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A simple decomposition of the variance of output growth across countries

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This article outlines a simple regression-based method to decompose the variance of an aggregate time series into the variance of its components, which is then applied to measure the relative contributions of productivity, hours per worker and employment to cyclical output growth across a panel of countries. Measured productivity contributes more to the cycle in Europe and Japan than in the United States. Employment contributes the largest proportion of the cycle in Europe and the United States (but not Japan), which is inconsistent with the idea that higher levels of employment protection in Europe dampen cyclical employment fluctuations.

Keywords: intensive margin; extensive margin; productivity; business cycles; variance decomposition

JEL Classification: C32; E24; E32

I. Motivation

In 2008 and 2009, the global economy suffered from a severe recession. Different economies behaved quite differently. In the United States, GDP per working-age person fell by 3.5% in log terms, and in western Germany GDP per working-age person fell by 5.0%.[1] Employment per head fell by 4.5% in the United States and actually rose in western Germany by 0.1%. As one might imagine, hours per worker make up some of the difference. Hours per worker fell by 1.6% in the United States and by 2.9% in western Germany. Nonetheless, a large discrepancy remains. Measured output per hour grew at a 2.6% rate in the United States but shrank by 2.3% in western Germany. Nearly half of the fall in output in Germany came from a fall in measured productivity, not from a fall in measured labour input. France and the United Kingdom show a similar pattern to Germany, with productivity absorbing much of the fall in output. During the most recent episode, different countries appear to have adjusted to the fall in output along different margins.

This article discusses the degree to which these differences are systematic. As is well known, labour market institutions are very different across Europe, Asia and the Anglo-Saxon world, with continental European countries and Japan exhibiting a high degree of employment protection. Nonetheless, little cross-sectional work has been done on how this translates into macroeconomic outcomes. This article follows up on the work of Merkl and Wesselbaum (2011), who compared the fluctuations in labour input between the United States and Germany using a simple covariance decomposition developed by Fujita and Ramey (2009). Merkl and Wesselbaum found that fluctuations in the intensive margin of labour input do not contribute much to the cycle in either country; most fluctuations come on the extensive

[1] The data are from the OECD and from German national sources; they are described in Section III.

margin, and the extensive margin is as important in Germany as in the United States. This article extends these works in two ways – by showing that the Fujita–Ramey decomposition can be calculated simply through a series of regressions for any accounting identity and by showing that measured productivity is an important cyclical adjustment margin in Germany as it is in most of continental Europe and Japan. Nonetheless, employment remains as important of an adjustment margin in Europe as in the United States; the difference comes from a slightly less procyclical hour margin in Europe. At a macroeconomic level, European employment is generally not rigid, though Japanese employment is.

II. A generalization of the Fujita–Ramey Decomposition

The data reside in a mean-zero k-by-T matrix of time series y_t with a covariance matrix Σ with an accounting identity linking it to an aggregate x_t. The goal is to attribute movements in the aggregate to movements in the original series. Let $\mathbf{b}$ equal a $1 \times k$ matrix which links the columns of y to the scalar aggregate x. Then one could write

$$x_t = \mathbf{b}y_t \tag{1}$$

For each i, regressing y_i on x and then multiplying by b_i gives the coefficient

$$c_i = \left(\sum_{t=1}^{T} x_t x_t{}'\right)^{-1}\left(\sum_{t=1}^{T} x_t y_{it}'\right) b_i \tag{2}$$

which converges in probability to

$$c_i \underset{p}{\rightarrow} \left(E(x_t x_t')\right)^{-1}\left(E(x_t y_{it}')\right) b_i \tag{3}$$

Writing Equation 3 as a function of variances gives the limit of c_i as a function of the covariance matrix:

$$c_i \underset{p}{\rightarrow} \left(\mathbf{b}\sum \boldsymbol{b}'\right)^{-1}\left(\mathbf{b}\sum_i b_i\right) \tag{4}$$

The right-hand side is series i's contribution to the overall variance of the aggregate series (i.e. the variance of the aggregate series conditional on series i), while the left-hand side is the overall variance of the aggregate series. Furthermore, the c_i coefficients all sum up to 1. It is in this sense that the elements of c_i could be thought of as an accounting-based variance decomposition.

This decomposition is numerically identical to the one used by Fujita and Ramey (2009) when b is a vector of all ones. In that case the measured variance contribution c_i is given as a function of sample covariances, which is obvious from the regression formula:

$$c_i = \left(\sum_{t=1}^{T} x_t x_t'\right)^{-1}\left(\sum_{t=1}^{T} x_t y_{it}'\right) b_i = \frac{\operatorname{cov}(b_i y_{it}, x_t)}{\operatorname{var}(x_t)} = \frac{\operatorname{cov}(y_{it}, x_t)}{\operatorname{var}(x_t)} \tag{5}$$

It is possible to vary b over time, which would be useful if one wished (for instance) to decompose fluctuations in a chain-weighted index of GDP. It is also possible to add other terms to the right-hand side of the regression to decompose fluctuations in an aggregate conditional on some other state variable.

III. Decomposing the Business Cycle Across Countries

This analysis uses data from the OECD's National Accounts and Annual Labor Force Statistics databases from 1970 to 2007, the latter date chosen in order to avoid the most recent crisis. The data cover 24 major economies after omitting transition economies and those whose data begin after 1991. Data for western Germany are constructed by the author to be as comparable as possible across time; data on output and employment come from the state-level economic accounts. Data on hours per worker come from the OECD, and unemployment data come from the national accounts, with adjustments made using the microcensus. The analysis uses annual data because country-level quarterly data are only sporadically available and are prone to transitory blips and discontinuities. The data are detrended in log levels using a Hodrick–Prescott (HP) filter with a smoothing parameter of 100.

The object of interest is growth in output per person aged 15–64 years. Output per person equals output per hour times hours per worker times the employment rate (on a labour force basis) times the labour force participation rate. In logarithms, this gives an accounting identity in the form of Equation 1, which also holds in first differences:

$$\Delta \log\left(\frac{Y}{N}\right) = \Delta \log\left(\frac{Y}{H}\right) + \Delta \log\left(\frac{H}{E}\right) + \Delta \log\left(\frac{E}{LF}\right) + \Delta \log\left(\frac{LF}{N}\right) \tag{6}$$

Table 1 shows the results of regressing the components on the right-hand side of Equation 6 on aggregate per capita output growth. It is instructive to split the table into the Anglo-Saxon economies, continental European economies and other economies, though discussion will concentrate on the 'big five' economies of France, Germany, Japan, the United Kingdom and the United States. The Anglo-Saxon economies with the exception of Australia and Ireland have much less procyclical productivity than the European and Japanese economies. In the United States, productivity only contributes to about 27% of the cycle and labour input four-fifths. Meanwhile, in France and Germany, productivity contributes 43% and 38% of the cycle, respectively. Japan is more European than Europe in this regard; productivity contributes 59% of the cycle there, while Korea looks more like the United States.

In the United States, hours per worker contribute 20% of the cycle and employment fluctuations contribute 52% of the cycle; the majority of those are driven by fluctuations in the employment rate and not by labour force participation. In the United Kingdom, hours per worker are much more important. The United Kingdom looks like an outlier in this regard; in most other countries, hours per worker do not contribute much to the cycle. Even in Japan, hours contribute only 19%. In most countries, the measured intensive margin of labour adjustment is not particularly important, and in fact, in France and Germany, it is less important than in the United States.

The employment adjustment margin is the most important in the United States as in most countries; there, most employment adjustment happens in the employment rate with an additional small contribution coming from procyclical labour force participation. Fluctuations in the employment rate are about as important in the United Kingdom and continental Europe, with labour force participation providing more of a margin of adjustment in France and Germany. Some countries, like Japan, see little fluctuation in employment rates at all. Labour force adjustment in Europe is generally more procyclical than in the United States, which is interesting but difficult to interpret. Nonetheless, comparing the big five countries shows that the extensive margin of

Table 1. Contribution of different components of cyclical output growth in 24 countries

Country	Begin	End	Y/H	H/E	E/LF	LF/N	SD (Y/N)
Anglo-Saxon economies							
Australia	1978	2007	0.53	0.20	0.24	0.03	0.016
Canada	1970	2007	0.20	0.15	0.44	0.21	0.019
Ireland	1983	2007	0.56	–0.01	0.39	0.06	0.022
New Zealand	1986	2007	0.06	0.18	0.33	0.43	0.021
United Kingdom	1970	2007	0.10	0.43	0.32	0.15	0.018
United States	1970	2007	0.27	0.20	0.43	0.09	0.019
Continental European economies							
Austria	1995	2007	0.54	–0.22	0.24	0.44	0.010
Belgium	1983	2007	0.21	0.16	0.54	0.09	0.012
Denmark	1970	2007	0.36	0.16	0.53	–0.05	0.018
Finland	1970	2007	0.40	0.02	0.47	0.10	0.024
France	1970	2007	0.43	0.12	0.27	0.18	0.014
Germany (West)	1970	2007	0.38	0.10	0.27	0.24	0.016
Greece	1983	2007	0.85	0.11	0.07	–0.02	0.015
Iceland	1970	2007	0.80	–0.06	0.12	0.15	0.027
Italy	1980	2007	0.49	–0.09	0.04	0.56	0.012
Netherlands	1987	2007	0.48	–0.05	0.30	0.27	0.013
Norway	1970	2007	0.44	0.05	0.20	0.31	0.013
Portugal	1986	2007	0.33	0.09	0.34	0.25	0.021
Spain	1970	2007	–0.06	0.13	0.66	0.26	0.016
Sweden	1970	2007	0.33	0.11	0.40	0.16	0.016
Switzerland	1991	2007	0.33	–0.02	0.22	0.46	0.015
Other economies							
Japan	1970	2007	0.59	0.19	0.08	0.14	0.018
Korea (South)	1980	2007	0.28	0.23	0.30	0.18	0.029
Turkey	1970	2004	1.32	–0.05	–0.09	–0.18	0.039

Notes: This table contains the results of regressing each component of cyclical growth in output per person aged 15–64 years on aggregate cyclical growth. All variables are logged, HP-detrended with a smoothing parameter of 100 and then taken in first differences. HP, Hodrick–Prescott.

employment adjustment is roughly equal in importance in all countries except Japan. Employment comprises 48% of the cycle in the United Kingdom, 51% in the United States, 45% in France and 51% in Germany. Only in Japan is the extensive margin relatively unimportant, at 22% of the cycle.

There are some countries where the estimates seem a little bit suspect, namely, Spain and Turkey. In Spain, total labour input accounts for more than the entire cycle, and measured productivity is in fact countercyclical. In Turkey, the estimates are even stranger; all forms of labour input are actually countercyclical. Estimates from these countries should not be taken too literally.

IV. Conclusion

This article has shown how the Fujita–Ramey decomposition can be calculated as a series of linear regressions, applying it to a cross-country panel in order to determine the form that cyclical fluctuations take in different countries. In much of Europe and Japan, productivity is more procyclical than in the United States, while employment is as procyclical in Europe and much less procyclical in Japan. Such behaviour is inconsistent with the idea that labour market rigidities substantially dampen employment fluctuations in European economies, though the Japanese labour market does appear to be very rigid at a macroeconomic level.

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