging terms,

$$\text{(A16)} \qquad g_{A^*} = g_{Y^*} - \frac{rK}{Y^*}g_K - \frac{wL}{Y^*}g_L - \frac{r_U U}{Y^*}g_U.$$

Some important differences between this equation and the growth accounting equation in (5) are that shares are computed in terms of total output now, not measured output. Productivity A^* reflects that. Now the unmeasured investment output is included in Y^* instead of treated separately:

$$\text{(A17)} \qquad g_A - g_{A^*} = \frac{F(K,L)}{Y}dA - F^*\frac{(K,U,L)}{Y^*}dA^*$$

$$= (g_Y - g_{Y^*}) + (rKg_K + wLg_L)\left(-\frac{1}{Y} + \frac{1}{Y^*}\right) + \frac{r_U U}{Y^*}g_U$$

$$= (g_Y - g_{Y^*}) - (rKg_K + wLg_L)\left(\frac{\eta}{Y}\right) + \frac{r_U U(1-\eta)}{Y}g_U.$$

If η is constant, then $(dY/Y) - (dY^*/Y^*) = (dY/Y) - ((dY/(1-\eta))/(Y/(1-\eta))) = 0$, and the difference in productivity growth terms is simply

$$\text{(A18)} \qquad g_A - g_{A^*} = (-\eta)\frac{(rKg_K + wLg_L)}{Y} + (1-\eta)\frac{r_U U}{Y}g_U.$$

This is the result in equation (9) without the final term.

We relax the assumption of constant η. If $\eta = \eta_t$ and changes over time, we need an additional term for the difference in growth rates between measured and total output. The steps are as follows:

$$\text{(A19)} \qquad Y^* = Y + I_U,$$

$$Y^* = \frac{Y}{1-\eta_t},$$

$$\text{(A20)} \qquad \ln Y^* = \ln Y + \ln\frac{1}{1-\eta_t},$$

$$\text{(A21)} \qquad \ln\frac{Y^*}{Y} = -\ln(1-\eta_t),$$

$$\text{(A22)} \qquad \ln\frac{Y}{Y^*} = \ln(1-\eta_t),$$

so then we have

$$\text{(A23)} \qquad d\ln Y - d\ln Y^* = d\ln(1-\eta_t),$$

or in growth terms since change in logs is the growth rate,

$$\text{(A24)} \qquad g_Y - g_{Y^*} = g_{1-\eta_t} = \frac{d(1-\eta_t)}{1-\eta_t}.$$

This reflects the change in intangible output share's contribution to the productivity growth differences in (9). If η_t goes up (down), this last term will be negative (positive):

$$\text{(A25)} \quad g_A - g_{A^*} = (-\eta_t)\frac{(rKg_K + wLg_L)}{Y} + (1-\eta_t)\frac{r_U U}{Y} g_U + g_{(1-\eta_t)}.$$

Note that this is also equivalent to an earlier line with respect to deriving equation (6) (reproduced below):

$$\text{(A26)} \quad g_A - g_{A^*} = \left(\eta_t g_y - \eta_t g_{I_U} - \eta_t\left(\frac{rK}{Y} g_K + \frac{wL}{Y} g_L\right) + (1-\eta_t)\frac{r_U U}{Y} g_U\right)$$

in the case that we have the following relationship:

$$\text{(A27)} \quad g_{1-\eta_t} = \eta_t\left(g_y - g_{I_U}\right).$$

Solving for this growth rate,

$$\text{(A28)} \quad g_{1-\eta_t} = \frac{d(1-\eta_t)}{1-\eta_t} = \frac{-d\eta_t}{1-\eta_t} = \frac{-d\frac{\phi I_U}{Y^*}}{1-\eta_t} = \frac{-1}{1-\eta_t}\left(\frac{\phi dI_U}{Y^*} - \frac{\phi I_U dY^*}{(Y^*)^2}\right)$$

$$= \frac{-1}{1-\eta_t}\left(\left(\frac{\phi I_U}{Y^*}\right)\left(\frac{dI_U}{I_U}\right) - \left(\frac{\phi I_U}{Y^*}\right)\left(\frac{dY^*}{Y^*}\right)\right)$$

$$= \left(\frac{\eta_t}{1-\eta_t}\right)\left(g_{Y^*} - g_{I_U}\right),$$

we also have that

$$\text{(A29)} \quad Y^* = Y + \phi I_U,$$

$$\text{(A30)} \quad dY^* = dY + \phi dI_U,$$

$$\text{(A31)} \quad \frac{dY^*}{Y^*} = \frac{YdY}{Y^*Y} + \frac{\phi dI_U}{Y^*}\frac{I_U}{I_U},$$

$$\text{(A32)} \quad g_{Y^*} = \frac{Y}{Y^*} g_Y + \frac{\phi I_U}{Y^*} g_{I_U} = (1-\eta_t) g_Y + \eta_t g_{I_U}.$$

Therefore combining results above,

$$\text{(A33)} \quad g_{1-\eta_t} = \frac{\eta_t}{1-\eta_t}\left(g_{Y^*} - g_{I_U}\right) = \frac{\eta_t}{1-\eta_t}\left((1-\eta_t) g_Y + \eta_t g_{I_U} - g_{I_U}\right)$$

$$= \frac{\eta_t}{1-\eta_t}\left((1-\eta_t) g_Y - (1-\eta_t) g_{I_U}\right) = \eta_t\left(g_Y - g_{I_U}\right),$$

as we wanted to show, proving that equations (6) and (9) are equivalent.[31]

Another formulation mirrors our empirical analysis. Recall the formula for adjusted TFP substituting the residual share net of labor and other measured components for the capital share of output with the assumption that $g_U = g_K$:

$$(A34)\quad g_{A^*} = (1-\eta_t)\left(g_Y - g_K - \left(\frac{wL}{Y}\right)(g_L - g_K) - \left(\frac{r_U U}{Y}\right)(g_U - g_K) + \left(\frac{\phi I_U}{Y}\right)(g_{I_U} - g_K)\right),$$

$$g_{A^*} = (1-\eta_t)\left(g_A - \left(\frac{r_U U}{Y}\right)(g_U - g_K) + \left(\frac{\phi I_U}{Y}\right)(g_{I_U} - g_K)\right),$$

$$g_Y = \left(\frac{1}{1-\eta_t}\right)g_{A^*} + g_K + \frac{wL}{Y}(g_L - g_K) - \frac{\phi I_U}{Y}(g_{I_U} - g_K).$$

This implies

$$(A35)\quad g_{A^*} = (1-\eta_t)\left(g_Y - \left(\frac{wL}{Y}\right)g_L - \left(1 - \frac{wL}{Y}\right)g_K + \frac{\phi I_U}{Y}(g_{I_U} - g_K)\right).$$

With $\phi I_U / Y = \eta/(1-\eta)$ (supressing time subscripts),

$$(A36)\quad g_{A^*} = (1-\eta)\left(g_Y - \left(\frac{wL}{Y}\right)g_L - \left(1 - \frac{wL}{Y}\right)g_K + \frac{\eta}{1-\eta}(g_{I_U} - g_K)\right),$$

$$g_{A^*} = (1-\eta)\left(g_Y - \left(\frac{wL}{Y}\right)g_L - \left(1 - \frac{wL}{Y}\right)g_K\right) + \eta(g_{I_U} - g_K),$$

$$g_{A^*} = (1-\eta)(g_A) + \eta(g_{I_U} - g_K).$$

This shows that the adjusted productivity growth is a convex combination of the standard productivity growth and the difference between the growth rate of unmeasured intangible investment and ordinary capital. The weights sum to one, and the weight on the second term is simply the true output share of unmeasured intangible investment (eta).

Clearly, increasing the true output share of unmeasured intangible investment puts more weight on the investment growth of intangibles, but also on the growth of the capital stock. We will (creatively) call the investment growth piece the "investment effect" and the capital stock growth piece the "capital stock effect." Without

[31] We thank an anonymous referee for helping us to clarify this approach.

the assumption of equal growth rates of different types of capital stocks, we have a similar formula:

$$g_{A^*} = (1-\eta)\left(g_A - \left(\frac{r_U U}{Y}\right)(g_U - g_K)\right) + \eta\left(g_{I_U} - g_K\right). \tag{A37}$$

B. *Derivation of Firm-Level Intangible Multipliers*

To incorporate adjustment costs and multiple capital varieties, we modify equation (1) following Lucas (1967). Production is now

$$Y = pAF(\mathbf{K}, L, \mathbf{I}). \tag{A38}$$

Assume perfect competition between firms and constant returns to scale in factor inputs. Now the production function incorporates an investment vector $\mathbf{I}$ with market price vector $\mathbf{z}$ such that the total cost of investment in one unit of capital goods is $(\mathbf{z} - \mathbf{pF_I})$. Here, F represents the final output net of adjustment costs and is assumed nonincreasing and convex in all types of $\mathbf{I}$. This reflects that adjustment costs grow increasingly costly for larger $\mathbf{I}$. In other words, the firm must forgo an increasing amount of output as its rate of capital investment increases. This helps explain why firms cannot, for example, instantaneously replicate the capital stocks of their competitors without incurring larger costs.

We can relate firm investment behavior to market value using this production function.[32] We assume perfect competition and constant returns to scale. For the price-taking firm, market value equals the sum of the capitalized adjustment costs. Let the jth entry of vectors $\mathbf{K}, \mathbf{I}, \boldsymbol{\delta}, \boldsymbol{\lambda}$ correspond to the jth capital variety. For simplicity's sake, there is only one type of labor. The firm must solve

$$\max_{\mathbf{I}, L}\left[\int_0^\infty \pi(t)\zeta(t)dt\right] = V(0), \tag{A39}$$

where

$$\pi(t) = pAF(\mathbf{K}, L, \mathbf{I}) - wL - \mathbf{z}'\mathbf{I}$$

and

$$\frac{dK_j}{dt} = I_j - \delta_j K_j \quad \forall j = 1, 2, \ldots, J.$$

That is, K_j is the capital stock of type j (indexes capital variety), L is labor, $\zeta(t)$ denotes the compound discount rate at time t, and δ_j is the depreciation rate of capital of type j. As in Yang and Brynjolfsson (2001), F is assumed nondecreasing

[32] See, for example, Hayashi (1982), Wildasin (1984), and Hayashi and Inoue (1991).

and concave in $\mathbf{K}$ and L. With homogeneity of degree one for F, we get the solution to the maximization of the Hamiltonian at time 0:

$$H(\mathbf{K}, L, \mathbf{I}, A) = \left(pAF(\mathbf{K}, L, \mathbf{I}) - wL - \mathbf{z}'\mathbf{I}\right)\zeta(t) + \sum_{j=1}^{J} \lambda_j \left(I_j - \delta_j K_j\right) \tag{A40}$$

with constraints

$$\frac{\partial H}{\partial \lambda_j} = \dot{K}_j = I_j - \delta_j K_j \quad \forall j \in \{1, 2, \ldots, J\}, \forall t \in [0, \infty],$$

$$\frac{\partial H}{\partial K_j} = -\dot{\lambda}_j = pF_{K_j}\zeta(t) - \lambda_j \delta_j \quad \forall j, \forall t,$$

$$\frac{\partial H}{\partial \mathbf{I}_j} = 0 = \left(pF_{\mathbf{I}_j} - z_j\right)\zeta(t) + \lambda_j \quad \forall j, \forall t,$$

$$\frac{\partial H}{\partial L} = 0 = \left(pF_L - w\right)\zeta(t) \quad \forall t,$$

$$\lim_{t \to \infty} \boldsymbol{\lambda}(t)\mathbf{K}(t) = 0,$$

leading to an equation for the value of the firm:

$$V(0) = \sum_{j=1}^{J} \lambda_j(0)\, K_j(0). \tag{A41}$$

Equation (A41) (the same as equation (15)) shows that the value of the firm at $t = 0$ is the sum over all varieties of the capital stock quantities multiplied by the "shadow price" of investment of the respective varieties. In our context, this shadow price reflects adjustment costs.[33]

Assuming all asset stocks are measured correctly and market prices correctly represent the value of claims on publicly traded firms, equation (A41) suggests that a regression of firm value on dollar quantities of asset varieties will yield a coefficient vector that represents the average present value of one unit of each type of capital. In a frictionless efficient market, that vector would be a series of ones for all assets, i.e., assets are priced precisely at replacement cost. In the presence of adjustment costs and hidden yet correlated intangible investments, however, the coefficient will be greater than one. Sunk adjustment costs can be considered intangible assets as well.

[33] Following equation (6) in Hall (2000), if λ_j represents the marginal q value (incremental market value created divided by asset replacement cost), then the marginal adjustment cost for the firm (set by the firms' competitors) at its chosen capital investment rate is set by

$$c'\left(\frac{k_t - (1 - \delta)\, k_{t-1}}{k_{t-1}}\right) = q_t - 1 = \lambda_t - 1,$$

where $c'(\cdot)$ is the marginal adjustment cost function and δ is the depreciation rate of capital. In this case, there are no unmeasured intangible correlates, only adjustment costs of investment. Our framework below allows for both adjustment costs and unmeasured intangibles. In that case, the sum of these two elements is our λ value. (One interpretation of this summation is that capitalized convex adjustment costs are, in effect, a nonlinear component of correlated intangible investments.)

Therefore, the ratio $(\phi_i + z_i)/\mathbf{z}_i = \lambda_i/z_i$ for some asset type i reflects the hidden intangible asset value per unit of observable installed capital for the firm.

C. *The J-Curve in Productivity Levels*

For an ordinary Cobb-Douglas production function, the productivity-level differences are dependent on η (the ratio of unmeasured intangible output to total output), U (unmeasured intangible capital inputs), and $r_U U/Y^*$ (the output share of unmeasured intangible capital inputs where $Y^* = Y + \phi I_U$). Equation (A42) shows the standard case, and equation (A6) shows the unmeasured intangibles-adjusted case:

$$\text{(A42)} \qquad \ln Y = \ln(1-\eta)Y^* = \ln(A) + \left(\frac{rK}{Y}\right)\ln(K) + \left(\frac{wL}{Y}\right)\ln(L),$$

$$\text{(A43)} \qquad \ln Y^* = \ln\left(\frac{Y}{1-\eta}\right) = \ln(A^*) + \left(\frac{rK}{Y^*}\right)\ln K + \left(\frac{wL}{Y^*}\right)\ln L + \left(\frac{r_U U}{Y^*}\right)\ln U.$$

The difference in log levels is a pass-through of the difference in output inclusive of unmeasured intangibles and ordinary output:

$$\text{(A44)} \qquad \Delta_{\log\_level} = \ln(TFP_{standard}) - \ln(TFP_{corrected}) = \ln(A) - \ln(A^*).$$

The difference is trivially equal to zero if all output is measured. The condition for the productivity-level gap to be equal to zero otherwise sets the unmeasured intangible output derived from measured inputs to be equivalent to the capital service flows from unmeasured intangible input, added to the difference in logs of measured output and total true output, as below:

$$\text{(A45)} \qquad \left(\left(\frac{rK}{Y^*}\right)\ln(K) + \left(\frac{wK}{Y^*}\right)\ln(L)\right)\frac{\phi I_U}{Y} = \left(\frac{r_U U}{Y^*}\right)\ln(U) + \ln(1-\eta).$$

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