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Quadratic Voting: How Mechanism Design Can Radicalize Democracy†

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Vickrey (1961) launched mechanism design as a project to implement Lerner's (1944) "socialist" dreams of welfare optimality absent a benevolent and omniscient planner. When Vickrey became a Nobel Laureate, he announced that he planned to bring this vision to the broader public using the "bully pulpit" the prize afforded him, but he sadly died days later. Furthermore, while Vickrey's ambitions gained some early adherents (Tideman and Tullock 1976), experience (Ausubel and Milgrom 2005; Rothkopf 2007) and Vickrey's own warnings suggested his exact scheme was usually impractical.

Thus, while mechanism design transformed economic theory, the novel mechanisms it suggests have largely been treated as a series of insightful theoretical curiosities of little practical relevance or applied in quite narrow settings, for example to spectrum and internet advertising auctions (Milgrom 2004; Edelman, Ostrovsky, and Schwarz 2007). In a book that will come out as this article does (Posner and Weyl forthcoming), one of us aims to revive the more radical ambitions of mechanism design and suggest they may offer a solution to the economic, political, and social crisis of our times.

Perhaps the most glaring element of that crisis has been the struggles of democracies to resolve majority-minority conflicts and the resulting ascendancy of "populist" tyrannies of the majority or plurality. This oldest and most persistent problem of democracy seems particularly to lend itself to Vickrey's approach both because of its sweeping scope and because the basic problem seems to be precisely the lack of markets.

One-person-one-vote (1p1v) rations a single unit of influence on any collective decision to each voter, preventing potential Pareto-improving trade. When we complain about the failure of standard democracy to account for intensity of preference or knowledge (Hirschman 1982), we mean that it blocks trades where some citizens would be willing to give up their influence on some issues to gain more influence on others. Furthermore, given that the standard market system of exchange may be derived as a limit of Vickrey's procedure in a large society (Tideman and Plassmann 2017), it seems natural that applying a similar logic to public goods could yield a solution.

Yet, to our knowledge, prior to our work on the subject no simple and flexible pricing system for collective decisions that could apply to common settings such as binary decisions had been proposed.[1] Groves and Ledyard (1977) and Hylland and Zeckhauser (1980) suggested closely related market systems for choosing continuous public goods, but groups of voters rarely directly choose continuous public goods. Here we propose a simple version of Hylland and Zeckhauser's scheme for the common setting of communities making repeated binary collective decisions. As with the market system for private goods, it is the large population limit (this time for public goods) of Vickrey's scheme and we will show that it is the unique way to price votes that achieves optimality in a "price-taking" model analogous to the conditions under which markets achieve optimality.

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[1] Slightly after the first draft of this paper circulated, Goeree and Zhang (2017) proposed a closely related mechanism, though one applicable only in a narrower scope.

I. Price-Theoretic Model

Consider a society of N voters $i = 1, \ldots, N$ where many binary collective decisions (e.g., referenda or choice of leaders) arise. To create the opportunity for market trade, each voter is endowed with a large stock of "voice credits" that they may spend influencing the outcome of these decisions. As is common in the analysis of markets for private goods (Willig 1976), we assume there are enough issues and that each is sufficiently inconsequential that every voter has a quasi-linear "continuation value" for retaining voice credits for future votes. As in the theory of fair resource allocation (Varian 1974), we assume that voice credits have been distributed in a manner (such as equal division) considered fair by the relevant society in the sense that maximizing total equivalent continuation value in units of voice credits defines social optimality.

Consider some particular decision. Call the value that voters would receive, in units of voice credits, for seeing alternative A prevail over alternative B $2u_i$, with negative values indicating a preference for B over A. The community votes to determine which alternative is implemented, with each voter choosing a continuous number of votes v_i either positive or negative depending on which alternative she favors. A is implemented just when $\sum_i v_i \geq 0$. Each voter pays a cost $c(v_i)$ voice credits for her votes where c is differentiable, convex, even, and strictly increasing in $|v_i|$ to a central clearing house. We describe c as a *vote pricing rule*.

A game-theoretic analysis would specify beliefs of each voter about the distribution of values and derive an equilibrium strategy. We conduct such an analysis, which involves narrow assumptions about beliefs and behavior and statistical subtleties, elsewhere (Lalley and Weyl 2017). We focus here instead on a price-taking approximation (Weyl forthcoming) that appears to be valid more broadly (Weyl 2017) and shows simply the core logic.

We thus assume that each voter, in deciding how many votes to purchase, weighs the marginal cost of an additional vote against the perceived chance the vote will be *pivotal* in swinging the election. The price-taking assumption we adopt, originally proposed by Mueller (1973) and Laine (1977), is that all voters agree on this *marginal pivotality* of votes p on this issue (though of course they may differ across issues). Under this assumption, a rational voter will choose v_i to maximize $2u_i p v_i - c(v_i)$. By our assumption that vote credits are distributed fairly, society wishes to implement A exactly when $\sum_i u_i \geq 0$. We say a vote pricing rule is *robustly optimal* if, for every $p > 0$, N, and vector u, each price-taking voter i chooses votes v_i^* so that $\sum_i v_i^*$ has the same sign as $\sum_i u_i$.

II. Uniqueness

Our main result is that the set of robustly optimal vote pricing rules are precisely the set of quadratic rules.

THEOREM 1: *A vote pricing rule is robustly optimal if and only if it is quadratic.*

PROOF:

See our online Appendix.

The central idea of the omitted formal proof is that quadratic functions are the only ones with linear derivatives and thus the only functions where a voter buying votes equates her marginal benefit and cost at a number of votes proportional to her value. Consider the class of vote pricing rules $c(x) = x^a$ for $a > 1$. The first-order condition for voter optimization is, by differentiation,

$$2pu_i = a\left(v_i\right)^{a-1} \Rightarrow$$

$$v_i = \operatorname{sign}(u_i)\left(\frac{2p}{a}\right)^{\frac{1}{a-1}} |u_i|^{\frac{1}{a-1}}.$$

If $a = 2$ this leads to v_i^* proportional to u_i and thus robust optimality. For any other a, v_i^* is not proportional to u_i and thus the costly voting rule will be suboptimal for some arrangements of values and p.

In contrast, consider the extremes of $a \to 1$ (as voting cost approaches linear) and $a \to \infty$ (as voting cost becomes very convex). In the former case, the power on u_i determining v_i^* becomes infinite and thus voters with only slightly greater values will vote infinitely more. Linear voting thus leads to dictatorship of the most intense voter, as in Casella, Llorente-Saguer, and Palfrey (2012). This reflects the common intuition against vote trading: allowing it will lead to capture by the most intense special interests.

In the case of $a \to \infty$, the power on u_i approaches 0 so all voters buy exactly one vote in the direction of their preference as in 1p1v. In this sense, QV is an optimal intermediate point between the extremes of dictatorship and majority rule. It is the one vote pricing rule under which voters who intend only their own gain are led, as if by an invisible hand, to advance the interests of society. We refer to voting based on a quadratic pricing rule as Quadratic Voting (QV) for obvious reasons.

III. Practical Promise

While price-taking is a useful heuristic, it is unlikely to literally hold in game theoretic models of voter behavior. In other work, we have considered a variety of these models. In Lalley and Weyl (2017) we analyze the most canonical case from the game theoretic literature, in which voters' values are drawn independently and identically from a known value distribution and act as rational, risk-neutral expected utility maximizers. We show that under appropriate conditions and in all symmetric Bayes-Nash equilibria in large populations the price-taking assumption approximately holds for almost all voters and, thus, that the welfare losses from QV decay generically at a rate $\frac{1}{N}$ as the population grows. In another parallel to markets, this is the same rate of convergence of private goods markets toward efficiency as competition grows.

The structure of game theoretic equilibrium is quite subtle and thus our rigorous mathematical proof of this result is long and technical. However, taking the approximations that we rigorously prove hold in this case to hold, appropriately adjusted, in other cases and using numerical methods allows analysis of QV for other environments. In Chandar and Weyl (2017) one of us, with another coauthor, considers the baseline model in smaller populations for a range of distributions numerically and finds that the welfare lost from QV relative to the optimum is very small (rarely more than a few percentage points) while that lost under 1p1v may easily be near 100 percent.

In Weyl (2017) one of us uses these approximations to allow for collusion, uncertainty about the value distribution, and the possibility that voters are not perfectly rational and consequentialist. In some of these settings QV performs better than in our baseline and it rarely has more than a very small welfare loss, again in contrast to 1p1v. Furthermore, laboratory experiments on QV (Goeree and Zhang 2017; Cárdenas, Mantilla, and Zárate 2014) are largely consistent with these robustness conclusions. While participants do not play precisely as game theoretic equilibrium would predict, votes are generally linearly proportional to values with some noise, leading to outcomes much closer to optimal than 1p1v.

Each of these results has important limitations on its own; some are in stylized, or specific experimental, settings, while others are approximate or numerical. Together, however, we believe they suggest significant promise as they illustrate the robustness of this simple yet (approximately) optimal voting rule. This contrasts with other voting rules which are either optimal, but widely seen as too fragile to apply in practice, or which do not aim at optimality.

In the first category, the most famous proposal was Vickrey's original scheme as refined by Clarke (1971) and Groves (1973), the first fully optimal mechanism. However, this approach is extremely sensitive to collusion by even a very small group (Ausubel and Milgrom 2005). It also requires large and highly uncertain expenditures of money that make it heavily dependent on using real money rather than voice credits (Greenberg, Mackay, and Tideman 1977), a possibility that may be unjust given the inequality of wealth in most societies (Laurence and Sher 2017; Ober 2017). As noted in our introduction, to address these problems, Groves and Ledyard (1977) and Hylland and Zeckhauser (1980) proposed a mechanism closely related to QV, but only applicable to continuous public goods problems, which rarely arise in practical applications. Other optimal mechanisms proposed in the literature are even less robust (Weyl 2017). Given this, practical attention has primarily focused on mechanisms that do not aspire to even approximate optimality, such as ones that do not allow expression of preference intensity (Brams and Fishburn 1978) or are like QV but with a linear costly voting function (Casella 2005; Hortala-Vallve 2012) and thus induce dictatorship of the most intense voters.

QV aims to combine the simplicity and practicality of this latter group of designs with an approximation to the optimality of the first. This has allowed applications to polling and survey research (Quarfoot et al. 2017; Holland 2017), where evidence from initial studies shows significant improvements over existing methods of preference elicitation. Higher-stakes applications to politics (Posner and Weyl 2015) or corporate governance (Posner and Weyl 2014) remain far more speculative and are not advisable without further experimentation at smaller scales.

IV. Radical Democracy, Radical Markets

However, cautious but increasingly ambitious experimentation seems warranted given the potential of QV to create a truly radical democracy in several senses. First, and most mechanically, QV may also be viewed as individuals receiving votes equal to the square root or radical of the voice credits they spend. Second, QV gets to the roots of what the framers of modern democratic institutions, such as Alexander Hamilton, the Marquis de Condorcet, and Jeremy Bentham, viewed as the aim of democracy: to ensure the state serves the general happiness of the people maximally. Third, QV radically expands the rights of citizens to fully and freely express their political views, liberating them from the straightjacket of 1p1v influence rationing. In this sense, it shows how mechanism design can suggest fundamental and yet also practical reforms of social institutions.

Yet Vickrey's vision has nearly as much to give to markets for private as public goods. A follower of Henry George, Vickrey was skeptical of private property and the central argument of his 1961 classic was that auctioning assets ensured their efficient allocation. On the other hand, Myerson and Satterthwaite (1983) show allocative efficiency is impossible beginning from private ownership. One can imagine many assets (other than personal property) being continuously auctioned for rental according to Vickrey's favored English auction, much as occurs with internet advertising slots. However, rather than the revenue being appropriated by a web platform, it could be returned to the public in equal shares as a social dividend, much as envisioned by George. Such a *Vickrey Commons* would on the one hand take the logic of the free market to its furthest extreme, while on the other hand abolish private property and most inequality. It is the natural private goods markets analog of the radical market logic of QV.

As with the direct application of Vickrey's schemes to collective decisions, a Vickrey Commons would face a host of practical challenges and variations on it, such as that proposed by Weyl and Zhang (2017), are of greater practical relevance. Yet as a thought experiment it, along with QV, illustrates the potential to break the deadlocked ideologies of left and right that plague our society. For mechanism designers to realize this potential, they will have to broaden their horizons beyond both narrow applications and impractical theory, to learn to be at once idealistic and pragmatic, to deploy rigor flexibly, and to account for sociological, psychological, legal, and other factors that they have traditionally shied away from given their analytical intractability. Here we tried to illustrate briefly the challenges and promise of such an approach.

REFERENCES

Ausubel, Lawrence M., and Paul Milgrom. 2005. "The Lovely but Lonely Vickrey Auction." In *Combinatorial Auctions*, edited by Peter Cramton, Yoav Shoham, and Richard Steinberg, 17–40. Cambridge, MA: MIT Press.

Brams, Steven J., and Peter C. Fishburn. 1978. "Approval Voting." *American Political Science Review* 72 (3): 831–47.

Cárdenas, Juan Camilo, César Mantilla, and Román David Zárate. 2014. "Purchasing Votes without Cash: Implementing Quadratic Voting Outside the Lab." https://www.aeaweb.org/aea/2015conference/program/retrieve.php?pdfid=719.

Casella, Alessandra. 2005. "Storable Votes." *Games and Economic Behavior* 51 (2): 391–419.

Casella, Alessandra, Aniol Llorente-Saguer, and Thomas R. Palfrey. 2012. "Competitive Equilibrium in Markets for Votes." *Journal of Political Economy* 120 (4): 593–658.

Chandar, Bharat K., and E. Glen Weyl. 2017. "Quadratic Voting in Finite Populations." http://ssrn.com/abstract=2571026.

Clarke, Edward H. 1971. "Multipart Pricing of Public Goods." *Public Choice* 11 (1): 17–33.

Edelman, Benjamin, Michael Ostrovsky, and Michael Schwarz. 2007. "Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords." *American Economic Review* 97 (1): 242–59.

Goeree, Jacob K., and Jingjing Zhang. 2017. "One Man, One Bid." *Games and Economic Behavior* 101: 151–171.

Greenberg, Joseph, Robert Mackay, and Nicolaus Tideman. 1977. "Some Limitations of the Groves-Ledyard Optimal Mechanism." *Public Choice* 29 (2S): 129–37.

Groves, Theodore. 1973. "Incentives in Teams." *Econometrica* 41 (4): 617–31.

Groves, Theodore, and John Ledyard. 1977. "Optimal Allocation of Public Goods: A Solution to the 'Free Rider' Problem." *Econometrica* 45 (4): 783–810.

Hirschman, Albert O. 1982. *Shifting Involvements: Private Interests and Public Action.* Princeton, NJ: Princeton University Press.

Holland, Alisha C. 2017. "Square Miles: The Spatial Politics of Mass Infrastructure." Unpublished.

Hortala-Vallve, Rafael. 2012. "Qualitative Voting." *Journal of Theoretical Politics* 24 (4): 526–54.

Hylland, Aanund, and Richard Zeckhauser. 1980. "A Mechanism for Selecting Public Goods when Preferences must be Elicited." Unpublished.

Laine, Charles R. 1977. "Strategy in Point Voting: A Note." *Quarterly Journal of Economics* 91 (3): 505–07.

Lalley, Steven P., and E. Glen Weyl. 2017. "Nash Equilibria for Quadratic Voting." https://arxiv.org/abs/1409.0264.

Laurence, Ben, and Itai Sher. 2017. "Ethical Considerations on Quadratic Voting." *Public Choice* 172 (1-2): 195–222.

Lerner, Abba P. 1944. *The Economics of Control: Principles of Welfare Economics*. New York: MacMillan.

Milgrom, Paul. 2004. *Putting Auction Theory to Work*. Cambridge, UK: Cambridge University Press.

Mueller, Dennis C. 1973. "Constitutional Democracy and Social Welfare." *Quarterly Journal of Economics* 87 (1): 60–80.

Myerson, Roger B., and Mark A. Satterthwaite. 1983. "Efficient Mechanisms for Bilateral Trading." *Journal of Economic Theory* 29 (2): 265–81.

Ober, Josiah. 2017. "Equality, Legitimacy, Interests, and Preferences: Historical Notes on Quadratic Voting in a Political Context." *Public Choice* 172 (1-2): 223–32.

Posner, Eric A., and E. Glen Weyl. 2014. "Quadratic Voting as Efficient Corporate Governance." *University of Chicago Law Review* 81 (1): 251–72.

Posner, Eric A., and E. Glen Weyl. 2015. "Voting Squared: Quadratic Voting in Democratic Politics." *Vanderbilt Law Review* 68 (2): 441–500.

Posner, Eric A., and E. Glen Weyl. Forthcoming. *Radical Markets: Uprooting Capitalism and Democracy for a Just Society*. Princeton, NJ: Princeton University Press.

Quarfoot, David, Douglas von Kohorn, Kevin Slavin, Rory Sutherland, David Goldstein, and Ellen Konar. 2017. "Quadratic Voting in the Wild: Real People, Real Votes." *Public Choice* 172 (1-2): 283–303.

Rothkopf, Michael H. 2007. "Thirteen Reasons Why the Vickrey-Clarke-Groves Process is Not Practical." *Operations Research* 55 (2): 191–97.

Tideman, Nicolaus, and Florenz Plassmann. 2017. "Efficient Bilateral Taxation of Externalities." *Public Choice* (1-2): 109–30.

Tideman, T. Nicolaus, and Gordon Tullock. 1976. "A New and Superior Process for Making Social Choices." *Journal of Political Economy* 84 (6): 1145–1159.

Varian, Hal R. 1974. "Equity, Envy, and Efficiency." *Journal of Economic Theory* 9 (1): 63–91.

Vickrey, William. 1961. "Counterspeculation, Auctions, and Competitive Sealed Tenders." *Journal of Finance* 16 (1): 8–37.

Weyl, E. Glen. Forthcoming. "Price Theory." *Journal of Economic Literature*.

Weyl, E. Glen. 2017. "The Robustness of Quadratic Voting." *Public Choice* 172 (1-2): 75–107.

Weyl, E. Glen, and Anthony Lee Zhang. 2017. "Depreciating Licenses." https://ssrn.com/abstract=2744810.

Willig, Robert D. 1976. "Consumer's Surplus Without Apology." *American Economic Review* 66 (4): 589–97.