On the Statistics of Individual Variations of Productivity in Research Laboratories*

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In the following pages a co-winner of the 1956 Nobel Prize in Physics presents a novel study of one of today's most precious commodities—scientific productivity. The author not only measures the variations that exist between different research workers, he also explains these differences and draws some specific conclusions about the relationship of salary to productivity. PROCEEDINGS readers will find this an especially timely and significant discussion, particularly in view of the present widespread concern about manpower shortages and proper utilization of scientific personnel.—*The Editor*

***Summary*—It is well-known that some workers in scientific research laboratories are enormously more creative than others. If the number of scientific publications is used as a measure of productivity, it is found that some individuals create new science at a rate at least fifty times greater than others. Thus differences in rates of scientific production are much bigger than differences in the rates of performing simpler acts, such as the rate of running the mile, or the number of words a man can speak per minute.**

On the basis of statistical studies of rates of publication, it is found that it is more appropriate to consider not simply the rate of publication but its logarithm. The logarithm appears to have a normal distribution over the population of typical research laboratories. The existence of a "log-normal distribution" suggests that the logarithm of the rate of production is a manifestation of some fairly fundamental mental attribute. The great variation in rate of production from one individual to another can be explained on the basis of simplified models of the mental processes concerned. The common feature in the models is that a large number of factors are involved so that small changes in each, all in the same direction, may result in a very large change in output. For example, the number of ideas a scientist can bring into awareness at one time may control his ability to make an invention and his rate of invention may increase very rapidly with this number.

A study of the relationship of salary to productivity shows that rewards do not keep pace with increasing production. To win a 10 per cent raise a research worker must increase his output between 30 and 50 per cent. This fact may account for the difficulty of obtaining efficient operation in many government laboratories in which top pay is low compared to industry with the result that very few highly creative individuals are retained.

I. INTRODUCTION

EVERYONE who has been associated with scientific research knows that between one research worker and another there are very large differences in the rate of production of new scientific material. Scientific productivity is difficult to study quantitatively, however, and relatively little has been established about its statistics. In this article, the measure of scientific production I have used is the number of publications that an individual has made.

The use of the number of publications as a measure of production requires some justification. Most scientists know individuals who publish large numbers of trivial findings as rapidly as possible. Conversely, a few outstanding contributors publish very little. The existence of such wide variations tends to raise a doubt about the appropriateness of quantity of publication as a measure of true scientific productivity. Actually, studies quoted below demonstrate a surprisingly close correlation between quantity of scientific production and the achievement of eminence as a contributor to the scientific field.

The relationship between quantity of production and scientific recognition has been studied recently by Dennis,[1] who considered a number of scientists who have been recognized as outstanding. As a criterion of eminence for American scientists, he has used election to the National Academy of Sciences; his study is based on 71 members of the National Academy of Sciences who lived to an age of 70 or greater and whose biographies are contained in the Biographical Memoirs of the Academy. He finds that all of these people have been substantial contributors to literature with the range of publications extending from 768 to 27, the median value being 145. (Based on a productive life of approximately 30 years, this corresponds to an average rate of publication of about 5 per year, a number to which I shall refer in later parts of this discussion.) Dennis concludes that relatively high numbers of publications are characteristic of members of the National Academy of Sciences. He conjectures that of those who have achieved the lesser eminence of being listed in American Men of Science, only about 10 per cent will have a

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[1] Wayne Dennis, "Bibliography of eminent scientists," *Sci. Monthly*, vol. 79, pp. 180–183; September, 1954.

publication record exceeding the 27; which represents the minimum publisher of the 71 listed in Biographical Memoirs of the National Academy of Sciences. He has also studied eminent European scientists and comes to essentially the same conclusion. In fact his study goes further and shows that almost without exception heavy scientific publishers have also achieved eminence by being listed in the Encyclopedia Britannica or in histories of important developments of the sciences to which they contributed.

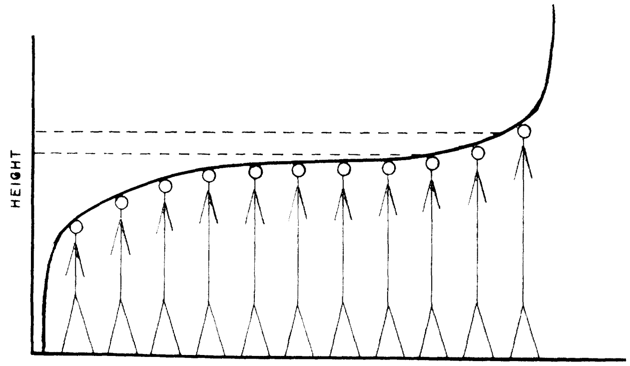
It should be remarked that in Dennis' work, he includes more routine types of contributions (such as popular articles) than are generally associated with scientific eminence. However, it may still be appropriate to quote a few of the statistics obtained by Dennis for people who certainly classify in the genius class of the scientific publishers. Among these Dennis refers to: Pasteur with 172 publications, Faraday with 161, Poisson with 158, Agassiz with 153, Gay-Lussac with 134, Gauss with 123, Kelvin with 114, Maxwell with 90, Joule with 89, Davy with 86, Helmholtz with 86, Lyell with 76, Hamilton with 71, Darwin with 61, and Riemann with 19. Riemann, who was the least productive, died at the age of 40. At his rate of publication, he would probably have contributed at least another 10 or 20 publications had he lived to the age of 70. Even with 19, he was in the top 25 per cent of the 19th century scientists referred to in Dennis' study.

The chief conclusion reached in this article is that in any large and reasonably homogeneous laboratory, such as, for example, the Los Alamos Scientific Laboratory and the research staff of the Brookhaven National Laboratory, which are included in this study, there are great variations in the output of publication between one individual and another. The most straightforward way to study these variations is to list the number of individuals with zero, one, two, etc., numbers of publications in the period studied. This compilation may then be plotted as a distribution graph [see Fig. 2(b) for an example]. In some cases, however, the data are too meager for a smooth trend to be seen easily and another form of presenting the data is more convenient.

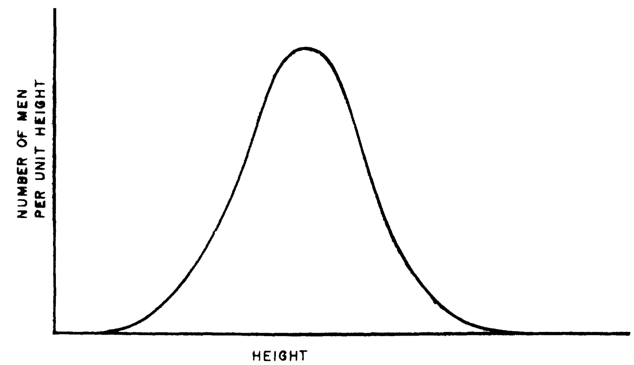
The form used for most of the data presented in this paper is the *cumulative distribution graph.*

Such a graph can be illustrated in terms of the distribution of the height of a regiment of men. If the men are lined up in order of increasing height at a uniform spacing, then, as shown in Fig. 1(a), there will be a steady increase in height from the shortest man to the tallest man. There will usually be a few men who are exceptionally short, a few men who are exceptionally tall. For the majority of the men the height will vary relatively uniformly along the line of the men. In general, one should thus expect an S-shaped curve with an inflection point near the middle of the distribution.

Such a curve is closely related to the distribution in height shown in Fig. 1(b), which represents the number of men whose height lies in any particular interval of height. This can be obtained from Fig. 1(a), as is represented there, by drawing two lines bracketing a certain interval in height and counting the number of men lying in this range. Fig. 1(b) represents a smooth curve drawn through such a distribution. It can, in fact, be obtained from Fig. 1(a) by drawing a smooth curve through the distribution in height and differentiating the number of men as a function of the height.



(a)



(b)

Fig. 1—The cumulative-distribution graph and the normal-distribution curve. (a) The cumulative-distribution graph represented by men arranged in order of height at uniform spacing. (b) A "smoothed" distribution curve, of normal form, such as might be obtained from (a) by finding the number of men in each small increment of height.

For many natural phenomena and in particular for those in which the measured quantity varies due to the additive effects of a large number of independently varying factors of comparable importance, a Gaussian or normal distribution, like that of Fig. 1(b), is obtained. Conversely, if distribution is normal, then the cumulative distribution graph will have the symmetrical S-shaped characteristic in Fig. 1(a), the middle flat portion corresponding to large numbers of cases in the central range, and the rapid convergence of the extremes to their asymptotes corresponding to the scarcity of cases which deviate much from the mean value.

One of the new results of this study, presented below in Section IV, is that the data on rates of publication can be well represented by a normal distribution when treated in a certain fashion. Some possible explanations for this observation are discussed in Section VI.

II. A Study of Publication Records

As a first example, I shall discuss the statistics of the publications of a group of people in the Los Alamos Scientific Laboratory. This sample of approximately 160 people was selected on the basis that the individuals were professionally mature and located in laboratories whose activities are of such a nature that there is some probability that workers in them might contribute to a physical or electrical engineering publication. Such publications are abstracted in *Science Abstracts A* and *B*, respectively. The publication record for each individual was ascertained by looking through the author index of *Science Abstracts* for the years 1950 to 1953, inclusive.

From these data, a cumulative-distribution graph constructed like that shown in Fig. 1(a) is obtained by listing the men in order of their publications. It is found that approximately half of the individuals have no publications at all. Then there are about 30 individuals with one publication, 20 individuals with two publications and so on. The cumulative-distribution curve shown in Fig. 2(a) has little resemblance to the simple S-shaped curve shown in Fig. 1(a). For one thing it is concave upwards throughout. For another it shows too many individuals with publication rates higher than seven in four years compared to the shape of the curve up to that rate. The distribution curve, shown in Fig. 2(b), is not normal, but instead is essentially hyperbolic in form.

Replotting this same data in Fig. 3 on a logarithmic scale for the number of publications results in a line which does look much more like a portion of the cumulative-distribution graph for a normal distribution. The line is not a smooth curve, of course, but rises in steps. However, a smooth curve drawn through the steps has an approximately linear portion, corresponding to the linear portion of Fig. 1(a), followed by an abrupt turn up at the high end corresponding to the relatively small number of people who on the logarithmic scale have exceptionally large rates of publication.

It is one of the chief conclusions of this study that the more or less normal distribution of the logarithm of rate of publication is characteristic of the statistics of the scientific creative process. Perhaps the most important feature of this conclusion is that the rate of publication increases approximately exponentially from individual to individual, taken in order of increasing rate, and that the differences in rate between low and high producers are very large. The conclusion that the exponential character of the distribution is fundamental to the creative process gains support from the fact that certain other hypotheses intended to explain it as some sort of artifact can be examined and rejected.

In subsequent sections we shall refer to the normal distribution of the logarithm as *log-normal distribution.*

III. Some Basic Data on Rates of Publication

One of the first hypotheses called the "organization hypotheses" put forward to explain how the log-normal

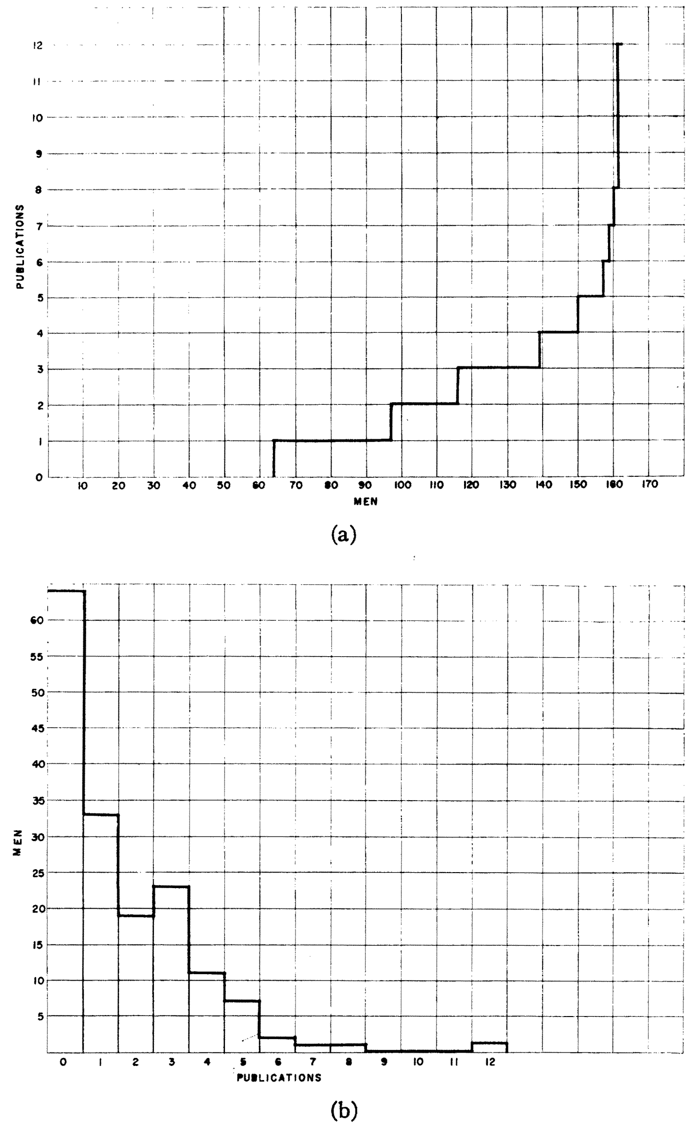


Fig. 2—Distribution of rate of publication (number of entries in *Science Abstracts A and B* in four years) at Los Alamos. (a) Cumulative distribution. (b) Distribution (number of men with each rate of publication).

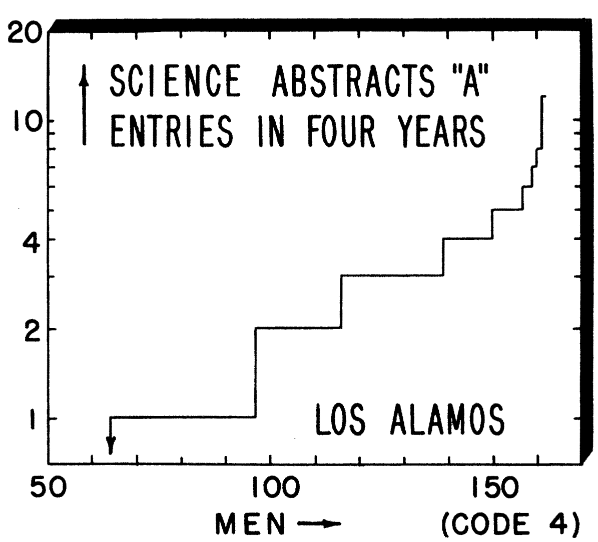


Fig. 3—Cumulative distribution on logarithmic scale for number of publications at Los Alamos.

distribution arises was that it is a consequence of the organization of research activities in large, modern laboratories. In such laboratories, physical scientists frequently make use of very complicated apparatus and large nuclear machines. As a result of this collaborative effort they frequently publish papers jointly, the number of authors varying from two to five or six in ordinary cases. The "organization hypothesis" endeavors to use joint authorship to explain the exponential character as follows: As a consequence of the size of the teams who work together, an individual who has some supervisory or organizational responsibility may contribute to the activities of many men and be listed as a co-author on many papers. As a result, a relatively few people will appear as co-authors of a very large number of papers and this group can be better included in a log-normal distribution than in a normal distribution.

This "organizational hypothesis" can be disposed of by several arguments, some of which are quite instructive. One of these arguments is based on the observation that the exponential aspects of the cumulative-distribution graph is independent of the particular organizational features of the laboratory considered and is a general characteristic of all laboratories studied in this article. For example, the organizational situation in some of the laboratories of the National Bureau of Standards would not lead to large numbers of publications by supervisors. For one Division of the National Bureau of Standards, records were available of the total number of publications and patents made by the individuals in this Division during a period of several years. These data are shown in Fig. 4. It is seen from this figure that the data lie on a relatively smooth exponentially increasing trend followed by a rapid turn-up corresponding again to a few individuals with exceptionally high publication records. Since the organization of activities is quite different in the Bureau of Standards from what it is at Los Alamos while the distribution curve is the same, the "organizational hypothesis" can be discarded.

The "organizational hypothesis" can also be rejected by studying the effect of joint authorship on the distribution of rate of publication. We shall illustrate this argument using data from the Brookhaven National Laboratory. There are approximately 180 members of the research staff of the Brookhaven National Laboratory. The "total" number of entries plotted as a cumulative distribution for these people is shown as the line marked "total" in Fig. 5. Since Brookhaven operates in a fashion rather similar to Los Alamos, it might be expected that the "organizational hypothesis" would apply equally well here. In order to test this, two other lines have been constructed on Fig. 5.

The bottom line, marked "solo," has been obtained by discarding all publications having more than one author. It is seen that a relatively small fraction of the people have made "solo" publications. However, it should be noted that the most prolific publishers of these have published at nearly half the maximum rate for the "total" line. On the other hand, a large number of people who appear as co-authors in the "total" distribution have no "solo" publications whatever. This fact shows that the rapidly rising part of the line is due

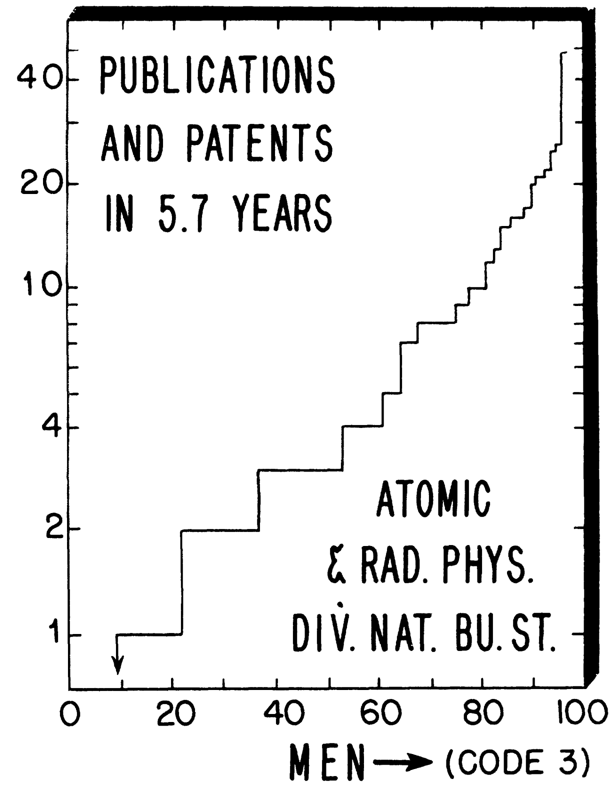


Fig. 4—Cumulative distribution on logarithmic scale for publications and patents for Atomic and Radiation Physics Div., National Bureau of Standards, for a period of 5.7 years.

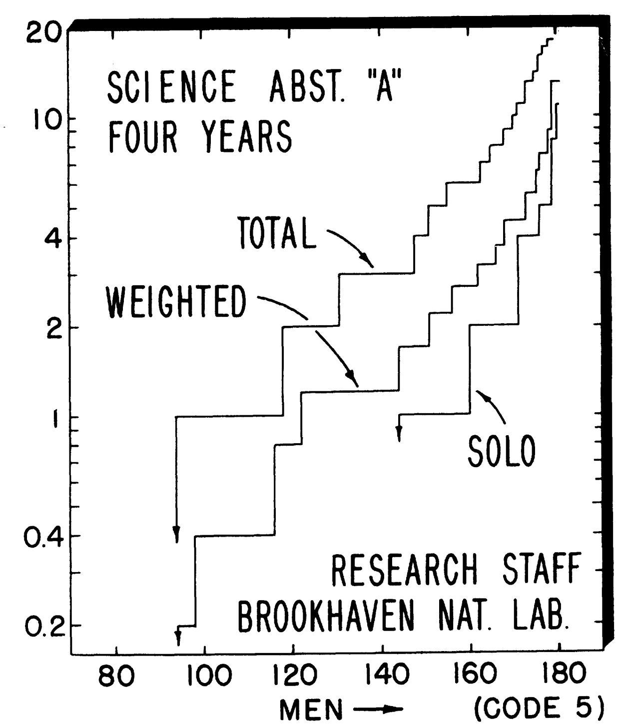


Fig. 5—Cumulative distributions on logarithmic scale for 3 cases at Brookhaven National Lab.

largely to people who are capable of producing "solo" publications, a conclusion contrary to the expectation based on the "organizational hypothesis." In fact, the evidence is that publication of about half of the people is supported by the more productive ones who would be capable of publishing at relatively high rates strictly on their own.

The middle line marked "weighted" is obtained by dividing the credit for multiple-author publications equally among the various authors. For example, each man on a four-author publication receives a contribution of 0.25 publication. The "weighted" line again shows the steadily increasing trend and does not permit an undue credit to be given to people who, through organizational position, may appear as a joint author on a large number of publications. This furnishes further support for the thesis that the exponential trend of the cumulative distribution is a fundamental characteristic of the distribution of productivity among the members of the laboratory rather than some organizational artifact.

Another possible explanation which can also be discarded is that the distribution of degree of publication from one person to another is a consequence of the distribution in age of the population considered. In principle, some such distribution might be obtained as a result of distribution in age since people on the average have a maximum in their publication rate at an age of about 35. The distribution of publication in age has been studied by Lehman.[2] Some of Lehman's results for rate of publication as a function of age are shown in Fig. 6. Very similar results are obtained for other geographical samples. Actually, what Lehman has studied is not simply publication record but "creative production." He judges creative production by references found in histories of science and other similar sources. Since the distribution of workers in the laboratories considered in this study shows a fairly uniform distribution from age 25 to age 50, it is difficult to see how the variation in productivity with age as shown in Fig. 6 could result in a very small fraction of people with exceptionally high publication rates: from Fig. 6, we would estimate that the maximum publication rate would be perhaps twice the publication rate of the median man. In contrast to this, the studies shown for Figs. 2, 3, and 4 correspond to maximum publication rates substantially more than ten times that of the median man.

IV. The Log-Normal Distribution of the Rate of Publication

The conclusion is thus reached that the exponential variation of productivity in the cumulative distribution graph is a characteristic feature of the statistics of productivity in a research laboratory. This conclusion receives further support from an additional analysis of the

[2] H. G. Lehman, "Men's creative production rate at different ages and in different countries," *Sci. Monthly*, vol. 78, pp. 321–326; May, 1954.

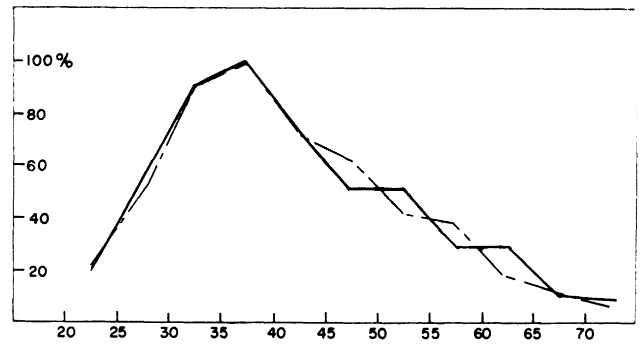


Fig. 6—Creative production rate in science and mathematics vs age for (solid line) nationals of 14 different countries other than Russia, England, France, Italy, Germany, and the U.S.A. and for (broken line) U.S.A.

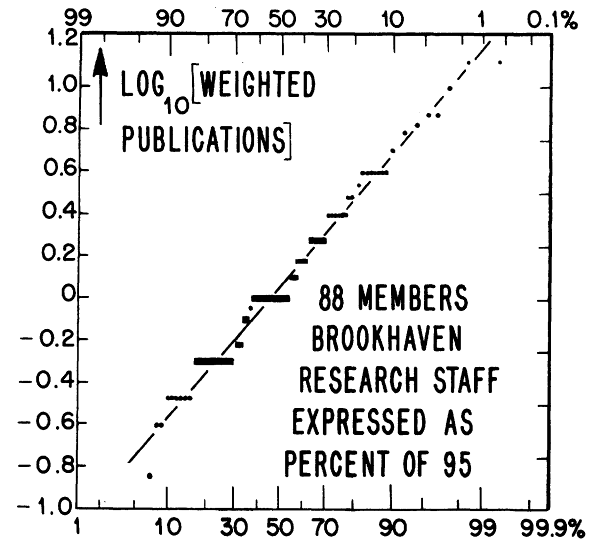


Fig. 7—Cumulative distribution of logarithm of "weighted" rate of publication at Brookhaven National Lab. plotted on probability paper.

data which show that the logarithm of the rate of publication can be well represented as a normal distribution in the cases studied.

The validity of the assumption of a normal distribution can be tested by making use of so-called "probability paper." On such paper, the cumulative number of men is expressed on a percentage scale. This percentage scale is so distorted as to increase the spread on the scale at percentages near the extreme distribution. This results in stretching out the ends of the cumulative-distribution graph of Fig. 1(a) so that it becomes a straight line, provided the distribution itself is normal.

Such a test has been applied to the weighted rate of publication for the Brookhaven Laboratory shown in Fig. 5. The result is shown in Fig. 7. It is seen that a straight line can be drawn in a very satisfactory way through the data with the exception of the two extreme men. It should be noted that in many cases so many men were assigned the same publication number that they have been represented as solid blocks on the diagram rather than as individual points. This grouping together is a genuine effect in the case of people who

published one "solo" publication during the period studied and thus have a logarithm of zero and those who have appeared on two publications or as a co-author of a single two-author publication and appear at logarithms of 0.3 and −0.3. Some of the other groupings have resulted artificially from the means of handling the statistics: for simplicity in listing the people, the scale of possible publications was divided into intervals and those whose publication rates fell in these intervals were grouped together. If this had not been done, the data would fall more closely along a straight line, *i.e.*, the "fit" to the normal distribution would be better.

Fig. 7 illustrates strikingly the range of variation in rate of publication—a factor of 40-fold between lowest 10 per cent and highest 5 per cent.

The fit shown on Fig. 7 is based on the assumption that the research staff of Brookhaven may be divided into two parts, one part containing 95 members who have some likelihood of publishing physics papers referenced in *Science Abstracts A* and 85 others with negligible likelihood of making such publications. The number 95 was found by trial and error to give the best straight line in Fig. 7. This arbitrary procedure does have justification in terms of the distribution of activities in the Brookhaven Research Staff. In fact if the list of members of the Research Staff at Brookhaven is examined name by name, it is found that many are biologists, medical physicists, and the like whose fields are not covered by *Science Abstracts*. The final conclusion is that all but 101 names are considered extremely unlikely to make publications abstracted in *Science Abstracts A*. Since the difference between 101 and 95 is negligible in respect to other uncertainties in the study, we may conclude that for the publishing part of the population the rate of publication is well represented by a normal distribution on the logarithmic scale, or for brevity, a *log-normal* distribution.

Generally similar fits are obtained for the Los Alamos data and for the National Bureau of Standards data. Furthermore, the data on "total" and "solo" entries in *Science Abstracts A* can be fairly well fitted by log-normal distributions. The fit is very "jumpy," however, since the only possible values for publication rates are integers. On the basis of the rather limited investigation that I have carried out to date in regard to the distributions for "solo" and "total" rates of publication, it appears that these also have log-normal distributions except that the rates of publication differ from the "weighted" rates by factors of 0.6 and 1.6, respectively.

It would be interesting to compare the statistics of science departments in universities with those of the large laboratories studied above. This has not yet been done except for the limited data on the Physics Department of Columbia University shown in Fig. 8. In spite of the smallness of the sample, the general trend of the data is such as to give confidence that the log-normal distribution will also hold in such cases.

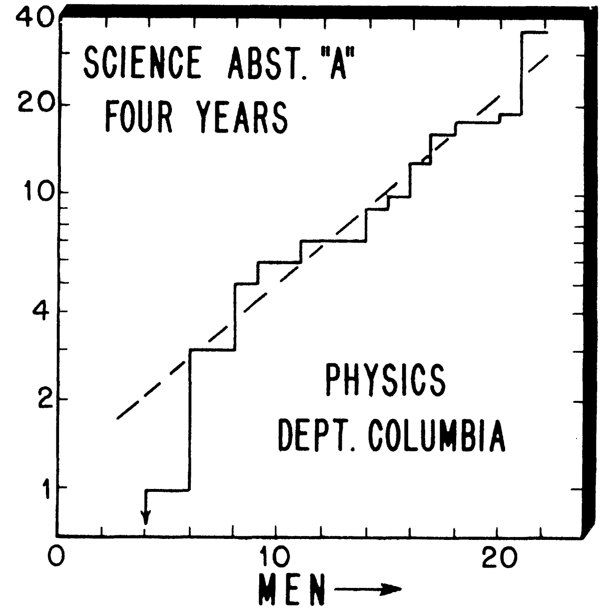


Fig. 8—Cumulative distribution on logarithmic scale for publications of the Physics Dept., Columbia Univ., for 4 years.

V. A Study of Patent Activity

Another measure of creative technical production, which is relatively readily available for study, is patent activity. Shown in Figs. 9 and 10 (opposite) are cumulative-distribution curves for patents for two large laboratories in the fields of electrical apparatus and communications. All of the data correspond essentially to "solo" publications since the number of joint patents is very small compared to individual patents.

It is instructive to compare patents with publications. Such a comparison is presented in Fig. 11 for a selected group of 60 men from one of the laboratories considered in Figs. 9 and 10. The most significant factor to note is that on the logarithmic scale, the patent distribution is markedly steeper.

VI. Speculations on the Origin of the Log-Normal Distribution

The very large spreads in productivity, for example the variation by nearly one hundred fold between extreme individuals in Fig. 7, are provocative of speculation. Most rates of human activity vary over much narrower limits, for example, pulse rates outside the two to one range from 50 to 100 per minute are extremely rare. Very few individuals walk at speeds outside the range of 2 to 5 miles per hour. In competitive activities involving trained and selected people, such as running the mile, the variation is much smaller, the ratio of speed for the mile between world's record and good high school performance being probably less than 1.5.

In the study presented here the individuals are presumably specially selected by natural ability and specially trained to accomplish scientific production. Yet the spread in rates is enormously greater than it is for the more physical activities discussed above. I believe

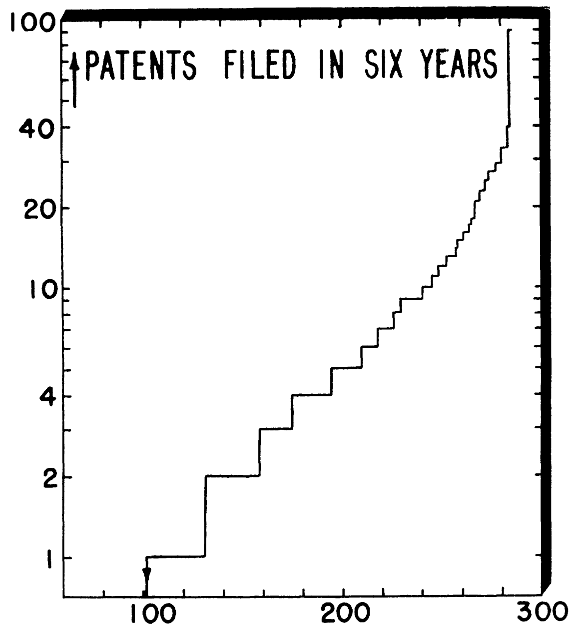


Fig. 9—Cumulative distribution on logarithmic scale for patents at a large industrial laboratory.

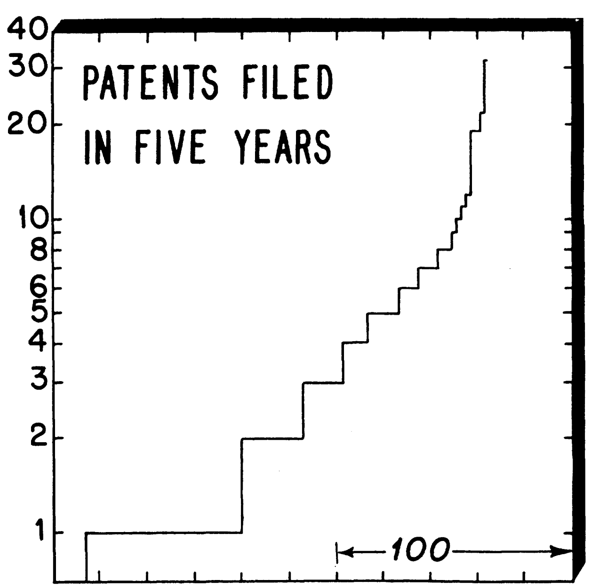


Fig. 10—Cumulative distribution on logarithmic scale for patents at another large industrial laboratory.

that it is possible to explain to some degree how such large variations in rate may occur in terms of certain characteristics of the creative scientific process. The basis of the explanation is that the large changes in rate of production may be explained in terms of much smaller changes in certain attributes. I shall illustrate this in terms of a simplified example of the inventing process.

In order to make an invention for which the United States Patent Office will issue a patent, it is, in general, necessary to conceive a new combination of features and

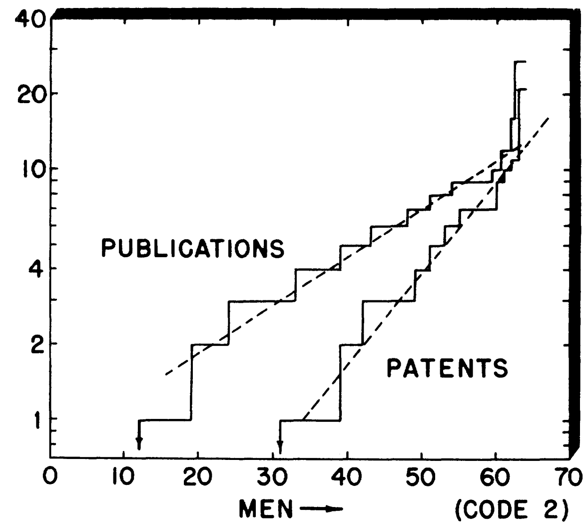


Fig. 11—Comparison of patent and publication activity for a group of research workers at a large industrial laboratory.

to appreciate how this combination may be useful. Let us suppose that the inventor perceives that he has made an invention when he appreciates the relationship between some number of ideas. For example, the automobile self-starter might have been conceived by recognizing the relationship of the following 4 ideas: the idea that a means of starting the engine without using human, muscular strength would be useful, the idea that the necessary energy could be held in reserve in a storage battery, the idea that a relatively small high speed electric motor could be used to turn the larger gasoline engine at starting speed, and the idea that the electric motor could be subsequently disengaged in order to avoid rotating it at excessive speeds.

Now let us suppose that there is some attribute of the human brain which allows an individual to be aware of "m" ideas and their relationships.[3] Then it follows that a man with $m=3$ will never invent the self-starter in the form discussed above whereas a man with $m=4$ can do so. A man with a higher value of m is much more likely to make the invention than a man with $m=4$. In fact, it may be established, by use of the formulas for permutations and combinations, that men with $m=5$, 6, and 7 can hold the 4 essential ideas in awareness (together with 1, 2, or 3 irrelevant ideas) in 5, 15, and 30 times as many ways as the man with $m=4$. This shows that a variation of 50 per cent in "brain capacity" ($m=4$ to $m=6$) can produce an increase in invention rate of 15-fold for inventions requiring the interaction of 4 ideas.

It may be instructive to illustrate the considerations presented above by an example which can be shown in detail. Suppose out of realm of idea associated with some field of endeavor an invention can be made by

[3] N. Rashevsky, "Mathematical Biophysics," University of Chicago Press, Chicago, Ill., ch. 29; 1938, presents very similar reasoning. His results are expressed in the form of equations rather than by numerical examples and lead to somewhat more general conclusions than those presented here.

holding ideas "1" and "2" in mind and seeing the relationship between them. Then a man with $m=2$ can make the invention in two ways as represented below:

$$(1, 2) \text{ and } (2, 1).$$

But a man with $m=3$ can think of these two ideas and some irrelevant idea x in six ways:

$$(1, 2, x),\ (2, 1, x),\ (1, x, 2),\ (2, x, 1),\ (x, 1, 2),\ (x, 2, 1).$$

Thus for every case in which the $m=2$ man can think of the idea, there are 3 ways in which the $m=3$ man can do it. Thus the $m=3$ man has 3 times as many chances to make the invention.

Evidently this advantage increases rapidly with the increasing complexity of the problem. For a 10-idea invention an 11-idea man has an 11-fold advantage over a 10-idea man; that is a 10 per cent increase in "mental capacity" produces a 1100 per cent increase in output. It is my impression that this sensitivity to the interaction of many factors in mental creativity is the key to the large variations in output found in this study. According to this explanation, the log-normal distribution in productivity then results from a normal distribution, over a relatively small range (say $m=8$ to $m=12$ in the model considered), of some attribute which controls productivity in a very sensitive way.

Still another way of rationalizing the log-normal distribution may be based upon the hypothesis that the interacting mental factors are of several different kinds rather than several of one kind, as in the case of several ideas as discussed above. For example, consider the factors that may be involved in publishing a scientific paper. A partial listing, not in order of importance, might be: 1) ability to think of a good problem, 2) ability to work on it, 3) ability to recognize a worthwhile result, 4) ability to make a decision as to when to stop and write up the results, 5) ability to write adequately, 6) ability to profit constructively from criticism, 7) determination to submit the paper to a journal, 8) persistence in making changes (if necessary as a result of journal action). To some approximation, the probability that a worker will produce a paper in a given period of time will be the product of a set of factors F_1, F_2, etc. related to the personal attributes discussed above. The productivity of the individual would then be given by a formula such as

$$P = F_1F_2F_3F_4F_5F_6F_7F_8. \tag{1}$$

Now if one man exceeds another by 50 per cent in each one of the eight factors, his productivity will be larger by a factor of 25. On the basis of this reasoning we see that relatively small variation of specific attributes can again produce the large variation in productivity.

The factor explanation discussed above also has an appeal from the point of view of the log-normal distribution. According to the formula, the logarithm of the product is the sum of the logarithms of the several factors. If we suppose that these factors vary independently, then to a good approximation their sum will have a normal distribution, and so, consequently, will the logarithm of the productivity. It seems to me that this is at present the most attractive explanation for the log-normal distribution.

In closing this section mention should be made of an attempt to fit the data by assigning to each individual a single parameter describing his creative potential. This parameter was referred to as "mental temperature" when the original lecture was given. It was introduced in analogy with the quantity β or $1/kT$ which occurs in the equation for rates of chemical reaction or thermionic emission. According to this hypothesis an individual i is characterized by a value β_i. In a situation s his rate of production is determined by a rate constant P_s and a barrier U_s, so that his rate of production is

$$P(i, s) = P_s \exp(-U_s\beta_i). \tag{2}$$

The rate constant P_s probably depends on β_i but in a relatively insensitive way, so that to a first approximation this dependence can be neglected.

On the basis of this equation, the difference between the two curves of Fig. 11 is to be attributed to a U value 1.7 higher for patents than for publications.

There appears to be a tantalizing possibility of establishing scales for U and β by comparing publications and patents and one laboratory with another. One might, for example, assume that the distribution of β values is the same in two laboratories having the same pay scales and similar working conditions. Then if U is chosen as unity for one activity in one of these, the scale of U can be chosen for the other cases in terms of the ratio of slopes like those of Fig. 11. Approximate values of P_s can be chosen by assuming that $\beta=0$ represents a situation in which the worker never lacks an idea to publish or an invention to patent so that his rate of production is limited by the mechanics of the situation. Such cases might correspond approximately to the most outstanding publishers in Dennis' study. On this basis P_s values of the order of 10 per year for either publications or patents might be chosen. I have made some attempts to establish scales of this sort but they are not well enough developed to warrant inclusion here.

VII. The Relationship Between Salary and Productivity

From the point of view of the economics of running a research laboratory, it is important to know the relationship between salary and productivity. For example, if the better paid men are more productive than their fellows in greater proportion than the increase in pay, then they are a sound investment. On the other hand, if they are less productive per salary dollar, then it may be wiser to hire relatively fewer of these outstanding people.

The question just posed is to some degree academic—anyone who has had to do with managing research knows that progress depends largely on a relatively

small number of exceptionally able individuals. He also knows that these people are usually substantially better paid than their fellows. How much better one can afford to pay outstanding people and still find them profitable is a quantitative question faced by many organizations during periods of rapid build up. The findings in this section throw some light on this question, the conclusion being that, in general, scientific productivity is so much greater for the outstanding people that in the current scientific labor market, it is unlikely that they will be overpaid.

It is clear, of course, that increasing salary of an individual will usually not increase his productivity much, if at all. In some cases it may even have the opposite effect by reducing incentive. What is studied here is the statistical relationship between salary and productivity as established by existing pay roll procedures. If any causal relationship is important in this connection, it is that high productivity of an individual causes the management to give him high rewards.

Before considering the method of investigating the statistical relationship between salary and productivity, it may be worth-while to say something about salary in general. In determining the salary of an individual in a research laboratory, the management takes into account many factors. Only one of these factors is considered in the previous parts of this study, namely, the rate of scientific production as measured by total numbers of publications or patents. This factor is probably rarely considered in a quantitative way. Instead, the usual procedure is for a group of people charged with supervising research workers to gather together and discuss the relative merit of the individuals. In such considerations, quantitative measures of the individual's contributions are seldom referred to. There probably does not exist at the present time any valid analysis of the various factors that are considered and their relative importance. Among them may be mentioned, however, the originality and importance of publications which are made. Thus quality as well as quantity is brought into account. Other factors which are certainly considered are the ability of an individual to carry out the techniques of his work, whether these be of a theoretical nature involving pencil and paper or the manipulation of apparatus; the ability to contribute to the solutions of problems of other workers in the organization; the ability to produce cooperation among other workers; the ability to attract productive candidates to the organization; the ability to influence the activities of other workers along lines which are more wisely chosen than they would choose themselves with respect to the goals of the organization as a whole; the ability to carry out activities which enhance the prestige of the organization. These and many other factors are generally considered in determining a man's "merit" and thus deciding what salary he should receive.

The assumption of this article is that merit and salary are somehow determined by the combination of such factors as those which we have discussed above. These factors are not closely correlated with each other, although it is probable that there is a tendency for outstanding ability in any one to be coupled with a probability of higher abilities in the others as well. The only attribute which has been studied here is simple quantitative productivity in the sense of publications and patents. If it is found that this attribute, which was studied purely for purposes of convenience, is strongly correlated with increasing salary, then it seems likely that the other attributes are also strongly correlated with salary.

It is not appropriate to consider simply the relationship of salary to productivity. The reason for this is that there is a general tendency of salary to increase with age, this being a recognition of increasing general judgment and experience with age as well as a socially acceptable procedure. Thus, in order to get a truly representative comparison of merit with productivity, it is necessary to correct for age. This procedure can be done in various ways; the one selected for this article being that associated with the concept of "merit quartiles."

The division of the population of a laboratory into "merit quartiles" may be illustrated with the aid of Fig. 12. This figure represents the salaries of a group of

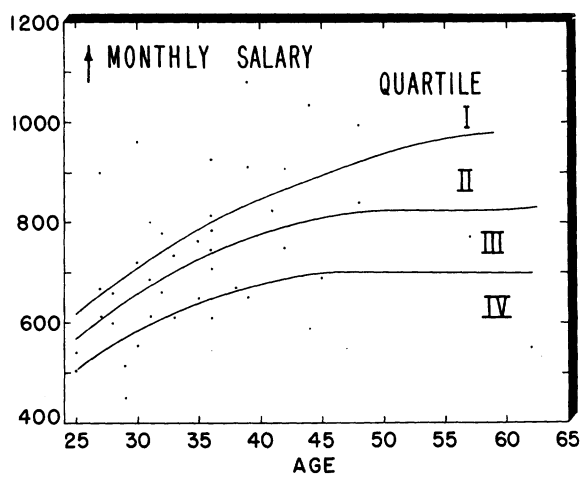


Fig. 12—Salary vs age (for a representative sample only of individuals) in a laboratory considered in this study with lines dividing the distribution into "merit quartiles"). Effective about October, 1954.)

individuals in a laboratory covered in this study. Each individual is represented by a point on the figure which shows his salary and his age. Three lines have been drawn on the figure dividing it into four groups of individuals, called quartiles. The procedure for drawing these lines is as follows: in each relatively small age interval the population of the laboratory is divided into halves such that half of the group gets more than the median salary and half less. Then the upper and lower halves are similarly divided into 2 equal parts so that each age interval is divided into 4 quartiles. This procedure is carried out for the various age intervals and then a smooth curve is drawn. These smooth

curves are drawn in such a way that at each age interval, approximately $\frac{1}{4}$ of the population of the laboratory lies in quartile I and approximately $\frac{1}{4}$ in each of II, III, and IV. Thus the people in the first or top quartile have approximately the same age distribution as those in the second, etc. Furthermore, all of the people in the top quartile obtain higher salaries than those in the second quartile at the same age.

These merit quartiles furnish a basis for dividing the laboratory into parts in accordance with salary but chosen in such a way that the age distribution in each part is similar. Thus any effect of varying productivity with age affects all the quartiles about the same way.

Fig. 13 shows a similar plot for the individuals in a U. S. government laboratory operated under Civil Service. It is to be noted that the highest salaries at any age range are substantially lower than those in the other non-Civil Service laboratory. The difference would be even more striking if the higher paid executive types of an industrial laboratory could also be shown.

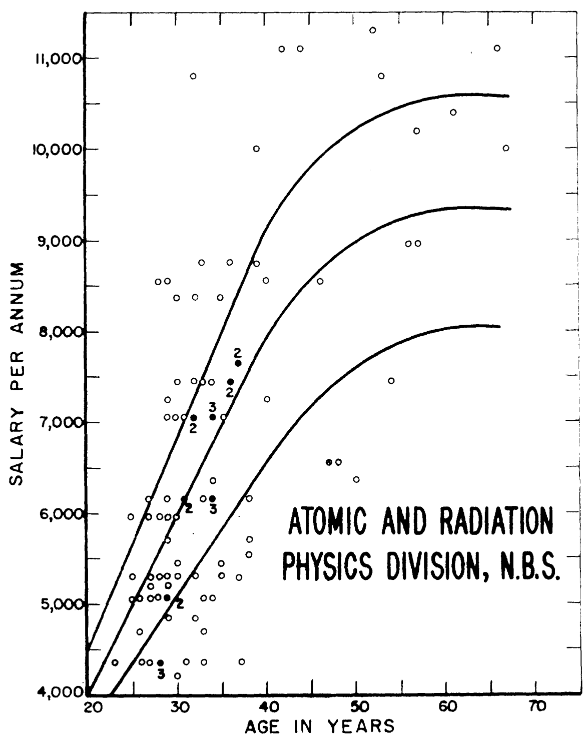


Fig. 13—Salary vs age for Atomic and Radiation Physics Div., National Bureau of Standards, together with "merit quartiles" divisions. (Effective about October, 1954.)

The use of merit quartiles, deciles, or similar divisions is playing a progressively more important role in salary administration.[4] One of the great advantages of the merit scale is that it provides an intuitively satisfactory way of ranking the individuals in an organization. The same would not be true if the men were ranked simply according to salary; thus a very able young man at a relatively low salary would be obviously out of place in company with an older group of average ability (but with more experience) at the same salary and it would be difficult to get any sense of order from such listing. On the other hand, a group of supervisors can come to agreement and reach decisions surprisingly easily about merit rank between people whose ages and salaries may differ by large amounts. I do not believe that it is evident in any *a priori* sense that such agreement would be expected; it appears rather an interesting and useful experimental result. In a sense, it is a surprising result since, as discussed above in this section, such diverse factors are considered in making the judgment. The agreement as to merit ranking by a supervisory group does not, of course, imply that the worth of the individual is truly assessed in any absolute sense. However, the large degree of consistency does imply that a useful and impartial tool for salary administration exists.

In principle, an organization can establish a family of merit curves at each raise period (allowing for cost of living adjustments, changing competition, etc.). The new salary for a man whose merit rank is correctly appraised can then be simply read off his location on the new curves. It sometimes happens, due perhaps to accidents of recruiting or due to changing skills on the part of the worker, that a revision of merit rating occurs. It is generally felt that only a fraction, say 50 per cent, of the correction should be made in any one raise since this will tend to smooth out fluctuations in the salary system.

A set of quartiles like those shown in the two previous figures have been prepared for the research staff of the Brookhaven National Laboratory. For each one of these quartiles, which contain about 46 men each, the publication records have been compiled as cumulative-distributions. These are not presented as graphs with steps since there are so many cases of overlap that the lines for different quartiles are very hard to separate. Consequently, smoothed distribution curves have been drawn through the steps in the manner illustrated in Fig. 1(a). The resulting curves are shown as Fig. 14.

From Fig. 14 it is readily seen that approximately the same numbers of people in quartiles I and II published, but that the amount of publication of the high publishing members of quartile I was larger by almost a factor of 2 than for the corresponding people of quartile II. Quartile III contains some individuals having high rates of publication and a smaller fraction of people publishing. The total amount of publication in quartile IV was substantially less than quartile III.

Similar diagrams have been made for other laboratories but there is no great uniformity in their characteristics. However, there is a very general trend which holds for all cases considered. This trend is for the average rate of publication per individual to increase steadily from quartile to quartile, being highest for the first or best paid quartile.

From the type of spread which is observed in Fig. 14,

[4] Employee interest is also high. For example, merit curves have been deduced from polls of employees of Bell Tel. Labs. by the Conf. of Prof. Tech. Personnel Inc., P.O. Box 625, Summit, N. J.

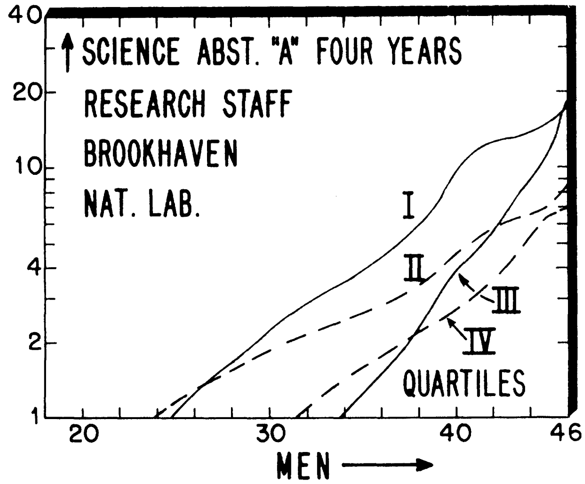


Fig. 14—Cumulative distributions (shown as smooth lines) for the four "merit quartiles" of the research staff of Brookhaven National Lab.

it is evident that publication per se is not given heavy weight in determining merit in terms of salary at Brookhaven. It is evident that something like 10 or 15 per cent of the individuals in quartiles III and IV exceed the publication records of about 50 per cent of the people in quartiles I and II. However, this is not sufficient to give them in terms of salary a recognition equal to those of quartiles I and II. Thus it follows that other factors certainly are being considered in determining salary.

From the general shape of the curves shown in Fig. 14, a very crude sort of an estimate can be made of the number of additional factors which must be taken into account in determining salary provided these factors are assumed to have importance approximately equal to amount of publication. For example, if we compare quartiles I and II we see that only 10 per cent, approximately, in quartile I exceed the maximum production of people in quartile II. This suggests that there might be something of the order of 10 other factors involved in weighting the people of quartile I, each one of these 10 other factors contributing to a group of about 10 per cent who exceed the performance of individuals in quartile II. Evidently this type of reasoning does not apply in the same way to quartiles I and III, but the fact that something between $\frac{1}{10}$ and $\frac{1}{4}$ of quartiles III and IV exceed most of the people in quartile I in terms of amount of publication suggests an analysis might lead to the conclusion that in determining subjectively the merit rating of an individual, salary reviewers act as if there were something like 4 to 10 factors of comparable importance to amount of publication.

I shall now return to the question taken up in the beginning of this section, namely, the quantitative relationship between salary and productivity. For the various laboratories considered in the study, sets of quartiles have been drawn and the average amount of production determined for each quartile. This information is gathered together in Fig. 15. The data have been expressed in terms of rate of activity in publication or patents per man-year. For the publications the total number of publications was used (not "solo" or "weighted"). It is observed that in all cases there is a monotonic increase in rate of activity with quartiles, increasing towards the highest paid quartile, quartile I. The actual spread in amount varies by a factor of about 9 for the most rapidly varying case and by a factor of a little over 3 for the most slowly varying case.

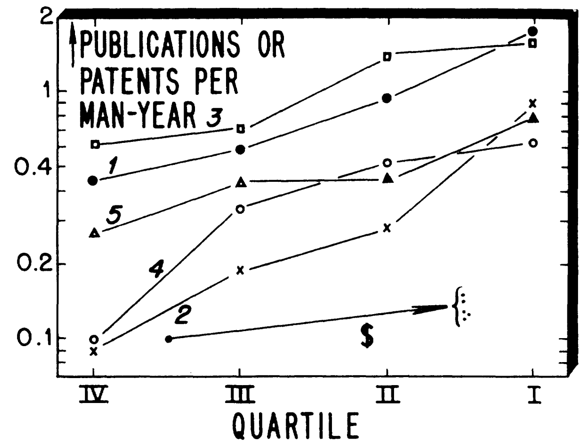


Fig. 15—Relationship between productivity and quartile number and salary and quartile number for several laboratories.

A comparison with salary is also indicated in the figure. The ratio of salary at the dividing line between quartiles III and IV to that between quartiles I and II at age 35 is also shown on the figure. Five cases have been considered and there are somewhat different spreads in salary for these. The line represents a sort of weighted average of the change in salary.

It is clear from inspection of the figure that in progression from quartile to quartile there is much less increase in salary than in productivity, in fact productivity lines rise 3 to 5 times as steeply as the salary lines. In other words, statistically an increase of 30 to 50 per cent in productivity is necessary for an individual to obtain an increase in salary of 10 per cent. However, as the reasoning given in connection with Fig. 14 shows, increase in scientific productivity alone is not sufficient to produce the increase in merit rating. In fact, coupled with the 30 to 50 per cent increase in productivity, there probably must be comparable increases in other kinds of contribution. In other words, the individual probably must become 30 to 50 per cent better in all respects in order to receive a recognition corresponding to a 10 per cent increase in salary.

VIII. Relevance to Civil Service Salary Scales

I should like next to discuss the relevance of these findings to the problem of Civil Service scientists in government laboratories. In addition to relatively low salaries positions in government laboratories are less attractive than those in industry or in universities. This is especially true in laboratories in the military estab-

lishments where periodically changing direction by officers who are not experienced in directing research frequently leads to morale problems. These problems have been thoroughly explored and reported in detail in the recent report[5] of the Riehlman committee of Congress. Clearly it is important to retain in these laboratories some highly-qualified, strong-minded, inspired leadership in order to prevent research effort from becoming thoroughly second grade.

This brings us to the most important conclusion in the study, and one which might possibly furnish a basis for action. The top salaries in government laboratories are substantially lower than both those in industry and in universities, at least for people in the latter whose line of work involves undertaking summer assignments and doing consulting. Even if there were no disadvantages aside from salary, the limits of salary set by Civil Service scales probably have a most severe effect on the leadership and originality available in government laboratories. Although these attributes have not been studied quantitatively, all of the findings in this article are consistent with the idea that leadership and originality increase very rapidly with salary just as do rate of publication and rate of invention. Cutting off the top of the salary scale at, say, \$12,000 per year as compared to \$18,000, will mean a reduction of productivity of 3 to 8 fold, according to the statistics deduced in connection with Fig. 15. Statistically, for the higher salaried man the return per dollar of salary is two to five times as great so far as individual productivity is concerned. If leadership qualities vary in a way similar to productivity, the return from increased salary will be enormously greater since an effective leader may substantially improve the output of many men.

In closing, I should emphasize that there are outstanding exceptions to most statistical results. Government laboratories do succeed in retaining a few outstanding individuals. These are unfortunately exceptions rather than the rule. Because of the present top limits on Civil Service salaries for scientists, the taxpayer's dollar is buying less research value than it should. A policy of having more highly paid positions might well double the return per dollar. It might also contribute significantly to offsetting the lead which the U.S.S.R. has currently gained in numbers of technical degrees granted in universities per year.

[5] Organization and Administration of the Military Res. and Dev. Programs, Twenty-fourth Intermediate Rep. of the Committee on Government Operations; August 4, 1954.

IX. Acknowledgment

The preparation of this paper has been made possible by the cooperation of a number of individuals and organizations. In particular, I would like to thank Dr. Lauriston S. Taylor and Mrs. Shea Kruegel of the National Bureau of Standards, Dr. L. J. Haworth of Brookhaven National Laboratory, Dr. John K. Herzog of Los Alamos Scientific Laboratory, and Dr. C. H. Townes of Columbia University. I would like also to acknowledge certain anonymous help. The appearance of this article in the Proceedings of the IRE results from a suggestion by E. W. Herold.

Correction

J. R. Wait and H. H. Howe, authors of "The Waveguide Mode Theory of VLF Ionospheric Propagation," which appeared on page 95 of the January, 1957, issue of Proceedings of the IRE, have brought the following corrections to the attention of the editors.

In (2), $(h/\lambda)^{1/2}$ should be replaced by (h/λ) and $S_n^{3/2}$ should be replaced by $\epsilon_n S_n^{3/2}$ where $\epsilon_0 = 1$, $\epsilon_n = 2 (n \neq 0)$.

In (3), (λ/n) should be replaced by (λ/h).

In Fig. 1, the abscissa labeled h should be L.