EIGHT CONSIDERATIONS FOR MATHEMATICALLY

Since its founding by Julian C. Stanley in 1971, the Study of Mathematically Precocious Youth (SMPY) at Johns Hopkins University has strongly advocated subject-matter acceleration for students who are *extremely* talented in mathematics. SMPY staff members have conducted many research studies showing the benefits of such acceleration (e.g., Brody & Benbow, 1987; Brody, Lupkowski, & Stanley, 1988).

Although helping talented students move faster in an attempt to find a good "fit" between their high abilities and the school mathematics curriculum is favored, it should be noted that acceleration has been misused. Too often students speed through textbook after textbook. This constitutes an abuse of acceleration and inhibits studying mathematics in depth.

Many practitioners advocate the use of enrichment because it enables students to study mathematics in greater breadth than in the regular math class. While enrichment activities may relieve boredom, enrichment alone does not provide the substantive, continuous, hierarchical stimulation needed by students *extremely* talented in mathematics. For example, one mathematics enrichment program requires students to complete numerous "problem-solving" worksheets. Whereas problem-solving and other enrichment activities are of value for all students, they are not the optimal means of attaining in-depth programming for extremely talented students. For these students, a systematic accelerated curriculum, balanced with appropriate enrichment activities, provides the speed and depth needed.

This article focuses on how accelerative and enrichment options complement each other to provide appropriate challenges for talented students. The following eight important points are presented for parents, teachers, and mathematically talented students to consider in planning an educational program:

1. Allow extremely talented elementary students time to develop the mathematical maturity needed to study algebra. Often parents and educators contact SMPY about *mathematically brilliant* youths in the age range 4-11 or so and they mention having the child study algebra right away. They usually are urged to go more slowly because students that young, no matter how brilliant, are unlikely to have a thorough background and firm foundation in general mathematics, the stucture of the number system, arithmetical problem solving, or even Piagetian formal operational thinking.

2. Extremely few elementary students will have the necessary cognitive structures already well enough developed to do more abstract mathematics such as second and third year algebra, geometrical proofs, trigonometry, analytic geometry, and calculus effectively and in ways that will give them intellectual satisfaction. They may be like the person who can walk fairly well on his hands, but greatly prefers to use his feet when not demonstrating mastery of the acrobatic stunt. For example, a child who excels at computation may use this mechanical skill to solve difficult problems without understanding the underlying concept. Mathematics as a stunt to please parents or educators is not likely to inculcate in the doer a love for the subject. Too much too early can cost the youngster pleasure in the subject, and the nation promising mathematicians or scientists. The authors have seen that happen a number of times.

3. For the mathematically brilliant youth, acceleration may provide the best educational option. Although in points 1 and 2 the reader is

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cautioned against rushing into fairly abstract mathematics, acceleration may be the option of choice for the extremely mathematically talented youth. For a small percentage of children, moving ahead in mathematics and related subjects such as physics and computer science more rapidly than their classmates is the only way to provide the best fit for their educational needs.

Identification of Other Abilities

Identification of exceptional mathematical talent usually occurs in conjunction with the identification of other abilities. Some mathematically apt boys and girls have much better verbal ability, mechanical ability, spatial relations ability, nonverbal reasoning ability, etc., than do others. These are relevant to the pace and level of subject matter ideal at a given age. Also quite important and *somewhat* distinct from the above is tested intelligence ("IQ"), especially as measured at age 6-8 or so by a skilled tester using individually administered tests. The central office of many sizable school systems is usually equipped to provide achievement, aptitude, and intelligence testing, but parents, as taxpayers, may have to insist that an assessment be done. Otherwise, private, certified psychologists (who should usually have a Ph.D. degree) may be needed. This type of assessment can be rather expensive, but it may be worth the cost, especially when the psychologist helps parents and educators develop an individualized educational plan. SMPY advocates testing that results not only in identification of strengths, but also in specific educational programming.

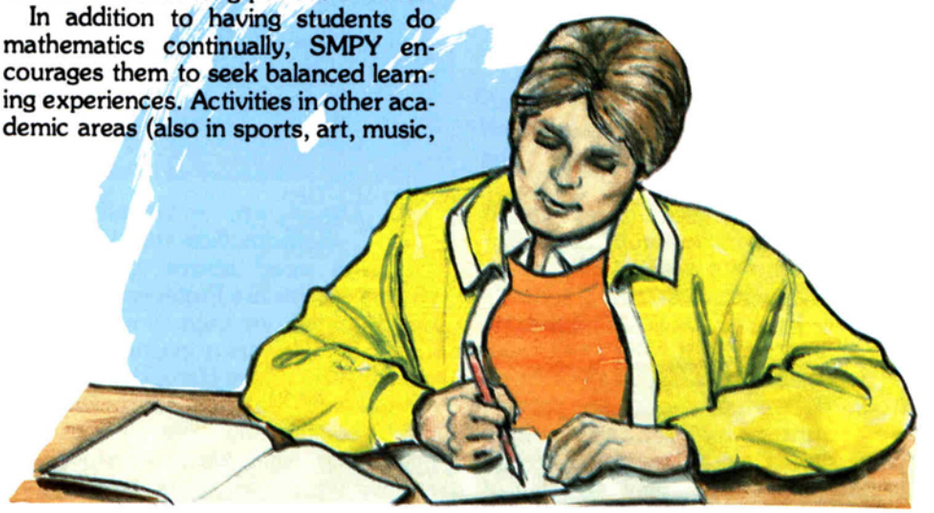
Accelerative options may include entering school early, skipping an entire grade or advancing in math only. (For 13 ways to accelerate, see Benbow, 1979.) An excellent way to advance fast but efficiently in a subject is SMPY's individually-paced and mentor-guided program (Lupkowski & Assouline, in preparation; Lupkowski, Assouline, & Stanley, submitted; Stanley, 1978, 1979, 1986). Called the Diagnostic Testing-Prescriptive Instruction (DT-PI) model, it can be applied at any age level and provides an efficient mechanism for challenging extremely talented youth. Employing the DT-PI model in elementary school leads to the first course in algebra without undue haste.

4. The mathematically brilliant youth should be kept on a steady diet of highly satisfying mathematics at his or her appropriate level of mental functioning. This does not necessarily mean racing through the standard sequence in truncated periods of time. There is no need to study mathematics intensely every day; one weekly two-hour session with a mentor may provide the challenges and stimulation an unusually talented student needs. Pacing of this sort helps avoid a situation in which a student will not have the opportunity to study mathematics for long periods of time.

In addition to having students do mathematics continually, SMPY encourages them to seek balanced learning experiences. Activities in other academic areas (also in sports, art, music, drama, dance, student government, community service, etc.) should supplement accelerated mathematics.

5. The talented elementary student who moves ahead extremely fast in the mathematical sequence is likely to be catapulted beyond the offerings of the school system long before he or she graduates from high school. Usually, the youth who hurries ahead in mathematics will have to slow down too much at some phase, perhaps not even taking mathematics courses until at the right grade level to resume the sequence. However, if there is an excellent college nearby where the secondary student can readily take regular college courses part-time without jeopardizing his or her high school education, this may not be a problem.

Conventionally, the progression is Algebra I-III, geometry, trigonometry, analytic geometry, at least two courses of calculus, linear algebra, differential equations, probability theory and statistics, and the various branches of "pure" mathematics such an analysis, higher algebra, mathematical logic, number theory, and topology.



6. Teachers, mentors, clubs, and competitions can enrich an accelerated mathematics curriculum for talented youths. Skilled mathematics teachers can offer supplemental problems that are more advanced than typical students can handle. A mathematics mentor can enrich or supersede the youth's mathematics curriculum and provide suitable pacing. The books by Lenchner (1983) and Saul, Kessler, Krilov, and Zimmerman (1986) and issues of *Arithmetic Teacher* and *Mathematics Teacher* are good places to begin finding challenging problems for talented students.

Participation in clubs and contests offers students an enriching opportunity to develop their mathematical maturity and a chance to meet other mathematically talented students. A mathematically talented youth should consider every opportunity to hone his or her talents in competitions, from the Mathematical Olympiads for Elementary Schools and Mathcounts in junior high school to striving to become a member of the United States team in the annual International Mathematical Olympiad, and other major international events. In addition to moving ahead in mathematics and other subjects, students can study and understand the material at a deeper level than is typical. One young man who participated in the International Mathematical Olympiad said, "The whole thing has given me a much stronger feeling for math, . . .a very strong foundation of elementary math. Stronger in some ways, probably, than many mathematicians who didn't spend so much time in elementary math" (Dauber, 1988, p. 10).

7. Summer programs offer varied opportunities for able students to forge ahead in mathematics. The truly mathematically talented youngster whose special abilities are recognized early should be made ready by age 12 or 13 to complete precalculus—i.e., through analytic geometry—quickly and well. This may be accomplished by attending one of the regional, residential three- or six-week summer programs conducted in various parts of the country by Johns Hopkins University, Duke University, Northwestern University, the University of Denver, Iowa State University, Arizona State University, California State University at Sacramento, the University of Wisconsin at Eau Claire, and other universities, colleges, or state departments of public instruction. Following the completion of precalculus excellently, the student fortunate enough to be in or near a high school that offers a good Advanced Placement Program Level BC calculus course may consider enrolling in it, even though far younger than the typical high school senior in such courses. This usually works well.

Qualifying Scores for Summer Programs

A student younger than 13 with no formal training in algebra who earns a score of 500 or above on the mathematical portion of the SAT probably has the cognitive skills needed to master algebra and the courses that follow. A score of 500 is the 49th percentile for college-bound male high school seniors and the 64th percentile for college-bound females. The summer programs mentioned in Point 7 usually require students to earn this score in order to attend.

Students who participate in these accelerative programs need to make inquiries regarding high school credit for the special summer classes as well as the availability and scheduling of high school courses to complement the courses completed in the summer program. *Before* taking one of the summer mathematics courses, students must be certain that it will be possible for them to continue with mathematics throughout high school in the school system, or at a local college, or with a mentor. The professionals of SMPY also encourage students to supplement their mathematics education by taking courses in physics, computer science, chemistry, and biology.

8. There are more-advanced "pure" mathematics institutes for students aged about 14-18. Two such programs are Professor Arnold E. Ross's famed one each summer at the Ohio State University in Columbus and the excellent one at Hampshire College in Amherst, Massachusetts. They require considerable love for mathematics and eagerness to immerse oneself in it (eat, sleep, and breathe mathematics) for a long period of time—six to eight weeks. This is not for everyone who finds regular school mathematics easy, but it is the golden opportunity for a few dedicated young students.

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The goals for these youngsters are proper pacing, proper sequencing, plenty of stimulation, time for planning and contemplation, appropriately planned challenges, and continual reinforcement of worthwhile achievements. These goals can be accomplished best without unseemly haste. Don't plunge the quite-young student precipitously into algebra, set theory, number theory, or the like. Let those subjects come in the natural sequence as his or her talents unfold. Take the long view that leads to steadily increasing achievement and deep intellectual satisfaction.

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