ELSEVIER

Human Movement Science 20 (2001) 791–805

HUMAN MOVEMENT SCIENCE

www.elsevier.com/locate/humov

# Extending Fitts' law to a three-dimensional pointing task

Atsuo Murata *, Hirokazu Iwase

*Department of Computer Sciences, Faculty of Information Sciences, Hiroshima City University, 3-4-1 Ozukahigashi, Asaminami-ku, Hiroshima 731-3194, Japan*

Accepted 4 September 2001

**Abstract**

An attempt was made to extend Fitts' law to a three-dimensional movement (pointing) task to enhance its predictive performance in this domain. An experiment was conducted in which 10 subjects performed three-dimensional pointing movements under the manipulation of target size, distance to target and direction to target. As expected, the duration of these three-dimensional movements was rather variable and affected markedly by direction to target. As a result, the variance in the movement times produced was not satisfactorily explained by the conventional Fitts' model. The conventional model was extended by incorporating a directional parameter into the model. The extended model was shown to better fit the data than the conventional Fitts' model, both in terms of $r^2$ and the standard error of the residual between the measured movement time and the value predicted by model fit. 

*PsycINFO classification:* 2330; 4010

*Keywords:* Fitts' law; Human movement; Two-dimensional pointing task; Three-dimensional extension of Fitts' model; Index of difficulty; Movement direction

* Corresponding author. Tel.: +81-82-830-1616; fax.: +81-82-830-1616.
*E-mail address:* murata@cs.hiroshima-cu.ac.jp (A. Murata).


PII: S0167-9457(01)00058-6

## 1. Introduction

This study concerns the modeling of the duration of goal-directed movements in three-dimensions. In particular, it represents an attempt to extend Fitts' law to a three-dimensional pointing task. According to Fitts' law (Fitts, 1954, 1964; Welford, 1960), which only applies to one-dimensional movements (see Fig. 1(a)), the time needed to reach a target is a function of the target width and the distance to the target in the direction of movement. In the context of one-degree-of-freedom tasks, movement time has been successfully modeled using Fitts' formula.

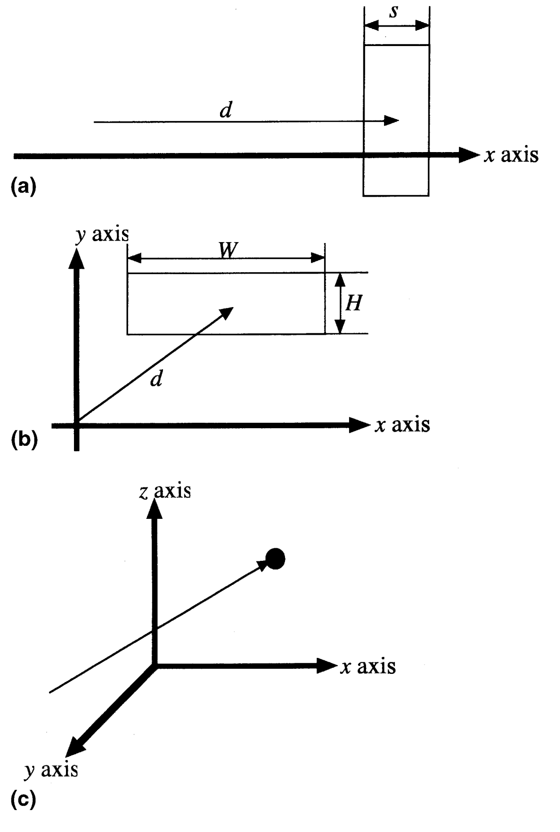


Fig. 1. Definition of one-, two-dimensional and three-dimensional pointing tasks. (a) One-dimensional task: movement along a single axis. (b) Two-dimensional task: movement in a (two-dimensional) plane. (c) Three-dimensional task: movement in three-dimensional space.

Crossman and Goodeve (1983) discussed Fitts' law from the viewpoint of a control-theoretic model of hand movements based on a reinterpretation of Fitts' original model. This alternative model, which involves a continuous velocity control of hand position, fitted the movement data equally well. Fitts' original model was also reinterpreted from the viewpoints of movement amplitude, movement velocity and movement acceleration (Carlton, 1979; Jagacinski, Repperger, Moran, Ward, & Glass, 1980; Langolf, Chaffin, & Foulke, 1976). All of these studies were aimed at modeling one-dimensional movements.

Although, Fitts' law only applies to one-dimensional movements, it has also widely been applied to two-dimensional pointing tasks on interactive computing systems, such as mouse movements implemented on a computer (see Fig. 1(b); cf. Boritz, Booth, & Cowan, 1991; Card, English, & Burr, 1978; Epps, 1986; MacKenzie, 1989, 1992; MacKenzie, Sellen, & Buxton, 1991; Radix, Robinson, & Nurse, 1999). In line with these studies, there have been some attempts to extend Fitts' law to two-dimensional tasks (MacKenzie & Buxton, 1992; Murata, 1996, 1999), which were aimed at improving the predictive power of modeling the pointing time of mouse operations.

Jagacinski and Monk (1985) discussed a two-dimensional generalization of Fitt's law in terms of multidimensional scaling and found that the pattern of movement was intermediate to the predictions of Euclidean and City-block models of movement space. Boritz et al. (1991) showed that there was a relationship between movement direction and movement (pointing) time and that this relationship depended on whether the user was right- or left-handed. However, no attempts were made in these studies to extend the conventional Fitts' model by incorporating the direction of movement. Ware and Lowther (1997) used Fitts' law in an experiment on target acquisition comparing a one-eyed two-dimensional cursor with a three-dimensional cursor in a fish tank virtual reality (VR) environment. They showed that the two-dimensional selection method using a one-eyed cursor was preferable to the three-dimensional method. However, this study did not examine pointing characteristics in a real-world three-dimensional movement.

Evidently, in order to describe three-dimensional pointing movements three axes are required ($x, y$, and $z$, see Fig. 1(c)). When performing discrete aiming movements limb displacements are achieved by generating adequately scaled and timed activity in both agonist and antagonist muscles (Rosenbaum & Krist, 1996; Van Galen & De Jong, 1995). The control over the amplitude and the duration of the forces generated becomes more complicated with an increase of the dimensionality of the task or the number of degrees of freedom related to the participating muscles and joints. To perform a three-dimensional pointing task higher muscular force is required, leading to more variable movement trajectories and, hence, more variable pointing times (Van Galen & De Jong, 1995). Based on these insights into discrete aiming movements, we expected that the three-dimensional pointing movements studied in the present experiment would be susceptible to the effects of direction and that, as a consequence, Fitts' law would be violated. In particular, we expected that the duration of three-dimensional pointing movements would be more variable and be affected more markedly by movement direction than the duration of one-dimensional and two-dimensional pointing tasks.

The purpose of the present study was to examine this hypothesis and, when it would be confirmed empirically, to extend the original Fitts' law to three-dimensional pointing tasks. To anticipate, we realized this goal as follows. First, we used the conventional Fitts' model to predict movement time data collected in a three-dimensional pointing task under the manipulation of target size, distance to target and direction to target. The fit was suboptimal due to the variance present in the data and the dependency of movement time on movement direction. Based on these results, an extended three-dimensional model of Fitts' law was proposed, which was shown to describe the data markedly better than the conventional Fitts' model.

## 2. Methods

### *2.1. Subjects*

Ten Hiroshima City University male undergraduates (21–24 years of age) participated in the experiment. The subjects were all right-handed and naïve with regard to the purpose of the experiment.

### *2.2. Apparatus*

A three-dimensional position measurement system (3Space Fastrak, Polhemus) was used. The system was connected to a personal computer (A1-N3T520J5, Panasonic) via the RS232 I/O port of the measurement system. The Fastrak instrumentation consisted of an electronic unit, a single transmitter and 1–4 receivers.

The Fastrak system uses electromagnetic fields to determine the position and orientation of a remote object. The technology is based on the generation of near-field, low-frequency, magnetic field vectors from a single assembly of three co-located, stationary transmitter antennas. The magnetic-field vector is detected by a single assembly of three co-located, remote-sensing receiver antennas. If only one receiver is used, then the magnetic field vector can be sampled at the highest frequency (120 Hz). Using two receivers reduces the sampling frequency to one-half, three to one-third, and four to one-fourth of the highest sampling frequency. The instrument measures with the specified accuracy (8 mm RMS error for $x$, $y$ and $z$ receiver positions) when the receivers are located within 76 cm of the transmitter. Operation at distances up to 305 cm is possible with reduced accuracy.

In the present study only one receiver was used allowing for maximal temporal resolution (120 Hz). The recorded signals were entered into a mathematical algorithm that computed the receiver's position and orientation relative to the transmitter. To avoid system error, we took every possible precaution to prevent any magnetic interference with the recording. During the experimental session, the position of the receiver was measured on-line.

### 2.3. Task

The receiver (sensor) was tightly attached to the right index fingernail of the subject. The starting point was placed at the same height as the center of the board, as shown in Fig. 2, and translated away from the board according to the distance condition. The subject was required to place his right index finger at the starting point before the experimenter gave him the signal to start the movement. The subject's task was to point with the right index finger to the target specified by the experimenter.

The board shown in Fig. 2 was placed vertically on the desk (see Fig. 3). A two-dimensional circle was used as the target rather than a three-dimensional sphere to equalize the distance between the starting point and the target for each directional condition with equal values of target size and distance. If a sphere had been used, it would have been difficult to define the target zone and to relate depth to the task. Three-dimensional pointing in this study means that the movement of the pointer (tip of the index finger) is performed in a three-dimensional space and measured along three axes.

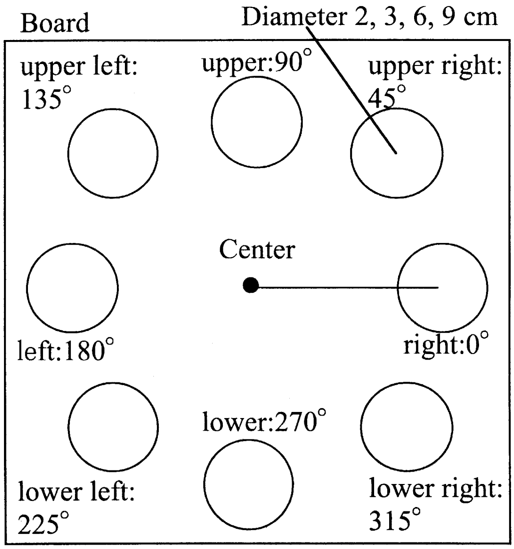


Fig. 2. Array of targets on the board used in the experiment.

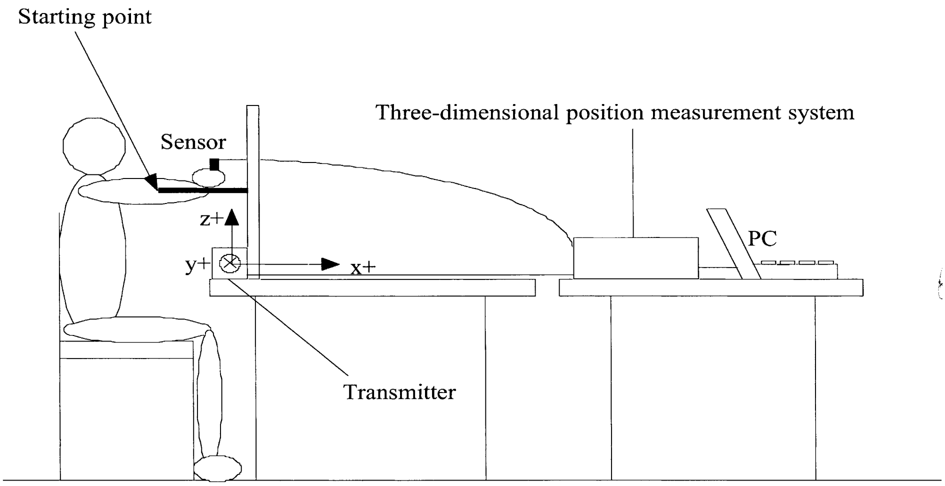


Fig. 3. Outline of experimental setup (see text for details).

## *2.4. Design and procedure*

Three within-subject factors were varied in the experiment, viz. target size (four levels: 2, 3, 6 and 9 cm), distance between starting point and target (four levels: 38.59, 42.01, 45.59 and 50.33 cm), and direction to the target from the center of the board ($\theta$) [eight levels: 0° (right), 45° (upper right), 90° (upper), 135° (upper left), 180° (left), 225° (lower left), 270° (lower) and 315° (lower right)]. According to the distance condition, the length of the pole shown in Fig. 3 was changed (20, 26, 32 and 38 cm). The movements were performed within 76 cm of the transmitter and could thus be measured with the highest degree of accuracy provided by the apparatus (i.e., 8 mm).

After having signed an information consent statement, each subject was tested. The subjects were required to sit in front of the board. Each subject performed a total of 128 ($4 \times 4 \times 8$) movement conditions. The order of performance of the 16 target size by distance combinations was randomized across subjects. Within each target size by distance combination, the order of pointing to the eight directions was randomized as well. For each condition, the $x$, $y$ and $z$ coordinates of the pointing movement were measured (see Fig. 3 for an illustration of the experimental setup).

The location of the chair was adjusted for each subject so that the effect of the subject's reach was minimized. The posture assumed by the subjects during task performance was deemed a crucial factor in the experiment. It is known that different muscles and joints are employed in different movements or movement strategies and that performance is affected as a result of this (Rosenbaum, Engelbrecht, Bushe, & Loukopoulos, 1993). The subjects were therefore required to keep their posture constant and to use the same muscles and joints (i.e,. the same movement strategy) during the task. As the selected subjects were about equally tall (mean height: $176 \pm 2$ cm), they could use and maintain the same posture and movement strategy during the task. The above experimental conditions were selected in such a way that all subjects performed the task from a similar posture with the same muscles and joints.

The subjects were instructed to carry out the task as accurately and as quickly as possible. The time when the coordinate of the fingertip began to change from the coordinate of the starting point was used as a criterion for movement onset. The halting criterion, that is, the criterion indicating the end of the movement (trial), was as follows. When the tip of the index finger reached the board, as shown in Fig. 2, the coordinate of the fingertip reached the same value and was saturated. The time when the coordinate began to converge was regarded as the end of the pointing task. The movement (pointing) time was obtained using the convergence of the time series of the $x$ coordinate. If the $x$ coordinate was within or on the target circle, then the trial was regarded as successful. All other cases were designated as error trials.

## 3. Results

### 3.1. Fitting the data to the conventional Fitts' model

First, the data were modeled using the following conventional Fitts' model (MacKenzie, 1989):

$$mt = a + b\log_2(d/s + 1.0), \tag{1}$$

where $mt$ represents the time to move the right index finger from the starting point to the target, and $d$ and $s$ are the distance from the starting point to the target and the size (diameter) of the target, respectively. The term $\log_2(d/s + 1.0)$ is the index of difficulty carrying the unit of bits. Finally, the parameters $a$ and $b$ are empirical constants to be determined through linear regression.

The error trials were excluded from the analysis. For the remaining successful trials, the mean $mt$ was calculated for each index of difficulty, pooled over all direction conditions and subjects. The $r^2$ of the linear regression between mean $mt$ and index of difficulty was 0.561 (see Fig. 4). (Note that $r^2$ measures the gain in accuracy obtained by using the correlation for prediction instead of guessing (Gravetter & Wallnau, 1999; Howell, 1999; Kurtz, 1999)). The 95% confidence intervals of the slope and intercept for the regression line were [0.083, 0.114] and [0.539, 0.650], respectively, while the

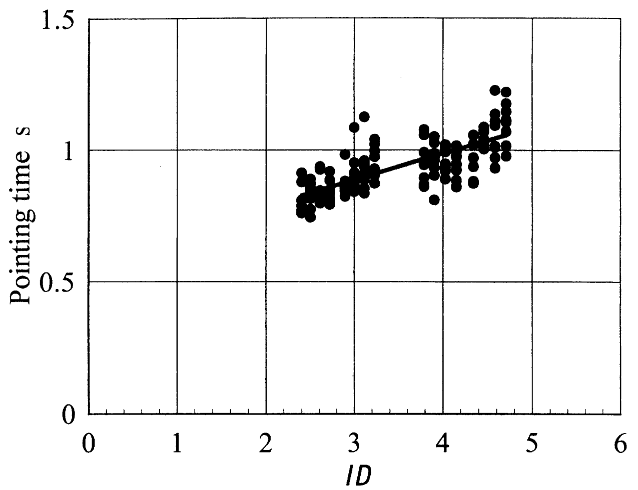


Fig. 4. Relationship between index of difficulty (ID) and mean pointing time (averaged over subjects).

standard error of the difference between the measured movement time and the value predicted by linear regression was 0.006 seconds. On the basis of this relatively poor fit it can be concluded that there is still substantial room for improving upon the conventional Fitts' model when it comes to the description of three-dimensional movements.

### 3.2. Extending Fitts' law to a three-dimensional pointing task

To find a meaningful extension of the conventional Fitts' model to three-dimensional pointing movements, we went on to examine the expected relationship between movement time and direction to target. Fig. 5 shows how the mean movement time *mt* (averaged over all *d* and *s* conditions and all subjects) varied across the eight levels of $\theta$. The subjective impression from this figure that *mt* depended on direction was confirmed in a one-way ANOVA, which revealed a significant main effect of $\theta(F(7,63) = 11.993$, $p < 0.01)$. To identify the source(s) of this statistically significant effect, a multiple comparison post hoc test was performed (i.e., Fisher's Protected Least Significant Difference test) using a conservative significance level of $p < 0.01$. On this test, a tendency was found for *mt* to be longer for the upper condition than for the seven other conditions. Furthermore, the following comparisons were significant: *mt* was shorter for the lower condition than for

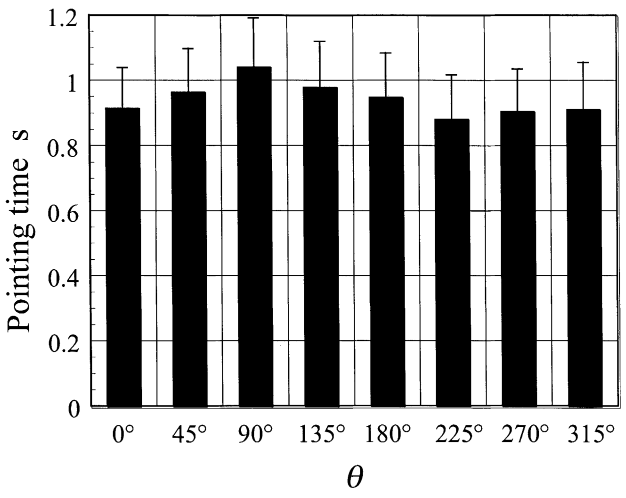


Fig. 5. Mean pointing time (averaged over subjects) as a function of $\theta$. The vertical bars indicate the interindividual standard deviations.

the left and upper-left conditions; *mt* was longer for the upper-right condition than for the right, lower and lower-left conditions; *mt* was shorter for the lower-left condition than for the left and upper-left conditions, and *mt* was shorter for the lower-right condition than for the upper-left condition. Finally, movements to the right tended to be longer than movements to the upper-left. These results indicate the presence of a systematic relationship between movement time and direction to the target. These imply that a model taking direction into account will lead to a better performance model than the conventional Fitts' model.

Strictly speaking, the movement direction is not identical to $\theta$ as it also depends on the distance between the starting point and the center of the board. However, for a given target distance, the actual direction of movement varies solely with $\theta$. Therefore, we judged that the movement direction to the eight target locations could be taken into account by incorporating $\theta$ into the *ID* in Eq. (1). Based on the discussion, the *ID* was revised using the following formula:

$$ID_3 = \log_2(d/s + 1.0) + c \sin\theta, \tag{2}$$

where $c$ is an arbitrary constant to be determined through linear regression.

For several values of $c$, the relationship between $ID_3$ and movement time *mt* was established by means of linear regression. Although movement direction has some effects on the movement time in two-dimensional tasks (Boritz et al., 1991; Jagacinski & Monk, 1985), there are few explicit models that take these effects into account. However, movement direction seems to be an important factor in performance modeling, especially the modeling of three-dimensional pointing tasks. Fig. 6 shows the $r^2$ for the data in Fig. 5 as a function of $c$. The highest $r^2$ (0.726) was found for $c = 0.5$. The fit to the experimental data was improved by using the index of difficulty $ID_3$, which incorporates the effect of direction on *mt*, and by using the value of $c$ producing the highest $r^2$ (Fig. 7). The optimal values of $c$ differed for the experimental data of the individual subjects.

The fit to the obtained movement times was better when extended three-dimensional modeling was applied. A one-way (modeling method: conventional Fitts' law vs extended three-dimensional modeling) ANOVA performed on the $r^2s$ for each subject revealed a significant main effect of modeling method ($F(1,9) = 7.99,\ p < 0.05$). The 95% confidence intervals of the slope and the intercept for the regression line for the extended model

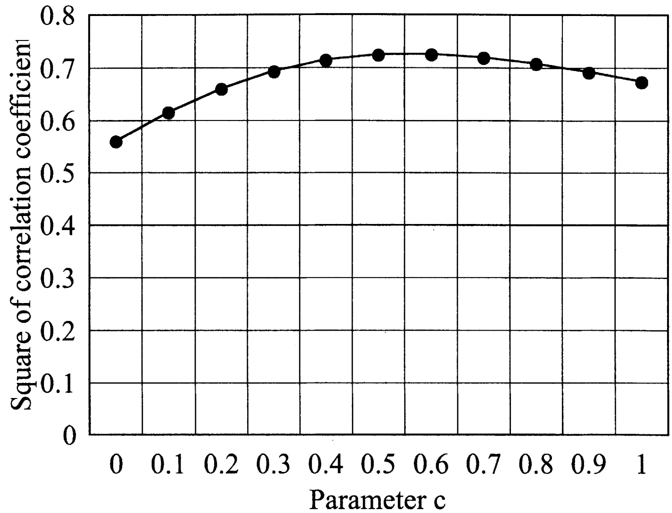


Fig. 6. Squared correlation coefficients as a function parameter $c$ for fitting the data in Fig. 4 to the extended Fitts' model.

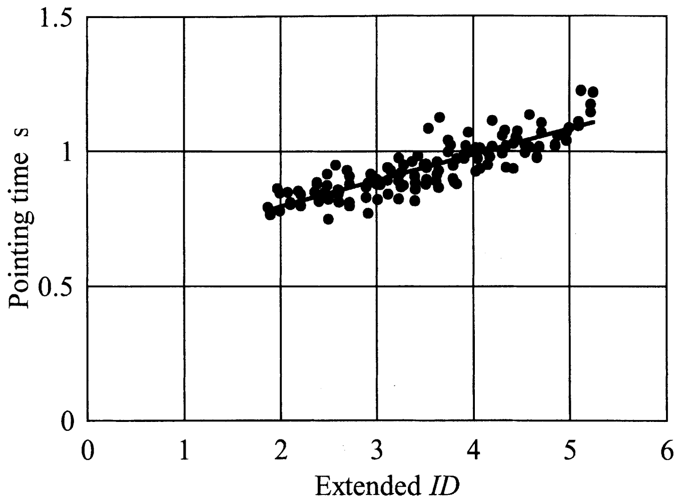


Fig. 7. Mean pointing time (averaged over subjects) as a function of the index of difficulty ($ID_3$) in the extended Fitts' model ($r^2 = 0.726$, $c = 0.5$).

were [0.086, 0.107] and [0.562, 0.638], respectively. These confidence interval are narrower than for the conventional Fitts' model. The standard error of the difference between the measured movement time and the value predicted by the fit to the extended model was 0.0047 seconds, which is smaller than that (0.0060 seconds) of the fits to conventional Fitts' model. A similar one-way ANOVA as used for the $r^2$ values showed that this difference was significant ($F(1,9) = 6.98$, $p < 0.05$). Collectively, these results clearly indicate that the extended model of Fitts' law better predicts the duration of three-dimensional (pointing) movements than the conventional Fitts' model.

## 4. General discussion

Typically, in two-dimensional pointing tasks on computer-based systems, the correlation coefficient $r$ between the index of difficulty and pointing time is much larger than that shown in Fig. 4 (cf., e.g., Jagacinski & Monk, 1985; MacKenzie, 1989, 1992; Boritz et al., 1991; Murata, 1996). Jagacinski and Monk (1985) and Boritz et al. (1991) investigated how directional mouse movement affected pointing time and showed that the pointing times differed significantly across conditions of directional movement. In these studies, however, the performance modeling was conducted by pooling all directional mouse movements. Nevertheless, the contribution ($r^2$) was much higher than in the present study (0.990 vs 0.726). This suggests that the effect of directional movement is not very prominent in two-dimensional computer-based pointing tasks. Hence, it appears reasonable to model the pointing time in such tasks with the conventional Fitts' model.

In the present experiment, movement time was affected significantly by movement direction (cf. Fig. 5). Movement times to targets in the upper directions (upper, upper left and right) were to tended to be longer than movement times to targets in the lower directions (lower, lower left and lower right). Thus, contrary to two-dimensional computer-based pointing tasks, the conventional Fitts' model cannot adequately explain the variance in movement time in a real-world three-dimensional pointing task, as it does not take the direction of movement into account (cf. Fig. 4).

The control over the amplitude and the duration of the forces applied during a movement becomes more complicated with the increase of the dimensionality of the task or the degrees of freedom related to the participating muscles and joints. The more variable movement trajectories and higher muscular forces caused by a more complicated system for controlling the $x$, $y$ and $z$ positions in a three-dimensional pointing task lead to more variable pointing times. The experimental data appear to validate the research hypothesis and highlight the necessity of constructing performance models that take the effects of movement direction into account.

The movement (pointing) time for each subject and the mean value of all subjects had forms that could be described by a sinusoidal wave. These characteristics were taken into account in recasting the index of difficulty according to Eq. (2). Specifically, a directional term ($\sin\theta$) was added to the conventional index of difficulty $ID$ to reduce the variance in the modeling. On the basis of analyses of the contribution ($r^2$), standard error and confidence interval of the data fitting, it was demonstrated that the predictive power of the extended performance model was superior to that of the conventional Fitts' model (compare Figs. 7 and 4).

These results support the idea that the extended performance model constitutes a promising route for modeling the movement time of three-dimensional pointing tasks. More efforts to attain an even higher predictive power are necessary. This might be accomplished by using, for example, the effective target width proposed by Murata (1999). Furthermore, the movement trajectories in a three-dimensional space should also be examined in more detail to obtain a higher predictive performance.

In the present study eight levels of directional movements were used. In future research, more graded levels of direction will be used in order to confirm the reproducibility of our results. In other words, the relationship between the movement time and the direction of movement must be confirmed using an experimental paradigm with more than eight levels of direction of movement. In this context it will be important to model the directional effects in terms of the actual movement directions as such rather than in terms of a variable which only correlates with these. Furthermore, the subjects in the present study were all right-handed. Future research may investigate modeling that involves right-handed subjects performing pointing tasks with their left hands and left-handed subjects participating in the experiments. Finally, in the present experiment the pointing was conducted, for the sake of simplicity, by using a two-dimensional target in a three-dimensional space. In future research, pointing movements to three-dimensional targets should be examined, requiring a method for defining the target size for a sphere and bringing in depth to the task.

## References


Boritz, J., Booth, K. S., & Cowan, W. B. (1991). Fitts' law studies of directional mouse movement. In J. Neilson (Ed.), *Proceedings of graphics interface '91* (pp. 216–223). Toronto: CIPS.

Card, S. K., English, W. K., & Burr, B. J. (1978). Evaluation of mouse, rate-controlled isometric joysticks, step keys, and text keys for selection on a CRT. *Ergonomics*, *21*, 601–613.

Carlton, L. G. (1979). Control processes in the production of discrete aiming responses. *Journal of Human Movement studies*, *5*, 115–124.

Crossman, E. R. F. W., & Goodeve, P. J (1983). Feedback control of hand-movement and Fitts' Law. *Quarterly Journal of Experimental Psychology*, *35A*, 251–278.

Epps, B. W. (1986). Comparison of six cursor control devices based on Fitts' Law models. In *Proceedings of the human factors society 30th annual meeting* (pp. 327–331). Santa Monica, CA: Human Factors Society.

Fitts, P. M. (1954). The information capacity of the human motor system in controlling the amplitude of movement. *Journal of Experimental Psychology*, *47*, 381–391.

Fitts, P. M (1964). Information capacity of the human motor system in controlling the amplitude of movement. *Journal of Experimental Psychology*, *67*, 103–112.

Gravetter, F. J., & Wallnau, L. B. (1999). *Essentials of statistics for the behavioral sciences* (3rd ed.). Pacific Grove: Duxbury Press.

Howell, D. C. (1999). *Fundamental statistics for the behavioral sciences* (4th ed.). Pacific Grove: Brook/Cole Publishing Company.

Jagacinski, R. J, & Monk, D. L. (1985). Fitts' law in two dimensions with hand and head movements. *Journal of Motor Behavior*, *17*, 77–95.

Jagacinski, R. J., Repperger, D. W., Moran, M. S., Ward, S. L., & Glass, B. (1980). Fitts' law and the microstructure of rapid discrete movements. *Journal of Experimental Psychology: Human Perception and Performance*, *6*, 309–320.

Kurtz, N. R. (1999). *Statistical analysis for the social sciences*. Boston: Allyn Bacon.

Langolf, G. D., Chaffin, D. B., & Foulke, J. A. (1976). An investigation of Fitts' law using a wide range of movement amplitudes. *Journal of Motor Behavior*, *8*, 113–128.

MacKenzie, I. S. (1989). A note on the information-theoretic basics for Fitts' law. *Journal of Motor Behavior*, *21*, 323–330.

MacKenzie, I. S. (1992). Fitts' law as a research and design tool in human–computer interaction. *Human–Computer Interaction*, *7*, 91–139.

MacKenzie, I. S., & Buxton, W. (1992). Extending Fitts' law to two-dimensional tasks. In I. R. Katz, & R. Mack (Eds.), *Proceedings of CHI'92 conference on human factors in computing systems* (pp. 219–226). New York: ACM.

MacKenzie, I. S., Sellen, A., & Buxton, W. (1991). A comparison of input devices in elemental pointing and dragging tasks. In L. Marks, & M. B. Rosson (Eds.), *Proceedings of CHI'91 conference on human factors in computing systems* (pp. 161–166). New York: ACM.

Murata, A. (1996). Empirical evaluation of performance models of pointing accuracy and speed with a PC mouse. *International Journal of Human–Computer Interaction*, *8*, 457–469.

Murata, A. (1999). Extending effective target width in Fitts' law to a two-dimensional pointing task. *International Journal of Human–Computer Interaction*, *11*, 137–152.

Radix, C. R., Robinson, P., & Nurse, P. (1999). Extension of Fitts' law to modeling motion performance in man–machine interfaces. *IEEE Transactions on System, Man, and Cybernetics – Part A: Systems and Humans*, *29*, 205–209.

Rosenbaum, D. A., Engelbrecht, S. E., Bushe, M. M., & Loukopoulos, L. D. (1993). Knowledge model for selecting and producing reaching movements. *Journal of Motor Behavior*, *25*, 217–227.

Rosenbaum, D. A., & Krist, H. (1996). Antecedents of action. In H. Heuer, & S. W. Keele (Eds.), *Handbook of perception and action. Vol. 2: Motor skills* (pp. 3–69). New York: Academic Press.

Van Galen, G. P., & De Jong, W. P. (1995). Fitts' law as the outcome of a dynamic noise filtering. *Human Movement Science*, *14*, 539–571.

Ware, C., & Lowther, K. (1997). Selection using a one-eyed cursor in a fish tank VR environment. *ACM Transactions on Computer–Human Interaction*, *4*, 309–322.

Welford, A. T. (1960). The measurement of sensory-motor performance: Survey and reappraisal of twelve years progress. *Ergonomics*, *3*, 189–230.