A combinatorial auction mechanism for airport time slot allocation

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A sealed-bid combinatorial auction is developed for the allocation of airport time slots to competing airlines. This auction procedure permits airlines to submit various contingency bids for flight-compatible combinations of individual airport landing or take-off slots. An algorithm for solving the resulting set-packing problem yields an allocation of slots to packages that maximizes the system surplus as revealed by the set of package bids submitted. The algorithm determines individual (slot) resource prices which are used to price packages to the winning bidders at levels guaranteed to be no greater (and normally smaller) than the amounts bid. Laboratory experiments with cash motivated subjects are used to study the efficiency and demand revelation properties of the combinatorial auction in comparison with a proposed independent slot primary auction.

1. The problem of allocating airport slots

■ In 1968 the FAA adopted a high density rule for the allocation of scarce landing and take-off slots at four major airports (La Guardia, Washington National, Kennedy International, and O'Hare International). This rule establishes slot quotas for the control of airspace congestion at these airports.

Airport runway slots, regulated by these quotas, have a distinguishing feature which any proposed allocation procedure must accommodate: an airline's demand for a take-off slot at a flight originating airport is not independent of its demand for a landing slot at the flight destination airport. Indeed, a given flight may take off and land in a sequence of several connected demand interdependent legs. For economic efficiency it is desirable to develop an airport slot allocation procedure that allocates individual slots to those airline flights for which the demand (willingness to pay) is greatest.

Grether, Isaac, and Plott (hereafter, GIP) (1979, 1981) have proposed a practical market procedure for achieving this goal. Their procedure is based upon the growing body of experimental evidence on the performance of (1) the competitive (uniform-price) sealed-bid auction and (2) the oral double auction such as is used on the organized stock and commodity exchanges. Under their proposal an independent primary market for slots at each airport would be organized as a sealed-bid competitive auction at timely intervals. Since the primary market allocation does not make provision for slot demand interdependence, a computerized form of the oral double auction (with block transaction ca-

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We are grateful to the National Science Foundation for research support under a grant to the University of Arizona (V.L. Smith, principal investigator).

pabilities) is proposed as an "after market" to allow airlines to purchase freely and sell primary market slots to each other. This continuous after market exchange would provide the institutional means by which individual airlines would acquire those slot packages which support their individual flight schedules. Thus, an airline that acquired slots at Washington National which did not flight-match the slots acquired at O'Hare could either buy additional O'Hare slots or sell its excess Washington slots in the after market. Although GIP's proposed after market permits airlines to exchange slots freely and thereby acquire the appropriate slot packages, it suffers from two disadvantages.

(1) Individual airlines may experience capital losses and gains in the process of trading airport slots in the after market. Thus, an airline with an excess of A slots and a deficiency of B slots may discover in the after market that the going price of B slots is unprofitably high (for that particular airline), while excess A slots can be sold only at a loss.
(2) It costs resources to trade in the after market. Hence, to the extent that slots are not allocated to the appropriate packages in the primary market, the cost of participating in the combined primary-after market mechanism is increased.

Ideally, the primary market would allocate slots in the appropriate packages initially with the after market performing only two functions (i) marginal corrections in primary market misallocations, and (ii) slot allocation adjustments due to new information not available at the time of the primary auction. Thus, a sudden grounding of all DC-10 aircraft would leave Continental Airlines with a surplus of O'Hare runway slots, which could be sold in the after market to airlines not affected by the DC-10 grounding.

In this article, we address the problem of designing a "combinatorial" sealed-bid auction to serve as the primary market for allocating airport slots in flight-compatible packages for which individual airlines would submit package bids. The objective is to allocate slots to an individual airline *only* in the form of those combinations and subject to those contingencies that have been prespecified by the airline.

2. The auction optimization mechanism

■ To increase the overall efficiency of the slot allocation mechanism suggested by GIP (1979), and to decrease its reliance on an after market, we have developed an optimization model with the following features for use in a computer-assisted primary sealed-bid auction market: (a) direct maximization of system surplus in the criterion function; (b) airport coordination through consideration of resource demands in logically packaged sets; (c) scheduling flexibility through contingency bids on the part of airlines.

Consider the following integer programming problem:

$$(P)\begin{cases}\text{Maximize} & \sum_j c_j x_j \\ \text{Subject to:} & \sum_j a_{ij} x_j \leq b_i \ \forall\, i, \\ & \sum_j d_{kj} x_j \leq e_k \ \forall\, k, \\ & x_j \in \{0, 1\};\end{cases}$$

where

$i = 1, \ldots, m$ subscripts a resource (some slot at some airport);
$j = 1, \ldots, n$ subscripts a package (set of slots) valuable to some airline;
$k = 1, \ldots, l$ subscripts some logical constraint imposed on a set of packages by some airline;

$$a_{ij} = \begin{cases} 1 \text{ if package } j \text{ includes slot } i, \\ 0 \text{ otherwise;} \end{cases}$$
$$d_{kj} = \begin{cases} 1 \text{ if package } j \text{ is in logical constraint } k, \\ 0 \text{ otherwise;} \end{cases}$$
e_k = some integer ≥ 1,
c_j = the bid for package j by some airline.

The contingency bids expressed in the set of logical constraints have one of two format types: "Accept no more than p of the following q packages." or "Accept package V only if package W is accepted." The first type is identical in format to any of the resource constraints. For example, suppose an airline bids c_a on package a, c_b on package b, and specifies either a or b, but not both. The added constraint is then written $x_a + x_b \leq 1$. The second type can be converted to this format through a simple variable transformation. For example, suppose an airline bids as in the previous example, but specifies b only if a. By creating the package ab with $c_{ab} = c_a + c_b$, package b can be eliminated from consideration. The added constraint becomes $x_a + x_{ab} \leq 1$.

In a manner analogous to the parallel independent slot auctions suggested by GIP (1979), sealed bids (c_j) for packages (j) and the contingency constraints (k) specified by each airline are used to parameterize the model and determine an "optimal" primary allocation. The problem which results is recognized as a variant of the set packing problem with general right-hand sides. It can be solved, as was done for the experiments reported in Section 3, with a specialized algorithm developed by Rassenti (1981). A problem of the enormous dimensions dictated by even a four-city application (perhaps 15,000 constraints and 100,000 variables) will present a significant challenge for the finest configuration of hardware and software available. Fortunately, a practicable solution within 1 or 2% of the linear optimum, and very often the optimum itself in the discrete solution set, is almost assuredly achievable in a reasonable amount of time.

Given the solvability of P and its potential for ensuring an efficient primary allocation, there remain several questions: how to induce bidding airlines to reveal their true values; how to price allocated slots; and how to divide income among the participating airports. We suggest a resolution of these concerns with the following procedure: (1) Determine a complete set of marginal (shadow) prices, one for each slot offered. (2) Charge any airline whose package j was accepted in the solution to P a price for j equal to the sum of the marginal prices for the slots in that package. This provides the uniform price feature that has demonstrated good demand revelation behavior in single commodity experiments (GIP, 1979). (3) Return to any airport whose slot i was included in some accepted package j an amount equal to the marginal price for i. Such a scheme will guarantee that the price paid for an accepted package is less than (or rarely equal to) the amount bid for that package.

If problem P were a linear program, the determination of the suggested set of shadow prices would be a trivial and well-solved matter. Discrete programming problems, however, present special difficulties with respect to shadow pricing. Consider, for example, a discrete project selection problem with a single resource constraint:

$$(K)\begin{cases} \text{Maximize} & 5X_1 + 3X_2 + 6X_3 + 5X_4 + 6X_5 + 3X_6 + 4X_7 + 3X_8 + 2X_9 + X_{10} = Z; \\ \text{Subject to:} & 3X_1 + 2X_2 + 6X_3 + 7X_4 + 9X_5 + 5X_6 \\ & \quad + 8X_7 + 8X_8 + 6X_9 + 4X_{10} \leq 24; \qquad X_j \in \{0, 1\}\ \forall j = 1, \ldots, 10. \end{cases}$$

If the choice space for X_j is relaxed to its linear programming equivalent, $0 \leq X_j \leq 1$, then the solution is trivially given by $(Z, X_1, X_2, \ldots, X_{10}) = (23, 1, 1, 1, 1, .66, 0, 0, 0, 0, 0)$. The critical return rate, $\lambda = 6/9$ for project 5, is the optimal Lagrangian multiplier or shadow price for the resource. But the discrete problem has the optimal solution (21, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0), and obviously no critical ratio exists which separates projects that are chosen from those that are not. Figure 1 illustrates these solutions.

In the traditional sense, Lagrangian multipliers for an integer program may not exist;[1] that is, no set of prices will support the optimal division of packages into accepted and rejected categories. Therefore, it is possible that a package bid that is greater than its shadow resource cost will be rejected, while another package bid that is also greater than its shadow cost is accepted. In the experiments reported in Section 3, we provided subject bidders with a guideline explanation of these cases. This allows subjects to select strategic or best reply (Cournot) responses if they wish.

With this problem in mind, the following two pseudo-dual programs to P were developed to define bid rejection prices (problem D_R) and acceptance prices (problem D_A) that will serve as bidding guidelines for individual agents.

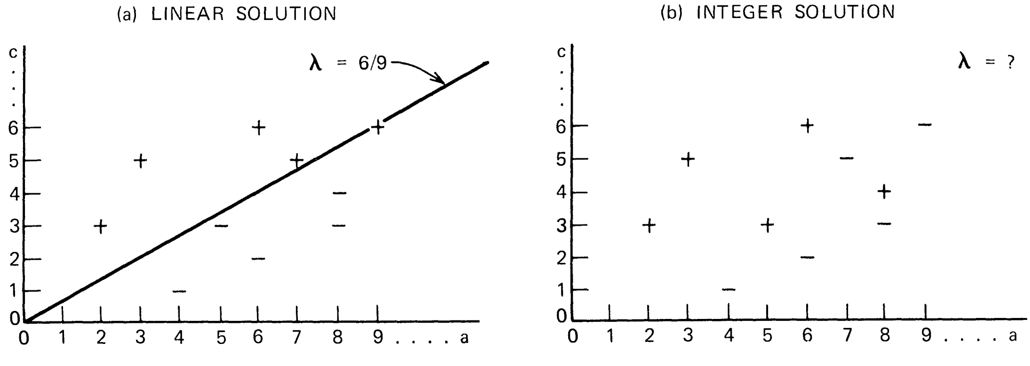
$$(D_R)\begin{cases} \text{Minimize} & \sum_R y_r \\ \text{Subject to:} & \sum_i w_i a_{ij} \le c_j \ \forall\, j \in A, \\ & y_r \ge c_r - \sum_i w_i a_{ir} \ \forall\, r \in R, \\ & y_r \ge 0, \qquad w_i \ge 0; \end{cases}$$

where

the optimal solution to P is $\{x_j^*\}$;
the set of accepted packages is $A = \{j | x_j^* = 1\}$;
the set of rejected packages is $R = \{r | x_r^* = 0\}$;
the set of lower bound slot prices (prices charged) to be determined is $\{w_i^*\}$;
the amount by which a rejected bid exceeds the market price (if at all) is y_r.

$$(D_A)\begin{cases} \text{Minimize} & \sum_A y_j \\ \text{Subject to:} & \sum_i v_i a_{ir} \ge c_r \ \forall\, r \in R, \\ & y_j \ge \sum_i v_i a_{ij} - c_j \ \forall\, j \in A, \\ & y_j \ge 0, \qquad v_i \ge 0; \end{cases}$$

FIGURE 1



[1] Wolsey (1981) presents a "state of the art" discussion of price functions in integer programming.

where

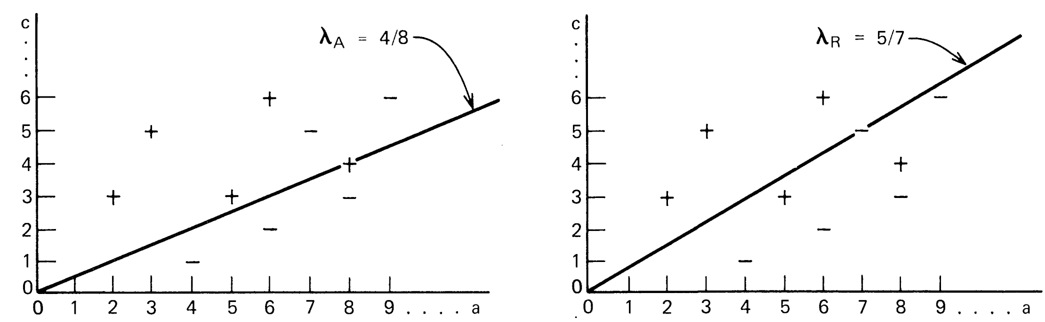
the set of upper bound slot prices to be determined is $\{v_i^*\}$;
the amount by which an accepted bid is below the upper bound slot prices (if at all) is y_j.

Problem D_A is the complement of D_R with respect to the accept-reject dichotomy. If unambiguous separating prices exist, the solutions to D_A and D_R coincide. In Figure 2 the analogous pseudo-dual problems for the project selection problem K above are schematically solved for the obvious upper and lower bound return ratios.

The following categorization of bids can now be made. (i) If a bid was greater than the sum of its component values in the set $\{v_i^*\}$, it was definitely accepted. (ii) If a bid was less than the sum of its component prices in the set $\{w_i^*\}$, it was definitely rejected. (iii) All bids in between were in a region where acceptance or rejection might be considered independent of relative marginal value and determined by the integer constraints on efficient resource utilization. The bids in category (iii) correspond to the core of the integer programming problem P. They comprise a small percentage of all bids and are known to decrease in relative number as problem size increases. Figure 3 gives the regions analogous to (i), (ii), and (iii) for the project selection example.

What can be said about the theoretical incentive properties of this proposed computer assisted sealed-bid auction? Certainly it is *not* generally incentive compatible; that is, if any bidder desires to acquire multiple units of any given package or multiple units of the same slot, then the door is open to the possibility of strategically underbidding the true value of certain packages (Vickrey, 1961). However, strategic behavior is fraught with risks for the individual, even in simple multiple unit auctions for a single commodity, because individuals do not know the bids and the true valuations of their competitors.[2]

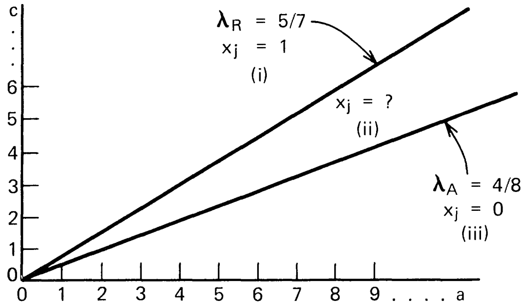
FIGURE 2



[2] A referee suggests that "if this mechanism were implemented, it would be used over a long period of time with substantial sums at stake and one would expect some investment in learning about the use of strategic maneuvers." Some comment on this view is important because it represents a widely shared belief among economists. We think it is at least as likely—the evidence seems to suggest that it is more likely—that the airlines would compete away any rents which they now capture from airport slot resources. In fact, a very reasonable hypothesis, given the immense uncertainty as to what airport slot combinations are actually worth (these are well defined in our experiments), might be that airline bids would actually exceed those levels that would sustain long-run profitability. If the recent, and continuing, vigorous price competition among airlines in passenger ticket sales is any indication, then this last hypothesis is quite likely to be supported. Braniff Airlines has just filed for Chapter 11 bankruptcy in an environment in which "it has been widely asserted that part of Braniff's financial ills stemmed from steep fare cuts to raise ridership—at the expense of profit" (*Wall Street Journal*, May 17, 1982, p. 4). In less than a week after Braniff's collapse, Midway Air announced entry and "set the fare between Chicago and Dallas-Fort Worth at a cut rate of \$89 one way, despite industry hopes that the collapse of Braniff would end extensive fare wars" (*Wall Street Journal*, May 17, 1982, p. 1). These field observations of noncooperative behavior are consistent with the results of several hundred experiments in which

FIGURE 3

CATEGORIZATION FOR PROBLEM K



We conjecture that this is why GIP do not observe significant underrevelation of demand in laboratory experiments with one commodity auctions.

Since the combinatorial auction we suggest for the airport slot problem is far more complex than any of the single commodity auctions that have been studied, we would expect to observe at least as much demand-revealing behavior in our auction as in the others. Since this is both an open and a behavioral question, we devised a laboratory experimental design to compare our combinatorial auction procedure with the procedure proposed in GIP (1979).

3. Experimental results

■ Eight experiments were conducted using students with economics and engineering backgrounds. Each experiment consisted of a sequence of market periods in which objective economic conditions remained constant in successive periods for the six participants. The first period was always considered a learning period (no payoff), and the number of periods completed was time dependent (3-hour limit). Individual subjects were paid the difference between the assigned redemption value of packages and the prices paid for packages in the market (Smith, 1976). Subjects' earnings varied between \$8 and \$60, depending on individual endowed valuations and subject and group bidding behavior.

A 2 × 2 × 2 experimental design was employed. The three factors were: (1) GIP (control) versus our RSB (treatment) primary auction; (2) subjects "experienced" versus "inexperienced;" (3) combinatorial "complexity" (easy versus difficult) of resource utilization.

The GIP mechanism employed copied that suggested and used by GIP (1979). The first two RSB experiments used marginal package pricing and the "easy" combinatorial design, while the second two used the marginal item pricing scheme described in Section 2 and the "difficult" combinatorial design.[3] The term "experience" indicates previous participation in either a control or treatment version of an experiment, but even "inexperienced" subjects generally had some experience with other less complicated decisionmaking experiments such as a single commodity auction. Combinatorial "complexity" refers to the degree of difficulty a subject would encounter in attempting to shuffle

two to four sellers compete away all except competitive rents in a variety of distinct pricing institutions. In the experiments reported here subjects were quite active in attempting manipulative strategies early in each experiment. These strategies tended to be abandoned over time, as indicated by the tendency for resource prices (Table 3) to increase in successive periods.

[3] Appendix A contains the instructions and forms used in the experiments in which the primary market priced items marginally. The instructions and forms used in our GIP experiments and in the two RSB experiments that priced packages marginally can be obtained by writing Smith at the University of Arizona. Appendix B contains the contrasting package valuation designs used in the "easy" and "difficult" combinatorial treatment.

slots from one package use to another for the purpose of making a redemption claim or a purchase or sale in the after market. It has two components: the amount of package repetition among various agents and the amount of item repetition within any agent's packages.

Table 1 gives the observed efficiency after primary and secondary markets for each cell of the design. The trial (zero) period results are not listed. Efficiency is defined as total subject payoff (realized system surplus) divided by total theoretical payoff (system surplus computed by assuming full demand revelation). The data support several important hypotheses. The overall efficiency of the RSB mechanism is generally greater than that of the GIP mechanism. It is achieved without the uniformly strong dependence on the secondary market displayed by the GIP mechanism. The ability of subjects to include contingency bids in appropriate situations, though not included in the experiments conducted, should serve to accentuate this effect. Market experience seems a significant factor in determining the efficiency of either mechanism. Learning is in evidence during the multiperiod course of each experiment. The RSB mechanism, however, seems to require less learning and displays quicker achievement of high efficiency. This fact suggests that the RSB mechanism will adapt more efficiently to changing economic conditions—an important criterion in judging the performance of any mechanism.

The sample size is too small, with one observation per cell, to test for the significance of each "treatment." But by aggregation across all treatments except the bidding mechanism, we can report a nonparametric sign test of the null hypothesis that the fourth period difference in efficiency between RSB and GIP is equally likely to be positive or negative, as against the research hypothesis that there is a positive difference. From Table 1, for the primary market, all four paired differences are positive, and the null hypothesis is rejected at $p = .0625$. In the after market, three of the four paired differences are positive, and the null hypothesis can be rejected only at $p = .25$. This is consistent with our prior expectation that the principal advantage of RSB over GIP is to improve primary market allocation sufficiently to make after market exchange unnecessary. Thus, comparing after market efficiency with primary market efficiency in period 5 across all RSB experiments, we observe no difference.

The more detailed breakdown of surplus presented in Table 2 adds further support to these observations. Uniformly more agents were in debt after the GIP primary market. On several occasions, an agent who needed to participate in multilateral trades for gains in the GIP secondary market was caught short of completion (e.g., GIP-inexperienced-easy period 1). This is a difficulty to be reckoned with in any independent slot marketing scheme.

Speculation is defined as the purchase of a package by an individual for whom the package has no redemption value. Such a purchase can be profitable only through resale at a higher price in the after market. Because of the extremely high efficiency realized in the primary RSB allocation, speculative behavior is very risky (e.g., RSB-inexperienced-easy period 5 and difficult period 4). Primary prices and allocations are too near optimal to yield much speculative profit, and combinatorial problems are encountered with after market multilateral trade. Table 2 also reinforces the notion that experience is less important in the RSB mechanism. Under the more incentive-compatible conditions of RSB, there was 95% demand revelation for nonspeculative bids in the final period.

The difficulty with determining item values by independent auctions is emphasized by Table 3, which traces prices by period. The absolute deviation from theoretical values is much larger for the GIP mechanism. In fact, the task of estimating item value from package redemption value proved too much for inexperienced GIP subjects under difficult conditions where a market collapse occurred. Unless there is demand overrevelation (bidding in excess of value), it is impossible for this situation to occur in the RSB mechanism, where the optimization routine makes an "intelligent" pricing decision.

TABLE 1 Efficiency, by Period and Treatment Condition, in the Primary and After Market

	GIP						RSB					
	Period	Primary Market, P	After Market, A	Period	Primary Market, P	After Market, A	Period	Primary Market, P	After Market, A	Period	Primary Market, P	After Market, A
Experienced Subjects	1	0.904	0.974	1	0.609	0.730	1	0.832	0.923	1	0.986	0.986
	2	0.871	0.920	2	0.695	0.864	2	0.898	0.944	2	0.978	0.987
	3	0.871	0.953	3	0.709	0.851	3	0.935	0.973	3	0.985	0.985
	4	0.907	0.983	4	0.752	0.903	4	0.971	0.971	4	0.985	0.985
				5	0.804	0.919	5	0.986	0.986	5	0.986	0.986
				6	0.795	0.969				6	0.991	0.993
Inexperienced Subjects	1	0.853	0.861	1	0.721	0.917	1	0.884	0.923	1	0.951	0.965
	2	0.778	0.942	2	0.726	0.831	2	0.918	0.951	2	0.860	0.940
	3	0.650	0.865	3	0.602	0.798	3	0.936	0.977	3	0.976	0.979
	4	0.685	0.911	4	0.408	0.829	4	0.967	0.977	4	0.931	0.931
	5	0.763	0.907	5	0.463	0.923	5	0.869	0.870	5	0.984	0.984
				6	0.465	0.902						
		Easy			Difficult			Easy			Difficult	

TABLE 2 **Breakdown of Surplus by Market, Period, and Treatment Condition (no. agents in debt/negative agents' surplus/positive agents' surplus)**

	GIP		RSB			
Period	Primary	After	Primary	After		
1	1/ −.17/36.11	0/ 0.00/45.20	1/−1.18/33.66	0/ .00/44.57	Experienced Subjects	Easy
2	1/ −4.15/33.64	1/ −.63/36.70	0/ .00/40.69	0/ .00/46.86		
3	4/ −5.11/10.70	1/ −1.77/18.24	1/−1.83/43.64	0/ .00/46.79		
4	3/ −352/15.55	0/ .00/21.32	0/ .00/41.75	0/ .00/41.75		
5			0/ .00/43.04	0/ .00/43.04		
6						
1	0/ .00/18.95	1/ −3.71/23.66	1/ −.39/27.18	0/ .00/31.92	Inexperienced Subjects	
2	3/−10.54/ 6.99	2/ −2.99/21.19	0/ .00/32.02	0/ .00/36.39		
3	3/−30.00/ 3.06	3/ −9.54/11.18	0/ .00/30.12	0/ .00/35.66		
4	4/−34.30/ 2.86	3/ −5.84/ 4.31	0/ .00/31.83	0/ .00/33.22		
5	4/−15.90/ 3.57	2/ −6.36/13.13	1/−5.17/25.13	1/−5.17/25.29		
6						
1	5/−11.18/ 1.86	3/ −4.63/10.36	0/ .00/34.52	0/ .00/34.52	Experienced Subjects	Difficult
2	4/ −7.65/ 7.54	1/ −1.19/32.44	0/ .00/28.46	0/ .00/29.58		
3	4/ −8.84/11.84	1/ −.13/20.80	0/ .00/21.56	0/ .00/21.56		
4	2/−12.30/14.43	1/ −2.70/23.52	0/ .00/19.53	0/ .00/19.53		
5	2/ −2.75/ 9.38	2/ −.41/21.28	0/ .00/18.94	0/ .00/18.94		
6	3/−10.27/ 9.71	0/ .00/21.02	0/ .00/15.04	0/ .00/15.21		
1	3/−31.20/ 1.67	3/ −7.39/ 2.04	1/ −.71/26.72	0/ .00/27.78	Inexperienced Subjects	
2	4/−37.68/ .00	4/−27.07/ 2.32	3/−6.29/ 7.28	1/−1.93/12.79		
3	5/−60.51/ .00	5/−36.69/ .34	0/ .00/11.30	0/ .00/11.59		
4	5/−87.40/ .00	5/−35.27/ .00	1/−6.51/13.74	1/−6.51/13.74		
5	4/−77.05/ .00	3/−23.13/ 3.09	0/ .00/15.21	0/ .00/15.21		
6	3/−18.82/19.26	0/ .00/54.65				

TABLE 3 Market Prices by Period under Difficult Conditions (using marginal item pricing in RSB mechanism)

	GIP						RSB						
	Item Prices						Item Prices						
Period	*A*	*B*	*C*	*D*	*E*	*F*	*A*	*B*	*C*	*D*	*E*	*F*	
1	2.30	2.50	2.00	2.10	1.00	2.00	2.27	2.56	2.11	2.39	1.06	1.73	
2	2.31	2.75	2.00	2.26	1.00	2.00	3.07	2.58	2.24	2.67	.94	1.93	
3	2.50	2.76	2.00	2.01	1.00	2.00	3.10	3.03	2.78	2.63	.95	1.88	Experienced Subjects
4	3.00	3.00	2.00	2.26	.50	2.26	3.35	2.90	2.95	2.70	1.15	1.60	
5	3.00	3.05	2.00	2.50	.25	2.50	2.56	3.19	2.98	2.74	1.17	1.81	
6	3.00	3.25	2.50	2.76	.50	2.00	2.88	3.13	3.23	2.57	1.63	1.97	
Theoretical Prices	2.66	3.16	3.09	2.66	2.51	2.49	2.66	3.16	3.09	2.66	2.51	2.49	
1	3.00	2.75	2.50	3.00	2.75	3.00	1.96	3.64	2.44	2.29	.71	2.08	
2	3.20	3.20	3.00	3.10	2.90	3.00	2.25	3.80	2.50	3.32	1.25	2.13	
3	3.50	4.00	3.30	3.50	2.00	2.90	3.24	3.93	2.78	2.97	1.29	1.47	Inexperienced Subjects
4	4.00	5.00	4.00	3.30	1.50	2.50	2.89	3.78	2.73	3.03	1.59	1.58	
5	4.51	5.00	4.51	3.20	1.00	1.00	2.76	3.49	2.80	2.95	1.86	1.39	
6	4.51	.01	3.50	.01	.01	.01	2.78	3.47	2.77	2.79	2.02	1.97	
Theoretical Prices	2.66	3.16	3.09	2.66	2.51	2.49	2.66	3.16	3.09	2.66	2.51	2.49	

Finally, combinatorial complexity seems to lower GIP mechanism efficiency by a significant amount, while the performance of the RSB mechanism appears not to deteriorate and perhaps even to improve. This is to be expected if we are correct in our conjecture that the decision costs potentially associated with this factor are borne by the computer in the RSB mechanism.

4. Alternatives for implementing an airport slot auction

■ To our knowledge, this study constitutes the first attempt to design a "smart" computer-assisted exchange institution. In all the computer-assisted markets known to us in the field, as well as those studied in laboratory experiments, the computer passively records bids and contracts and routinely enforces the trading rules of the institution. The RSB mechanism has potential application to any market in which commodities are composed of combinations of elemental items (or characteristics). The distinguishing feature of our combinatorial auction is that it allows *consumers* to define the commodity by means of the bids tendered for alternative packages of elemental items. It eliminates the necessity for producers to anticipate, perhaps at substantial risk and cost, the commodity packages valued most highly in the market. Provided that bids are demand revealing, and that income effects can be ignored, the mechanism guarantees Pareto optimality in the commodity packages that will be "produced" and in the allocation of the elemental resources. The experimental results suggest that: (a) the procedures of the mechanism are operational, i.e., motivated individuals can execute the required task with a minimum of instruction and training; (b) the extent of demand underrevelation by participants is not large, i.e., allocative efficiencies of 98–99% of the possible surplus seem to be achievable over time with experienced bidders. This occurred despite repeated early attempts by inexperienced subjects to manipulate the mechanism and to engage in speculative purchases.

The problem of allocating airport time slots requires improved methods (GIP, 1981), and the problem has grown from bad to worse in the aftermath of the recent strike attempt by the air traffic controllers. We think the RSB mechanism, or some variant that might be developed from it, has potential for ultimate application to the time slot problem. But as we view it, before such an application can or should be attempted, at least two further developments are necessary. First, at least two additional series of experiments need to be completed. Another series of laboratory experiments should be designed, using larger numbers of participants, resources, and possible package combinations. The subjects in these new experiments should be the appropriate operating personnel of a group of cooperating airlines. Depending on the results of such experiments, the next step might be to design a limited scale field experiment with only a few airports and airlines.

Second, there should be extensive discussion and debate within the government, academic, and airline communities concerning alternative means of implementing the combinatorial auction. There is a wide range of choice here. Our discussion, as well as the reported experiments, were based on the assumption that airline bids would be denominated in U.S. currency and that the revenue would be allocated to the airports. There are, however, other alternatives; we offer just a few to stimulate discussion. (1) If it is believed that airport revenue should not be based on the imputed rents from scarce time slots, then bids in the combinatorial auction could be denominated in "slot currency" or vouchers issued in fixed quantities to each airline. These vouchers could be freely bought and sold among the airlines but would only be redeemable in time slots. (2) Alternatively, each airline could continue to be given some "historical" allocation of slots, with the RSB mechanism modified to become a two-sided sealed bid-offer combinatorial auction. In such an auction each airline would submit package bids for slots to be purchased, and package offers of slots to be sold. Under this form of implementation, the rent imputed to airport slots would of course be retained as "revenue" by the airlines.

(3) An important question not addressed in either the GIP or RSB procedures is the pricing of airline seats, which directly affects the willingness-to-pay for airport slots. We would suggest that the idea of a computerized continuous double (bid-offer) auction of seats be considered along with the combinatorial auctioning of slots. All the major airlines are computerized down to the boarding gate, so that the computerized trading of seats may be technically feasible, and could provide a more flexible means of increasing airline revenue while lowering passenger cost through improved load factors. (4) Finally, we should note that we think there may be an inherent contradiction in the attempt to allow free (deregulated) entry by the airlines, but not permit free entry by airports. Ultimately, a pricing system for airport slots, which returns revenue to the airports, could allow not only for package bids from the airlines, but slot price offers from the airports, with each airport subject to competition from new regional, suburban, and national airports.

Appendix A

Instructions

RSB instructions. This is an experiment in the economics of market decisionmaking. Various research organizations have provided funds for the conduct of this research. The instructions are simple, and if you follow them carefully and make good decisions, you might earn a considerable amount of money, which will be paid to you in cash after the experiment. In this experiment we are going to conduct two kinds of markets to distribute six distinct items among you in a sequence of periods or market days. The six distinct items are represented by the letters: A, B, C, D, E, F. At the end of the experiment we shall redeem (that is, buy) certain packages of items you have acquired during each period. The amounts to be paid to you as an individual can be determined from your payoff sheet included with the instructions. The payoff tables may differ among individuals. This means that the patterns of payments differ and the monetary amounts may not be comparable. The first market is the primary market and is of the sealed-bid type. In this market each of you may bid to buy items offered in fixed quantities. The second market will be a secondary market of the oral-bid-offer type. In this market you may buy or sell items obtained in the primary market to one another if you wish. Alternatively, you may simply keep what you have for the experimenters to redeem. In all sales, whether to the experimenters or to other participants, you may keep any profits you earn. For each sale you make, your profits are computed as follows: your earnings = sale price − purchase price.

□ **Redemption values.** In your folder there is a sheet labelled "Redemption Values." This sheet indicates the amount the experimenter will pay you for given packages of items at the end of the period. Suppose for example you ended the period with $2A$, $2C$, and $2F$ items, and your redemption values were as follows:

package	items included						value
	A	B	C	D	E	F	
1	1	1	1				1.20
2	1		1				.40
3	1		1			1	1.20
4	1					1	.72
5			1			1	.70
6	1					1	.60.

Since you may claim each item in only one package, you may legitimately claim for the set [*AC*, *CF*, *AF*] which will redeem .40 + .70 + .72 = \$1.82. But the set [*ACF*, *CF*] with one leftover item *A* is a better claim since it redeems 1.20 + .70 = \$1.90. In this case your period profit may have been increased by previously selling off the leftover item *A* in the secondary market.

□ **Primary market.** Each period there will be a limited number of units of each kind of item available. As a buyer you purchase packages of one or more units by submitting bids which may be accepted or rejected. You will decide each period how many bids to submit for which packages in what amounts. Suppose you wish to bid for packages *AF* at .72, *AF* at .48, and *AC* at .37. Then your bid sheet for the primary market should look like this:

package	items included						value
	A	*B*	*C*	*D*	*E*	*F*	
1	1					1	.72
2	1					1	.48
3	1		1				.37.

Bids are accepted or rejected each period as follows. The bid sheets are collected from all buyers. All bids are fed into a computer program which selects the set of bids which are most valuable without violating any constraint on the number of units of each item available. The program also gives two sets, low and high, of item unit values. Each accepted bid represents the purchase of one package at a total price equal to the sum of its low item values. Your purchase price will always be less than or equal to your bid price, since any bid less than the sum of its low item values was definitely rejected. Any bid greater than the sum of its high item values was definitely accepted. Consider the above set of bids. Suppose the low and high item unit values for *A*, *C*, and *F* were given as (.25, .10, .32) and (.25, .16, .34). Then package 1 was definitely accepted since $.72 > .25 + .34$. The market price for the package *AF* was $.25 + .32 = .57$. Package 2 was definitely rejected since $.48 < .57$. Package 3 might have been rejected or accepted since $.25 + .10 < .37 < .25 + .16$. At the close of the primary market bid sheets will be returned to each buyer indicating which of his bids were accepted. The low and high sets of item values will be posted.

□ **Secondary market.** The secondary market provides an opportunity to buy additional units or sell units from the inventory acquired in the primary market. This is an oral auction. You may announce a bid (offer) to buy (sell) any package of one or more items for a specified amount. This bid (offer) will be placed on the board until it is accepted by some other participant or you cancel it. You are free to make as many bids and offers as you wish. Many may remain unaccepted but you are free to keep trying. *Note*: You may not sell what you do not have in inventory. Each purchase or sale in which you participate should be recorded on a separate line in the sequence of occurrence on your secondary market balance sheet. From your final inventory at the end of the secondary market, you specify a set of item packages that you want to redeem for cash.

□ **Profits.** Period profit is calculated as: profit = redemption revenue + sales revenue from secondary market − purchase costs from both markets. After each period has ended, make the appropriate entries on your payoff sheet. The experimenters will pay you all you have earned during all periods at the conclusion of the experiment.

Primary Market Agent #1 Period #1

Agent's Bids for Packages

Package	A	B	C	D	E	F	Bid	Accepted Yes or No	Market Price
1									
2									
⋮									
8									
# Units								Total Cost	
	Totals for Auction Accepted Bids Only								WWW

Secondary Market Agent # 1 Period # 0

Note: Before the secondary market begins, copy the # of units of each item bought in the primary market into the spaces labelled inventory for transaction 0.

Transaction	Package Dealt: Items Included A	B	C	D	E	F	Sold For $	Bought At $	Inventory: # Units Each Item A	B	C	D	E	F
0														
⋮														
6														
Total Sales YYY														
Total Costs XXX														
Final Inventory														

Redemption Values Agent # 1 Period # 0

Note: Before claiming redemptions, copy your final inventory from the secondary market into the following table. Make sure all packages you intend to redeem are covered by this inventory. Remember that any unit of a given item can only be used in one package.

Item	A	B	C	D	E	F
# Units						

Items Included

Package	A	B	C	D	E	F	Value	Claimed Yes or No
1	1	1					6.27	
2			1	1			5.77	
3	1				1		5.06	
4	1		1			1	8.25	
5		1	1		1		8.34	

Total Value of Redemptions = *ZZZ*

Payoff Sheet Agent # 1

Your profit from each period is calculated as follows:
Period Profit = Redemption Value (*ZZZ*) + Sales in Secondary Market (*YYY*)
− Costs in Secondary Market (*XXX*) − Costs in Primary Market (*WWW*)
After each period concludes, make the proper entries in the following table:

Period	Red. Value *ZZZ*		Sales Sec. *YYY*		Costs Sec. *XXX*		Costs Pri. *WWW*		Profit
0		+		−		−		=	
1		+		−		−		=	
2		+		−		−		=	
⋮		+		−		−		=	
6		+		−		−		=	
	Total Profit Over All Periods						*PPP*		

I acknowledge receipt of the above amount (*PPP*) from the experimenters:

Appendix B

Agent value information

■ **Easy resource utilization design.**

Agent	Package	Value	Item *A*	Item *B*	Item *C*	Item *D*	Item *E*	Item *F*
1	1	5.98	1	1				
1	2	9.46	1	1	1			
1	3	5.17		1	1			
2	4	6.32	1			1		
2	5	6.63		1	1			
2	6	9.51	1	1	1			
3	7	8.77	1	1				1
3	8	5.95	1		1			
3	9	5.15		1	1			
3	10	8.85	1	1	1			
4	11	5.46	1		1			
4	12	9.83	1		1			1
4	13	5.69	1	1				
4	14	6.03		1	1			
5	15	6.42	1	1				
5	16	4.50	1				1	
5	17	4.98		1	1			
5	18	9.13	1		1		1	
5	19	4.76	1		1			
6	20	5.76	1		1			
6	21	8.02		1	1	1		
6	22	4.39		1	1			
6	23	9.45	1		1			1
6	24	6.17	1		1			
6	25	5.20	1	1				
# Units Demanded			18	15	18	2	2	3
# Units Available			13	11	15	1	2	3

□ Difficult resource utilization design.

Agent	Package	Value	Item *A*	Item *B*	Item *C*	Item *D*	Item *E*	Item *F*
1	1	6.27	1	1				
1	2	5.77			1	1		
1	3	5.06	1				1	
1	4	8.25	1		1			1
1	5	8.34		1	1		1	
2	6	5.31	1	1				
2	7	5.56			1	1		
2	8	5.76	1		1			
2	9	6.44		1			1	
2	10	5.84			1		1	
2	11	8.86	1				1	1
3	12	5.17	1	1				
3	13	5.76			1	1		
3	14	8.87	1		1		1	
3	15	9.40		1	1			1
4	16	5.98	1	1				
4	17	6.27			1	1		
4	18	5.78	1					1
4	19	5.78		1		1		
4	20	5.56				1		1
4	21	8.61		1			1	1
5	22	5.60	1	1				
5	23	5.82			1	1		
5	24	5.65		1				1
5	25	8.34		1		1	1	
5	26	7.82	1			1		1
6	27	5.07	1	1				
6	28	5.65			1	1		
6	29	8.33		1		1		1
6	30	9.59	1			1	1	
# Units Demanded			14	14	12	12	9	9
# Units Available			7	7	7	7	7	7

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