PROBABILITY, STATISTICAL DECISION THEORY, AND ACCOUNTING

HAROLD BIERMAN, JR.*

THERE has recently been a revolution in statistics which is soon going to affect the decisions which accountants have traditionally considered their own reserve (in fact it has already affected the area of inventory control), and it might well affect accounting theory itself. This revolution has been given the title, statistical decision theory.

Statistical decision theory is concerned with making decisions under uncertainty. We shall define uncertainty as being a situation where the underlying probability model is not known. Tossing a fair coin fairly is an example of a probability model which is known. There is .5 probability of a head and a .5 probability of a tail. However, if a person took a coin out of his pocket and threw it in the air, the coin might not be perfectly fair, or with enough practice his pitching arm could be taught tricks. With either event, the underlying probability model is not known and the process of placing a bet on the toss of a coin is the type of problem to which one may apply statistical decision theory.

One characteristic of decision making under uncertainty is that the decision maker will not always make the correct decision. For one thing we are working with a probability model, and secondly, we do not know the characteristics of the model. If the coin is fair, and it is tossed fairly, we know there is .5 probability of a head on any one toss. But if we choose heads we will be wrong if a tail appears, and there is .5 probability of a tail. When we do not know the probability of a head and the probability of a tail the decision process is even more complex.

The revolution in statistics has two aspects which are of primary interest to accountants:

a. The use of subjective probabilities in making decisions.
b. The incorporation of cost or loss functions in the decision process.

Taken together these two items form the backbone of present day statistical decision theory.

Subjective probabilities. There is no uniform agreement among statisticians (or philosophers) that subjective probabilities should be used in making decisions. But it has been argued that, in order to make decisions consistent with a person's basic beliefs, the probabilities that a person believes to surround the possible outcome should be incorporated into the analysis.

The use of subjective probabilities requires the decision maker to indicate the likelihood of the occurrence of the possible events. For example, returning to the coin example, if an individual took a coin out of his pocket and tossed it in the air, each of us could place a prior subjective probability on the event "heads." Depending on the degree of trust in the individual possessing the coin, some would say the probability of a head was .5, but others would prefer either 1.0 or zero probability depending on whether they thought that the coin was fair, two-headed or two-tailed.

Loss functions. Classical statistics concerned itself with the likelihood of a hypothesis being true or false, but did not

* Harold Bierman, Jr., is Professor of Accounting at Cornell University. He is the author of a text on managerial accounting and of a book on capital budgeting. This paper was the basis of a talk given at the Northeast Regional Group meeting of the AAA on October 28, 1961.

attempt to attach losses (or gains) connected with the possible decisions. Thus the classical statistician would talk of a .95 confidence level, but would not incorporate the economic or psychological consequences of using this level of confidence as the basis of the decision. Statistical decision theory incorporates the decision maker's reaction to the occurrence of all possible events for each possible act. It then applies a decision rule in evaluating the evidence in order to choose one act which is the best act for the decision maker.

Among the decision rules that are available are:

Minimax
Maximax
Maximum likelihood
Equally likely
Bayes decision rule

It is not feasible to analyze here each of the rules given, but it should be recognized that there are weaknesses and difficulties associated with each rule. The rule which is currently receiving a large amount of support for application to a wide variety of problems is the Bayes decision rule. For each act we multiply the gain (or loss) for each possible event by the probability of the event, summing the products to obtain the expected value of the act. The act with the highest expected value is the optimum act. It should be noted that the values summed should be in the form of utilities and not money for the Bayes decision rule to be theoretically sound. For certain decisions it is reasonable to use monetary measures as approximations of the utility measures.

The following simplified example illustrates the use of the Bayes decision theory.

Example: The Smith and Jones public accounting firm in conducting an audit encountered a complex transaction which would cost $2,000 to investigate. If the transaction is bad and it is not audited (i.e. discovered by Smith and Jones) it would cost the firm $1,000,000. However, the partners agree that the probability of it being faulty transaction is one in a thousand (i.e. .001). The following analysis may be made:

Possible Acts	*States of Nature*	
	Transaction is Acceptable	*Transaction is Unacceptable*
Audit	−2,000	−2,000
Do nothing	0	−1,000,000
Probabilities of States	.999	.001
Expected cost of auditing:	$2,000	

Expected cost of doing nothing is $1,000. (equal to .001×1,000,000)

Based on the expected monetary value of the alternatives the cost is minimized by doing nothing. However the utility measures of the consequences of the two acts and the states of nature should be taken into consideration. Assume the following utility function:

Dollars	*Utility Measures*
−1,000,000	−8,000
−2,000	−5
0	0

The expected utility of the two acts is:

Expected utility of auditing:	−5
Expected utility of doing nothing: (equal to .001×8,000)	−8

Based on the utility analysis Smith and Jones should audit the transaction. The large negative utility associated with the loss of $1,000,000 may be interpreted as the disutility of going bankrupt. The use of the expected utility as the basis of the decision (using subjective probabilities in taking the expectation) is an application of the Bayes decision rule. In some situations there will be some past experience which

may be the basis of the probabilities which are assigned the possible acts, and frequently a sample may be taken which will give evidence which will be the basis of a revision of the original subjective probabilities.

The significance of decision theory to accounting theory. Statistical decision theory has application to that portion of accounting theory which is concerned with definitions of assets and liabilities. The concept of expected value is very much relevant to both of these definitions.

Should the accountant record liabilities when the amount to be paid is not certain? Since conservatism is generally accepted, it is not controversial to show a liability even where the amount is not certain. For example, taxes payable is shown even where the income has not yet been earned (as with accounting for taxes and the effects of accelerated depreciation).

Let us consider the controversial reserve for contingencies. Assume events have occurred which indicate the company will have a liability in the future, but the amount of the liability is not currently determinable. It could be argued that the expected value of the liability should be indicated as a liability even though the exact amount is not defined (for strategy purposes this might not be smart if there is a lawsuit pending). As long as the basic triggering event has occurred the accountant is willing to indicate the liability (and the expense or loss, if there is one) even where the economic transaction is not completed. Thus the accountant is willing to make use of expectations when this is convenient to the pose of conservatism. However, occasionally the accountant will carry this practice to extremes.

Many financial reports contain an item called reserve for possible losses in connection with investment in foreign operations. Is this procedure justified? If the triggering event has not taken place, assuming a loss is certain to occur is not justified. A loss is only one possible event and it has some probability less than one. The likelihood of making gains should also be taken into consideration in the computation of the valuation of the investment. Thus after Castro has taken over the private property of corporations operating in Cuba there is justification in writing down assets and having an expected loss. There is little justification in setting up a similar contra account for investments in Australia or Canada. The same type of analysis argues against setting up reserve accounts for possible price declines of inventory. The price is just as likely to move up as down, and setting up a contra asset account for possible price decline is no more justified than writing up the valuation because the price may go up (the expected value of the inventory is equal to the current market price).

The accountant has been reluctant to recognize gains and write up assets before complete realization has taken place (with realization being poorly defined). However, if marketable securities have risen in price then the expected value of these securities has also risen and it is reasonable to record this fact. The expectation should be based on the probabilities of a disinterested third party, not the probabilities of a person who may benefit from a biased appraisal. For example, there is some justification in a situation involving marketable securities to take the expectations of the market place, i.e., the market price of the security, except where there is reason to believe that the market price does not truly reflect the value of the investment. If we interpret the point of realization to mean the moment in time when the probability of collecting cash becomes high, then it is more reasonable to recognize the increment in the value of marketable securities as it occurs, than it is to recognize revenue when a sale is made to a

person of questionable credit standing.

The use of mathematical expectations would also affect the recording of oil discoveries for an oil company. With an oil company the expectation of profits is different after the discovery of oil than it was before the discovery. The expected value of the property is different. In like manner the value of timber land with full grown trees is different than the same land with newly planted seedlings. The arguments that these accretions in value are not realized are not relevant since the realization takes place through time as the probabilities of ultimately receiving cash change, and the amount of cash which is realizable at any moment in time changes.[1]

Obviously the suggestion being made here to record certain expectations would make accounting less neat than it currently is since it would remove the requirements of objective evidence and realization in a variety of transactions. On the other hand, the accounting information resulting may well be more useful. Would the recording of expected values give too much of a free hand to management to influence the accounting results? The answer is clearly yes; thus there is much need for independent review and certification of the reports. The expectations cannot be those of a biased party. The incorporation of expectations in financial reports would broaden the tasks of the public accountant. For example, he would have to have access to expert opinions as to the value of oil in an oil field (both the amount of oil and its economic value). The resulting accounting reports would be of increased usefulness to the investing community.

Does this mean that the accounting reports as we know them today should be abandoned? A more desirable procedure would be to retain the present reports based on objective evidence and supplement them with reports incorporating subjective judgments. If accountants do not present reports of this nature, knowledgeable investors will obtain the information from other sources. Unfortunately these sources may not be independent of mind. Rather than leaving the interesting part of accounting to non-accountants, I would prefer accountants to agree that there is not "one accounting system" but rather several acceptable accounting systems each capable of contributing significantly to our understanding of economic events as they affect operating entities.

The primary contribution of statistical decision theory to accounting theory is that it offers a theoretical justification for the recording of certain transactions where there is incomplete information. The expected value concept based on subjective probabilities can and should be the justification for recording the transaction. We are currently doing just that in the accounting treatment of the bad debt allowance; we should also do it in recording changes in value which have occurred (for example, finding oil), rather than relying blindly on the concept of "realization." The concept of "expected value" should find its place in the terminology of accounting.

Statistical decision theory and accounting-type decisions. The tools of decision theory are also of interest in several areas of cost accounting. The definitions and uses of such terms as practical capacity, normal activity, and normal spoilage are made much more useful when expressed in statistical terms. That is, in terms of the expected value and standard deviation of the probability distribution. Activity measures which incorporate the probability of the occurrence are more useful than those which just indicate a dollar figure.

[1] The use of mathematical expectations (i.e., computation of expected monetary value) may not always be appropriate. It may be necessary to include in the analysis consideration of the variance of the probability distribution, or more exactly additional characteristics of the probability distribution.

Normal spoilage and abnormal spoilage are close to being meaningless expressions until we define normal spoilage as the expected spoilage and add information about the probability distribution of spoilage, such as the standard deviation.

However, it is in the area of decision making that the biggest impact of statistical decision theory is being felt. Accounting-type decisions are decisions which are normally made by a financial officer, or decisions to which the finance officer contributes a significant amount of the information and participates in the decision process.

Statistical decision theory enables the decision maker to incorporate both his opinions of the future into the decision being made and also the gains or losses associated with these decisions. This may sound trivial and it is frequently assumed that decision makers always do both of these things. But consider this situation: classical statistics is not only being widely used but is being recommended in current journal articles. Several issues of the *Journal of Accountancy* in 1961 have carried articles on the subject of sampling in auditing. None of the articles have incorporated the cost function of obtaining information or the loss function associated with not obtaining information. Until this is done the statistical analyses being made in the area of auditing will be incomplete and misleading.

The primary decision area which was once readily accessible to the accountant but is now almost completely lost is inventory control. The techniques of statistical decision theory are being used in inventory control complete with such exotic terms as Poisson Process, Markov Chain, simulation and dynamic programming, and it is a brave accountant who dares to indicate interest in this area. It will not be long before these techniques and others will be spreading into other areas where relatively primitive tools are being used to solve accounting-type decisions.

Below are several illustrations of problems to which tools of statistical decision theory may be applied. In each of these areas the decision maker could come a little closer to making a reasonable decision than he can make using the traditional approach.

Break-even analysis: especially the decision as to whether or not to change the price. Incorporating the expected level of sales and the standard deviation of the distribution is more of a help in explaining the relative reasonableness of courses of action, than just two breakeven charts.

Bond refunding: including the probability of interest rates going down further adds another dimension to the problem and makes the solution more meaningful.

Capital budgeting: including uncertainty (the probability of different events, and utilities, i.e., the reactions of the investor to the occurrence of the events) is a significant step towards rational decision making.

Cash and marketable security administration: there are a whole string of interesting decisions here, including the size of the line of credit, the amount of cash carried with banks, the decision to invest in marketable securities, and the decision as to the amount of cash to be borrowed when borrowing is indicated.

Cost investigation: when should a cost variance be investigated? What are the costs of investigation and the costs of not investigating? What is the probability of the variance being caused by random factors?

Conclusions. The schools offering the Ph.D. have responsibility to see that their graduates are better equipped than the present generation to solve the complex problems of the business community. One of the tools available, and which will be widely used in the future, is the tool of quantitative analysis, including the very important tool of statistical decision theory.

Teachers of prospective practitioners of the art of business administration have to

instill an appreciation of quantitative skills so that the businessmen of the future are receptive to the ideas that will be generated in industry and in the academic community.

The suggestions made in this paper are evolutionary steps toward better financial information and better decision-making. Accountants cannot ignore the fact that such quantitative tools exist.