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STIGLER'S DIET PROBLEM REVISITED

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We review Stigler's diet problem, its impact on linear programming and operations research, and we determine minimum cost diets using updated nutritional and cost data. We also discuss how Stigler's diet problem formulation and its extensions have, over the years, influenced dietitians and nutritionists in their search for more wholesome but cost-effective diets.

In his book *The Man Who Ate Everything*, Jeffrey Steingarten (1998), the food editor for *Vogue* magazine, notes (p. 33):

> Years ago I read somewhere that the absolutely cheapest survival diet consists of peanut butter, whole wheat bread, nonfat dry milk and a vitamin pill.

Later on, in Steingarten's search for a more satisfying and varied diet and the answer to the question "What is the absolutely cheapest subsistence diet, and can it be turned into something palatable?" he states (p. 43):

> The problem looked like child's play. All I needed was a list of all foods, five thousand or ten thousand of them; nutritional information about each food and its cost; a personal computer with a statistics program installed; and somebody to type the first two things into the third. The mathematical problem is generally referred to as linear programming, and the routine commonly used to solve it is the Simplex Method, *which somebody once tried to teach me in graduate school long, long ago* (emphasis added). You simply ask the computer to choose a group of foods that collectively satisfy your list of nutritional requirements while absolutely minimizing the overall cost. It's like the simultaneous equations we learned to solve in high school, but much more complicated. Yet with a personal computer, the whole problem should take just a few minutes to solve. I planned to patent the answer as the Simplex Subsistence Diet.

The reader will, of course, recognize Steingarten's problem as the classical linear-programming problem with the eponymous title of Stigler's diet problem (Stigler 1945). Stigler, a future Nobel laureate in economics, working in pre-linear programming times, used a set of simultaneous (9×77) linear inequalities (greater than or equal to) to find a low-cost diet that met nutrient and caloric requirements. In this paper, we review Stigler's diet problem, its impact on linear programming and operations research, and we determine minimum cost diets using updated nutritional and cost data. We also discuss how Stigler's diet problem formulation and its extensions have, over the years, influenced dietitians and nutritionists in their search for more wholesome but cost-effective diets.

1.0. STIGLER'S DIET PROBLEM

Stigler posed the following problem: For a moderately active man (economist) weighing 154 pounds, how much of each of 77 foods should be eaten on a daily basis so that the man's intake of nine nutrients (including calories) will be at least equal to the recommended dietary allowances (RDAs) suggested by the National Research Council in 1943, with the cost of the diet being minimal? RDAs "… are the levels of intake of essential nutrients that, on the basis of scientific knowledge, are judged by the Food and Nutrition board to be adequate to meet the known nutrient needs of practically all healthy persons" (National Research Council 1989). Stigler's RDAs of interest were calories, protein, calcium, iron, vitamin A, thiamine, riboflavin, niacin, and ascorbic acid. Table 1 lists the values of these RDAs for the man in question. Stigler obtained the nutrient content of his 77 foods from a 1940 U.S. Department of Agriculture (USDA) publication as well as from a privately printed 1944 book (Chatfield and Adams 1940, Bowes and Church 1944). Stigler (1945, p. 308) commented on "… the tentativeness of these figures," and lamented "… the almost infinite complexity of a refined and accurate assessment of nutritive value of a diet."

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Table 1. 1943 RDAs for a moderately active 154-pound man.

Nutrient	RDA
Calories	3,000 kcalories
Protein	70 grams
Calcium	0.8 grams
Iron	12 milligrams
Vitamin A	5,000 IU
Thiamine (Vitamin B_1)	1.8 milligrams
Riboflavin (Vitamin B_2)	2.7 milligrams
Niacin	18 milligrams
Ascorbic Acid (Vitamin C)	75 milligrams

The basic linear-programming diet problem model is given by Min cX, subject to $AX \geq b$, $X \geq 0$, where c is a vector of prices for the foods X, each column of the matrix A contains the nutrient content of the corresponding food, and the vector b is the set of lower bounds for the RDAs. Stigler's limit of 77 foods was because of the need for associated price data, and such data were available for these more or less representative foods for both 1939 and 1945. As Stigler (1945, p. 309) commented, "It is beyond question that with a fuller list the minimum cost of meeting the National Research Council's allowances could be reduced, possibly by a substantial amount." The food prices, collected by the Bureau of Labor Statistics, were averages across many large cities. Stigler's "minimum" cost diets, using average prices gathered in August of each year, are given in Table 2. Note that total costs are *annual* costs and the food quantities are for the *full* year (to be divided into 365.25 days). Much has been said about the inadequacy of Stigler's minimal subsistence diets in terms of palatability, variety, and overall adequacy. From an operations research perspective, Stigler's diet problem is a prime example of an OR model that faithfully describes the real-world situation but whose solution validity is close to zero. As Stigler (1945, p. 312) cautioned, "No one recommends these diets for anyone, let alone everyone." Also, Lancaster (1992, p. 61) in her article describing the evolution of the diet model into the more acceptable menu-planning approach, observed, "The solution to the least-cost diet is the equivalent of the human dog biscuit." Credit must be given to the diet problem, however, in that it was the precursor of a wide variety of successful linear-programming applications dealing with cattle and chicken feed, fertilizer, and general ingredient mix problems. As we shall see, the concept of a minimum cost diet for humans does serve as a baseline for governmental funding and school lunch planning.

Table 2. Stigler's annual diets, August 1939 and 1944.

	August 1939		August 1945	
Food	Annual Quantity	Annual Cost	Annual Quantity	Annual Cost
Wheat Flour	370 lb.	$13.33	535 lb.	$34.43
Evaporated Milk	57 cans	3.84	—	—
Cabbage	111 lb.	4.11	107 lb.	5.23
Spinach	23 lb.	1.85	13 lb.	1.56
Dried Navy Beans	285 lb.	16.80	—	—
Pancake Flour	—	—	134 lb.	13.08
Beef Liver	—	—	25 lb.	5.48
Total Annual Cost		$39.93		$59.88
Total Daily Cost		$0.109		$0.135

Stigler used trial and error, and mathematical insight and agility to solve his (9×77) set of inequalities. Based on cost and nutrient content, he was able to "weed" the original 77 foods down to 15 as the eliminated foods were dominated by those in the list of 15. (The 15 foods had no meat except beef liver and excluded all sugars, beverages, and patented cereals.) Stigler's diet for 1939 data cost $39.93 per year (daily cost of $0.1093) and included varying amounts of wheat flour, evaporated milk, cabbage, spinach, and dried navy beans (Stigler 1945). Stigler's 1939 diet problem was the first "large-scale" problem that was solved using the simplex method (Dantzig 1963, 1990). In 1947, nine clerks, using hand-operated desk calculators, pivoted away for 120 clerk-days and found the linear-programming (LP) minimum cost of $39.69 (daily cost of $0.1087). Stigler knew what he was doing! The LP solution foods were from Stigler's reduced list and were the same except that a small amount of beef liver in the LP solution caused 57 cans of evaporated milk to disappear. These solutions had excesses (surplus) in niacin, thiamine, protein, and iron. If no excesses are allowed (equality constrained problem), the yearly cost increases to $49.40 (daily cost of $0.1352) and includes varying amounts of wheat flour, cornmeal, evaporated milk, peanut butter, lard, beef liver, cabbage, potatoes, spinach, and dried navy beans (corn meal, peanut butter, and lard were not in Stigler's reduced list of foods). These nine foods represent an optimal basic feasible solution to the (9×77) equality system (Dantzig 1963). Adjusted for inflation to April 1998, the cost of the 1939 LP diet increases from $39.69 to $466.69. Table 3 contrasts the Stigler and the LP solutions for 1939.

2.0. RELATED DIET PROBLEM FORMULATIONS

Over the years, many others have developed and solved diet problems that attempt to replicate or extend Stigler's basic structure. Each researcher brings something new: updated prices, new RDA values, or recalculated nutrient content of foods. Some are described in Smith (1963), who calls them "purely nutritional models" because they have the objective to be as inexpensive as possible while meeting the caloric and nutrient recommendations (the same goal as in Stigler's model). With this simple objective, it is difficult to form acceptable diets by just combining individual foods. Selecting and suboptimizing by food groups, bounding quantities from above and below, and using preference weights were all ideas suggested and tested soon after Stigler's diet was combined with linear programming (Gass 1958). We next cite two such studies.

Table 3. Stigler and linear programming annual diets, August 1939.

Food	Stigler—1939 Annual Quantity	Stigler—1939 Annual Cost	LP Simplex-Excess 1939 Annual Quantity	LP Simplex-Excess 1939 Annual Cost	LP Simplex-No Excess 1939 Annual Quantity	LP Simplex-No Excess 1939 Annual Cost
Wheat Flour	370 lb.	$13.33	299 lb.	$10.78	184 lb.	$6.62
Evaporated Milk	57 cans	3.84	—	—	246 cans	16.48
Cabbage	111 lb.	4.11	111 lb.	4.10	91.1 lb.	3.37
Spinach	23 lb.	1.85	23 lb.	1.83	2.96 lb.	0.24
Dried Navy Beans	285 lb.	16.80	378 lb.	22.29	—	—
Beef Liver	—	—	2.57 lb.	0.69	20.8 lb.	5.57
Peanut Butter	—	—	—	—	0.67 lb.	0.12
Corn Meal	—	—	—	—	135 lb.	6.21
Lard	—	—	—	—	90.9 lb.	8.91
Potatoes	—	—	—	—	82.9 lb.	1.88
Total Annual Cost (1939 $)		$39.93		$39.69		$49.40
Total Daily Cost (1939 $)		$0.1093		$0.1087		$0.1352
Total Daily Cost (1998 $)		$1.285		$1.278		$1.590

One of Smith's diet problems had 13 constraints, including bounds on fat and carbohydrate intake and an upper bound on total calories. He used a USDA handbook as a source for the nutritive contents for 73 foods. These foods were those that were commonly used by 176 families surveyed in the Lansing, Michigan area in 1955. The food prices were averaged across the surveyed families. The RDAs were set for a family of two 45-year-old adults and 18-year-old daughter who were assumed to be healthy, active, and living in a temperate climate. Smith's optimal diet included milk (fresh, homogenized, plain), oleomargarine, fresh carrots, fresh potaotes, pork (picnic ham, cured butts), and flour (white, enriched) and cost $0.336 per person per day or $122.76 per person per year (adjusted to $745.13 per person per year for April 1998).

Beckmann (1960) reported on a model and solution using the same nine nutrients as Stigler, but with RDA values updated to 1958. He used a different set of foods and different sources for their nutrient content, with food prices as found in Providence, Rhode Island in the fall of 1959 (Beckmann was at Brown University). He solved two problems, both for a 45-year-old male, one at the 3000-calorie level and the other for 2200 calories. The optimal 3000-calorie Beckmann diet included soybean meal, beef liver, lard, and frozen orange juice and had a daily cost of $0.216 per day or an annual cost of $78.99 (adjusted to $440.46 for April 1998).

3.0. STIGLER'S DIET PROBLEM REVISITED

In our revisit to Stigler's diet problem, we first took his original data and replicated the LP results for 1939 using the LINDO LP solver. We next updated these data to reflect price changes, revised values of the RDAs, and current evaluations of the nutrient content of the 77 foods. We shall refer to this problem with these new values as the *updated Stigler diet problem.*

Since 1939, the National Research Council has extended the number of nutrients that should be included as RDAs when determining a diet. We used these RDAs in developing a set of *extended Stigler diet problems* that include the full set of RDAs and the updated values of the prices and nutrient content of the 77 foods. For the extended problems, we set upper bounds for those nutrients that are known to be toxic or undesirable above certain levels. The extended set of RDAs is given in Table 4. Further, although not incorporated into the diet constraints, we did check the result-

Table 4. 1989 RDAs (tci = total calcium intake).

Nutrient	RDA 25–50 Years Old: Male	RDA 25–50 Years Old: Female	Upper Bound
Calories (kcal)	2,900	2,200	
Protein (g)	6.3	50	126
Calcium (g)	0.8	0.8	2.5
Iron (mg)	10	15	19,750
Vitamin A (μg RE)	1,000	800	15,000
Thiamine (mg) (Vitamin B_1)	1.5	1.1	
Riboflavin (mg) (Vitamin B_2)	1.7	1.3	
Niacin (mg)	19	15	10,000
Ascorbic Acid (mg) (Vitamin C)	60	60	
Vitamin D (IU)	200	200	1000
Vitamin E (mg-α-RE)	10	8	
Vitamin K (μg)	80	65	
Vitamin B_6 (mg)	2	1.6	209
Folate (μg)	200	180	20,000
Vitamin B_{12} (μg)	2	2	
Phosphorous (mg)	800	800	2 × tci
Magnesium (mg)	350	280	
Zinc (mg)	15	12	15
Iodine (μg)	150	150	
Selenium (μg)	70	55	5,000

ing diets to see if they meet the minimum and maximum bounds of other nutrients of interest as well as the bounds for dietary concerns such as cholesterol. These items and their values are shown in Table 5 (National Research Council 1989).

Stigler's RDAs were for an active 154-pound man, assumed from his article to be Stigler himself. The NRC RDAs are given for different sex and age groups, with median heights and weights for each group. Today's 154-pound man would fall into the 15–18-year-old or 19–24-year-old age group. Stigler, who was born in 1911, would have been 33 years old at the time he did his study in 1944–1945. We decided to use the RDAs for a 25–50-year-old-man, although this age group has the median weight of 174 pounds. This weight is a bit higher than Stigler's male, but we believe that it was his intention to pick an average male for his diet, and the 25–50-year-old group best fits that objective. We also solved the updated diet problem for a women in the 25–50-year-old age group. In both cases, we use calorie requirements for a light to moderately active man or woman. Here "light" activity includes "… walking on a level surface 2.5 to 3 miles per hour, garage work, electrical trades, carpentry, restaurant trades, house cleaning, child care, golfing, sailing, and table tennis"; "moderate" activity includes "… walking 3.5 to 4 mph, weeding and hoeing, carrying a load, cycling, tennis, and dancing" (National Research Council 1989). The average OR man or woman is, we believe, included in these definitions.

Our ability to measure accurately the nutrient content of foods has improved greatly since the mid 1940s. Further, we now find that many foods are fortified (e.g., corn flakes, one of Stigler's foods). The new nutrient content for the updated problems was taken from Pennington (1998), which is the 17th edition of one of the sources used by Stigler. Most of the data used are cited as coming from the current U.S. Department of Agriculture handbooks, while some—like corn flakes and peanut butter—came from food companies and trade associations. In an attempt to make the problem more realistic, we used nutritive content of foods in the form they would most likely be eaten; for example, cooked rice instead of raw rice, the latter being used by Stigler.

Table 5. 1989 suggested nutrient intakes (TCI = total calorie intake).

Nutrient	Lower Bound	Upper Bound
Pantothenic Acid	4 mg	
Sodium	500 mg	
Potassium	2,000 mg	18,000 mg
Copper	1.5 mg	
Manganese	2 mg	5 mg
Fat		0.3 × TCI
Saturated Fatty Acids		0.1 × TCI
Carbohydrates	0.125 × TCI	
Polyunsaturated Fatty Acids	0.07 × TCI	0.1 × TCI
Dietary Fiber	12 g	
Cholesterol		0.125 × TCI

The 77 foods that Stigler included in his diet were all included in the commodity list for which the Bureau of Labor Statistics (BLS) collected retail price information. Our hope was that the BLS would still be collecting prices for the Stigler list. However, the current BLS list includes around 30 of the 77 foods, with the list changing depending on what foods are included in the Consumer Price Index. To be consistent with respect to time and place, and upon the recommendation of a USDA economist, we fell back on the strategy used by Smith and Beckmann: We determined prices from the Giant Foods supermarket chain in the Washington, DC area for April 1998.

There were problems with trying to make an exact match between Stigler's foods and the current listing of foods. Stigler's list came from the BLS publication *Retail Prices of Foods, 1923–1936*, which includes average price quotations of 51 large cities in 1939 and 56 cities in 1944. It proved difficult to exactly match the foods in the old BLS list and those cited in the new Pennington list. Again, in our attempt to be more realistic than Stigler, where Stigler had raw eggs, we had to decide between some form of a cooked egg, e.g., fried, hard-boiled, scrambled. In such cases, we chose the one that was "healthier" or easier to prepare. Our resulting list of 77 foods then had to be matched with the foods available at the Giant. If the food selected from Pennington was a brand name, we priced it accordingly. Otherwise, we chose the least expensive brand in the least expensive form. Our pricing task was somewhat relieved as the Giant posts prices per measured unit. A listing showing the food matches between Stigler, Pennington, and Giant is given in Appendix A. In essence, a food Stigler matched to a food in Pennington for its nutrient content was in turn matched to a Giant food for its price.

If we were to start the minimum subsistence diet from scratch, we would attempt to repair some of the obvious flaws faced and described by both Stigler (1945) and Dantzig (1963, 1990). First, the new list of foods would be more extensive. There are many inexpensive, nutrient-rich foods that could be included. Also, Stigler's list includes foods that are uncommon in the kitchens of the 1990s, e.g., lard, Crisco. Certainly, one would now favor vegetable oil or olive oil. Second, it was recognized from the start that nutrient consumption and nutrient usage may not be linear for all, if any, nutrients. The Stigler diet model assumes that there are no interactions between foods nor interactions between nutrients. That is, the quantity of a nutrient consumed by eating a specified amount of a certain food is exactly the quantity of that nutrient that will be used by the body. This is the basic assumption that allows us to cast the diet problem as a linear-programming problem. We know of no other diet evaluation process that does otherwise.

4.0. SOLUTIONS TO THE UPDATED AND EXTENDED STIGLER'S DIET PROBLEMS

As the new data sets were entered into spreadsheets, it was convenient to use the linear-programming spreadsheet solver add-in to solve our modified diet problems. The basic mathematical structure is given by the following linear-programming model:

$$\text{Min} \quad X$$

$$\text{subject to} \quad d \le AX \le b,$$

$$X \ge 0,$$

where d is a vector of nutrient RDA lower bounds and b is a vector of nutrient RDA upper bounds. As the entries (a_{ij}) of matrix A contain the number of grams per dollar for nutrient i in food j, the variables $X = (x_j)$ are then the dollar values of food j to buy. The objective function is then just the minimization of the total sum of the food purchases. We solved diet problems for five different sets of data: (1) updated Stigler's problem for a 25–50-year-old man and (2) for a 25–50-year-old woman; (3) the extended Stigler problem where we incorporate constraints for all of the current RDAs for a 25–50-year-old man and (4) 25–50-year-old woman; and (5) Stigler's problem using current RDAs and food nutrient contents, but with 1939 food prices. Also, for each of these problems, we solved for a minimum cost diet where no excess nutrients were allowed. Recall that we use 365.25 days in a year to compute the annual costs. We next discuss the solution to each problem.

Updated Stigler's Diet Problem

Here we give the optimal solution diets using the nutrients considered by Stigler but current data for RDAs, food prices, and food nutrient content. Lower bounds were used for the following nutrients: calories, protein, vitamin A, vitamin C, thiamine, riboflavin, niacin, calcium, and iron. Upper bounds were used for the following nutrients: protein, vitamin A, niacin, calcium, and iron. In the first two solutions described below, we allow for excess nutrients within the upper bounds; while in the last two solutions, we allow no excess nutrients, that is, the lower bounds are met exactly.

25–50-year-old man (excess nutrients). The optimal solution diet for a 25–50-year-old man has an annual cost of $412.26 ($1.13 daily). Each day it consists of 5.68 cups of wheat flour, 17.5 fluid ounces of milk, 0.710 of an orange, and 0.335 of a carrot. The annual costs are shown in Table 6 and the corresponding annual quantities are shown in Table 7. For the extended set of 1989 RDAs (Table 4) and suggested 1989 nutrient intakes (Table 5), the following minimum requirements are not satisfied by this diet: polyunsaturated fatty acids, vitamin B_6, vitamin B_{12}, pantothenic acid, sodium, potassium, magnesium, zinc, copper, iodine, vitamin E, and vitamin K. No upper limits were exceeded. This diet provides servings (or portions of servings) from the following food groups established by the USDA: milk group (milk), vegetable group (carrots), fruit group (oranges), grain product group (wheat flour). Of the foods in this diet, only wheat flour is common with Stigler's reduced list of foods.

25–50-year-old woman (excess nutrients). The optimal solution diet for a 25–50-year-old woman has an annual cost of $354.05 ($0.97 daily). Each day it consists of 4.12 cups of wheat flour, 18.3 fluid ounces of milk, 0.718 of an orange, and 0.229 of a carrot. The annual costs are shown in Table 6 and the corresponding annual quantities are shown in Table 7. The following minimum requirements are not satisfied with this diet: polyunsaturated fatty acids, vitamin B_6, sodium, potassium, magnesium, zinc, copper, iodine, vitamin E, and vitamin K. No upper limits were exceeded. This diet provides servings (or portions of servings) from the following food groups established by the USDA: milk group (milk), vegetable group (carrots), fruit group (oranges), grain product group (wheat flour). Of the

Table 6. Optimal updated and extended excess diets: annual costs (April 1998 prices).

	Stigler Update		Extended Stigler	
Food	25–50-Year-Old Male	25–50-Year-Old Female	25–50-Year-Old Male	25–50-Year-Old Female
Wheat Flour	$228.53	$165.72	$52.93	$45.33
Rolled Oats	—	—	$50.69	$62.00
Milk	$138.52	$145.20	$126.81	$132.03
Peanut Butter	—	—	$123.38	$105.86
Lard	—	—	$83.06	$29.57
Beef Liver	—	—	$0.50	$0.33
Bananas	—	—	$95.82	$64.84
Oranges	$37.47	$37.84	$4.34	$12.35
Cabbage	—	—	$19.94	$15.82
Carrots	$7.74	$5.29	$7.25	$5.05
Potatoes	—	—	$31.44	$31.27
Pork and Beans	—	—	$55.11	$30.82
Total Cost (Annual)	**$412.26**	**$354.05**	**$651.27**	**$535.27**
Total Cost (Daily)	**$1.1287**	**$0.9694**	**$1.7831**	**$1.4655**

Table 7. Optimal updated and extended excess diets: annual quantities.

Food	Stigler Update		Extended Stigler	
	25–50-Year-Old Male	25–50-Year-Old Female	25–50-Year-Old Male	25–50-year-Old Female
Wheat Flour (lb.)	571	414	132	113
Rolled Oats (lb.)	—	—	85.9	105
Milk (qt.)	198	207	181	189
Peanut Butter (lb.)	—	—	49.8	42.7
Lard (lb.)	—	—	76.2	27.1
Beef Liver (lb.)	—	—	0.251	0.166
Bananas (lb.)	—	—	162	110
Oranges (lb.)	74.9	75.7	8.68	24.7
Cabbage (lb.)	—	—	39.9	31.6
Carrots (lb.)	19.4	13.3	18.2	12.7
Potatoes (lb.)	—	—	62.9	62.5
Pork and Beans (lb.)	—	—	108	60.4

foods in this diet, only wheat flour is common with Stigler's reduced list of foods.

We next give the optimal solution diets using the nutrients considered by Stigler but current data for RDAs, food prices, and food nutrient content, but do not allow for excess nutrients (the constraints are equalities). We include constraints for the following nutrients: calories, protein, vitamin A, vitamin C, thiamine, riboflavin, niacin, calcium, and iron.

25–50-year-old man (no excess nutrients). The optimal solution diet has an annual cost of $522.30 ($1.43 daily). The diet consists of wheat flour, milk, peanut butter, lard, bacon, roasting chicken, oranges, carrots, and molasses. The annual costs and quantities are shown in Table 8. The following minimum requirements are not satisfied with this diet: carbohydrates, polyunsaturated fatty acids, dietary fiber, vitamin B_6, pantothenic acid, zinc, copper, manganese, iodine, selenium, vitamin E, and vitamin K. No requirements are exceeded. This diet provides servings (or portions of servings) from all six of the food groups established by the USDA: fats, oils, and sweets group (peanut butter, lard, molasses), milk group (milk), meats and beans group (bacon, roasting chicken), vegetable group (carrots), fruit group (oranges), grain product group (wheat flour). Of the foods in this diet, only wheat flour is common with Stigler's reduced list of foods.

25–50-year-old woman (no excess nutrients). The optimal solution diet has an annual cost of $461.72 ($1.26 daily). The diet consists of wheat flour, milk, beef liver, roasting chicken, oranges, carrots, pork and beans, sugar, and molasses. The annual costs and quantities are shown in Table 8. The requirements for the following nutrients are not met by this diet: polyunsaturated fatty acids, dietary fiber, vitamin B_6, vitamin B_{12}, folate, phosphorus, zinc, copper, iodine, selenium, vitamin D, vitamin E, and vitamin K. This diet provides servings (or portions of servings) from all six of the food groups established by the USDA: fats, oils, and sweets group (sugar, molasses), milk group (milk), meats and beans group (beef liver, roasting chicken, pork and beans), vegetable group (carrots), fruit group (oranges), grain product group (wheat flour). Of the

Table 8. Optimal updated no excess diets (April 1998 prices).

Food	20–50-Year-Old Male		25–50-Year-Old Female	
	Annual Quantity	Annual Cost	Annual Quantity	Annual Cost
Wheat Flour	99.2 lb.	$39.69	71.3 lb.	$28.50
Milk	190 qt.	$133.03	127 qt.	$88.94
Peanut Butter	13.4 lb.	$33.18	—	—
Lard	143 lb.	$155.45	—	—
Beef Liver	—	—	0.43 lb.	$0.85
Bacon	16.7 lb.	$16.52	—	—
Roasting Chicken	72.7 lb.	$57.42	78.5 lb.	$61.99
Oranges	75.2 lb.	$37.62	73.6 lb.	$36.82
Carrots	19.1 lb.	$7.62	14.2 lb.	$5.66
Pork and Beans	—	—	76.4 lb.	$38.96
Sugar	—	—	210 lb.	$92.43
Molasses	14.0 qt.	$41.77	36.1 qt.	$107.57
Total Cost (Annual)		**$522.30**		**$461.72**
Total Cost (Daily)		**$1.4300**		**$1.2641**

foods in this diet, only wheat flour and beef liver are common with Stigler's reduced list of foods.

Extended Stigler's Diet Problem

This problem uses current data for RDAs, food prices, and food nutrient content with more nutrients than Stigler considered. Lower bounds were incorporated for the following nutrients: calories, protein, vitamin A, vitamin D, vitamin E, vitamin K, vitamin C, thiamin, riboflavin, niacin, vitamin B_6, folate, vitamin B_{12}, calcium, phosphorus, magnesium, iron, zinc, iodine, and selenium. In addition, upper bounds were formulated for the following nutrients: protein, vitamin A, vitamin D, niacin, vitamin B_6, folate, calcium, phosphorus, iron, zinc, and selenium. The resultant linear-programming model has 31 constraints and 77 foods.

25–50-year-old man (extended). The optimal solution diet has an annual cost of $651.27 ($1.78 daily). Each day it consists of: 1.31 cups of wheat flour, 1.32 cups of rolled oats, 16.0 fluid ounces of milk, 3.86 tablespoons of peanut butter, 7.28 tablespoons of lard, 0.0108 ounces of beef liver, 1.77 bananas, 0.0824 of an orange, 0.707 cup of shredded cabbage, 0.314 of a carrot, 0.387 of a potato, and 0.530 cup of pork and beans. The annual costs are shown in Table 6 and the corresponding annual quantities are shown in Table 7. The following minimum requirements are not satisfied with this diet: carbohydrates, polyunsaturated fatty acids, and copper. The upper limit for manganese was exceeded. This diet provides servings (or portions of servings) from all of the six food groups established by the USDA: fats, oils, and sweets group (peanut butter, lard), milk group (milk), meats and beans group (beef liver, pork and beans), vegetables group (cabbage, carrots, potato), fruit group (bananas, oranges), grain product group (wheat flour, rolled oats). The following foods from this diet are common with Stigler's diet reduced list: wheat flour, beef liver, cabbage, and potatoes.

25–50-year-old woman (extended). The optimal solution diet has an annual cost of $535.27 ($1.47 daily). Each day it consists of: 1.13 cups of wheat flour, 1.61 cups of rolled oats, 1.67 fluid ounces of milk, 3.31 tablespoons of peanut butter, 2.59 tablespoons of lard, 0.00724 ounces of beef liver, 1.20 bananas, 0.234 of an orange, 0.561 cup of shredded cabbage, 0.219 of a carrot, 0.384 of a potato, and 0.297 cup of pork and beans. The annual costs are shown in Table 6 and the corresponding annual quantities are shown in Table 7. The following minimum requirements are not satisfied with this diet: polyunsaturated fatty acids and copper. The upper limit for manganese was exceeded. This diet provides servings (or portions of servings) from all of the six food groups established by the USDA: fats, oils, and sweets group (peanut butter, lard), milk group (milk), meats and beans group (beef liver, pork and beans), vegetable group (cabbage, carrots, potato), fruit group (bananas, oranges), grain product group (wheat flour, rolled oats). The following foods from this diet are from Stigler's reduced list: wheat flour, beef liver, cabbage, and potatoes.

When the no excess nutrient condition was applied to the extended diet problem, there were no feasible diets for either the man or woman. For all our diets, it is cheaper to feed a woman than a man.

Semi-Updated Stigler's Diet Problem

In this version of the problem, we solved Stigler's original problem using current data for the RDAs (for a 25–50-year-old light to moderately active man) and for the updated nutritive values of foods, but with 1939 food prices. The problem had only the nine inequality constraints, one each for the nine nutrients considered by Stigler. The constraints in this problem are the same as the constraints in the updated Stigler's problem discussed above.

To solve this problem, some food units of measurement needed to be converted from what Stigler used. The prices collected in April 1998 from the Giant supermarket were all given per pound, except for the prices for milk, evaporated milk, cream, corn syrup, and molasses, which were in quarts. Some of the conversions were quite clear. For example, the unit for the following items in the 1939 data was ounces (oz.): wheat cereal, corn flakes, hominy grits, raisins, cocoa, and chocolate. These were readily converted to pounds. More involved conversions dealt with foods in cans (a $14\frac{1}{2}$-ounce can of evaporated milk was used by Stigler), eggs (a dozen was the unit measure used by Stigler), and carrots (a bunch was Stigler's unit).

The optimal solution to the semi-updated excess Stigler diet problem was $40.92 annually in 1939 ($481.16 in April 1998). The annual costs and quantities are shown in Table 9 and Table 10, respectively. The semi-updated excess diet included wheat flour, evaporated milk, cabbage, and sweet potatoes—a different diet than Stigler's original, which included spinach and navy beans but no sweet potatoes. The following minimum requirements are not satisfied with this diet: polyunsaturated fatty acids, vitamin B_6, vitamin B_{12}, pantothenic acid, sodium, potassium, magnesium, zinc, copper, iodine, vitamin D, and vitamin E. The upper limit for manganese was exceeded. This diet provides servings (or portions of servings) from the following USDA food groups: milk group (evaporated milk), vegetable group (cabbage, sweet potatoes), grain product group (wheat flour). All the foods in this diet are also on Stigler's reduced list of foods. As shown in Table 9, the annual cost of this diet is just $0.99 more than the annual cost of Stigler's 1939 diet and $1.23 more than the annual cost of Dantzig's 1939 simplex LP solution diet. Tables 9 and 10 also show the semi-updated no excess solution.

5.0. APPLICATIONS OF THE MINIMUM COST DIET

One might wonder about the real-world use of the minimum cost diet as originally posed by Stigler. We have already noted how monotonous this diet would be if consumed on a regular basis. Stigler (1945) compared the

Table 9. Solutions to Stigler's original and semi-updated diet problems: annual costs for August 1939.

Food	Stigler (Original)	LP Simplex Excess Nutrients (Original)	LP Simplex No Excess Nutrients (Original)	LP Simplex Semi-Updated Excess Nutrients	LP Simplex Semi-Updated No Excess Nutrients
Wheat Flour	$13.33	$10.78	$6.61	$19.98	$2.90
Corn Meal	—	—	$6.22	—	—
Rolled Oats	—	—	—	—	$3.79
Evaporated Milk	$3.84	—	$16.48	$14.46	$16.01
Peanut Butter	—	—	$0.12	—	$11.42
Lard	—	—	$8.91	—	$11.15
Beef Liver	—	$0.69	$5.57	—	$0.35
Cabbage	$4.11	$4.10	$3.37	$4.64	$3.33
Potatoes	—	—	$1.88	—	$2.19
Spinach	$1.85	$1.83	$0.24	—	—
Dried Navy Beans	$16.80	$22.29	—	—	—
Sweet Potatoes	—	—	—	$1.84	$1.52
Total Daily Cost (1939 $)	**$0.1093**	**$0.1087**	**$0.1352**	**$0.1120**	**$0.1442**
Total Daily Cost (1998 $)	**$1.285**	**$1.278**	**$1.590**	**$1.317**	**$1.695**
Total Annual Cost (1939 $)	**$39.93**	**$39.69**	**$49.40**	**$40.92**	**$52.56**
Total Annual Cost (1998 $)	**$469.52**	**$466.69**	**$580.87**	**$481.16**	**$619.08**

yearly cost of his diet ($39.93) with the costs of diets proposed by "professional dieticians." The diets cited all cost two to three times as much as his minimum cost diet. He explained the differences in terms of palatability, variety, prestige value of various foods, and other cultural facets of consumption. This, he felt, led the dieticians to emphasize meats and the inclusion of sugar. (In this regard, Stigler 1939, p. 314 commented, "Tax supported bureaucrats and professors may also have another reason for certain of their practices.") Without making drastic modifications to the basic diet problem, such minimum cost diets will never prove to be acceptable. In his discussion of the diet problem, Gass (1958) noted that any new model would have to enable more foods to be in the minimum solution (as is the case of our extended Stigler problem), involve taste preferences (with weights that can be introduced into the objective function), and by forcing foods in by lower bounds. Also, another approach would be to subdivide the problem into smaller diet problems, each of which would involve a single class of food. This may be a reasonable approach using the six food groups of the USDA's Food Guide Pyramid (Pennington 1998).

Stigler's diet problem and its modifications are ideal for the classroom. In constructing such a model, the student learns about the rigors of data collection (its magnitude and need for accuracy and consistency), the need for validating a model in a real-world setting, and the need to run various scenarios and perform sensitivity studies. Gass (1985) suggests student exercises using the eight basic RDAs and calories and for constructing food

Table 10. Solutions to Stigler's original and semi-updated diet problems: annual quantities.

Food	Stigler (Original)	LP Simplex Excess Nutrients (Original)	LP Simplex No Excess Nutrients (Original)	LP Simplex Semi-Updated Excess Nutrients	LP Simplex Semi-Updated No Excess Nutrients
Wheat Flour (lb.)	370	299	184	555	80.6
Corn Meal (lb.)	—	—	135	—	—
Rolled Oats (lb.)	—	—	—	—	53.4
Evaporated Milk (cans)	57	—	246	215	239
Peanut Butter (lb.)	—	—	0.67	—	63.8
Lard (lb.)	—	—	90.9	—	114
Beef Liver (lb.)	—	2.57	20.8	—	1.29
Cabbage (lb.)	111	111	91.1	125	89.9
Potatoes (lb.)	—	—	82.9	—	96.6
Spinach (lb.)	23	23	2.96	—	—
Dried Navy Beans (lb.)	285	378	—	—	—
Sweet Potatoes (lb.)	—	—	—	36.1	29.8

mixes for cats and dogs. More recently, Bosch (1993) reports on his classroom-oriented diet problem in terms of selecting a menu from McDonald's (also see Erkut 1994, Liberatore and Nydick 1999). The NEOS (1999) Web site, http://www.mcs.anl.gov/otc/Guide/SIREV/, contains data and formats that enable students to try their hand at forming a diet that satisfies given levels of RDAs.

Obvious deficiencies in the minimum cost diet have led investigators to apply the power of linear and integer programming to the development of more realistic models of human diets (minimum cost feed diets for cattle and chicken seem to be devoured by the recipients; Jewell 1960). In particular, pioneering work in menu planning, or computer-assisted menu planning, is described in Balintfy (1975). The history of this progress is reviewed in Lancaster (1992). There are, however, some ongoing applications of modified diet models that can trace their lineage back to Stigler and linear programming. We next describe two such applications: (1) the thrifty Food Plan, a subsistence dietary approach developed by the USDA Agricultural Research Service that is used to determine the number of food stamps to issue to Food Stamp Program recipients; and (2) the rules imposed by the USDA on school dietitians and related diet/menu software that ensures that public-school students will receive nutritious school lunches.

The Thrifty Food Plan

The Thrifty Food Plan (TFP) was originally developed by the Agricultural Research Service (ARS) in 1974–1975 (Kerr et al. 1984). As noted in ARS (1975): "This plan specifies the amounts of foods of different types (food groups) that families might use to provide nutritious diets for family members." Here, chief dietary concerns, besides the usual RDA and related constraints, were palatability and variety. The TFP model finds "… the combination of food groups … that represents as little change from the food consumption pattern as required to meet the nutritional goals at a given cost" (ASR 1975). The basis for current family food consumption patterns was obtained by surveying households in 1965–1966, with an update conducted 1983. Suggested diets are given for the different sex–age categories determined by the National Research Council for the given RDAs. The Center for Nutrition Policy and Promotion is currently revising the TFP model so it will include the most recent dietary guidelines (1989 RDAs) and recommendations (food pyramid).

A quadratic programming model was used in the development of the TFP. The goal of the model was to minimize change in the food consumption patterns, where "change" was measured in terms of squared weighted deviations from the amount of food groups in the consumption pattern. Each of the foods determined by the survey fell into one of 15 food groups, with the foods in each group having similar nutritive values. (Note that these are not the food groups established by the USDA.)

Constraints relative to each group were incorporated in the model. Except for the groups fats/oils, sugars/sweets, and soft drinks, no more than twice the amount of the food consumption pattern for each food group was allowed, and no less than half of the food consumption pattern for each food group was allowed. For the fats and sugars, no more than the established food consumption pattern was allowed. No more than half the food consumption pattern amount of soft drinks were allowed in the diet. Also, upper and lower limits on the ratio of the amount of flour to the amount of leavening agents and seasonings were imposed.

The 1974 values of the RDAs were used. The model set lower limits for 5% above the RDA for the following nutrients: calories, protein, calcium, iron, vitamin A, thiamine, riboflavin, niacin, vitamin B_{12}, and vitamin C. These exaggerated upper limits allow for some waste of edible food. The model set lower limits of 80% of the RDA for magnesium and vitamin B_6 and set an upper limit for calories of 10% above the RDA. An upper limit was also set for fat intake: No more than 40% of the calories came from fat. A cholesterol limit was also imposed in that no weekly diet for any person could contain more than four eggs (ARS 1975).

The TFP model does not minimize cost but minimizes a deviation measure from present consumption patterns. Cost is included and limited by a constraint. Maximum costs are set for each sex–age category. To determine these maximum costs, two "preplans" were computed first. One preplan diet was the least-cost diet and the other was a diet with no cost limit. These preplans were determined using the quadratic programming model. Equitable costs were determined for the categories by subtracting a constant proportion of the difference between costs for the two preplans from the cost of the more expensive preplan. The proportion used was set to result in the per-capita cost of the "economy plan" (ARS 1975).

The *monthly cost* of the 1975 thrifty plan for a 20–54-year-old male was $49.20 ($162.16 in April 1998), and for a 20–54-year-old female the cost was $39.90 ($131.50 in April 1998) (USDA 1983). The annual costs were $590.40 ($1945.88 in April 1988) and $478.80 ($1578.06), respectively. The diet is given in terms of the number of units from each food group that should be consumed to obtain sufficient nutrients within the budget of the thrift plan. The weekly diet consists of a set of foods that offers a variety of foods and appears to be an improvement with respect to palatability. We summarize the weekly thrift-plan diets and associated monthly costs for men and women in Table 11.

Balintfy (1979), in his search for the cost of a diet that yields a decent subsistence, introduces a model that maximizes a quadratic utility function subject to the usual linear diet restrictions and a budget level constraint. His approach was applied to the 15 food groups described above for 1965 and 1975 food prices. The method was further extended and applied to more current data in Balintfy et al. (1996).

School Lunch Menu Planning

Since its inception in 1946, the school lunch meal plan has maintained the basic rule: "… schools must offer chil-

Table 11. Weekly Thrift Plan Diets 1975—monthly costs.

Food Group	Male 20–54-Year-Old	Female 20–54-Year-Old
Milk, Cheese, Ice Cream	2.57 quarts	2.81 quarts
Meat, Poultry, Fish	3.03 pounds	2.41 pounds
Eggs	4	4
Dry Beans and Peas, Nuts	0.44 pounds	0.27 pounds
Dark-Green, Deep-Yellow Vegetables	0.39 pounds	0.52 pounds
Citrus Fruit, Tomatoes	1.80	1.86
Potatoes	2.02 pounds	1.51 pounds
Other Vegetables, Fruit	3.69 pounds	3.39 pounds
Cereal	0.89 pounds	0.90 pounds
Flour	0.92 pounds	0.67 pounds
Bread	2.29 pounds	1.41 pounds
Other Bakery Products	1.33 pounds	0.67 pounds
Fats, Oils	0.95 pounds	0.57 pounds
Sugar, Sweets	0.86 pounds	0.57 pounds
Accessories: Coffee, Tea, Soft Drinks, Juices, etc.	1.24 pounds	1.18 pounds
Monthly/ Annual Cost 1975	**$49.20/$590.40**	**$39.90/$478.80**
Updated April 1998	**$162.16/$1,945.88**	**$131.50/$1,578.06**
Monthly/Annual Cost 1997	**$118.40/$1,420.80**	**$106.80/$1,281.60**
Updated April 1998	**$119.88/$1,438.53**	**$108.13/$1,297.59**

dren five food items from four food components." (USDA 1998). These components are: (1) a serving of meat or meat alternate, (2) bread alternate (at least one each day and a total of eight servings over the course of a week), (3) two different fruits or vegetables, and (4) fluid milk. This is called the Type A lunch pattern or the traditional meal pattern. A USDA study, School Nutrition Dietary Assessment, found that although school lunches were meeting the nutritional basic rule, the lunches surveyed exceeded "recommended levels of fat and saturated fat established by the Dietary Guidelines" (USDA 1975).

Analyzed and revised every five years since their development in the late 1970s, the Dietary Guidelines for Americans are issued by the USDA and the Department of Health and Human Services. In 1990, the Dietary Guidelines recommended that people eat a variety of foods; maintain a healthy weight; choose a diet with plenty of vegetables, fruits, and grain products; and use sugar and sodium in moderation. The Dietary Guidelines also recommended diets that are low in fat, saturated fat, and cholesterol so that over time fat comprises 30% or less of caloric intake and saturated fat less than 10 percent of total calories for persons two years of age and older (USDA 1995).

On May 29, 1996, President Clinton signed Public Law 104–149, The Healthy Meals for Children Act, which states that schools may use any reasonable approach to menu planning that will achieve compliance with the nutrition standards, as long as the approach conforms to guidelines issued by the USDA.

Schools have until school year 1998/99 to comply with the most current recommendations of the Dietary Guidelines for Americans. School lunches, measured over one week, must provide one-third of the RDAs for protein, vitamin A, vitamin C, iron, calcium, and calories. School lunch planners need only consider these nutrients because studies have shown that these are the nutrients in which U.S. children are most likely to be deficient. Also, if the RDAs for these nutrients are met, the probability is high that the RDAs for other nutrients will also be met. In addition to the RDA constraints, no more than 30% of the total calories in the lunches may come from fat and no more than 10% of the total calories may come from saturated fat (USDA/CNP 1998). One of the ways school systems can meet these objectives is by using computer-based software with the generic name of NuMenus.

NuMenus "... is a computer-based menu planning system which allows menus to be planned without conforming to specific food components or quantity requirements. Approved software analyzes the nutrient content of foods prepared for school meals and enables the menu planner to adjust portion sizes and food components as needed to achieve compliance with nutrition standards. While menu planners are not bound by strict component and quantity requirements, they must, nonetheless, ensure that children are offered an entree, milk, and at least one other food item" (USDA 1998).

In January 1994, the USDA's Food and Nutrition Service published specifications for software that can be used to implement NuMenus (USDA 1994). Software companies that wish to provide a menu planning system to schools must first have it approved by the USDA. There are currently ten approved systems (USDA/NuMenus 1998).

6.0. SUMMARY

The statement and early solution of Stigler's diet problem predates much of what we now call operations research. Its influence on OR, however, has been early and extensive; its contributions cut across OR theory, computation, and application. The hand-computed solution of Stigler's original (9×77) problem, done in 1947, helped prove that the simplex method would work in practice. The concept of a diet or blending problem led the way to many minimum-cost applications: cattle and chicken feed, chemical and fertilizer blending. The inadequacy of the Stigler's diet problem to produce a nutritious and palatable human diet caused researchers to extend the approach to menu planning, which in turn raised new research questions in integer and goal programming. The diet problem has proved to be a pedagogical paradigm. Students at all levels readily understand the problem. The linear-programming model of the basic diet problem can be used to explain just about all linear programming assumptions and concepts (additivity, proportionality, nonnegativity, sensitivity analysis, duality). Student field projects involving the diet problem illustrate the difficulties of data collection (accuracy, consistency). It is a classic example of a "correct" mathematical model of a real-world problem that does not produce a valid solution.

Stigler's pre-linear programming approach to the modeling of a human diet—that is, the recognition that a diet can be expressed as constrained linear system (one that assumes additivity and proportionality intake of dietary nutrients)—forms the basis for how we now (1) evaluate the nutritional content of diets for school children, (2) plan menus for institutions (hospitals, jails), and in general, (3) manage food-systems. Although the minimum-cost diets we determined in our extended Stigler's diet model were more varied and more nutritious than Stigler's original diet, we do not suggest that such diets be followed. They do, however, represent low-cost baseline diets against which other nutritionally correct diets can be compared.

The diet conscious reader may wish to pick up the challenge posed by Steingarten (1998, p. 45):

> Now it's your job and mine to make something delicious out of our Simplex Subsistence Diet. Just remember: This is as close to the theoretically cheapest diet that will keep you alive and well nourished. Even if we add an extra ounce of sugar, a cup of coffee, and a little olive oil to make our lives more scrumptious, we can still beat the USDA and Thrifty Food Plan at its own game.

The so-challenged reader may wish to try Chef Daniel Boulard's menu for "swiss chard and bean soup with ricotta toast," which Steingarten includes in his book. He notes that this soup "… uncannily mirrors our Simplex Subsistence Diet." In 1993, it would have served four as a complete supper at $1.76 a person (Steingarten 1998, p. 45–47). *Bon appétit!*

APPENDIX A.

Stigler Name	Bowes & Church's Food Values Name	Giant Name
Wheat Flour	Wheat flour, white, all purpose, enriched	Super G stone ground whole wheat graham flour
Macaroni	Macaroni, enriched, cooked	Super G elbow macaroni (enriched)
Wheat cereal	Farina, cooked, enriched	Farina enriched creamy hot wheat cereal
Corn flakes	Corn flakes, Kellogg's	Kellogg's corn flakes
Corn meal	Cornmeal, yellow, degermed, enriched Quaker	Quaker yellow corn meal
Hominy grits	Corn grits, regular/quick, enriched, cooked	Quaker instant grits—original
Rice	White rice, long grain, enriched, cooked	Super G long grain white rice (bulk)
Rolled oats	Oats, regular/quick/instant, dry	Quaker Oats rolled oats (bulk)
White bread	Bread, white	Super G enriched white sandwich bread
Whole wheat bread	Bread, whole wheat	Home Pride wheat bread
Rye bread	Bread, rye, American	Super G seedless Jewish rye bread
Pound cake	Cake, pound, golden, from mix, Betty Crocker	Betty Crocker pound cake mix
Soda crackers	Cracker, oyster & soup, Keebler	Super G soup and oyster crackers
Milk	Milk, lowfat, 2% fat	Super G 2% milk fat reduced fat milk
Evaporated milk	Milk, evaporated, canned, Carnation	Carnation evaporated milk (canned)
Butter	Butter	Super G regular butter quarters
Oleomargarine	Margarine, Fleischmann's, stick	Fleischmann's original spread sticks
Eggs	Egg, chicken, boiled, hard/soft	Super G A medium eggs
Cheese	Cheese, cheddar	Super G Wisconsin sharp cheddar cheese
Cream	Light (coffee/table) cream	Super G table cream
Peanut butter	Peanut butter, creamy, Jif	Jif creamy peanut butter
Mayonnaise	Mayonnaise, Best Foods/Hellman's	Hellmann's mayonnaise
Crisco	Shortening, Crisco, regular/butter flavor	Crisco all-vegetable shortening
Lard	Animal fats, lard (pork fat), raw	Esskay lard
Sirloin steak	Top sirloin, separable lean, choice, broiled, 0″ fat trim	Top sirloin steak boneless choice (meat)
Round steak	Round, eye of, separable lean, roasted, choice, 0″ fat trim	Beef eye round steak choice (meat)

Continued.

Rib roast	Rib, small end (ribs 10-12), separable lean, choice, roasted, 1/4″ fat trim	Beef rib roast small end choice (meat) bone in
Chuck roast	Chuck arm pot roast, separable lean, braised, choice, 0″ fat trim	Beef chuck blade roast (w/bone)
Plate	Beef, flank, choice, separable lean & fat, braised, 0″ fat trim	Beef flank steak (meat)
Liver	Liver, beef, braised	Fresh baby beef liver
Leg of lamb	Lamb, domestic, leg, shank half, choice, roasted, separable lean	semi boneless lamb leg choice (meat)
Lamb chops	Lamb, domestic, rib, separable lean, choice, broiled	Lamb rib chops choice (meat)
Pork chops	Pork, center loin (loin chops/roasts), separable lean, roasted	Pork loin roast
Pork loin roast	Pork, center loin (loin chops/roasts), separable lean, roasted	Pork loin sirloin roast
Bacon	Bacon, cured, broiled/pan fried (unit = yield from 1 pound raw)	Generic sliced bacon
Ham	Ham, cured (fully cooked as purchased) lean (4–5% fat), roasted	Mash's fully cooked smoked ham
Salt pork	Belly, pork, raw	Hormel cured salt pork
Roasting chicken	Chicken, roaster, light & dark meat w/skin, roasted	All Natural fresh chicken
Veal cutlets	Veal, loin, separable lean braised	Veal cutlet—boneless (meat)
Salmon	Salmon, pink, canned, Libby's	Season pink salmon (canned)
Apples	Apple, raw, with skin	McIntosh apples
Bananas	Banana, raw	Bananas
Lemons	Lemon, raw	Lemons
Oranges	Orange, navel, raw	California navel oranges
Green beans	Green beans (snap beans) boiled	Green beans
Cabbage	Cabbage, green, raw	Green cabbage
Carrots	Carrots, raw	Wm. Bolthouse Farms cold water carrots
Celery	Celery, raw	Celery hearts
Lettuce	Lettuce, iceberg, raw	Iceberg lettuce
Onions	Onions, raw	Yellow onions
Potatoes	Potato, baked, with skin	US #1 baking potatoes
Spinach	Spinach, raw	Spinach
Sweet potatoes	Sweet potato baked, with skin	US #1 red sweet potatoes
Peaches	Peaches, canned, heavy syrup	Super G halves yellow cling peaches in heavy syrup
Pears	Pears, canned, heavy pack	Super G Bartlett pear halves in heavy syrup
Pineapple	Pineapple, canned, heavy syrup	Super G sliced pineapple in heavy syrup
Asparagus	Asparagus, canned	Super G all green asparagus cut spears
Green beans	Green beans (snap beans) canned	Super G cut green beans
Pork and beans	Pork and beans, in tomato sauce, canned	Super G pork and beans in tomato sauce
Corn	Corn, yellow, canned	Super G whole kernel golden corn
Peas	Peas, green, canned	Super G sweet peas
Tomatoes	Tomato, red, whole, peeled, canned	Super G California whole peeled tomatoes
Tomato soup	Soup, tomato	Campbell's tomato soup
Peaches, dried	Peaches, dried, sulphured	Ann's House of Nuts dried peaches (bulk)
Prunes, dried	Prunes, dried	Ann's House of Nuts whole prunes (bulk)
Raisins, dried	Raisins, seedless	Ann's House of Nuts seedless dark raisins (bulk)
Peas, dried	Peas, split, boiled	Goya green split peas
Lima beans, dried	Lima beans, baby, boiled, mature	Goya No. 1 grade baby lima beans
Navy beans, dried	Navy beans, boiled	Goya No. 1 grade navy beans
Coffee	Coffee, brewed	Super G coffee
Tea	Tea, brewed, black, 3 minutes	Super G tea bags
Cocoa	Cocoa, unsweetened, dry powder, Hershey	Hershey's cocoa
Chocolate	Baking chocolate, unsweetened	Hershey's baking chocolate—unsweetened
Sugar	Sugar, white, granulated	Super G granulated sugar (bulk)
Corn syrup	Syrup, corn, dark	Karo dark corn syrup
Molasses	Molasses	King Po-T-Rik molasses
Strawberry preserves	Jam/preserves	Super G strawberry preserves

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