Galileo's discovery of scaling laws

Mark A. Peterson[a)]
Department of Physics, Mount Holyoke College, South Hadley, Massachusetts 01075

(Received 26 September 2001; accepted 7 March 2002)

Galileo's realization that nature is not scale invariant motivated his subsequent discovery of scaling laws. His thinking is traced to two lectures he gave on the geography of Dante's Inferno.

[DOI: 10.1119/1.1475329]

I. INTRODUCTION

Galileo's last book was the *Two New Sciences*,[1] a dialogue in four days. The third and fourth days describe his solution to the long-standing problem of projectile motion, a result of obvious importance and the birth of physics as we know it. But this was only the second of his two new sciences. What was the first one?

Two New Sciences begins in the Venetian Arsenal, the shipyard of the Republic of Venice, with a discussion of the effect of scaling up or scaling down in practical construction projects, like shipbuilding. The discussants include two of Galileo's closest friends, Giovanni Francesco Sagredo and Filippo Salviati, and an Aristotelian philosopher Simplicio, who is perhaps Galileo's friend and sometime adversary Cesare Cremonini, although he is never identified by his real name. It is unlikely, but not impossible, that these three men actually met in life, as Sagredo was a nobleman of Venice, Cremonini was professor of philosophy at Padua, and Salviati, a nobleman of Florence and patron of Galileo, may have been Galileo's student at Padua. The dialogue form is not intended to represent a real meeting, but rather to provide a framework for developing ideas in a lively, engaging way.

The conversation on the First Day wanders into a dazzling variety of topics, but on the Second Day, following this day-long digression, it returns to a serious analysis of scaling, especially in the context of the strength of materials. According to the publisher's foreword, it is this topic that should be understood as the first of the two new sciences.[2] It was the publisher and not Galileo who gave the book its title, and Galileo was unhappy with it, perhaps because it seems irrelevant to much of the ingenious speculation of the First Day.[3] It is clear, though, that Galileo did assign enormous importance to the problem of scaling and the strength of materials, an importance that modern readers have found more than a little puzzling.

I will show that the key to much of what is strange in *Two New Sciences* is to be found in two rather neglected early lectures given by Galileo on the shape, location, and size of Dante's Inferno. The text of these lectures is readily available in the standard 20-volume *Opere* of Galileo among the ''literary'' writings in Volume 9.[4] My reconstruction of its actual significance, which is not at all literary, is the subject of this paper.

II. SCALING IN THE *TWO NEW SCIENCES*

Two New Sciences begins with the subject of scaling. Galileo's observations on scaling in general are ingenious and elegant, and entirely deserving of the prominent place he gives them. These ideas are basic in physics, and are introduced in most introductory physics texts under the heading of dimensional analysis. We could even say that modern renormalization group methods are just our most recent way to deal with problems of scale, still recognizably in a tradition pioneered by Galileo. It is true that Galileo didn't have the algebraic notation to do dimensional analysis the way we do, but his insights within a restricted arena are the same as ours.

An example of such an insight is ''the surface of a small solid is comparatively greater than that of a large one'' because the surface goes like the square of a linear dimension, but the volume goes like the cube.[5] Thus as one scales down macroscopic objects, forces on their surfaces like viscous drag become relatively more important, and bulk forces like weight become relatively less important. Galileo uses this idea on the First Day in the context of resistance in free fall, as an explanation for why similar objects of different size do not fall exactly together, but the smaller one lags behind.

Even though the idea is completely general, most of the discussion on scaling is directed specifically to the strength of materials. That subject is introduced immediately in *Two New Sciences* with the assertion that large ships out of water risk breaking under their own weight, something that is not a concern with small ships.[6] The same subject occupies most of the Second Day, which is devoted to the strength of beams. The question is put in an especially paradoxical way by Sagredo, who says,[6]

> ''Now, because mechanics has its foundation in geometry, where mere size cuts no figure, I do not see that the properties of circles, triangles, cylinders, cones and other solid figures will change with their size. If therefore a large machine be constructed in such a way that its parts bear to one another the same ratio as in a smaller one, and if the smaller is sufficiently strong for the purpose for which it was designed, I do not see why the larger also should not be [sufficiently strong] . . . .''

This argument is remarkable. It combines faith that (1) physics ''is'' geometry, and (2) the scale invariance of the theorems of geometry. For the purpose of this paper we will call it Sagredo's theory of scale invariance.

Two New Sciences modifies Sagredo's theory of scale invariance. Specifically, Galileo adds material properties to (1) above, while still preserving the essential geometrical nature of physics. As Salviati puts it much later, on the Second Day,[7]

> '' . . . these forces, resistances, moments, figures, etc., may be considered in the abstract, dissociated from matter, or in the concrete, associated with matter. Hence the properties which belong to figures that are merely geometrical and non-material must

be modified when we fill these figures with matter and therefore give them weight."

These modifications break scale invariance, but they do not change the essence of the theory: "Since I assume matter to be unchangeable and always the same, it is clear that we are no less able to treat this constant and invariable property in a rigid manner than if it belonged to simple and pure mathematics."[8]

A typical example of the new considerations that arise when "we fill these figures with matter" starts with the breaking of a wooden beam. If the beam breaks, all its longitudinal fibers break, and because their number is proportional to the cross-sectional area of the beam, the beam's strength in withstanding a longitudinal pull should be proportional to its cross-sectional area. If a supported beam is subject to a transverse force, however, the associated torque must be balanced by the first moment of the stress in the fibers, which introduces an additional factor of the diameter of the beam.[9] Thus the beam's strength in withstanding a transverse force should be proportional to its diameter cubed.[10] If the beam is made longer, the torque due to the same transverse force applied at a geometrically similar position is proportionately greater and the beam is proportionately weaker, and so forth. There follow Galileo's very famous observations on why animals cannot be simply scaled up, but rather their bones must become proportionately thicker as they get larger.[11]

All of this discussion is developed in a series of eight propositions with geometrical proofs. It is clearly the geometrical framework which makes these scaling laws a "science."[12] After the eight propositions and the discussion on animals, certain more detailed questions are considered, such as the problem of locating the weakest point in a beam, that is, where it would break, and the problem of finding a beam that would have no unique weakest point, but would be equally strong everywhere. These last topics are clearly separate from the original scaling theory of the eight propositions. They come after it, as applications of it, and they are not numbered, as if they had been added at a later time. These topics arise naturally when one juxtaposes the scaling theory with Aristotle's *Questions on Mechanics*,[13] as Galileo himself points out.[14]

A persistent oddity on the Second Day is a continuing preoccupation with beams that break under their own weight, the same phenomenon that began the whole discussion in the Venetian Arsenal, except that there it was ships that break under their own weight. Another oddity is the sudden change of subject on the First Day. The conversation, which had just barely gotten under way, is about the strength of ropes, made of fibers, but the question of nonfibrous materials is raised, such as metal or stone. Simplicio wants to know what gives such materials their strength. Just before this, and perhaps intended to motivate it, we had read about a marble column that broke under its own weight.[15] Although Salviati hesitates to take time to discuss this, Sagredo says, "But if, by digressions, we can reach new truth, what harm is there in making one now . . . ?"[16] and the conversation goes off for the rest of the day into speculations about the atomic theory of matter, cohesive forces among atoms, and questions of the infinitesimally small and infinitely large, among other subjects. The discussion is diffuse but occasionally brilliant, even if nothing definitive can be said about what holds matter together. The opening lines of the Second Day confirm, however, that the intention is to apply the scaling theory to all solid materials and not just to fibrous materials like wood. Taking everything together, there is an odd hint of a problem in the background that motivates all the discussions of the first two days, but that is never actually stated: some object of stone or metal that breaks under its own weight when it is scaled up. This unspoken problem is what the conversation seems to be circling around, without ever quite saying so.

In seeking to understand what the *Two New Sciences* is really about, we recall how it came to be written. Galileo was remarkably reticent about publishing—before the *Starry Messenger*[17] of 1610, the year he turned 46, he had published hardly a thing. His astronomical discoveries of that year brought him instant fame, and he parlayed this fame into his appointment as Mathematician and Philosopher to the Grand Duke of Tuscany. In the negotiations leading up to that appointment, he described the many books that he wanted leisure to write, based on the research he had been quietly doing at Padua, but these books did not appear. His life at the Tuscan court was fraught with controversy, and the books he actually wrote were reactions to events and new discoveries, the *Discourse on Bodies in Water*,[18] and the *Sunspot Letters*.[17] When the Holy Office declared the opinion of Copernicus formally heretical in 1616, Galileo's fondest project was closed off to him. He silenced himself for several years, and when he finally wrote again (*The Assayer*),[17] after much pressuring from his friends, it was part of a testy controversy that made him new enemies. The debacle over his great *Dialogue Concerning the Two Principal World Systems*[19] and the disastrous trial of 1633[20] should have been the end of his career. Yet in his old age he somehow found the strength to begin writing what would become the *Two New Sciences*. And from somewhere in his past he drew the scaling theory and placed it first.

Just where the scaling theory comes from is a bit mysterious, because there are few surviving references to it. However, it was certainly complete by 1612, the year Galileo published the *Discourse on Bodies in Water*.[18] The most vexing problem that Galileo deals with there is the problem of a thin lamina of material denser than water which nonetheless floats on the surface, supported by surface tension, as we would say now. At the very end of the Discourse, Galileo points out that if the lamina were supported at its perimeter, a thin sample (say of square shape and fixed thickness) would be more easily supported if it were cut into many smaller squares, because in reducing the size, the area of each piece, and hence its weight, would go down faster than the perimeter, which is the source of its support. Galileo presents this argument purely hypothetically, as contrary to fact, because his own idea of what supports the lamina is quite different. Peculiar details aside, it suffices for our purpose that this reasoning is an explicit use of the scaling theory as early as 1612.

An even earlier reference, although not as explicit, occurs in a letter from Galileo to Antonio de'Medici in 1609. This remarkable letter is, in effect, a one-page outline of the *Two New Sciences*, almost thirty years before it was printed! Galileo describes his most recent investigations, saying[21]

> "And just lately I have succeeded in finding all the conclusions, with their proofs, pertaining to forces and resistances of pieces of wood of various lengths, sizes, and shapes, and by how much they would be weaker in the middle than at the ends, and how much more weight they can sustain if the

weight were distributed over the whole rather than concentrated at one place, and what shape wood should have in order to be equally strong everywhere: which science is very necessary in making machines and all kinds of buildings, and which has never been treated before by anyone."

He then goes on to describe certain insights into the motion of projectiles. This letter makes it clear that the basic organization of *Two New Sciences* was already in his mind in 1609, and even seems to say that the results were at that time quite fresh. On closer inspection, though, one sees that the detailed results described in the letter belong to the last section of the Second Day, building on the basic results of the first eight propositions, which must therefore be even earlier material. That the projectile theory was still incomplete at this point, and also comes later in *Two New Sciences*, suggests (which is plausible in any case) that the order of treatment in *Two New Sciences* is chronological. That is, the scaling theory comes first there because it literally was first, and we will have to look earlier to find it.

The next suggestion for the origin of the scaling theory might come from *Two New Sciences* itself. The opening scene in the Venetian Arsenal is peculiarly effective—why not take it at face value? Galileo was obviously familiar with the Arsenal, and it is known that he was consulted there on ship design. What likelier place could there be to encounter problems of scale, in just the way that he says? This assumption could place the origin of the scaling theory as early as 1592, the year he became Professor of Mathematics at Padua. On closer reading, though, one sees that even if Sagredo is surprised by the failure of scale invariance in the shipyard, Salviati is not. He already understands it, and is immediately ready to explain it all to Sagredo. What the scene really says is that Salviati's mind (read Galileo's) was already prepared to understand what he saw in the Arsenal, because he had already done the scaling theory. This means we must search for the scaling theory still earlier, at the very beginning of Galileo's career, or perhaps even earlier than that.

III. THE LECTURES ON DANTE'S INFERNO

Galileo enrolled at the University of Pisa to study medicine at the age of 17, and dropped out at the age of 21. He spent the next several years studying mathematics independently, especially Euclid and Archimedes, and did tutoring in mathematics. While living at home he assisted his father in remarkable experiments on the pitch of plucked strings under known tension.[22] Fifty years later he briefly summarized their experimental results in *Two New Sciences*, the First Day.[23] He proved some theorems on centers of gravity in the style of Archimedes, and at the end of his life he also included those in *Two New Sciences*, as an appendix. It is clear that *Two New Sciences* contains some very early material.

The young Galileo hoped to make a reputation in mathematics with his theorems, and he sent them to a number of Italian mathematicians. He was fortunate to receive a favorable reply from Guidobaldo del Monte, Inspector of Fortifications to the Grand Duke of Tuscany, someone in a position to help him.[24] They corresponded. When the chair of mathematics at Pisa became open, someone, probably Guidobaldo but perhaps his even more illustrious brother Francesco, who had just been made a cardinal,[25] arranged for Galileo to be invited to address the Florentine Academy to give two lectures on mathematical topics.[4] It was in effect a seminar and job interview. Galileo's lectures charmed his audience. Within a few months the young dropout was Professor of Mathematics at Pisa.

Galileo's audience at the Florentine Academy was not a mathematical one. The Florentine Academy was a creation of the Medici dynasty (which had ascended to the nobility only in the immediately previous generation), and had as one of its chief functions the glorification of the Medici in every intellectual arena.[26] It was far more important for Galileo to play to this predilection of his audience than to display mathematical erudition. In the event he brilliantly combined a clear exposition of mathematics with a topic Florence loved to hear, their great poet Dante, and in particular, the geometry of Dante's Inferno, based on evidence from the poem. Although Galileo did introduce certain original material in his lecture, he did not call too much attention to it, and represented himself rather as describing two previous rival attempts to determine the plan of Hell. One of these was by Antonio Manetti, who was represented as a member now deceased of the Florentine Academy itself.[27] The other plan, a later attempt, was by Alessandro Vellutello, not a Florentine.[28] Galileo begins in a way that seems even-handed, but as the second lecture proceeds, he becomes more and more sarcastic about Vellutello's plan until in the end he seems to be defending the virtuous Florentine Manetti against the ridicule of the stupid and thoughtless Vellutello, to the delight of his audience, no doubt. This was how a Medici intellectual should defend the honor of Florence!

Only the large scale features of these plans need concern us here. Manetti's Inferno is a cone-shaped region in the earth, with its vertex at the center and its base on the surface, centered on Jerusalem. But because Galileo is a master of exposition, let him describe it:

> " . . . imagine a straight line which comes from the center of the earth (which is also the center of heaviness and of the Universe) to Jerusalem, and an arc which extends from Jerusalem over the surface of the water and the earth together to a twelfth part of its greatest circumference: such an arc will terminate with one of its extremities on Jerusalem; from the other let a second straight line be drawn to the center of the earth, and we will have a sector of a circle, contained by the two lines which come from the center and the said arc; let us imagine, then, that the line which joins Jerusalem to the center staying fixed, the other line and the arc should be moved in a circle, and that in such motion it should go cutting the earth, and move itself until it returns to where it started. There will be cut from the earth a part like a cone; which, if we imagine it to be taken out of the earth, there will remain, in the place where it was, a hole in the form of a conical surface, and this is the Inferno."

As an aside to the mathematically adept, Galileo gave the volume of this region, which he knew from his study of Archimedes.

The various levels of Manetti's Inferno are regularly spaced, for the most part, with 1/8 the radius of the earth between each level and the next. In particular the first level, Limbo, is at a depth of 1/8 the radius of the earth below the surface, and the shell of material down to this depth forms a cap of this thickness over the whole of Hell. Vellutello's

Inferno, by contrast, is much smaller, located near the center of the earth, and only about 1/10 the radius of the earth in height, making it, as Galileo is quick to say, ridiculously small, only 1/1000 the volume of Manetti's.

Near the end of his presentation, Galileo says

> "Here one might oppose that the Inferno cannot be so large as Manetti makes it, since, as some have suspected, it doesn't seem possible that the vault that covers the Inferno could support itself and not fall into the hole, being so thin, as is necessary if the Inferno comes up so high. And especially, beyond being no thicker than the eighth part of the radius of the earth, which is 405 miles more or less, some of it must be removed for the space of the Grotto of the Uncommitted, and even more must be removed [on the top] for the very great depth of the sea. To this one answers easily that such a thickness is more than sufficient; for taking a little vault which will have an arch of 30 braccias, it will need a thickness of about 4 braccias, which not only is enough, but even if you used just 1 braccia to make an arch of 30 braccias, and perhaps just 1/2, and not 4, it would be enough to support itself; and knowing that the depth of the sea is a very few miles, or better, even less than one mile, if we believe the most expert sailors, and assigning as many miles as seem necessary for the Grotto of the Uncommitted, a determinate measure not being given by the Poet, if this together with the depth of the sea comes to 100 miles, the said vault will still be very thick, and far more than is necessary to hold itself up."

Because in Galileo's units the earth's radius is about 3200 miles, and 1/8 of that is 400 miles, it is clear that he is describing a scale model of the roof of the Inferno, including a certain anteroom hollowed out of it, at a scale of about 1 braccia to 100 miles. A normal man is 3 braccias tall, so the model suggests a large domed roof, somewhat smaller than the famous Brunelleschi dome of the Florentine cathedral which, as Galileo says, is less than 4 braccias thick and supports itself beautifully. This is a convincing argument that Manetti's model can support itself—but only until you realize that the argument assumes scale invariance! Could you really scale it up by a factor of 100,000? Absolutely not! The scaled up version is effectively weaker by that enormous factor and would immediately collapse of its own weight.

IV. DISCUSSION

When Galileo realized his mistake, probably just a short time later, it must have struck him like a lightning bolt. In fact, this is just how Sagredo reacts, in strangely emotional language, at the beginning of *Two New Sciences* where Salviati asserts that nature is *not* scale invariant: "My brain . . . reels. My mind, like a cloud momentarily illuminated by a lightning-flash, is for an instant filled with an unusual light, which now beckons to me and which now suddenly mingles and obscures strange, crude ideas . . . ."[29]

We need look no further to know why the problem of scaling and the strength of materials had urgent meaning for Galileo. There is nothing hypothetical about this suggestion. It is clear in the record, with no ambiguity at all. He had made a gigantic blunder in the Inferno lectures, sufficient to turn his whole argument on its head, and with it his claim to be an intellectual champion of his country and his sovereign, on whom his young career depended.

It may be difficult, from a modern point of view, to take Sagredo's overwrought words seriously. Yet the situation was serious enough, apparently, to force Galileo to concentrate on the scaling problem and develop a new theory almost immediately, to judge from numerous internal clues. Then, instead of publishing the theory, he kept it to himself for almost fifty years! This delay is baffling, if we ask ourselves only how we might have acted. Rather, we must ask how an intellectual of Galileo's time would have acted and why.

The lectures themselves were evidently a success, so what was the crisis? Here we note that the context of the lectures was a dispute, a "duel," between Manetti and Vellutello. The lectures were actually a savage attack on Vellutello. To an extent that it is difficult to appreciate now, an intellectual career at a Renaissance court consisted in attacks and counterattacks, which often functioned as a kind of spectator sport.[30] This kind of ongoing dispute, with reputations on the line, was the very stuff of Italian intellectual life. Galileo's own career offers many examples. And here was the crisis. Nothing would be more natural than to anticipate another round in this dispute, a reply to Galileo from some partisan of Vellutello. Galileo should expect to be attacked—it went without saying—and he should have a good reply ready. But his position on scaling in the lectures was untenable. He had to develop a new position.

In this culture of attack and counterattack, it made a certain kind of sense to keep results secret until they were needed. The most famous example of this phenomenon is probably the solution of the cubic equation, which was held secret for decades until it was finally published by Cardano in 1545.[31] In the same way, Galileo may have developed the scaling theory and then not published it because the moment never actually arrived when he needed it.

It is not pure conjecture that Galileo would have followed such a strategy, keeping secret an important result until he needed it for self-defense. Indeed, in the controversy on floating bodies we see him doing exactly this. This controversy began when Galileo asserted, by Archimedes' principle, that ice must be less dense than water, because it floats. His Aristotelian opponents asserted that according to Aristotle, ice, being cold, must be more dense than water, and that ice floats, despite being more dense, because it is broad and flat. Galileo quickly disposed of this foolish argument, but then his opponents found an example of something (a chip of ebony) that really is denser than water and nonetheless floats, if you carefully put it on the water's surface (because it is broad and flat, they said). At this point neither side in the controversy had a satisfactory understanding of what they were seeing. Galileo's slightly odd response was to bring forward a method to prove Archimedes' principle by geometry, using some mechanical ideas of balance that had long fascinated him. This theory was clearly something he had kept ready for just such a moment, even though it did not really address the problem of the floating ebony chip. He made some new observations of this anomalous floating phenomenon (which we attribute mainly to surface tension) and stretched his geometrical theory to argue that Archimedes' principle could entirely account for it. This strategy worked, in the sense that he argued his opponents to a standstill. He never convinced them, but that was not the point. Rather, he had used his hitherto secret knowledge in a successful defense. As we noted earlier, Galileo also used the scaling theory defensively in this controversy, right at the end, as a clincher, so to speak.[32] Thus it is plausible that he had kept it for just such a purpose.

When Galileo finally published the scaling theory at the end of his life, he hints at the story of its discovery, including his original naivete about scale invariance. We have seen how Sagredo endorses scale invariance at the beginning of *Two New Sciences*, but in fact all three participants do so. On the Second Day, Simplicio, who is often slow to catch on, suddenly endorses scale invariance, as if he had just thought of it, and Sagredo has to point out that he had said it the day before, and this is what they have been talking about. At this point Salviati, who is Galileo's spokesman, says that he too had assumed scale invariance once, until various observations showed him the contrary.[33] In this way Galileo is commendably frank, truthfully representing his former opinion in the Inferno lecture. On the other hand, he never mentions the lecture itself, which would still have been awkward for the honor of the Florentine Academy. No one can blame him for this. Quite possibly by 1638 there was no one else alive who even remembered this detail of the original lecture, and there was no real need to recount this story. Thus the motivating problem of *Two New Sciences*, the collapse of Manetti's Inferno, is discreetly avoided, and another one, not particularly convincing, a ship that falls apart of its own weight, is invented in its place.

In fact, Galileo seems to have regarded the Inferno lectures as an embarrassment, and this may be another reason why he postponed publishing the scaling theory. His first biographer, Viviani, was unaware of the lectures, despite living in Galileo's house during his last years, collecting Galileo's stories, and devoting himself to Galileo's memory after his death. Because Viviani is the source of most stories about Galileo, subsequent biographies have also said little or nothing about the Inferno lectures, which were only rediscovered in the 19th century. Thus one can plausibly conclude that Galileo never told Viviani about them. This circumstance is rather odd when you think that they were very likely the pivotal opportunity that got his career started. A letter of 1594 from Luigi Alamanni also indicates that Galileo was unhelpful to someone who wanted to get a copy of them.[34] The letter says, ''About that lecture of Galileo, he is in Padua, and I have not been able to get it from him.''[35] There were copies with Bacio Valori in Florence, including one written out in Galileo's own hand, but he apparently did not volunteer this information.

There is another hint that Galileo's scaling theory was originally planned for defensive purposes. In working out the scaling theory, Galileo would have noticed a second problem in the Inferno lectures, besides the inevitable collapse of the roof. In the first lecture the dimensions of the lowest regions are determined by comparison with certain giants who have been placed there by Dante, embedded in ice. In the lecture Galileo assumed that these giants have the same proportions as normal men, and his only hesitation on this point was whether giants have the ideal human proportions favored by artists like Albrecht Dürer, who wrote on the subject, or whether they have proportions more like ordinary men. Manetti and Vellutello had differed on this point, and Galileo favored Manetti, of course. But it is a consequence of the scaling theory that giants couldn't have either of these proportions. If we look in *Two New Sciences* in the section on scaling in animals,[36] we find that in fact the animal of most concern is man: Galileo is concerned about *giant men*. He even quotes a poet describing a giant, although it is Ariosto, not Dante, and he suggests, although it is a rather far-fetched interpretation of the words, that Ariosto understands that giants would be misshapen.[36] This occurrence of poetic giants and their proportions in *Two New Sciences* is entirely to be expected if the context is the Inferno lectures. The Ariosto lines make sense as part of a prepared defense: not only could he expound the scaling theory, he could invoke Ariosto as a poet who understands it. Although this literary allusion might not impress a scientific audience, the audience that mattered was a courtly audience. And although they would not understand the geometry, they would undoubtedly applaud this trick, just as they had applauded his skillful combination of Dante and geometry in the original lecture. Nearly fifty years later, as he finally wrote the scaling theory down for a very different audience, these associations still persist. This is speculation, of course, but the reader is invited to find a more plausible explanation for what Ariosto's giant is doing in *Two New Sciences*.

The setting in the Venetian Arsenal seems to promise practical science, and the scene is so effective that it has functioned as a credential for Galileo ever since, showing him to be a practical man of science, right at home with the foreman on the shop floor. There can be no doubt whatever that Galileo really was an ingenious and skillful experimentalist, but this opening scene fails to suggest how theoretical the work on scaling actually was. The Arsenal fades away almost immediately on the First Day, never to reappear. The discussion on the Second Day, when the subject finally comes into focus, is about beams with rectangular and circular cross section because the aim is geometrical proof. And the result to which the whole development is tending is entirely theoretical, and is given no practical illustration, namely that *anything* would break of its own weight if it were sufficiently scaled up. This is the content of the culminating Propositions VII and VIII on the Second Day. Proposition VII, for example, is ''Among heavy prisms and cylinders of similar figure, there is one and only one which under the stress of its own weight lies just on the limit between breaking and not breaking: so that every larger one is unable to carry the load of its own weight and breaks; while every smaller one is able to withstand some additional force tending to break it.'' This 17th century version of ''the inevitability of gravitational collapse'' is what the Second Day is really about.

It is noteworthy that Galileo describes no experiments to verify the scaling theory, although experiments on the bending of a loaded beam would have been very simple for him, and entirely characteristic of his Padua years. This circumstance suggests that the scaling theory is early, and had no practical motivation.

The Inferno lectures begin by praising the skill and audacity of those discoverers who have measured the heavens and the surface of the earth, and point out how much more difficult it is to know the earth's interior, where it seems that no one can go and return—yet even this our Dante had done! It is an implied theme of the lectures that the geometer can do this too: by the use of geometry we see beyond the limitations of our senses, reasoning about otherwise inaccessible things. This insight is, in a way, the heart of physics as Galileo came to understand it. Yet in the Inferno lectures the idea occurs almost accidentally, as part of a glib and clever entertainment. The geometry is merely descriptive. There are no actual discoveries.

It is entirely different with the scaling theory. Suddenly geometry really *did* give the key to new discovery,[37] a discovery that Galileo was still proud to claim as his own at the end of his amazing career. This new insight by way of geometry must have come as a wonderful surprise. For all the faith that Galileo and a few other mathematical enthusiasts had in geometry, it was, in practice, the everyday tool of artisans and merchants, not the source of new insights. Archimedes, to be sure, had used geometry in this wonderful way, but no one imagined there could be another Archimedes. With the scaling theory, however, Galileo had something truly new, worthy of comparison with Archimedes, something that validated his faith in geometry and hinted at undreamed of successes to come.

Looked at this way, Galileo's lifelong reluctance to publish seems even more inexplicable, but perhaps this pattern began with the experience of the Inferno lectures. He seems to have done his best to make people forget the lectures, and he kept the scaling theory to himself. What he made public, at least in this case, was a source of trouble, while what he kept secret was a source of confidence. The unpleasantness of being vulnerable to attack is a lesson that he might have taken to heart then, and it is a view he expresses feelingly later on, on the basis of real experience (although without admitting vulnerability), in the opening lines of *The Assayer*.[17] Galileo frequently claims to have wonderful results that he has not yet revealed, things he has not yet chosen to disclose. We know that this was true through much of his career, and apparently it was true right from the start.

Finally, it is an irony that the first success of Galileo's mathematical physics, which is close to being the first success of mathematical physics at all, was a response to a problem that was not physical, but rather the collapse of an imaginary structure in a work of literature. This peculiar story, while probably not of much use in a standard physics class, has great appeal to an audience of students from the humanities—students to whom we should pay more attention. Like Galileo, we, as physics teachers, have both scientific and unscientific audiences. Like him, we should think about how to reach them.

ACKNOWLEDGMENTS

I would like to thank two anonymous referees for helpful suggestions and criticisms.

[a]Electronic mail: mpeterso@mtholyoke.edu

[1]Galileo Galilei, *Two New Sciences*, translated by Henry Crew and Alfonso de Salvio (Dover, New York, 1954). Originally published by Elzevir, 1638.

[2]Reference 1, pp. xx–xxi.

[3]Reference 1, p. xii. The full title is "Discourses and Mathematical Demonstrations concerning Two New Sciences pertaining to Mechanics and Local Motions." Galileo complained that the publishers had substituted "a low and common title for the noble and dignified one carried upon the title-page." His preferred title is unfortunately lost.

[4]Galileo Galilei, "Due lezioni all'Accademia Fiorentina circa la figura, sito e grandezza dell'Inferno di Dante," in *Le Opere di Galileo Galilei*, edited by G. Barbèra (Ristampa della Edizione Nazionale, Florence, 1933), Vol. 9, pp. 31–57. Translated by Mark A. Peterson, http://www.mtholyoke.edu/~mpeterso/classes/galileo/inferno.html.

[5]Reference 1, p. 90.

[6]Reference 1, p. 2.

[7]Reference 1, p. 112.

[8]Reference 1, p. 3.

[9]Archimedes' law of the lever, the balance of torques, was second nature to Galileo.

[10]We would now say the stress itself varies linearly across the beam, so that the correct dependence is the second moment of area, going as the diameter to the fourth power.

[11]All propositions in this paragraph are from the Second Day.

[12]Reference 1, p. 6.

[13]Aristotle, in *Minor Works*, translated by W. S. Hett (Harvard U.P., Cambridge, MA, 1936), pp. 330–411.

[14]Reference 1, p. 135.

[15]Reference 1, p. 5.

[16]Reference 1, p. 7.

[17]Galileo Galilei, in *Discoveries and Opinions of Galileo*, translated by Stillman Drake (Anchor Books, New York, 1957).

[18]Galileo Galilei, *Discourse on Bodies in Water*, translated by Thomas Salusbury (University of Illinois Press, Urbana, IL, 1960).

[19]Galileo Galilei, *Dialogue Concerning the Two Chief World Systems*, translated by Stillman Drake (University of California Press, Berkeley, CA, 1967).

[20]M. A. Finocchiaro, *The Galileo Affair* (University of California Press, Berkeley, CA, 1989).

[21]"Le Opere di Galileo Galilei," edited by G. Barbèra (Ristampa della Edizione Nazionale, Florence, 1933), Vol. 10, No. 207, pp. 228–230.

[22]Stillman Drake, *Galileo at Work* (Dover, Mineola, NY, 1978), pp. 16–17.

[23]Reference 1, p. 100.

[24]Reference 16, p. 13.

[25]Mario Biagioli, *Galileo Courtier* (University of Chicago Press, Chicago, 1993), pp. 30–31.

[26]Reference 16, pp. 106–107.

[27]In fact Manetti was a Florentine intellectual of the previous century, the biographer of Filippo Brunelleschi. He lived well before the founding of the Florentine Academy.

[28]Vellutello was from Lucca, a notable rival of Florence.

[29]Reference 1, p. 3.

[30]Reference 19.

[31]See, for example, the entertaining account in William Dunham, *Journey Through Genius* (Wiley, New York, 1990).

[32]The way he uses scaling goes as follows. Galileo's opponents argue that the lamina floats because it is broad and flat, and the water resists being cut. But where the water is cut is along the perimeter, so it is supported at the perimeter, and thus, by scaling, the lamina floats better if it is divided into smaller pieces, that is, it floats better if it is *less* broad and flat, contradicting their own position.

[33]Reference 1, p. 125.

[34]"Le Opere di Galileo Galilei," edited by G. Barbèra (Ristampa della Edizione Nazionale, Florence, 1933), Vol. 9, p. 7.

[35]Because he could not obtain the text, Alamanni provided his correspondent a summary of the lectures (five years after they were delivered!). "It consisted in this, that he reviewed the opinion of the Florentine Antonio Manetti concerning the site of Dante's Inferno, published in a book printed by the Giunti, and then he reviewed the opinion of Vellutello, a commentator on Dante, on the same subject, and comparing the one with the other, he showed that of Manetti to be better."

[36]Reference 1, pp. 130–131.

[37]See for example the exchange between Simplicio and Sagredo, Ref. 1, bottom of p. 137.