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Dr. Camilla Persson Benbow worked at the Study of Mathematically Precocious Youth (SMPY) at Johns Hopkins University for nine years. In the end she was its co-director along with Professor Julian C. Stanley, the founder of SMPY. In July 1985 Dr. Benbow began as an associate professor in the Department of Psychology at Iowa State University (ISU). A new branch of SMPY, called "*SMPY at ISU*," has been created at Iowa State University. SMPY at ISU carries out the SMPY longitudinal studies and is in the process of starting SMPY programs there. When Dr. Stanley completely retires, SMPY's activities will be based at Iowa State University under Dr. Benbow's direction.

Summary

SMPY's Model for Teaching Mathematically Precocious Students

One practical model for providing sound programming for most intellectually talented students can simply be accomplished by schools' allowing curricular flexibility. For over a dozen years, the Study of Mathematically Precocious Youth (SMPY) at Johns Hopkins has utilized already available educational programs to meet the needs of its talented students through educational acceleration. SMPY students are offered a "*smorgasbord*" of special educational opportunities from which to choose whatever combination, including nothing, best suits the individual. Some of the options are entering a course a year or more early, skipping grades, graduating early from high school, completing two or more years of a subject in one year, taking college courses on a part-time basis while still in secondary school, taking summer courses, and credit through examination. Clearly, SMPY utilizes already available educational programs to meet the special needs of talented students. Because this approach is extremely flexible, teachers or administrators can choose and adapt the various options in ways to fit their schools' unique circumstances and their students' individual abilities, needs, and interests.

Moreover, this method avoids the common criticism of elitism and costs little for a school system to adopt. Actually, the various accelerative and enriching options devised by SMPY may save the school system money. Yet this rather simple adjustment, i.e., advancing a gifted child in each school subject to the level of his/her intellectual peers, is rarely made because of bias against acceleration. It is important to note, however, that no research study to date has found properly effected educational acceleration detrimental, but rather the contrary.

SMPY's Model for Teaching Mathematically Precocious Students

Since 1971, the Study of Mathematically Precocious Youth (SMPY) at The Johns Hopkins University has systematically explored various possibilities for identifying and educating mathematically precocious secondary students. Out of this work several promising procedures have been developed. Dr. Julian C. Stanley, Professor of Psychology at Johns Hopkins and the founder and director of SMPY, deserves most of the credit for this SMPY model, which will be described in this chapter. Without his foresight, creative ideas and dedication, the findings presented could not have been made.

SMPY's Definition of Mathematical Precocity

It is conventional for new investigators to define or conceptualize giftedness before they start to work in this area. SMPY, however, has not concerned itself very much with conceptions of giftedness (Stanley & Benbow, 1986), even though it has been in existence since 1971. The staff of SMPY has had their reasons for this lack of action. The following quotation illustrates their position well:

What is particularly striking here is how little that is distinctly psychological seems involved in SMPY, and yet how fruitful SMPY appears to be. It is as if trying to be psychological throws us off the course and into a mire of abstract dispositions that help little in facilitating students' demonstrable talents. What seems most successful for helping students is what stays closest to the competencies one directly cares about: in the case of SMPY, for example, finding students who are very good at math and arranging the environment to help them learn it as well as possible. One would expect analogous prescriptions to be of benefit for fostering talent at writing, music, art, and any other competencies that can be specified in product or performance terms. But all this in fact is not unpsychological; it simply is different psychology" (Wallach, 1978, p. 617).

SMPY has, of course, an operational definition of giftedness, which is consistent with the above position. SMPY's indicator of mathematical talent or precocity is simply a high score at an early age on the mathematics section of the College Board Scholastic Aptitude Test (SAT-M). This may appear narrow. The staff of SMPY feel, however, that its elegance lies in its simplicity and objectivity. Moreover, few would argue that such an ability (to be described further below) does not indicate a high level of cognitive functioning. Although some students may be overlooked by this criterion, we identified more youths who reason exceptionally well mathematically than we could handle.

The Talent Search Concept

In order to identify mathematically talented students, SMPY developed the concept of an annual talent search and conducted six separate searches, in March 1972, January 1973, January 1974, December 1976, January 1978 and January 1979. During those years 9,927 intellectually talented junior high school students between 12 and 14 years of age were tested. Students attending schools in the Middle Atlantic Region of the United States were eligible to participate in an SMPY talent search only if they scored in the upper 5 percent (1972), 2 percent (1973 and 1974), or 3 percent (1976, 1978 and 1979) in mathematical ability (not computation or learned concepts)

I should like to thank Dr. Julian C. Stanley for helpful comments on an earlier version of this chapter.

on the national norms of a standardized achievement-test battery, such as the Iowa Test of Basic Skills, administered as part of their schools' regular testing program.

In the talent search, such students took the SAT-M and, except in 1972 and 1974, also the verbal (SAT-V) sections. These tests were designed to measure developed mathematical and verbal reasoning abilities, respectively, of above-average 12th-graders (Donlon & Angoff, 1971). Most of the students in the SMPY talent searches, however, were in the middle of the seventh grade and less than age 13. Few had received formal opportunities to develop their abilities in algebra and beyond (Benbow & Stanley, 1982a, b, 1983c). For example, we have found that among the top 10 percent of our talent search participants (i.e., those eligible for fast-paced summer programs in mathematics), a majority do not know even first-year algebra well. Thus, they must begin their studies with Algebra I.

Therefore, most of these students were demonstrably unfamiliar with mathematics from algebra onward, yet many of them were able to score highly on a difficult test of mathematical reasoning ability. Presumably, this could occur only by the use of extraordinary ability at the "*analysis*" level of Bloom's (1956) taxonomy. We concluded that the SAT-M must function far more at an analytical reasoning level for the SMPY examinees than it does for high school juniors and seniors, most of whom have already studied rather abstract mathematics for several years (Benbow & Stanley, 1981, 1983c). Moreover, because the test was so difficult and many students viewed the talent searches as a competition, our mode of identification also selected for high motivation.

Although it is not well known how precocious mathematical reasoning ability relates to "*mathematical reasoning ability*" of adults, SMPY has a protocol any researcher can reproduce (many have), that enables the selection of groups of individuals with high tested ability. Criticisms of whether we are measuring "*true*" mathematical reasoning ability are presently not germane. If a test can predict future achievement, it has value regardless of the exact nature of the aptitude measured. If the test does predict high achievement, then we may want to determine what it measures or what mathematical reasoning ability may be. SMPY's purpose is in part to determine the predictive validity of the SAT-M. Our work to date indicates that it does predict relevant criteria (e.g., Benbow & Stanley, 1983a). For example, SAT-M scores identified mathematically highly talented 11th-graders better than their mathematics teachers (Stanley, 1976).

Finally, SMPY has sought already-evident ability, rather than some presumed underlying potential that has not yet become manifest. Thus, we have not concerned ourselves with possible late bloomers. We are not even convinced that there exist many late bloomers in terms of ability. Although it is possible to find a student whose SAT scores improve greatly in one year, for example over 200 points more than other students his/her age, the chance is remote. We at SMPY feel that nearly all late bloomers are more the result of early lack of motivation or test sophistication than of suddenly developed ability.

Talent Search Results

Results from the six SMPY talent searches are shown in Table 1. Most students scored rather high on both the SAT-M and SAT-V. Their performance was equivalent to the average scores of a national sample of high school students. On the SAT-V, the boys and girls performed about equally well. The mean performance of 7th grade students on SAT-V was at the 30th percentile of college-bound 12th graders. On the SAT-M seventh grade boys scored at approximately the 37th percentile of college-bound senior males

Table 1
Performance of Students in the Study of Mathematically Precocious Youth in Each of the First Six Talent Searchers (*N* = 9927)

				SAT-M Scores[a]				SAT-V Scores[b]			
		Number		Boys		Girls		Boys		Girls	
Test Date	Grade	Boys	Girls	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
March 1972	7	90	77	460	104	423	75				
	8+	133	96	528	105	458	88				
January 1973	7	135	88	495	85	440	66	385	71	374	74
	8+	286	158	551	85	511	63	431	89	442	83
January 1974	7	372	222	473	85	440	68				
	8+	556	369	540	82	503	72				
December 1976	7	495	356	455	84	421	64	370	73	368	70
	8[c]	12	10	598	126	482	83	487	129	390	61
January 1978	7 and 8[c]	1549	1249	448	87	413	71	375	80	372	78
January 1979	7 and 8[c]	2046	1628	436	87	404	77	470	76	370	77

[a]Mean score for a random sample of high school juniors and seniors was 416 for males and 390 for females.

[b]Mean score for a random sample of high school juniors and seniors was 368 for males and females.

[c]These rare 8th graders were accelerated at least 1 year in school grade placement.

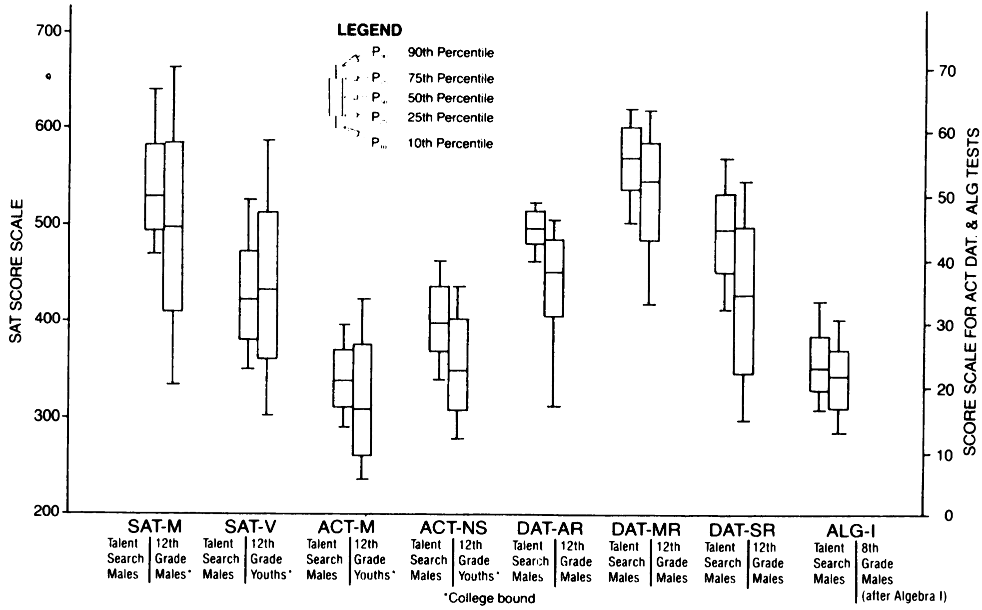
Taken from Stanley & Benbow (1983b).

and the seventh grade girls at approximately the 39th percentile of college-bound senior females. The eighth graders scored slightly better than the seventh graders, as would be expected.

Clearly, SMPY identified a group of mathematically precocious students who also tended to be highly able verbally. Cohn (1977, 1980) and Benbow (1978) found that mathematically talented students are also advanced in their other specific cognitive abilities and in their knowledge of science and mathematics (see Figures 1 and 2). SMPY students tended to have especially strong spatial, mechanical, and nonverbal reasoning abilities. Their performance was similar to students several years older than our talent search participants. Their verbal abilities were also superior, but less so than their mathematical abilities (as is predicted by regression towards the mean).

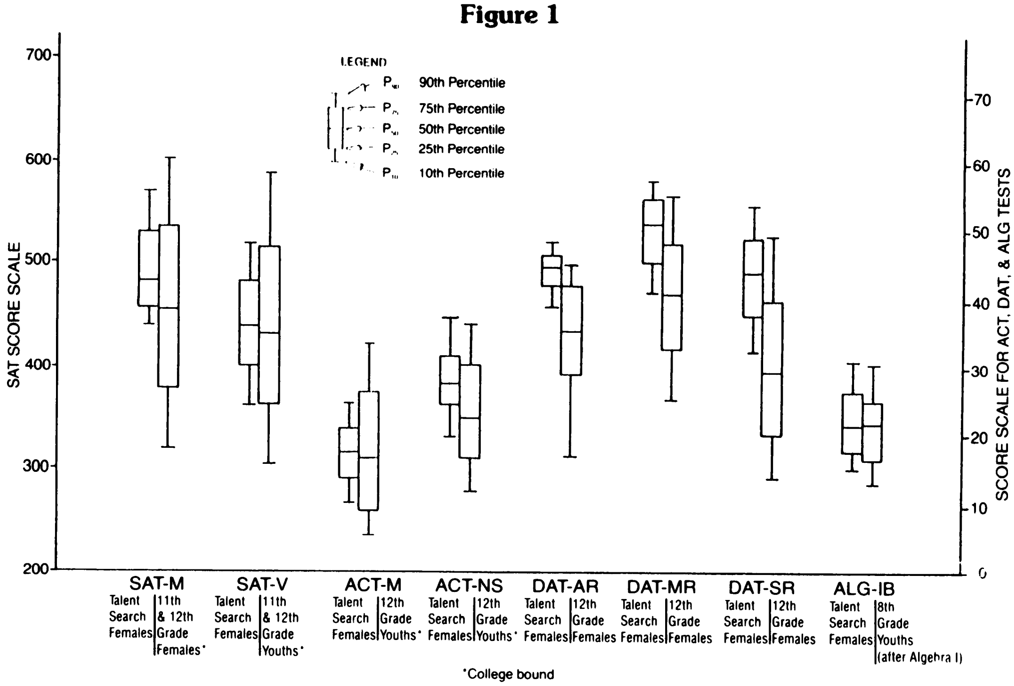
Renzulli (1978) has argued that giftedness is made up of three separate components: above-average ability, task commitment, and creativity. The students identified by SMPY exhibit two of the three qualities: high mathematical reasoning ability and motivation. An objective of SMPY is to provide the knowledge necessary to be creative and to determine if the SMPY participants then become creative as adults. As Keating proposed (1980), in order to be creative a person needs to have knowledge. Creativity cannot exist in a vacuum. Moreover, creativity is difficult to measure. For these reasons, SMPY has largely ignored using an explicit creativity measure as part of its identification procedure.

In addition, SMPY chose to focus on mathematical reasoning ability rather than general intelligence or IQ. The IQ is a global composite, perhaps the best single index of general learning rate. One can, however, earn a certain IQ in a variety of ways, e.g., by scoring high on vocabulary but much lower on reasoning, or vice versa. Therefore, it seemed to the staff of SMPY illogical and inefficient to group students for instruction or



Comparison of scores earned on eight cognitive tests by 7th grade MALES (N = 188) who participated in the December 1976 Talent Search and were called back for further testing with the scores earned by various normative groups of older youths. NOTE: Since the score scales are not equivalent across the different tests, compare the scores earned by the Talent Search males on a particular test with the scores earned by the normative group for that test only.

Figure 1



Comparison of scores earned on eight cognitive tests by 7th grade FEMALES (N = 90) who participated in the December 1976 Talent Search and were called back for further testing with the scores earned by various normative groups of older youths. NOTE: Since the score scales are not equivalent across the different tests, compare the scores earned by the Talent Search females on a particular test with the scores earned by the normative group for that test only.

Figure 2

special programs in mathematics mainly on the basis of overall mental age or IQ. Often this is done and students who lag behind are accused of being underachievers or not well motivated. The true reason often is that they simply have less aptitude for learning mathematics than some in the class who have the same IQ.

The first six talent searches (1972–1979) were conducted to seek young people who reason extremely well mathematically. This was, however, primarily a means to the end of finding suitable students on whom to develop educational principles, practices, and techniques that schools could then adapt to meet their own needs. As of the seventh talent search, conducted in January 1980, SMPY relinquished that important service function to the newly created agency at Johns Hopkins, the Center for the Advancement of Academically Talented Youth (CTY). CTY adapted and extended the talent search model to discover verbally and/or generally talented students, also. The effectiveness of this approach for these three areas has been proven by CTY thus far in seven massive talent searchers, 1980–1986, involving about 125,000 students.

SMPY's Four D's

The first book on SMPY's work (Stanley, Keating & Fox, 1974) was entitled ***Mathematical Talent: Discovery, Description, and Development.*** Since then we have added a fourth D, ***Dissemination*** of our findings, and abbreviated that title to MT:D[4]. ***Discovery*** is the identification phase during which the talent is found through the talent searches. ***Description*** is the phase during which the top students in the talent searches are tested further, affectively and cognitively. This leads to SMPY's main goal, ***development.*** During this phase mathematically talented students are continually helped, facilitated and encouraged. Each is offered a smorgasbord of special educational options (see Stanley & Benbow, 1982a) from which to choose whatever combination, including nothing, that best suits the individual. The staff of SMPY provides as much guidance as its resources permit.

Most studies of talent do not provide educational facilitation for those students identified as part of their investigations. From the start the SMPY staff was determined to steer a different course. Intervention on behalf of the able youths found took an important role. Thus, discovery and description were seen as essential only in that they lead to emphasis on accelerating educational development, particularly in mathematics and related subjects.

We chose to emphasize educational acceleration rather than enrichment. There were both logical and empirical reasons for this. Our rationale was that the pacing of educational programs must be responsive to the capacities and knowledge of individual children. As Robinson (1983) eloquently stated, this conclusion is based on three basic principles derived from developmental psychology. The first is that learning is a sequential and developmental process (e.g., Hilgard & Bower, 1974). The second is that there are large differences in learning status among individuals at any given age. Although the acquisition of knowledge and the development of patterns of organization follow predictable sequences, children progress through these sequences at varying rates (Bayley, 1955, 1970; George, Cohn, & Stanley, 1979; Keating, 1976; Keating & Schaeffer, 1975; Keating & Stanley, 1972; Robinson & Robinson, 1976).

The final such principle influencing SMPY's work is that effective teaching involves assessing the student's status in the learning process and posing problems slightly exceeding the level already mastered. Work that is too easy produces boredom; work that is too difficult cannot be understood. This Hunt (1961) referred to as ***"the problem***

of the match, which is based on the premise that "learning occurs only when there is an appropriate match between the circumstances that a child encounters and the schemata that he/she has already assimilated into his/her repertoire" (p. 268). Hunt notes that *"the principle is only another statement of the educator's adage that 'teaching must start where the learner is'* " (p. 268).

These three principles, as delineated by Robinson (1983), form the guiding premise behind SMPY's work. Its implication for education, as interpreted by SMPY, is that the pace of educational programs must be adapted to the capacities and knowledge of individual children. Clearly, gifted students are not at the same levels academically as their average-ability classmates. Moreover, what is offered in the regular classroom for all children cannot possibly meet this requirement.

SMPY has found adapting existing curricula rather than writing new curricula to be most productive in meeting this need. A side benefit of this approach is that it avoids the common criticism of elitism and costs little for a school system to adopt. Actually, the various accelerative and enriching options devised by SMPY may save the school system money.

Educational Options

The various options the staffs of SMPY and CTY have established as being effective and thus present to their students who express a desire for more rapid educational growth will be described in more detail in this section. They have been articulated earlier in such publications as Stanley and Benbow (1982a, 1983) and Benbow and Stanley (1983b). The main attraction of these dozen alternatives is that they are extremely flexible. Thus, teachers or school administrators can choose and adapt them in ways to fit their unique circumstances and their students' individual abilities, needs and interests.

1 The least unsettling alternative for many students is to have them take as many stimulating high school courses as possible, but yet enough others to ensure high school graduation. At the same time, the student takes one or two college courses a semester from a local institution on released time from school, at night or during summers. Thereby, the student graduates from high school with the added bonus of some college credit. Some of the college courses may even be used for high school credit as well. The individual can, therefore, enjoy the atmosphere of high school while being challenged intellectually.

2 In lieu of the above option, or in addition to it, it may be possible for a bright student to receive college credit for high school course-work through examination. The Advanced Placement Program, which has been sponsored by the College Board since 1955, offers able and motivated students the opportunity to study one or more college-level courses and then, depending on their examination results, to receive advanced standing in college, credit or both.

The program provides schools AP course descriptions in over 20 disciplines, such as biology, chemistry, mathematics, physics and computer science. These course descriptions are prepared by committees of school and college teachers and are revised biennially. The extensive guidelines for high schools to use in setting up and conducting AP classes can be obtained at a minimal cost by writing to ***College Board Publications Orders,*** Box 2815, Princeton, New Jersey 08541.

The committees responsible for the course descriptions also prepare a three-hour examination in each of the respective subjects except Studio Art, for which a portfolio of the student's art is used instead. The Educational Testing Service (ETS) administers these examinations each May. Readers from various schools and colleges then assemble to grade the examinations on a five-point scale: 5, extremely well qualified (or A+ in a college course); 4, well qualified (or A); 3, qualified; 2, possibly qualified; or 1, no recommendation. Each candidate's grade report, examination booklet and other materials in support of his application for advanced placement or credit are sent in July to the college he/she plans to enter. It is then up to the college to decide whether and how it will recognize his/her work. Scores of 4 and 5 on the five-point scale are usually accepted for credit by even the most selective colleges; often, even a 3 is accepted.

The staff of SMPY has encouraged high schools to offer AP courses that prepare students for these examinations and also provide much needed intellectual stimulation. For those small high schools where there are not enough students to fill AP classes, independent study arrangements for the few students ready for AP work could be instituted. Under the supervision of a teacher, students could study at the AP level of a topic following the guidelines of the AP syllabus. Such independent study arrangements should be in lieu of a class.

The rewards of conducting an AP class are rich. Gifted students become intellectually stimulated and thereby avoid boredom while they study at the college level. Successful students may also receive exemption from the first-year course in college so that they can move initially into more appropriately difficult materials there.

Do not, of course, confuse the AP exams with the College Board's Achievement tests. The former are at college level, whereas the latter cover the standard content of high school courses. With the occasional exception or foreign languages, students cannot usually receive any college credit for high scores on the achievement tests.

3 If an appropriate course is not available for a gifted student, have that student take correspondence courses at the high school or college level from a major university, such as Wisconsin or California. This approach requires so much self-discipline from the student, however, that frequently it is less than satisfactory. Nevertheless, this is another possible option for providing an appropriate education for the gifted, especially if a suitably motivating and pacing procedure can be set up. The student must not count on receiving college credit for such studies, however, unless arrangements have been made in advance with the appropriate department in the college or university at which he or she will matriculate.

4 The mechanism of choice when programming for gifted students may be subject-matter acceleration. For example, an individual may complete Algebra I and II in a single school year or during the summer. This can be accomplished by *"doubling up,"* by working with a competent mentor, or through fast-paced classes (Bartkovich & George, 1980; Bartkovich & Mezynski; 1981; Mezynski & Stanley, 1980). Since 1972 SMPY has pioneered the concept of fast-paced classes in several subject matters. These classes are now offered during the academic year and in the summer by CTY. During the summer of 1984, for example, CTY offered courses in precalculus, calculus, several sciences, computer science at three levels, American history at two levels, music theory, German, Latin, writing skills (four levels), etymologies, micro-economics, and probability and statistics. Many school systems have adapted the fast-paced class model for their own use (e.g., Lunny, 1983; Van Tassel-Baska, 1983). Instructions for setting up a fast-paced class can be found in Bartkovich and George (1980) and Reynolds, Kopelke and Durden (1984).

5 A school may attempt to condense grades 9–12 into three years for especially gifted students. Those students would graduate from high school a year early and thereby reach more quickly the intellectually stimulating courses available at college. Senior-year credits, such as English, may be taken during the junior year or during summer sessions. Another possibility is to take college courses that also specifically fulfill high school course requirements, such as supplanting high school calculus with a more advanced college course in calculus (see 10 below). The key to this alternative is a school exercising flexibility in allowing individual programs.

6 In some communities there are insufficient existing educational alternatives to stimulate a very bright student. In such a circumstance, it may be advisable to have a student attend an early entrance college or program in lieu of high school. The three most notable opportunities are Simon's Rock College of Bard College at Great Barrington, Massachusetts; the Freshman Program of the New School for Social Research in New York City; and the program run by Professor Nancy Robinson of the Child Development Research Group at the University of Washington, Seattle, Washington (Robinson, 1983). Exercising this option would require strong commitment on the part of the parents.

7 A skilled local mentor (not necessarily a teacher) may work privately with the student, pacing him or her in areas in which the student is most advanced (Stanley, 1979).

8 For some students it may be desirable to enter college at the end of the tenth or eleventh grade with or without the high school diploma. This may seem extreme, but actually it has become a fairly common practice for highly able students. In fact, a number of institutions have set up specific programs and procedures for applicants who wish to enter college at the end of the eleventh grade. Moreover, the rules of several state boards of education allow the substitution of one year or even one semester of college credit for one year of high school credit. Thus, the high school diploma may be awarded at the end of the first year of college.

The staff of SMPY usually recommends that the student earn some college credits, especially via AP examinations, before leaving high school. This makes the transition smoother when the student goes from high school to college early. For many bright students, leaving high school early with advanced standing via AP examination credits and/or college courses seems to be the preferable mode.

Many of SMPY's protegés have entered college early and done well (see ***Time***, 1977; Nevin, 1977; Stanley & Benbow, 1982b; Stanley & Benbow, 1983a). They attend or have attended a considerable percentage of the most selective universities and colleges. In SMPY's opinion, highly able, well-motivated, emotionally stable students can complete college by age 14 to 20, accruing considerable personal and academic benefit.

9 A quite simple strategy to use in meeting the needs of the gifted for advanced course-work is to allow students to take courses appropriate to their ability and achievement levels, regardless of their age. For example, allow an unusually mathematically able 7th-grader to study algebra, rather than having to wait until the 8th or 9th grade.

10 Encourage intellectually talented students to substitute college courses in mathematics for high school courses that are either unavailable or too elementary. It was not rare for SMPY's ablest, most motivated protegés to complete mathematics through the third semester of college calculus, differential equations, and/or linear algebra while still in high school. One intrepid youth finished the entire undergraduate mathematics curriculum of The Johns Hopkins University's Evening College, through complex variable theory and Fourier analysis, by age 16. Another did likewise at the University of Maryland.

11 Perhaps the most innovative option SMPY has pioneered for mathematically talented students is its fast-paced mathematics classes, where several years of mathematics are learned in one year (Fox, 1974; George & Denham, 1976; Bartkovich & George, 1980; Mezynski & Stanley, 1980; Bartkovich & Mezynski, 1981; Mezynski, Stanley, & McCoart, 1983). This approach has been adapted to the study of college physics and chemistry (Mezynski, Stanley, & McCoart, 1983), high school biology, chemistry, physics, and computer science (Stanley & Stanley, 1986), and the verbal areas (Durden, 1980; Fox & Durden, 1982).

12 Most youths who reason exceptionally well mathematically do not need the basic eighth-grade-level course in science. They normally know the concepts usually covered or can be taught them in a few weeks of review, using the DT-PI model (to be discussed in the next section). Thus, most mathematically and/or scientifically highly gifted eighth graders should begin their studies with biology. Using the DT-PI model or by teaching the course content at an accelerated pace, an instructor could easily cover biology in one semester and then chemistry in the second semester, or vice versa. Students would then advance to physics and computer science the following year. By the time the gifted student reaches tenth grade, he or she would be ready and have enough room in his/her schedule to study the sciences at the college level through the Advanced Placement Program *(see Option 2)*.

These are the main options offered to the mathematically talented students identified by SMPY. In discussions with the students, parents and the SMPY staff, an individual program is tailored for the students using a combination of options. This approach utilizes already available educational opportunities rather than designing new programs or rewriting curricula. As a result, it is politically viable and inexpensive. SMPY's approach may not be the best approach for educating the gifted child, but it is certainly the most practicable to help gifted students immediately. Longitudinal teaching teams, as proposed by Stanley (1980), may be a much better system, but would be difficult to implement. Furthermore, a different teaching approach than used with average ability students may be desirable to teach the gifted student basic material. SMPY has designed one such appropriate teaching method. It will be described in the next section.

SMPY's Instructional Approach

The extensive experience SMPY had in teaching mathematics at a fast pace to its students revealed that many of them already knew mathematical concepts not yet explicitly taught to them (Bartkovich & George, 1980; Bartkovich & Mezynski, 1981; Stanley, Keating, & Fox, 1974). Actual knowledge seemed somewhat dependent upon the individual's ability (Favazza, 1983). Moreover, the rate at which unknown mathematical concepts and principles were acquired was also a function of ability. These results verified the need for developing a teaching approach that could accommodate both the individual's idiosyncrasies in knowledge of mathematics and his/her rate of learning. The results of experimenting led to the DT-PI (Diagnostic Testing followed by Prescriptive Instructional) model (Stanley, 1978, 1979).

This individualized instructional approach, which can be used in both individual and group settings, is a strategy for teaching gifted students only those aspects of a subject they do not know at a rate dictated by their abilities. It is basically a sequential method of (1) determining the student's current level of knowledge using appropriate standardized tests; (2) pinpointing areas of weakness by analyzing items missed on a given test; (3) devising an instructional program that targets those areas of weakness and allows the student to achieve mastery of the level on a second form of the test; and (4) proceeding to the next higher level and repeating steps 1–3.

The DT-PI model has been used successfully with students as young as six years of age. It can be used to help the student master arithmetic or basic mathematics, precalculus, calculus, the sciences and other subjects such as the mechanics of standard written English. Not only teachers but also teachers' aides, mentors and qualified volunteers from the community can use this approach. It is an extremely flexible instructional model.

The diagnostic testing followed by prescriptive instruction (DT-PI) teaching method is an integral aspect of certain of the above options, especially numbers 7, 11 and 12. Below will be described step by step how to use this instructional approach with gifted students. The description is an adaption of Stanley (1978, 1979). Dr. Julian C. Stanley is the originator of the DT-PI model.

Step 1

Before using the DT-PI model, obtain an estimate of the level at which diagnostic testing should begin. Beginning diagnostic testing at the appropriate level is extremely important in order to avoid frustrating the examinees and thereby weakening motivation. An examinee should score at least half-way between the sheer chance score and a perfect score (which is generally the number of items of which the test consists) on the proper level of the measurement instrument. Usually, this will be approximately the 50th percentile of the age or grade group for which the test is most nearly optimum—that is, the score below which the scores of half of the examinees lie.

Three factors should be taken into account when estimating the level with which to begin. They are the student's standardized achievement and/or ability test performance, educational background and school curriculum. This assessment can be supplemented by remarks from the student's parents or the teacher's knowledge about the student.

With gifted children the level at which assessment commences will probably be considerably above their chronological age. To obtain an initial estimate of the student's ability, the staff of SMPY uses the SAT with 11- to 13-year-olds. Younger or less able students can have their abilities evaluated by the use of easier aptitude tests than the SAT, such as the School and College Ability Test (SCAT) or the Differential Aptitude Test (DAT). (In the appendix to this paper are names and addresses of the publishers of the various tests described.) It can also be useful to measure the student's specific abilities separately. Knowledge of his or her spatial, nonverbal and mechanical comprehension abilities are especially valuable.

In a manner similar to estimating where to begin testing with the Stanford-Binet Intelligence Scale, the examiner must use all available evidence to estimate the point where the student would score at the 85th percentile of the most stringent national norms of students having had that level of mathematics for one year. Such a level of performance indicates that the student already knows well that subject matter. On an Algebra I test, for example, this would be the 85th percentile of students having completed Algebra I. Diagnostic testing would begin at the **next** level up. Thus, if it is estimated that a student already knows Algebra I but not Algebra II, diagnostic testing would begin with Algebra II.

If the estimating procedure is successful, the testee should score around the 50th percentile of the first test administered. Then the procedure goes on to the next step. If, however, the student scores above the 85th percentile, material not yet known should be covered fast and well with a tutor *(Step 9)* before the next higher level of the subject-matter test is administered. Likewise, if the student scores below the 50th percentile of the first test taken, the examiner must go back and test at the previous level in order to insure mastery of that level. If the examinee then scores below the 85th percentile on the easier level of the test, instruction should begin with that level. Otherwise, the level first tested should be pursued.

In SMPY's and CTY's fast-paced mathematics classes for end of the year seventh graders who have scored at least 500 on SAT-M, diagnostic testing begins with Algebra I.

For diagnostic testing in mathematics, the staff of SMPY and CTY has relied on the Cooperative Achievement Tests in Mathematics (Arithmetic, Structure of the Number System, Algebra I, II, and III; Geometry, Trignometry; Analytic Geometry; and Calculus) and/or the Sequential Tests of Educational Progress *(STEP)* in mathematics (Mathematics Concepts and Mathematics Computation, several levels of each). All these were prepared by ETS in two or three essentially equivalent forms each. But other tests may be appropriate. For the teaching of science, the College Board achievement tests in biology, chemistry and physics have been utilized (address of publisher is in appendix). Other standardized tests may be as appropriate or useful.

We shall use the general case of mathematics to illustrate the process of applying the DT-PI model.

Step 2

After estimating where to begin, assess knowledge of mathematics in order to find "*holes*" in the student's background. Administer the determined level of the test to the student, observing carefully the instructions, especially time limits, and providing sufficient scratch paper and pencils.

a. Encourage the examinee to mark on the answer sheet every item that time permits, but to spend little time on those about which he/she has little knowledge.

b. Urge him/her to put a question mark next to the number of each item about whose answer he/she is uncertain. The testee should return to these for further scrutiny if time permits.

c. Notify the examinee when half the testing time has elapsed and also when only five minutes remain.

d. Do not answer any questions about the content of the items. Just say "*Do the best you can.*" Procedural questions, such as how or where to mark an item, may be answered quickly, but should have been covered before testing began.

Step 3

When the testing time expires, collect the answer sheet and score it immediately. Record the number answered correctly. Determine the percentile rank of the score on national norms. If this is at least the 50th percentile of students having had that level of mathematics for one year, but not beyond the 85th, proceed to the next step.

If the score is below the 50th percentile, repeat Step 2 with the next lower level test. As long as the student's score is at or above the 85th percentile on the lower test, continue with Step 4 for the test originally used (but also do Step 9 for the lower level test). If the score is between the 50th and the 85th percentile on the second test, proceed to Step 4, but use the lower level test. If the student scores below the 50th percentile on the second test, an even lower level test should be utilized and the whole process repeated. See the flow chart in Figure 3.

If the score was above the 85th percentile on the original test, repeat Step 2 for the next more difficult level.

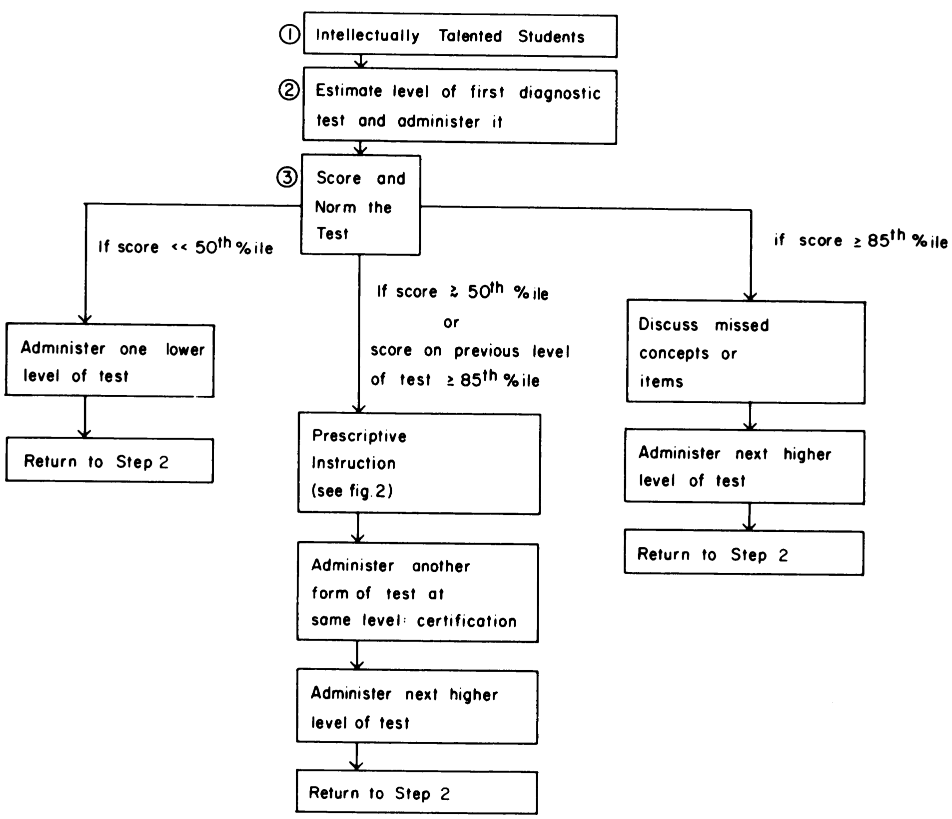


Figure 3. Diagnostic Testing Procedure

Step 4

Using the test that the examinee scored in the approximately 50th to 85th percentile range, give the examinee a list of the numbers of the items still missed on that test and have him/her try them again with unlimited time. Do not show the examinee the scored answer sheet or tell him/her how the missed items were marked. Just give the examinee the item numbers, the test booklet, and scratch paper on which to do those problems not answered correctly under the standard conditions.

Step 5

Those items the examinee still misses should be examined carefully by a mentor, especially to see how the pupil missed them both times; the same way, or a different way. If available, use an item-profile chart to determine which points the examinee does not understand. Item-profile charts are usually provided in the test's manual. If the student appears to have difficulties in more than two areas, it is useful to also administer an instructor-designed test to ensure sufficient knowledge. The purpose of such testing is to pick up those students who scored fairly well on the standardized achievement test because of their high mathematical reasoning ability, but yet do not know the subject as well as their score would indicate.

Step 6

By considering the points underlying the twice-missed items, by querying the examinee about questioned items he/she marked correctly and by further talking with the examinee, the mentor should be able to *"read the examinee's mind"* and devise an instructional program to perfect the examinee's knowledge of that level of mathematics. This should deal only with the points not yet understood. Especially, the mentor should not have his/her pupil work through the entire textbook, but instead do only suitable problems (especially the most difficult ones) concerning those topics not yet well known.

Step 7

This is mentor-paced instruction, not self-paced. The mentor stimulates the youth to move through the materials fast and well, providing help where needed.

Step 8

The goal is for the examinee to score almost perfectly on another form of the same test and also on other standardized tests at that same level. The staff of SMPY has used the 85th percentile as the mastery level.

Step 9

When the student achieves an 85th percentile on another form of the same level test, it is still beneficial to quickly go over the points missed by the student to clear up any misunderstandings. Similarly, this should be done for any test where an 85th percentile is obtained during diagnostic testing.

Step 10

After prescriptive instruction has been completed for one level of mathematics, the next higher level should be administered and Steps 2 through 9 be repeated. For example, after Algebra I has been taught in this way, proceed with Algebra II, and so on. See Figure 4.

For the **"prescriptive instruction"** one needs a skilled mentor. He or she should be intellectually able, fast-minded, and well versed in the subject area, considerably beyond that to be learned by the *"mentee(s)."* This mentor must not function didactically as an instructor, pre-digesting the course material and *"spoon-feeding"* the

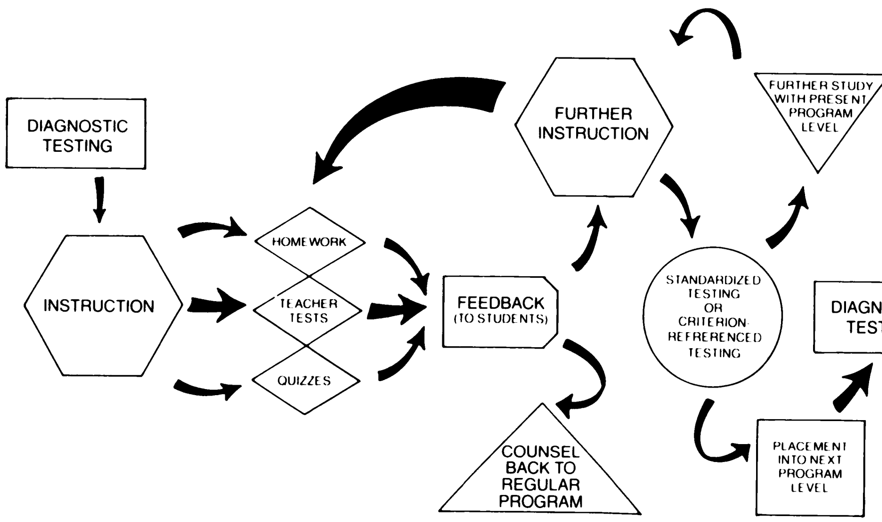


Figure 4. Evaluation process for a fast-paced mathematics class

This material was first published in* **New Voices in Counseling the Gifted, *Colangelo and Zaffron. Copyright 1979, Kendall Hunt Publishing Company.*

mentee. Instead, he or she must be a pacer, stimulator, clarifier and extender. The mentee must take responsibility for his or her own learning, especially via homework done carefully, fully and well between the meetings with the mentor. The mentor must ensure that all the homework is indeed done well.

Not all youths will want to work long under these conditions. The alternative for them is to find a *"tutor,"* someone who will *"teach"* him or her to a much greater extent than is the proper function of the mentor. Obviously, one can get ahead faster with a mentor than if a tutor is required.

The mentor need not be a trained teacher, nor need he or she even be older than the mentee (but much *"smarter,"* of course). SMPY has used a brilliant 10-year-old to serve as the mentor for a brilliant 6-year-old, and later as the 12-year-old (college-sophomore!) mentor for a 15-year-old tenth-grader. Usually, though, the mentor will be several years older than the mentee. Eleventh- or twelfth-graders or college students majoring in the relevant subject area may be excellent. So may older persons, if they are well-grounded in the modern mathematics and science and not slow-minded, pedantic or excessively didactic.

The length and frequency of sessions with the mentor is again an individual matter, depending upon the motivation, ability and time available from the student. Weekly sessions are preferable, but they may be more frequent, especially during summers.

Examples of SMPY's Instructional Approach

Example 1

Step 1. A father wrote in April about his son, 9¾ years old and in the fourth grade, including evidence of extreme mathematical precocity (i.e., SAT scores). The boy was studying algebra on his own, with some help from his parents.

Step 2. At age $9\frac{5}{6}$ this boy took the Cooperative Achievement Test, Algebra I, Form B, under standard conditions.

Step 3. He marked 30 of the 40 items correctly during the 40 minutes. He marked Nos. 17, 26, 27 and 37 incorrectly and omitted Nos. 21, 29, 32, 37, 38 and 39 (although having been encouraged to try all the items). On the most stringent norms his percentile rank was 43, meaning that he scored better than 43% of suburban eighth graders do after studying Algebra I for some 180 45- or 50-minute periods. His score of 30 exceeded the scores of 87% of eighth graders across the country who have studied Algebra I for a school year, and 89% of ninth graders.

Step 4. When given plenty of extra time to try again the 10 items he had missed, the boy worked 6 of them correctly.

Step 5. By studying missed items and consulting an item-profile chart, it seemed clear that the boy's main difficulties were with two topics, "*solution of linear equations*" and "*factoring and quadratic equations*." He was inefficient with the former and largely ignorant concerning the latter.

Steps 6–8. He was given specific, appropriate instruction before taking the other form (A) of this algebra test.

Step 9. He scored above the 85th percentile on the other form of the test but still missed a few items. These were quickly resolved.

Step 10. The process was repeated for Algebra II.

Example 2

Step 1. A third grade student was referred to us by his school because he seemed bright, especially so in mathematics. We administered the Revised Stanford-Binet Intelligence Scale to him and found that his IQ was 150. His strengths did appear to be in the non-verbal areas.

From a discussion with his parents and himself, we estimated his level of knowledge of mathematics. Taking his ability, achievement level and age into consideration, we felt that the STEP Series II Mathematics Computation Form 4A and Mathematics Basic Concepts Form 4A would be most appropriate. Level 4 is for upper elementary school students.

Steps 2–4. He was tested and his score on computation was 433, which placed him at the 52nd percentile of 5th graders tested in the spring. On the basic concepts test he achieved a converted score of 437, which placed him at the 59th percentile of 7th graders in spring or the 41st percentile of 8th graders. When given back his paper to work on, he made four more concepts problems correct on the 50 item test and six more computation problems on that 60 item instrument.

Steps 5–7. His weaknesses were determined, and these were worked on.

Step 8. After several months of mentoring, he was given form B of the same STEP tests. This time he scored in the 90th percentile of eighth graders.

Step 9. The missed items were discussed and explained.

Step 10. We went back to Step 3 and did diagnostic testing, using the next higher level of the STEP test. The instructional process was repeated.

Step 10. We then went back to Step 3 again to begin Algebra I. On the Algebra I test he scored at the 53rd percentile of suburban eighth graders having taken algebra for one year. The instructional process was repeated.

Example 3

Step 1. A young girl was brought to us by her parents. She was accelerated one year in grade placement and had taken Algebra I. Her SAT scores were 590 math and 600 verbal. Since she had completed Algebra I and had high SAT scores, we began testing with the Coop Algebra II test.

Steps 2–3. Her score on the Algebra II test was at the 95th percentile of students having already taken Algebra II for a whole year.

Step 4. We proceeded to Step 4 and cleared up any misunderstandings of the student. Afterwards we went back to Step 3 but now testing her with the Coop Algebra III test. There she scored at the 55th percentile of students having completed that course.

Steps 5–7. Using the profile chart and by talking to her, we determined which concepts were not fully understood and then set up an instructional program.

Step 8. After instruction, her score on the other form of the Algebra III test rose to the 95th percentile.

Steps 9–10. The missed points were covered, and we began geometry by going back to Step 3 and repeating the process. In geometry, however, we supplemented instruction with work on proofs. The ability to do proofs is not tested by the standardized achievement test and is not picked up easily. Because learning how to do proofs is so important in geometry, such additional instruction is necessary.

Although the DT-PI model seems appropriate only in an individual setting, it has been successfully used in a group approach, too. For example, during the summer of 1978 SMPY helped 12 of 33 post-seventh-graders of 1-in-1000 math aptitude to learn Algebra I–III, geometry, trigonometry and analytic geometry excellently in 40–48 hours! As beginning eighth graders they were ready to study college-level calculus (Bartkovich & Mezynski, 1981).

In the group setting students are first classified into various subgroups. Students receiving the same examination are tested together. Upon completion, scoring is immediately performed, and any further evaluation that is needed is determined and done. Using the results, an individual program is set up via the mechanisms described in the model. Students working at the same level (but not necessarily on the same topics) are put in the same class with a mentor. Each works at his or her own rate. There is a mentor available for approximately every 5 or 6 students. Sessions can be held every day, twice a week, or even once a week, but for several hours at a time.

CTY now conducts all the fast-paced mathematics classes that were pioneered by SMPY. Every summer they are offered in a residential setting or for commuter students. During the academic year Saturday commuter classes are conducted. Satellite programs in different regions of the country have also been set up. Moreover, other programs across the country have adopted the model, for example, the Talent Identification Project (TIP) at Duke University, Center for Academic Precocity (CAP) at Arizona State University-Tempe, Child Development Research Group at the University of Washington, and the staff of the Center for Talent Development at Northwestern University. Clearly the DT-PI model has been used successfully in diverse settings. It has also been used to teach biology, chemistry and physics. The staff of SMPY feel that the model has been field-tested sufficiently for us to recommend its adoption as a means to teach mathematics and science to intellectually talented students.

Long-term effects of SMPY participation

While it has been demonstrated that students participating in the various SMPY programs or options have benefited initially (Stanley, Keating, & Fox, 1974; Keating, 1976; Eisenberg & George, 1979; Fox, 1974; George & Denham, 1976; Bartkovich & Mezynski, 1981; Mezynski & Stanley, 1980; Mezynski, Stanley, & McCoart, 1983; Durden, 1980), it is important to determine the long-lasting effects. From the beginning, SMPY was intended to be a longitudinal study to investigate the development of intellectually talented students, as Terman did in his classic study, and also to evaluate the long-term effects of SMPY's educational interventions. Through SMPY's longitudinal studies, it has been shown that short-term benefits are also long-term.

The students in SMPY's first three talent searches have been studied approximately five years after initial contact. Their development was traced through high school (Benbow, 1981, 1983). Students who as seventh- or eighth-graders had scored at least 370 verbal or 390 math on the SAT (the mean scores of a national random sample of high school females) were sent an eight-page printed questionnaire. Over 91 percent of 2188 SMPY students participated by completing the survey. The general conclusion of the study was that SMPY students had fulfilled at least a considerable proportion of their potential in high school.

Relative to appropriate comparison groups SMPY students were superior in both ability and achievement, expressed stronger interests in mathematics and the sciences, were accelerated more frequently in their education, and were more highly motivated educationally, as indicated by their desire for advanced degrees from difficult schools. Over 90% were attending college, and approximately 60% of those were planning to major in the sciences. The results suggested strong relationships between mathematical talent of students in grade seven or eight and subsequent course-taking, achievements, interests, and attitudes in high school. SMPY's identification procedure was effective in selecting students in the seventh grade who achieve at a superior level in high school, especially in science and mathematics (Benbow, 1981, 1983). These students are now being surveyed one year after expected college graduation and will be followed-up throughout their adult lives.

In addition to studying the development of mathematically talented students, the longitudinal study provides useful data for evaluating lasting effects of SMPY's various methods in facilitating the education of its students. It was found, for example, that the successful participants in SMPY's first fast-paced precalculus classes achieved much more in high school and college than the equally able students who had not participated. They were also much more accelerated in their education than the non-participants. The former were satisfied with their acceleration, which they felt did not detract from their social and emotional development. Furthermore, there appeared to be no evidence to justify the fear that accelerating the rate of learning produces gaps in knowledge or poor retention (Benbow, Stanley, & Perkins, 1983). Similar results were found for those students who graduated from *college* before age 19 (Stanley & Benbow, 1983a; Benbow & Stanley, 1983a) and the less accelerated students in the follow-ups (Benbow, 1981, 1983). Most of the SMPY students felt that SMPY had helped them at least some, while not detracting from their social-emotional development (Benbow, 1981, 1983). This was true even for the students with whom the staff of SMPY had not had much contact.

Solano and George (1976) presented the initial findings from encouraging students identified by SMPY to take college courses on a part-time basis before entering college full-time. During the first five years of SMPY's existence, "131 students took 277 college courses and earned an overall GPA of 3.59, where 4 = A and 3 = B. . . . Community colleges are a great deal easier for these students than either colleges or universities. These youths experience little social or emotional difficulty in the college classroom" (Solano & George, 1976, p. 274). SMPY's extensive experience since then does not alter the above conclusions, except to urge that highly able students attend the most academically selective college in their locality.

Case Histories

To illustrate how we use curricular flexibility to provide an appropriate education for gifted students, some examples and three case histories will be provided. The three case histories are updated versions of those appearing in Stanley and Benbow (1983b), while the examples are borrowed from Stanley and Benbow (1986).

A seventh grade boy, who had an SAT-M score of 760, asked permission to enter the eighth-grade Algebra I class in February. Since he already had missed more than half the course, his request was denied. To prove his capabilities, he then insisted on being given a standardized test covering the first year of algebra. On this he made a perfect score, which is two points above the 99.5th percentile of national norms for ninth-grade students who have been in that type of class for a complete school year. Upon seeing this achievement, the teacher agreed with the boy that he was indeed ready to join the class. The boy realized, however, that even the Algebra I class would be too elementary for him. Thus, instead, he took a college mathematics course that summer, in which he easily earned a grade of A. Later, as a high-school senior he represented the United States well in the International Mathematical Olympiad contest.

At the end of the sixth grade a student took second-year algebra in summer school without having had first-year algebra; his final grade was A. He continued his accelerated pace of learning mathematics. Thus, by the end of the eighth grade he had earned credit by examination for two semesters of college calculus by correspondence from a major university, again receiving an A as his grade. At age 21 he graduated from a top university with triple majors in mathematics, physics, and humanities.

Another student learned two and one-half years of algebra well by being tutored while in the fifth and sixth grades. He continued, by means of mentoring, to master geometry at a high level. His tutor in geometry was a sixteen-year-old freshman at Johns Hopkins who was simultaneously taking honors advanced calculus (final grade, A), as well as other courses that most nineteen-year-olds would find extremely difficult.

A remarkable six-year-old boy living in California mastered two years of high-school algebra. At age seven he enrolled in a standard high-school geometry course. Since he found it too slow-paced, he decided to complete the book on his own before Christmas, while he also taught himself trigonometry. Before age $7^1/_2$ he had scored at the 99th percentile on standardized tests of Algebra I–III, geometry and trigonometry. His SAT-M score at age 7 was 670, the 91st percentile of college-bound male high-school seniors. This boy, however, is truly not a typical example of a gifted child. He may be the most precocious boy that SMPY has worked with. His main competition is an eight-year-old boy in Australia, who scored 760 (the 99th percentile) on SAT-M, even though he was unaccustomed to taking multiple-choice tests.

Several girls have accelerated their progress in mathematics considerably, though not as much as the boys discussed above [see Fox (1976) for a discussion of this point]. One of them graduated from high school a year early while being the best student in SMPY's second high-level college calculus class. She went on to earn a bachelor's degree in computer engineering from an outstanding university and then a master's degree in computer science and a Master of Business Administration degree.

To further illustrate what highly motivated and highly able young students can accomplish if given the curricular flexibility they need, three case histories will be delineated below. They are updated versions of those found in Stanley and Benbow (1983b).

Case History 1

Colin Farrell Camerer, who was born in December 1959, is the only son in a family of four children. His father, a college graduate, is a sales manager; his mother, a high-school graduate, is an executive secretary. Both parents are highly intelligent as judged from results of standardized testing. As an accelerated eighth-grader in SMPY's January–February 1973 Talent Search, Colin scored 750 on SAT-M and almost as highly on SAT-V. Through SMPY's first fast-paced mathematics class, which began when he had just finished the sixth grade, Colin learned $4^1/2$ years of precalculus mathematics chiefly on Saturdays, in a total of 14 months. SMPY recommended to him that he accelerate in school, which he was eager to do. Thus, he skipped grades 7, 9, 10 and 12 and then entered Johns Hopkins with sophomore standing through advanced Placement Program (AP) course work and college credits earned while attending the 8th and 11th grades. Despite his acceleration and emphasis on academics, he participated in a wide range of activities. In high school he was on the wrestling and TV quiz teams and participated in student government. At barely 17 years of age, Colin finished his work for the BA degree in quantitative studies at Johns Hopkins at the end of the first semester of the academic year 1976–77 after only five semesters (Stanley & Benbow, 1982b). During his undergraduate years, he was on the Hopkins varsity golf team and was described by a journalist as an **"all-rounder"** (Nevin, 1977). Colin held a variety of jobs while in college, including summer work as an associate editor of a weekly newspaper. In September 1977, while still 17 years old, Colin became a graduate student at the University of Chicago. He remained there, earning his MBA degree at 19 and completing all work for the Ph.D. degree in finance before age 22. In the meanwhile, he resurrected the student newspaper along with a friend. His hobbies include skiing, tennis, golf, horseracing and writing. Several letters written during graduate school indicated that he was very mature for his age. The content and style was similar to that expected of a student well into his twenties. While still 21 years old and with several research publications to his credit, he became an assistant professor of management at Northwestern University and a consultant to businesses. He is now an assistant professor at the Wharton School of Business at the University of Pennsylvania.

When Colin is asked about his acceleration, he feels very satisfied with it. He shudders at the thought of not having been given the curricular flexibility that he so desired and needed. As for his social and emotional development, he does not think that acceleration affected it. He views himself as a natural loner. He would not have socialized more if he had not been accelerated, perhaps less because of the frustrations he surely would have had to deal with.

Case History 2

Chi-Bin Chien is also among the brightest students identified by SMPY. In December 1975, a month after his 10th birthday, he took the SAT and scored 600 on SAT-V and 680 on SAT-M. A year later in SMPY's December 1976 Talent Search, he raised these scores to 710 and 750, respectively. A variety of intelligence test scores indicated an IQ of at least 200. A Chinese-American boy whose father is a professor of physics and whose mother has a master's degree in psychology, Chi-Bin has two younger siblings who are also extremely able and scored above 700 on SAT-M before age 13. Because of his father's persistent efforts he was given special educational opportunities in a private school. It was decided that this was not enough, however. Thus, Chi-Bin received some individual mentoring in mathematics, using the DT-PI model. Through the diagnostic testing, it was discovered that, even though Chi-Bin had taken only Algebra I in the fifth grade, by age 11 he knew Algebra II, Algebra III and plane geometry. Trigonometry and analytic geometry were taught to him in a few weeks. Through consultation with SMPY, it was decided that he should skip several grades while taking college courses on the side and Advanced Placement work. By age 12 Chi-Bin had completed his work for a diploma from an excellent public school in Palo Alto, California and calculus courses at Stanford. In the fall of 1978, while still 12 years old, Chi-Bin entered Johns Hopkins with sophomore standing. He had been accepted at Harvard and Cal Tech as well. In May of 1981 he received his baccalaureate at age 15, with a major in physics, general and departmental honors, the award in physics, a Churchill Scholarship for a year to study at Cambridge University in England, and a 3-year National Science Foundation Graduate Fellowship to work toward his Ph.D. in biophysics at the California Institute of Technology after returning from England. Chi-Bin is presently pursuing his studies at Cal Tech.

Case History 3

A third example is a remarkable girl who entered Johns Hopkins one year early with sophomore standing. In May 1980, near the end of her 11th grade, Nina Morishige, from a small town in Oklahoma, took five AP examinations in one week and scored four 5's and a 4. Thereby, she earned a full year of college credit at Johns Hopkins. Previously, as a tenth-grader she had won the state high school piano competition. Not only is Nina an academic and musical prodigy, she also shows leadership potential. This is evidenced by her having been elected governor of the high school political assembly, Girls' State, in Oklahoma. In September 1980, with a National Merit Scholarship and sophomore class standing, Nina became a full-time student at Johns Hopkins, choosing the University both for its accelerated mathematics program and for the opportunity to pursue piano studies at its Peabody Conservatory. At Hopkins she played the flute and violin, was a member of the women's varsity fencing team, completed her BA degree in mathematics with high honors, including election to Phi Beta Kappa, at age 18. A few months later she earned her master's degree in mathematics. She is probably the youngest American ever to win a Rhodes Scholarship, which provides two years of study at Oxford University. She is studying mathematics and science there and expects to receive her doctorate in mathematics before she returns to the U.S. Nina also won a Churchill Scholarship to Cambridge University for a year. Faced with this choice, she accepted the Rhodes. While studying for her doctorate degree, Nina has traveled all over Europe and Africa to further satisfy her thirst for learning.

These three examples are extreme cases of precocity, achievement and motivation. They illustrate well, however, what highly motivated and precocious students can achieve when given the curricular flexibility they so desperately require. Unfortunately, educators are often biased against acceleration, even though research has shown it to be one of the most viable methods for providing an appropriate education for the gifted (Daurio, 1979; Gallagher, 1975; Pollins, 1983; Robinson, 1983). No study to date has shown that acceleration is detrimental to social and emotional development (ibid.).

These extreme case histories also illustrate well how the various options devised by SMPY can be used together. The less able gifted student would not need as much acceleration and therefore would use fewer of the options or just one. The elegance of the SMPY model is that through its use an individual program can be tailored to meet the needs of each intellectually talented student.

Conclusions

A major conclusion is that academically advanced students need to be identified early and, through curricular flexibility, helped educationally in major ways. Rather than providing special programs within regular schools, it is more practical to allow students to advance to a level of the curriculum that is at their intellectual level. Thus, instead of having teachers of the gifted, we need educational coordinators for the gifted. These coordinators would plan with each student his or her educational program, using available opportunities. Stanley (1980) has also proposed longitudinal teaching teams in each subject area. Thereby, students could advance at their own pace within each

It is apparent that SMPY has encouraged acceleration for gifted students (see Stanley & Benbow, 1982a). Readers may wonder, **"Why hurry?"** One part of the answer is that boredom stifles interest, liking for these subjects and sharpness of thinking in them. Moreover, accelerated youths who reason extremely well mathematically will tend to go much further educationally, in more difficult fields and at more demanding universities, than if they were left age-in-grade (see Nevin, 1977; ***Time,*** 1977). They will tend to stay more directly in the mathematical, engineering and physical sciences and earn outstanding doctorates, master's degrees or baccalaureates before entering the job market at an early age. This enables them to be fully functioning professionals during their peak mental and physical years (see Lehman, 1953), when most of their equally able agemates are still students. Instead of receiving the doctorate at around 30 years of age, they will have it in the early 20's or even the late teens. Both creative contributions and other activities of the "*normal scientist*" (Kuhn, 1970) are likely to be enhanced greatly by the better base laid earlier and by the in-depth pursuit of important special fields.

Finally, Zuckerman (1977) found that a common thread among Nobel Laureates was their systematic, long-term accumulation of educational advantage. Accelerating a student's education would be one such advantage. Data from SMPY's longitudinal study have already shown how acceleration is an advantage that accumulates. Thus, SMPY's most salient finding from working with 85,000 gifted young students over a 13-year period is that school systems need far more curricular flexibility than most of them yet have. The staff of SMPY has extensively tried out various practicable, cost-effective ways to gain such flexibility.

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Appendix: Publishers of Various Tests

College Board Achievement Tests. The College Board, 888 Seventh Avenue, New York, New York 10102.

Cooperative Mathematics Tests. Addison-Wesley Publishing Company, Reading, Massachusetts 01867.

Differential Aptitude Tests. The Psychological Corporation, 304 East 45th Street, New York, New York 10017.

Scholastic Aptitude Test (SAT). The College Board, 888 Seventh Avenue, New York, New York 10102.

School and College Ability Test (SCAT). The College Board, 888 Seventh Avenue, New York, New York 10102.

Discussion Questions

1. Compare SMPY's operational definition of giftedness to Renzulli's concept of giftedness. What advantages or disadvantages result from using these types of definitions to determine giftedness rather than a high I.Q. score alone?

2. SMPY bases its educational programs on three principles of learning as outlined by Robinson. Discuss the effects on education if school systems were to adopt these principles on an overall scale.

3. This chapter outlines twelve educational alternatives for gifted students. What are the advantages or disadvantages of these options for the student? The student's family? The school system?

4. SMPY tailors an individual program for each student. Which of the twelve alternatives could be implemented by a school system on a regular basis?

5. SMPY's teaching method is the DT-PI model. What are the advantages or disadvantages of this method versus the teaching methods currently implemented in schools?

6. The DT-PI model has been used successfully for group teaching. How might school systems use this model for teaching both gifted and non-gifted students?