Educating Mathematically Precocious Youths: Twelve Policy Recommendations

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Mathematical reasoning ability is a scarce and probably dwindling national resource. The Study of Mathematically Precocious Youth (SMPY) estimates that only about 25,000 persons in each student age group reason mathematically well enough to become *outstanding* engineers, mathematicians, physical scientists, or quantitative social scientists. All the professions and vocations that need great quantitative ability must compete for this limited pool of talent.

Of course, one can make strenuous efforts to develop the mathematical reasoning ability of students who are not already excellent in this respect. Better initial teaching will probably help some, and maybe a great deal. It makes no sense, however, to allow youths who (for whatever reasons of environment and heredity) already have this valuable ability to languish in what are, for them, painfully slow-paced mathematics and science classes. Those who are both able and eager to forge ahead of their age group educationally deserve every reasonable opportunity to do so (Stanley, 1977, 1980a).

Consider youths who reason so well mathematically that, even before taking a course called "Algebra I," they score higher on a standardized algebra test than children who have taken the course for a full school year. About half of the 12-year-olds who score 500 or more on SAT-M would do that well. The percentage is small, but the number from a given age group is likely to be around 12,000.

Most students who have picked up a great deal of algebra knowledge while in general mathematics or "pre-algebra" have to serve the same 135–158 hours in a first-year algebra classroom as other students who are far less mathematically knowledgeable. Usually, a class is paced according to the needs of the average student, who at the outset knows little algebra and may have slight aptitude for learning it even when the material is presented slowly. The least mathematically able students in the class slow the pace even more. Picture youths who are highly apt mathematically and their dilemma at being incarcerated in this situation. Can they insist that the teacher move faster? Should they daydream? Become the class clown? Manicure homework in order to make perfect grades, even though they don't need the practice? "Bait" the teacher? Show off their knowledge? Skip the class? Ask to be allowed to attend class only for quizzes and examinations?

These children are not likely to escape this situation in ways that articulate with the rest of the curriculum, such as second-year algebra. Mathematically precocious students are seldom allowed to prove that they already know algebra well, and perhaps even better than the junior high school mathematics teacher, because school systems rarely use tests to discover aptitude. The child's entrance to first-year algebra may be a simple, 40-item group test that requires less than 45 minutes to administer and two or three minutes to score and norm. For what to us are obscure reasons, most educators believe that every child should have a year of algebra, no matter how unnecessary. Many teachers consider the time serving itself important.

In many areas of education such as music and sports we readily accept the principle of placement according to competence. The age-in-grade system is not preordained. Before World War II one frequently found a mixture of ages in the classroom, especially in the one-room schoolhouse. The age-in-grade system is a rather modern invention based on "socially promoting" the least able and holding back the ablest.

When pressed to explain why they defy the commonsense notion that students who already know a subject should not have to take it, many educators vaguely question the validity of the standardized test or say that, in ways not explained, being in the class will somehow be good for the student's

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social and emotional development. It is remarkable that some students, bored and frustrated in this way, nevertheless preserve an interest in mathematics through the long procession that continues with Algebra II and III, plane geometry, trigonometry, analytic geometry, and (for the small percentage who persevere) calculus; unfortunately, quite a few do not. Usually, each of these must be pursued for a full or half Carnegie unit, in all requiring about 5½ school years—more than 700 class hours. Most mathematically talented students (upper 3%) require far less time than that to learn this basic material (Bartkovich & Mezynski, 1981).

How can the time in class be shortened for students scoring 500 or above on SAT-M? SMPY has developed what it calls a DT → PI model: diagnostic testing followed by prescribed instruction (Stanley, 1978b, 1979). The basic principle is to determine what the knowledgeable student does *not* know about a given subject and then help him or her to learn just that, without having to take an entire course or wade through a textbook containing material already known. In this way, aided by skilled mentors (guides, not lecturers) and suitable materials, the youth is likely to master Algebra I in 15 hours or less. The mathematically brilliant student is unlikely to need much more than that for Algebra II, either. So, if the student's actual aptitude for learning mathematics this way proves to be high and he or she is eager to move ahead rapidly and willing to do homework well, the 4½-year process from Algebra I through analytic geometry can be shortened drastically. For example, 12 of the 33 highly selected boys and girls in SMPY's summer-of-1978 fast-paced precalculus program mastered up through analytic geometry in about 35 hours of instruction time, meeting 4½ hours per day, 1 day per week, for 8 weeks. From a less carefully selected group of 96 students, in 1979 5 did that well (Bartkovich & Mezynski, 1981). (Success rates in the intensive 3-week residential classes during the summers of 1980 and 1981 were even higher.)

Thus, in less than 6 percent of the typical minimum time, these 17 students were well prepared to start calculus, usually a 12th-grade subject, as eighth graders. Most of them did and they succeeded well. Later, several earned credit for two semesters of *college* calculus, usually by scoring high on the difficult Level BC of the College Board's Advanced Placement Program mathematics examination. These achievements opened new vistas to the students in mathematics and related subjects such as physics and computer science. They also provided time for other courses in the student's high school curriculum. In most cases, mathematically talented students chose to accelerate their progress through the school grades in order to enter college with advanced study. Some of these students entered college with advanced standing (e.g., as sophomores or, in at least two instances, juniors) because of APP examinations they had passed and/or college courses they had taken on a part-time basis while still in high school. This extends for them the time when the most creative achievements in one's professional field tend to be made (Lehman, 1953).

Furthermore, participating in a fast-paced mathematics program appears to have multiple effects. Not only is the time devoted to mathematics greatly reduced, but also the time used to complete one's education. In the 8-year follow-up of the students in SMPY's first fast-paced mathematics class, we found that the very successful students had achieved academically higher in high school and college than the equally able students who could not participate (Benbow, Perkins, & Stanley, 1982). This ties in well with Zuckerman's (1977) theory of the accumulation of advantage.

SMPY considers the DT → PI approach central to the proper utilization of our "smorgasbord of special educationally accelerative opportunities," from which students may chose *ad libitum* (Benbow, 1979; Stanley, 1978a). It is especially helpful in preventing mathematically talented youths from wasting time in beginning algebra, because that subject is almost trivially easy for many of them.

Policy Recommendations

On the basis of SMPY's 13 years of work with talented students and their longitudinal follow-up, we offer the following educational policy recommendations:

(1) *Students who are capable of achieving at a high level and are good prospects for educational facilitation should be identified early nationwide.* The use of the College Board's Scholastic Aptitude Test (SAT) with seventh and eighth graders is one such method (Stanley, 1977–78). It has not only been shown to be useful initially, but has also been validated over a long-term basis (Benbow, 1981). Moreover, duplication of the SMPY model has been done at Duke University for 16 states. This major effort, led by Robert N. Sawyer, began in September 1980.

Sanford J. Cohn has begun a Southwestern and Western extension of the SMPY model from the Department of Special Education at the Arizona State University, Tempe.

A program that begins with extremely young preschoolers was conducted for a number of years in the Department of Psychology of the University of Washington, Seattle, by the late Professor Halbert B. Robinson. It is now being continued by his associates (Robinson, 1982). This program has radically accelerative aspects, surpassing even SMPY's emphasis on educational acceleration.

(2) *Students should be allowed to take mathematics courses appropriate to their ability and achievement levels, regardless of their age.*

In many school systems, even the most mathematically talented students are not permitted to study algebra formally until the ninth grade. In others, students in college preparatory curricula find algebra available when they are eighth graders. It would be desirable to have the eighth-grade option all over the country. Large public and private junior high schools may need a section of Algebra I for unusually able seventh graders.

Being underage or in a lower grade than the one in which algebra (or any other type of course) is offered should not prohibit a student from beginning the subject. For example, in one of SMPY's special, single-school, fast-paced Algebra I classes where only able fourth through seventh graders were enrolled, the top student was a fifth grader. In just 37 hours he learned the subject better, as judged by the Cooperative Mathematics Test, than all but 1 in 67 *eighth* graders do after an entire school year of instruction (See Stanley, 1976, pp. 132–146).

Another way to improve the situation for students who reason extremely well mathematically is to let them take two courses (e.g., Algebra II and plane geometry) during the same school year.

(3) *Intellectually talented students should be able to substitute courses such as college algebra and calculus, taken as a part-time college student, for high school courses that are either unavailable or too elementary.* Moreover, all colleges and universities should be encouraged to permit such enrollment for credit. At present, many public and private schools do not have such a policy.

It is not rare for SMPY's ablest, most motivated protégés to complete mathematics through the third semester of college calculus, differential equations, and/or linear algebra while still in high school. One intrepid youth finished the entire undergraduate mathematics curriculum of The Johns Hopkins University's Evening College, through complex variable theory and Fourier analysis, by age 16. Solano and George (1976) presented the initial findings from encouraging students identified by SMPY to take college courses on a part-time basis before entering college full-time. During the first 5 years of SMPY's existence, "131 students took 277 college courses and earned an overall GPA of 3.59, where 4 = A and 3 = B. . . . Community colleges are a great deal easier for these students than either colleges or universities. These youths experience little social or emotional difficulty in the college classroom" (Solano & George, 1976, p. 274). SMPY's extensive experience since then does not alter the above conclusions.

(4) *Taking Advanced Placement Program (AP) examinations by highly able students should be encouraged in all possible ways.* Success on them usually provides inexpensive college credit, waiver of the college courses, and permission to enroll at the next level. Each May the Advanced Placement Program of the College Board administers these college-level tests in a variety of subjects. If a student earns a high score on one or more of these difficult examinations, many colleges will grant credit.

SMPY has found that AP-oriented, college-level classes meeting 2–2½ hours per week on 30 Saturdays or Sundays during the school year can help youths who are also taking the high school subject to score considerably better on the AP examinations (see Benbow & Stanley, 1978; Mezynski, McCoart, & Stanley, 1982; Mezynski & Stanley, 1980).

(5) *Some academically talented students should enter college as full-time students while still younger than the typical age, with or without having earned a high school diploma.* Many of the country's colleges, even some of the finest, however, refuse to accept an applicant for full-time enrollment without a high school diploma, some denying admission to high school graduates who are too underage, and some being reluctant to grant it. A few others welcome them. Much valuable time and motivation are probably irrevocably lost because of these restrictive age-in-grade policies. The age discrimination regulations of the United States Government should apply to a number of these situations. Also, state departments of education could authorize high schools to award diplomas to youths who leave high school a year or more early, after they complete a year of appropriate college courses satisfactorily.

Besides facilitating those who enrolled one year early, at age 17 rather than 18, SMPY has helped many to accelerate their high school education by from 1 to 7 years. The following are a few examples:

- A student entered Brooklyn College from the sixth grade in 1973 at age 11½, started a mathematics major with Calculus III, and graduated in 1977 summa cum laude at age 15 years, 3 months. Two years later he received a master's degree in mathematics from an Ivy League university and continued there working toward a Ph.D. degree in the same field.
- A student entered Johns Hopkins with sophomore status at age 12 and received, with high honors, a B.A. degree in physics at age 15; others entered at 13.
- A student entered an Ivy League university at 13.
- A student completed the first semester at a fine New England college at age 12 with all A's.
- A student who had earned 64 college credits part-time while still in high school came to Johns Hopkins at age 14 as a junior and received a B.A. degree in biology at age 16.

During recent years, half a dozen students graduated successfully from Johns Hopkins at age 17 (see Stanley & Benbow, 1981-82). Quite a few others finished there or elsewhere at 18, 19, or 20, some of them with master's degrees. Of the 632 entering students at Johns Hopkins in the fall of 1980, 72 were at least one year underage. These included a 13-year-old female sophomore.

Eisenberg and George (1978) studied the progress of 35 early entrants during their first four semesters of college and 24 early entrants during their first two semesters. They found that the early entrants performed as well as or better than their age-in-grade classmates. Acceleration did not detract from their social and emotional development.

(6) *The age restrictions on all the National Science Foundation (NSF) summer institutes should be lowered.* Until the summer of

1980, highly able youths had to wait until they had completed the 11th grade before becoming eligible for an NSF summer institute. This is almost surely counterproductive for them and the country. It seems foolish that Mark Kleiman was one of the eight representatives of the United States in the Mathematics World Olympiad for 2 years before he became a high school senior, and yet was technically ineligible for an NSF summer institute in mathematics for at least one of those years.

For the first time during the summer of 1980, however, a number of NSF summer institutes were designed specifically for students in junior high school. We at SMPY hail this as a major advance.[1]

(7) *NSF should require that at least half of the NSF summer institutes be highly accelerative.* At present there is a restriction that such institutes must not anticipate any of the subjects usually taught later in high school or college. This is counter to nearly everything that SMPY has learned from its studies of how to help mathematically able youth forge ahead faster and better than the usual lock-step, age-in-grade Carnegie-unit school curriculum permits (see Stanley, 1978a). NSF's total commitment to educational "enrichment" rather than acceleration is not consistent with the findings of Daurio (1979) or Robinson (1982) or with the research literature on enrichment and acceleration (see George, Cohn, & Stanley, 1979). As Robinson concluded, "the pace of educational programs must be adapted to the capacities and knowledge of individual children" (p. 1). Optimal rate of learning occurs when there is a match between the circumstances that a child encounters and the schemata that he or she has developed.

We hope that many of the summer institutes will be conducted at the college level to prepare a student better for taking College Board Advanced Placement Program examinations in biology, chemistry, mathematics (Level BC, calculus), and physics (Level C). Thereby, students would earn college credit for these courses. A student should have scored at the 75th percentile or better on the College Board achievement test in the subject to be studied before entering this type of class. Besides this minimum, other criteria such as substantial scores on the Verbal and/or Mathematical parts of the Scholastic Aptitude Test or the American College Testing Program battery might also be used.

The benefits of earning two semesters of college credit early would be important. The stimulation of the college-level summer program would probably also cause the student to proceed that fall at the next level of the course (e.g., a sophomore year biology course, organic chemistry, Calculus III, or sophomore year physics) or study other fields.

(8) *Students who complete both a bachelor's and a master's degree in eight semesters or less should be eligible for NSF fellowships.* Currently, they are not. By making some of the ablest students in the country ineligible, the NSF graduate fellowships program may be denying support to a number of the type of students it most desires to help become outstanding scientists. The NSF fellowships are for only 3 years. Most persons will need at least that long to complete a top-level Ph.D. degree program, even if they already have a master's degree.

(9) *Government agencies and private foundations should consider allocating more financial support for the descriptive and long-term follow-up aspects of longitudinal studies such as characterize SMPY's learning how its high-scoring talent search participants turn out in the year 2000.*[2] The need for "pure" research is great. Much funding emphasis is on clever new ideas of a game-like nature. Applied research is less glamorous, but for many purposes careful description and evaluation over a long period of time are essential. Such research can tell us more about how mathematical *reasoning* ability develops. To what extent is it founded on nonverbal reasoning ability such as that measured by the Raven Progressive Matrices? What heredity base, if any, does it have? How is it related to largely algorithmic mathematics, such as elementary calculus, versus "pure" mathematics, such as number theory, analysis, topology, and higher algebra? How important is general intelligence for using mathematical ability? These and a host of other questions cry out for research. It should not be done, however, in lieu of special educational attention to every child in the country who reasons especially well mathematically. The results of such research can become available only "in the long run." Meanwhile, we know plenty with which to help mathematically able youths right now.

(10) *Research should be conducted to discover why females tend to have less well developed mathematical reasoning ability than males and to discover possible remedies.* Eleven years of investigating by SMPY have revealed that among intellectually talented students large sex differences exist in mathematical reasoning ability (Benbow & Stanley, 1980, 1981). Moreover, this sex difference in ability seems to be related to sex differences in mathematics and science achievement in high school and at the beginning of college (Benbow & Stanley, in press[a], in press[b]). If talented girls tend to reason less well mathematically, spatially, and mechanically, is it any wonder that few of them find physics and engineering thrilling? Before urging bright girls who say they prefer biology or chemistry to shift into physics or engineering, on the assumption that they have been frightened away from them by the culture, it would seem sensible to investigate their cognitive abilities further. Of course, other aptitudes should also be included.

A great deal of work on vocational role models for mathematically able girls is being done by Fox (e.g., Fox, Brody, & Tobin, 1980). She and her associates are also studying the genesis of mathematical aptitude and achievement in girls (Fox, Brody, & Tobin, 1982). This last policy recommendation leads naturally into some suggestions for future research.

Future Research

(11) *Teaching gifted children how to use study time effectively should be a priority.* Many intellectually brilliant youths fall short of their potential because they are not good workers, even when the subject matter is at the right level and pace for them. They balk at thinking deeply even about topics that interest them, and they are unwilling to do homework carefully and well, particularly after having had a "free ride" for a number of years in school because of their brilliance. Homework, however, is crucial for success in SMPY's fast-paced and advanced classes. The SMPY staff often finds that the parents of these students are not good managers and motivators. They allow their children to put off their homework until the last minute and then to do it poorly. Often, there are no systematic study plans and routines in the home. We need to study carefully the setting in which each student works so that plans can be devised to prevent failing from lack of focused effort (Stanley, 1980b).

(12) *Research should be pursued on the causes of the great hostility toward precocious intellectual achievement that is endemic in this country and on ways to counteract it* (George et al., 1977). Why is a child violinist, composer, chess player, cinema star, or athlete lauded, whereas the child who excels mathematically or writes splendid poetry is sometimes regarded as a "freak"? This attitude may be stronger in the United States than in some other countries such as the Soviet Union and China. Whether or not it is, however, the deleterious influence on intellectual achievements is probably great. Furthermore, many people consider attempts to provide special educational opportunities for the intellectually talented as elitist. This, we believe, is based on a misconception: democracy does not mean that children must receive the same education, but instead that they should have equal opportunities to develop their abilities.

Conclusions

With these 12 policy recommendations, some of which have several parts, we conclude the presentation of certain educational implications that have grown out of SMPY's decade of work with many thousands of boys and girls who, identified when most of them were 12-year-old seventh-graders, reasoned extremely well mathematically. Students such as these form the major basis for our country's scientific and technological future. We can and must help them use their abilities far better than is permitted at present. Otherwise, the United States is likely to fall far behind the Soviet Union and several other countries in scientific research and technological development.

Moreover, as *Science & Engineering Education for the 1980's & Beyond* (National Science Foundation & Department of Education, 1980) emphasizes, the United States has reached a critical period with respect to its continuing supply of high-level, innovative scientists and engineers. Especially needed are persons who reason extremely well mathematically and promptly earn doctorates from top-flight universities in crucial fields such as petroleum engineering, computer science, chemical engineering, electrical engineering, mechanical engineering, operations research, systems analysis, and mathematical statistics. But if recent test-score declines are an indication, the nation's limited supply of people possessing developed high-order mathematical reasoning ability is shrinking. Furthermore, most persons who reason exceptionally well mathematically at age 12, for example, do not use that invaluable ability well. These policy recommendations can help to ensure that academically talented youths receive recognition, systematic preparation, and opportunities to become superbly educated and trained.

"But," one could ask, "what's the hurry?" We have studied that issue in depth (George, Cohn, & Stanley, 1979). One part of the answer is that boredom kills interest, appreciation for the subjects, and sharpness of thinking. Wouldn't interest be stimulated when in place of the mandatory time spent in beginning algebra in the eighth or (usually) the ninth grade, one needed only 0–25 hours with a good mentor working with a group of three to five math-able students? Even more dramatically, the staff of SMPY has often seen a student go from no credit even for first-year algebra to mastery of the entire precalculus sequence through college algebra, geometry, trigonometry, and analytic geometry (about 4½ years of school mathematics) in 40–48 hours, or in a 3-week residential program in the summer, or (when we were learning how to do this) in 100–120 hours of fast-paced instruction (see Bartkovich & George, 1980; Bartkovich & Mezynski, 1981). Not all highly able students complete the full sequence in such a short time, but the typical one masters about 2 years of school mathematics in 5 hours per day, 1 day per week, for 8 weeks or in the 3-week residential program, with far more zest than in a usual class.

These eager, accelerated youths will go further educationally, in more difficult fields and at the most demanding universities, than if they were left at age-in-grade (see Benbow, Perkins, & Stanley, 1982; Nevin, 1977; *Time*, 1977). They will tend to stay more directly in the mathematical, engineering, and physical sciences and get outstanding doctorates, master's degrees, or baccalaureates before entering the job market at an early age. This enables them to be fully functioning professionals during their peak mental and physical years (see Lehman, 1953), when most of their equally able agemates are still students. Instead of receiving a doctorate at age 30, they will earn it by their early 20's or even the late teens. Both creative contributions and those of the "normal scientist" (Kuhn, 1970) are likely to be enhanced greatly by the better base laid earlier and the indepth pursuit of important special fields. Furthermore, Zuckerman (1977) showed that the effects of being given special education are multiplicative. She describes it as the accumulation of advantage. In addition, this kind of educational acceleration saves the student time and frustration. It also saves the parents and the schools considerable money, because fewer years of education are required. For example, getting Advanced Placement examination credit for biology, calculus, chemistry, and physics usually earns the student sophomore standing and the chance to save one-fourth of the cost of his or her undergraduate education. Most of all, though, it produces earlier a far better, more fully prepared and presumably happier professional than would probably have resulted otherwise.

Footnotes

[1]This recommendation and Numbers 7 and 8 might become moot, so far as NSF is concerned, if its entire science education budget is eliminated. Branscomb (1981) implied that this would not occur: "the National Science Board [which helps set policies and priorities for NSF] . . . will develop appropriate programs for precollege science education" (p. 516). Anyway, these recommendations are applicable to whatever agency or agencies may fund summer institutes for high school students. In this time of decreasing quality of science and mathematics instruction in junior and senior high school, such institutes are sorely needed.

[2]We deeply appreciate The Spencer Foundation of Chicago's generous support of SMPY's activities ever since the study began in 1971.

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